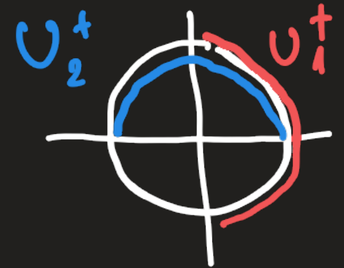
Ex 21.10 Atlas on S^1

$U_1^+ = \{(x,y) : x>0\}$, $U_1^- = \{(x,y) : x<0\}$, $U_2^+ = \{(x,y) : y>0\}$, $U_2^- = \{(x,y) : y<0\}$



Is this an oriented atlas on S^1?

$$
\begin{tikzcd}
 & U_1^+ \cap U_2^+ \arrow[dl, "\psi_1^+ \; \phi_1^+"'] \arrow[dr, "\phi_2^+ \; \psi_2^+"] & \\
(0,1) = \phi_1^+(U_1^+ \cap U_2^+) & & \phi_2^+(U_1^+ \cap U_2^+) = (0,1)
\end{tikzcd}
$$

$\frac{d}{dt}(1-t^2)^{1/2} = \frac{1}{2}(1-t^2)^{-1/2}(-2t)$

is negative for $t \in (0,1)$.

$t \longmapsto (\sqrt{1-t^2}, t) \longmapsto \sqrt{1-t^2}$

$t \longmapsto (\sqrt{1-t^2}, t) \longmapsto -\sqrt{1-t^2}$, so $\frac{d}{dt}\left(-(1-t^2)^{1/2}\right) > 0$ for $t \in (0,1)$.

Define new charts

$\psi_1^+ : U_1^+ \to (-1,1) : (x,y) \longmapsto y$

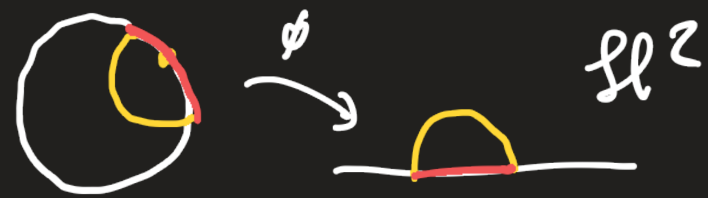
$\psi_1^- : U_1^- \to (-1,1) : (x,y) \longmapsto -y$

$\psi_2^+ : U_2^+ \to (-1,1) : (x,y) \longmapsto -x$

$\psi_2^- : U_2^- \to (-1,1) : (x,y) \longmapsto x$

Ex 22.8 Cylinder  with given orientation

Describe boundary orientations on the upper and lower boundary circle.



Ex 22.9 Let $D^{n+1} \subseteq \mathbb{R}^{n+1}$ with $\partial D^{n+1} = S^n \subseteq \mathbb{R}^{n+1}$.

Show $\omega = \sum_{i=1}^{n+1} (-1)^i x^i \, dx^1 \wedge \dots \wedge \widehat{dx^i} \wedge \dots \wedge dx^n$ is an orientation form on S^n.

Know $X = \sum_{i=1}^{n+1} x^i \frac{\partial}{\partial x^i}$ is an outward-pointing vector field along ∂D^{n+1}. Hence $\iota_X(dx^1 \wedge \dots \wedge dx^{n+1})$ is an orientation form on ∂D^{n+1}:

$$\iota_X(dx^1 \wedge \dots \wedge dx^{n+1}) = \sum_{i=1}^{n} (-1)^{i-1} dx^i(X) \, dx^1 \wedge \dots \wedge \widehat{dx^i} \wedge \dots \wedge dx^{n+1}$$

$$= \sum_{i=1}^{n+1} (-1)^{i-1} x^i \, dx^1 \wedge \dots \wedge \widehat{dx^i} \wedge \dots \wedge dx^{n+1}.$$