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SUPERSOLIDS AND SOUND WAVES: A PERTURBATIVE APPROACH TO CALCULATING FIRST AND SECOND SOUND IN SPATIALLY ORDERED BECS

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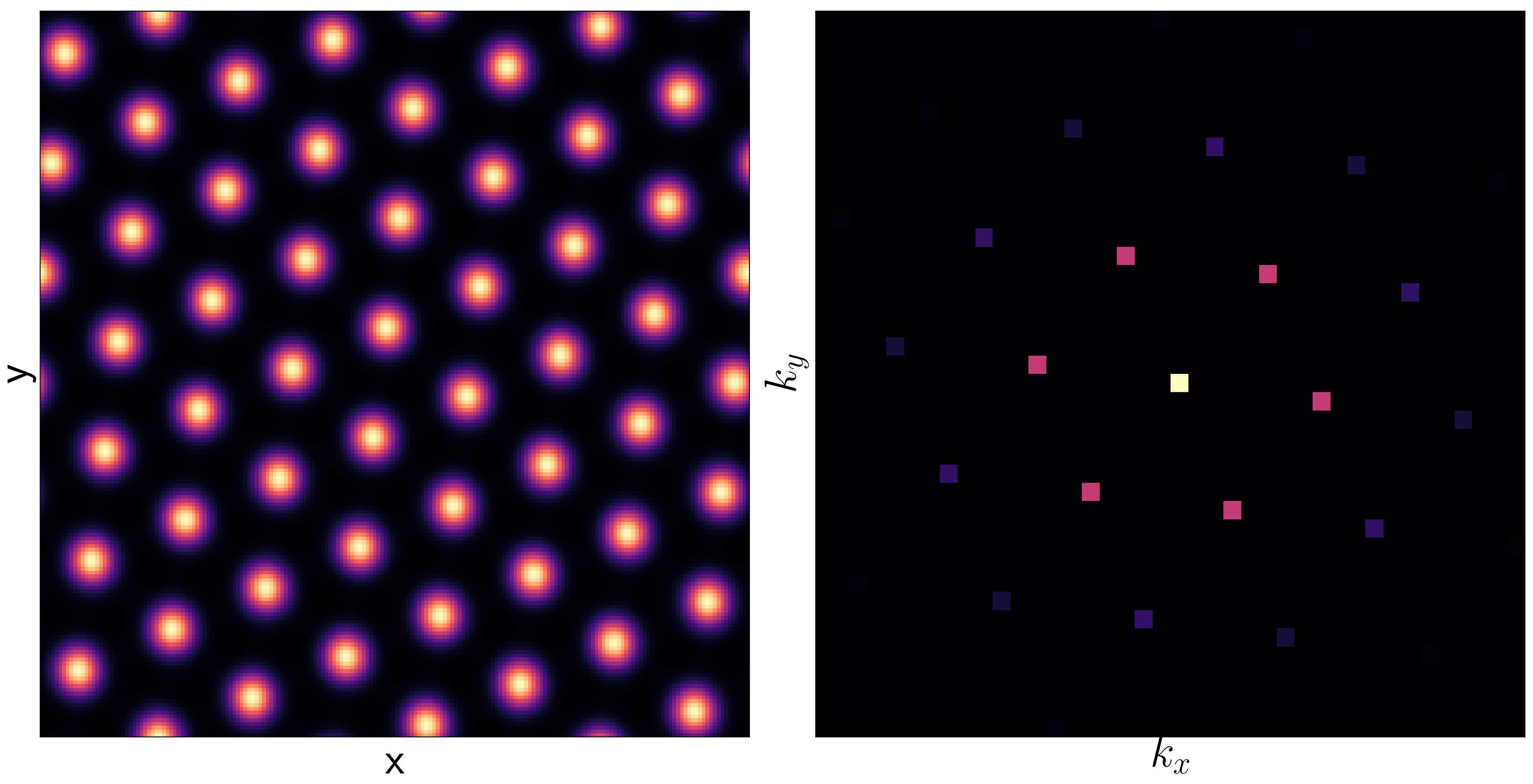
BACKGROUND

A **supersolid** is a phase which exhibits both **super**fluid and **solid** properties simultaneously, possessing long-range off-diagonal order leading to **non-viscous flow** as well as **spatial ordering**. This additional symmetry breaking allows for the existence of **extra Goldstone modes** with richer dynamics and more complex interactions than in either a superfluid or a solid. Recent experiments on **graphene substrates** [4] and **dipolar BECs** in optical traps [1, 2] have separately provided strong evidence of a phase of matter which either has Non-Classical Rotational Inertia (**NCRI**) or strong **non-local phase correlations**, respectively, whilst in the presence of spatial modulation.

FIGURE 1



A density distribution generated by the non-local Gross-Pitaevskii equation which admits a modulated phase, in real and Fourier space. The $\mathbf{k} \neq 0$ components are responsible for coupling between the Bogoliubov mode and the longitudinal elastic phonon via Bragg and Umklapp scattering.

ELASTIC THEORY

We begin by taking the non-local **Gross-Pitaevskii** model for a general interaction $U(\mathbf{r})$

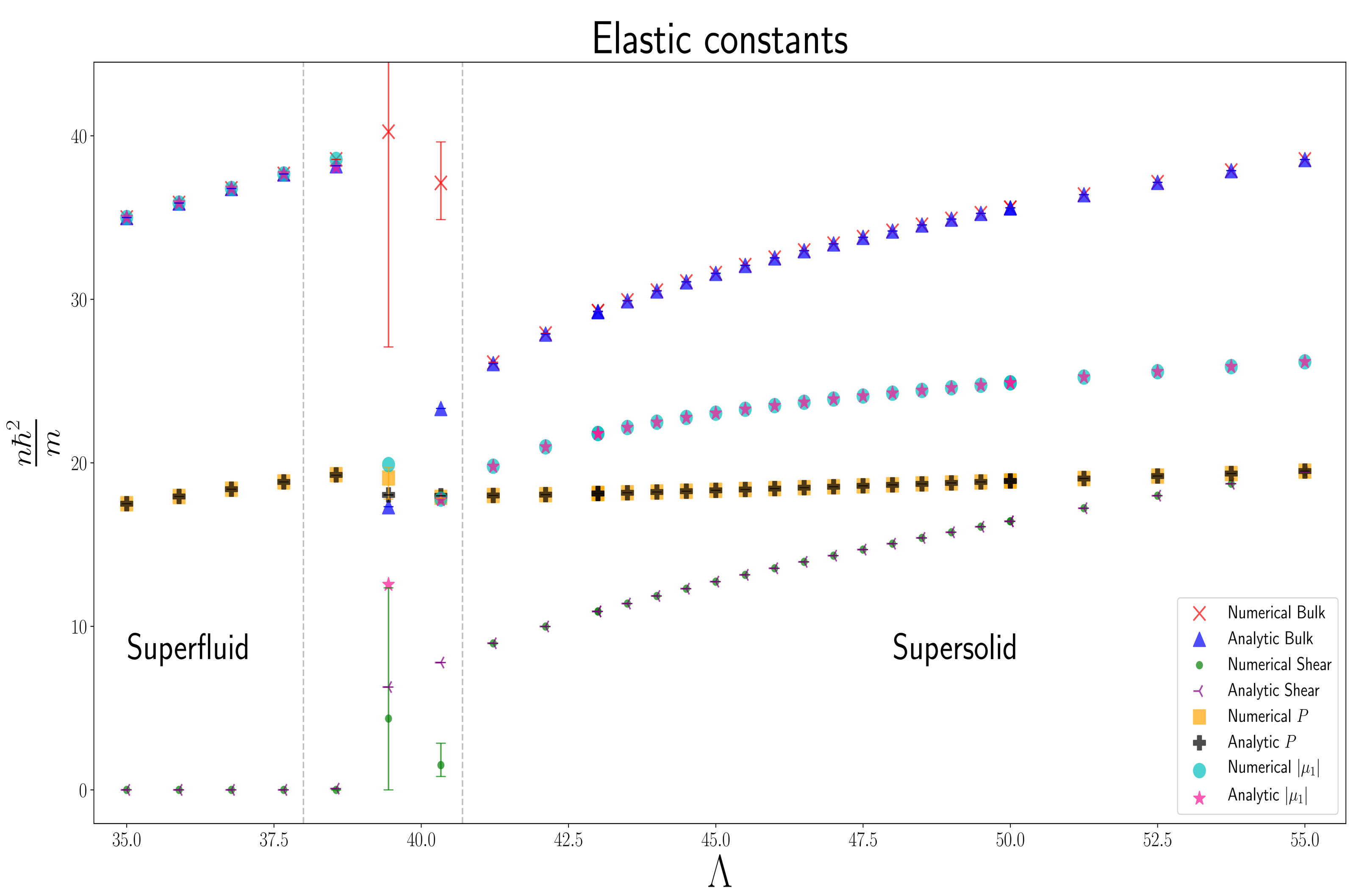
$$\mathcal{L} = -\int_{\Omega}\left[\hbar\rho\frac{\mathrm{d}\phi}{\mathrm{d}t} + \frac{\hbar^2}{2m}\left(\rho(\nabla\phi)^2 + \frac{(\nabla\rho)^2}{4\rho}\right) + \frac{\rho}{2}\int_{\Omega}U(|\mathbf{r}-\mathbf{r}'|)\rho(\mathbf{r}')\,\mathrm{d}\mathbf{r}'\right]\mathrm{d}\mathbf{r} \quad (1)$$

and apply an **elastic deformation** and homogenisation procedure building on work by Josserand et al [3]. We are able to express a coarse-grained effective Lagrangian as

$$\begin{aligned}\frac{L}{|\Omega|} = & -\mathcal{E} - \hbar n\partial_t\phi + \frac{\hbar^2}{2}\left(\frac{(\partial_t\phi)^2}{\mathcal{E}''} - \frac{(n-\varrho)}{m}(\nabla\phi)^2\right) - \hbar\left(n - \varrho - \frac{P'}{\mathcal{E}''}\right)\partial_t\phi\nabla\cdot\mathbf{u} \\ & + \frac{1}{2}\left(m\varrho(\partial_t\mathbf{u})^2 - \left(B_{iklm} - \frac{(P')^2}{\mathcal{E}''}\delta_{ik}\delta_{lm}\right)\partial_i u_k\partial_l u_m\right).\end{aligned} \quad (2)$$

with microscopically determined coefficients, a subset of which we are able to verify numerically.

FIGURE 2

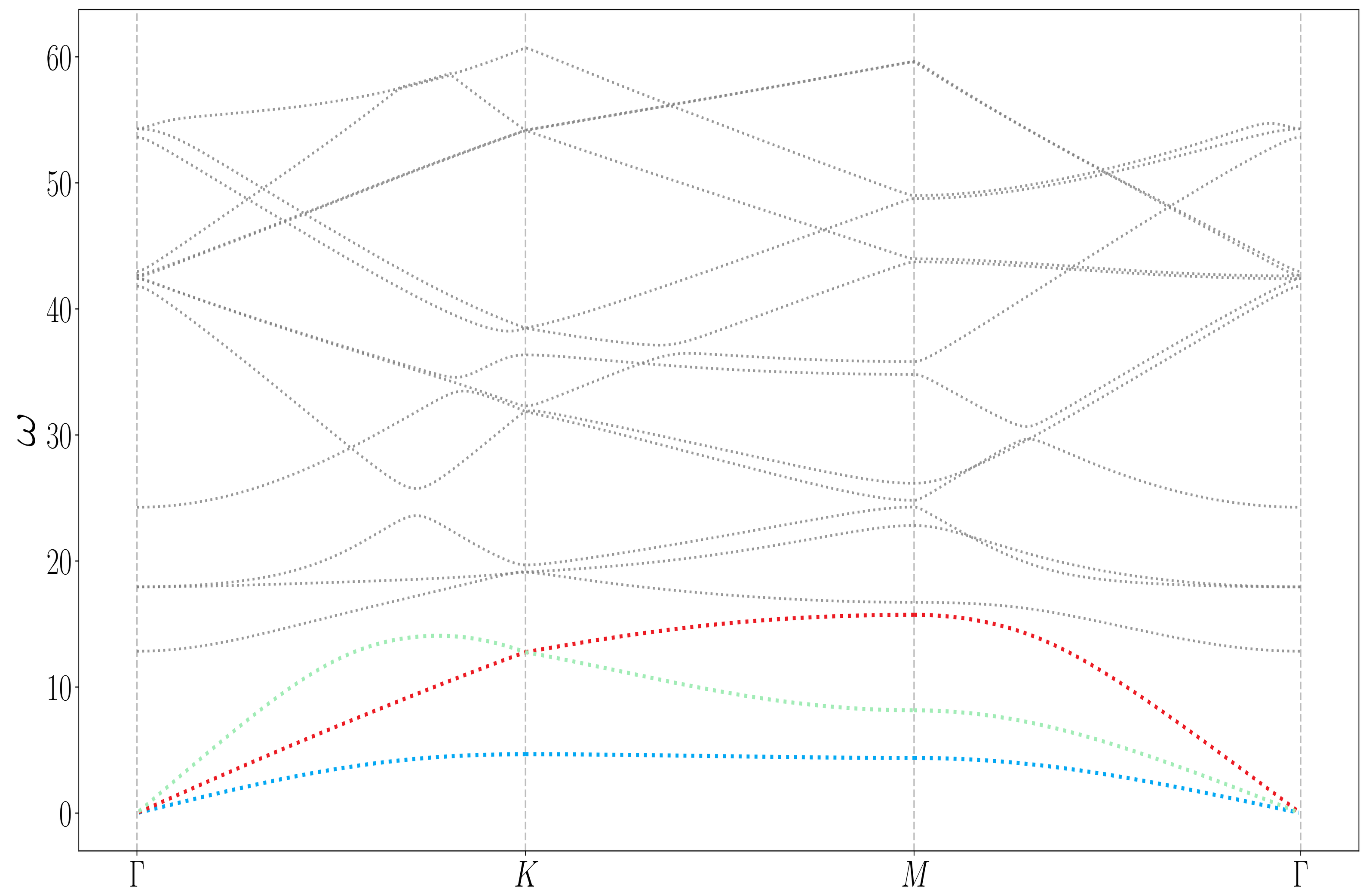


Numerical calculations of elastic constants agree excellently with analytics on either side of the transition but suffer from **instabilities** close to the phase boundary. The analytical theory can be used to **probe this near-transitory** region without error, as well as calculate **relevant physical quantities** that numerics are otherwise unable to produce.

BOGOLIUBOV FLUCTUATIONS

An alternative route to analysing supersolids is by obtaining the **dispersion relation** of **Bogoliubov fluctuations**. We can apply Bloch's theorem in our periodic system and obtain a **band structure** which should a-priori give us all the **Goldstone modes** and their **velocities**. We should expect both the elastic theory and the Bogoliubov theory to recover the exact same results.

FIGURE 3



The Bogoliubov-like longitudinal mode, elastic-like longitudinal mode and the shear transverse mode contained in a higher band structure. We find that both the homogenisation theory and the Bogoliubov theory generate the same velocities to within several significant figures.

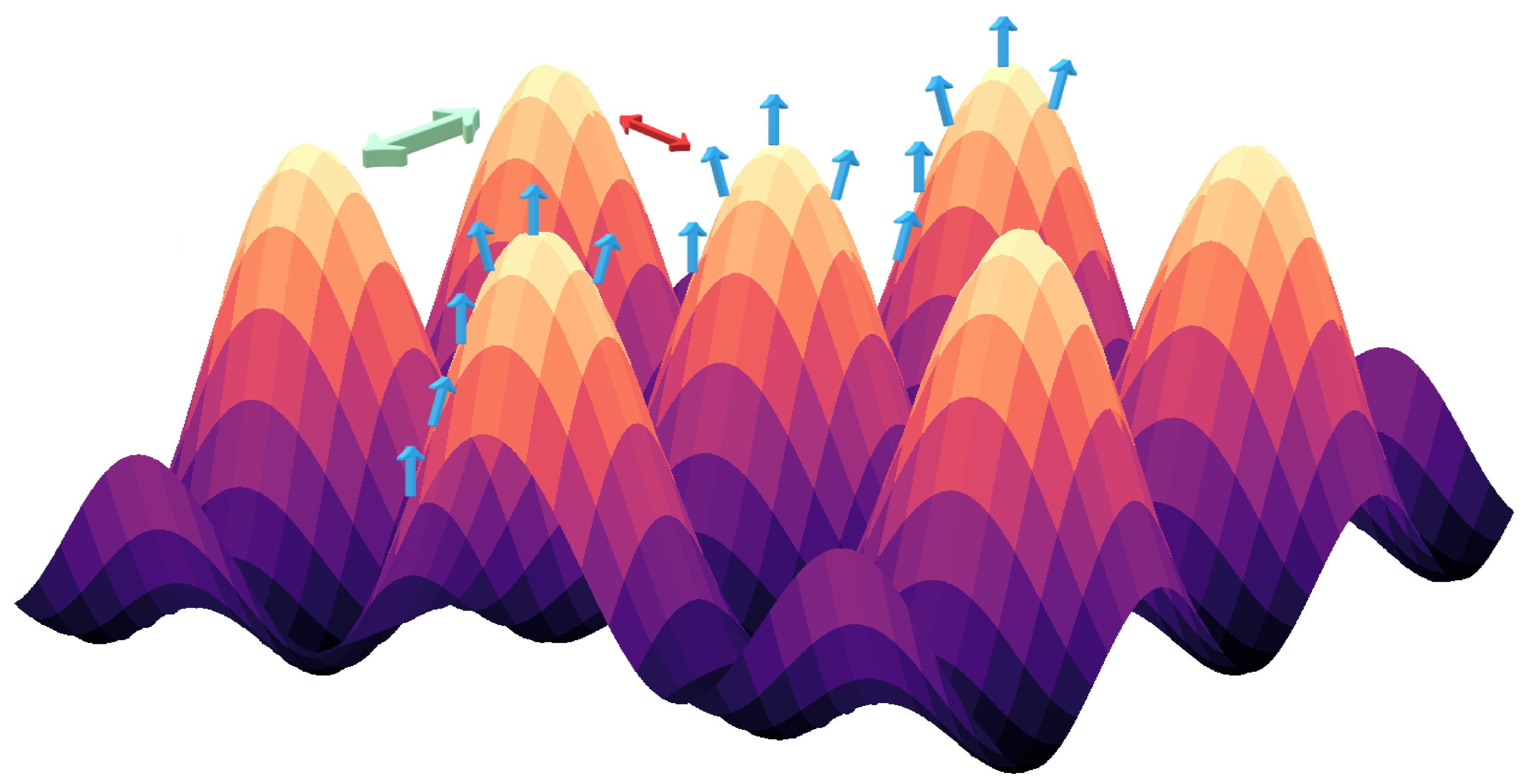
CONCLUSIONS

We have derived **two separate techniques** which analytically derive the behaviour of the Goldstone modes in different ways. The **band-structure** technique generates a dispersion relation at arbitrary **k**, whereas the **homogenisation theory** is only valid in the low-**k** limit but does produce several **physically observable parameters** directly. In the limit where the techniques can agree with each other (and with numerics), they do so excellently.

In this process we have found an **inertial density** ϱ which describes heuristically the fraction of the system that has an inertial mass, directly correlating with the experimental notion of **NCRI** and is in principle measurable.

We find the existence of the supersolid has **dynamically dressed the elastic response**, assigned an **inertial mass to the pattern**, and created one **fast and one slow longitudinal mode** as well as a **shear mode**.

FIGURE 4



The Bogoliubov mode, which is a phase-density wave, is shown in blue by arrows which correspond to the Argand diagram vector of the phase ϕ. The longitudinal and shear elastic modes vibrate the modulation of the system.

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