Ex. 4.2 $\quad p = (p^1, p^2, p^3) \in \mathbb{R}^3$, $\omega_p : T_p(\mathbb{R}^3)^2 \longrightarrow \mathbb{R}$,

$$\omega_p\left(\begin{bmatrix} a^1 \\ a^2 \\ a^3 \end{bmatrix}, \begin{bmatrix} b^1 \\ b^2 \\ b^3 \end{bmatrix}\right) = p^3 \det \begin{bmatrix} a^1 & b^1 \\ a^2 & b^2 \end{bmatrix}.$$

By definition $\omega \in \Omega^2(\mathbb{R}^3)$

Write ω in the standard form $\omega = \sum a_I \, dx^I$, i.e.

$$\omega = a_{12}\, dx^1 \wedge dx^2 + a_{13}\, dx^1 \wedge dx^3 + a_{23}\, dx^2 \wedge dx^3.$$

To find the a_{ij}, evaluate on (e_i, e_j)

$$\omega_p(e_1, e_2) = p^3 \qquad \omega_p(e_1, e_3) = 0, \quad \omega_p(e_2, e_3) = 0$$

Hence $\omega_p = p^3 \, dx^1 \wedge dx^2$.

Ex 4.3 Coordinates r, θ on $\mathbb{R}^2$, let $x = r\cos\theta$, $y = r\sin\theta$.

$$dx = \frac{\partial x}{\partial r} dr + \frac{\partial x}{\partial \theta} d\theta = \cos\theta \, dr - r\sin\theta \, d\theta$$

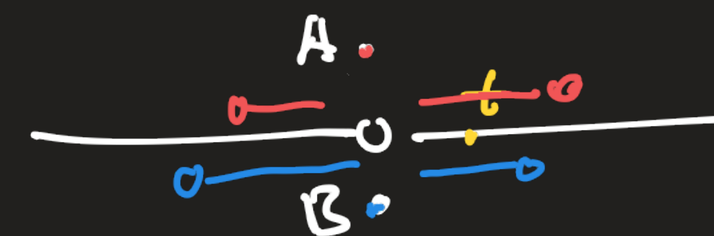
$$dy = \frac{\partial y}{\partial r} dr + \frac{\partial y}{\partial \theta} d\theta = \sin\theta \, dr + r\cos\theta \, d\theta$$

$$dx \wedge dy = r\cos^2\theta \, dr \wedge d\theta - r\sin^2\theta \, d\theta \wedge dr$$

$$= (r\cos^2\theta + r\sin^2\theta) \, dr \wedge d\theta = r \, dr \wedge d\theta .$$

Ex 5.1

$S = \mathbb{R} - \{0\} \cup \{A, B\}$



Basis for a topology on S:

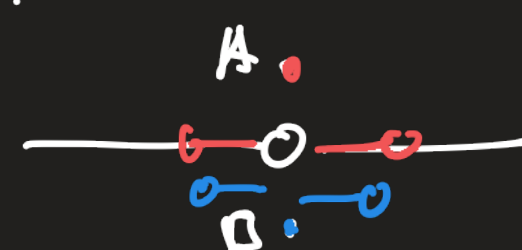
- All open subsets of $\mathbb{R} - \{0\}$.
- subsets $I_A(-c,d) = (-c,0) \cup \{A\} \cup (0,d)$, for $c,d > 0$.
- subsets $I_B(-c,d) = (-c,0) \cup \{B\} \cup (0,d)$, for $c,d \geq 0$.

(a) Claim: $h: I_a(-c,d) \to (-c,d)$ is a homeomorphism.

$$t \longmapsto t \quad \text{if } t \neq A$$
$$A \longmapsto 0$$

(b) Show S is locally Euclidean and second countable, but not Hausdorff.

Not Hausdorff



The intersection of a neighborhood of A with a neighborhood of B is always non-empty.

Ex. 5.2

$X =$ 

Claim: X is not locally Euclidean at q

Suppose $q \in U \subseteq X$ open and homeomorphic to a subset $U' \subseteq \mathbb{R}^n$, some n, $f: U \to U' \subseteq \mathbb{R}^n$.

Take out q to get a homeomorphism $\bar{f}: U - \{q\} \to U' - \{f(q)\}$.

Notice $U - \{q\}$ is not connected.

Since $\bar{f}$ is a homeomorphism, it follows from invariance of domain that $U' - \{f(q)\}$ has a part that is homeomorphic to an open subset of $\mathbb{R}^2$ and a part homeomorphic to an open subset of $\mathbb{R}$.

But cannot have $n = 2$ and $n = 1$.