Drawsgtree:
a tool for visualizing properties in the semigroup tree

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In these pages we illustrate examples drawn by the code `drawsgtree`.
The code `drawsgtree` can be downloaded from `https://github.com/mbrasamoros/drawsgtree`.
Contact `maria.bras@urv.cat` for any comment or questions.
Please, cite as [1].

COMMAND:

```
./drawsgtree -h
```

OUTPUT:

```
./sgroup [options]      generate a latex file with the semigroup tree
  -h                    display this help
  -g <int>              [mandatory option] maximum genus
  -m <int>              multiplicity
  -n [option]           node representation
     -n list               list of semigroup elements
     -n minimalgenerators  representation by minimal generator set
     -n gapset             representation by gapsets
                               (S. Eliahou, J. Fromentin: Gapsets and
                                  ↪ numerical semigroups, Journal of
                                  ↪ Combinatorial Theory, Series A, 2020)
     -n gapseedbitstream   representation with the gap bitstream and the
        ↪ seed bitstream
                               (M. Bras-Amoros, J. Fernandez-Gonzalez:
                                  ↪ Computation of numerical semigroups
                                  ↪ by means of seeds, Math of Comput,
                                  ↪ 2018
                                M. Bras-Amoros: On the seeds and the great
                                   ↪ -grandchildren of a numerical
                                   ↪ semigroup, Math of Comput, Accepted,
                                   ↪  2023)
     -n seedstable         representation by seeds tables
                               (M. Bras-Amoros, J. Fernandez-Gonzalez:
                                  ↪ Computation of numerical semigroups
                                  ↪ by means of seeds, Math of Comput,
                                  ↪ 2018
                                M. Bras-Amoros: On the seeds and the great
                                   ↪ -grandchildren of a numerical
                                   ↪ semigroup, Math of Comput, Accepted,
                                   ↪  2023)
     -n dyckhook           representation by augmented Dyck paths and Hook
        ↪  lengths
                               (M. Bras-Amoros, A. de Mier: Representation
                                  ↪  of numerical semigroups by Dyck
                                  ↪ paths, Semigroup Forum, 2007)
                                H. Constantin, B. Houston-Edwards, N.
                                   ↪ Kaplan: Numerical sets, core
                                   ↪ partitions, and integer points in
                                   ↪ polytopes, Combinatorial and
                                   ↪ Additive Number Theory, 2017)
     -n aperykunzposet     representation by Apery sets, Kunz coordinates,
        ↪  and posets
                               (E. Kunz: Uber die Klassifikation
                                  ↪ numerischer Halbgruppen, Regensburger
                                  ↪  Mathematische Schriften, 1987
                                J.C. Rosales, P.A. Garcia-Sanchez, J.I.
                                   ↪ Garcia-Garcia, M.B. Branco: Systems
```

```
                                    ↪ of inequalities and numerical
                                    ↪ semigroups, J. Lond. Math. Soc.,
                                    ↪ 2002
                                 N. Kaplan, K. O'Neill: Numerical
                                    ↪ semigroups, polyhedra, and posets I:
                                    ↪  the group cone, Combinatorial
                                    ↪ Theory, 2021)
-e [option]              edge distinction
   -e infinitechains     distinguish the infinite chains in the
      ↪ semigroup tree
                            (M. Bras-Amoros, S. Bulygin: Towards a
                               ↪ better understanding of the semigroup
                               ↪  tree, Semigroup Forum, 2009
                             M. Rosas-Ribeiro, M. Bras-Amoros: Infinite
                               ↪  chains in the tree of numerical
                               ↪ semigroups. Submitted, 2023)
   -e med                 distinguish the chains of MED semigroups
                            (J.C. Rosales, P.A. Garcia-Sanchez, J.I.
                               ↪ Garcia-Garcia, M.B. Branco: Numerical
                               ↪  semigroups with maximal embedding
                               ↪ dimension, Int. J. Commut. Rings,
                               ↪ 2003)
   -e pattern <sign1>a1<sign2>a2..<signn>
                           distinguish the semigroups admitting the (
                              ↪ strongly admissible) pattern <sign1>a1x1
                              ↪ +<sign2>a2x2+...+<signn>anxn
                            (M. Bras-Amoros, P.A. Garcia-Sanchez:
                               ↪ Patterns on numerical semigroups,
                               ↪ Linear Algebra App. 2006)
-etrim                   discard the non-distinguished edges together with
   ↪ all its descendants
-t [option]              alternative tree
   -t ordinarization
                            (M. Bras-Amoros: The ordinarization
                               ↪ transform of a numerical semigroup
                               ↪ and semigroups with a large number of
                               ↪  intervals, J. of Pure and App.
                               ↪ Algebra, 2012)
   -t quasiordinarization
                            (M. Bras-Amoros, H. Perez-Roses, J. M.
                               ↪ Serradilla-Merinero: Quasi-
                               ↪ ordinarization transform of a
                               ↪ numerical semigroup, Symmetry, 2021)
-incremental             incremental with genus
-inputfile               input file (not compiling without a calling file)
-vertical                vertical tree growing down
-plain                   plain representation of objects using less memory
-blackandwhite           graph without colors
-framednodes             frame each tree node
-d <float>               enlarge distance between generations by the
   ↪ specified factor
```

```
  -s <float>              enlarge distance between siblings by the specified
     ↪  factor
  -rotated                rotated 90 degrees
  -o <filename>           output file name
  0 N[1] N[2] ... N[k]    root at the semigroup {0,N[1],N[2],N[k],N[k]+1,N[k
     ↪ ]+2,...}

examples:  ./drawsgtree -g5 -n list
           ./drawsgtree -g7 -n list -incremental
           ./drawsgtree -g7 -n list 0 5 8 -s .37 -d 1.2
           ./drawsgtree -g4 -n minimalgenerators -vertical
           ./drawsgtree -g5 -n gapset -vertical
           ./drawsgtree -g7 -n gapseedbitstream -n list -plain
           ./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27
              ↪  30
           ./drawsgtree -g10 -n aperykunzposet 0 6 7 9
           ./drawsgtree -g8 -m4 -n dyckhook
           ./drawsgtree -g10 -e infinitechains
           ./drawsgtree -g10 -e infinitechains -d 3.
           ./drawsgtree -g42 -m6 -e infinitechains -etrim -d .2
           ./drawsgtree -g6 -e med -n minimalgenerators
           ./drawsgtree -g5 -e pattern 1+1-1 -n minimalgenerators -e trim
              ↪ -vertical
           ./drawsgtree -g10 -m4 -e pattern 1+1+1-1 -n minimalgenerators -
              ↪ d 2.3 -s 4.
           ./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
              ↪ gapseedbitstream -n aperykunzposet -framednodes
           ./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n
              ↪ aperykunzposet
           ./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n
              ↪ dyckhook
           ./drawsgtree -g7 -t ordinarization -n list
           ./drawsgtree -g7 -t quasiordinarization -n list
```

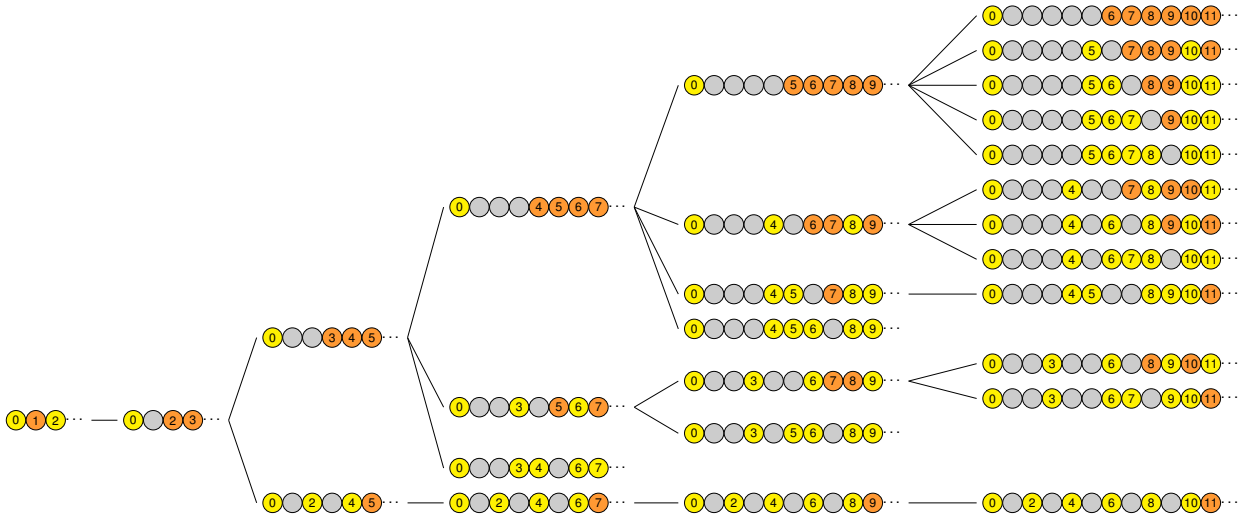
COMMAND:

```
./drawsgtree -g5 -n list -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
GENERATED FILE: inputfile-list-semigrouptree-5.tex
```

GENERATED GRAPH:



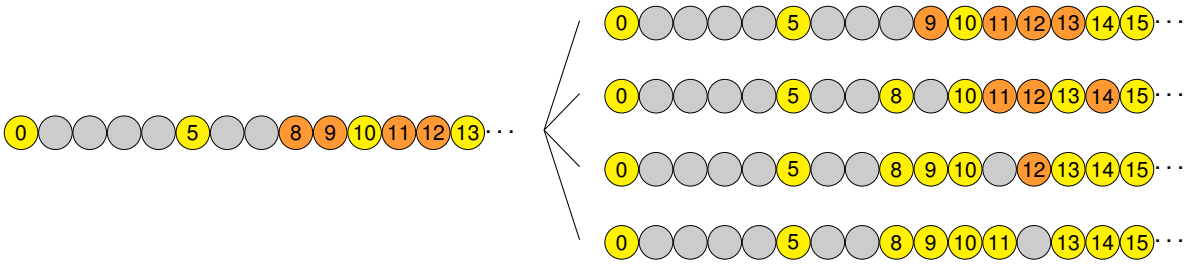
COMMAND:

```
./drawsgtree -g7 -n list 0 5 8 -s .37 -d 1.2 -inputfile
```

OUTPUT:

```
N[0]=0
N[1]=5
N[2]=8
[g=7] count=4 ng=39 [0 seconds]
GENERATED FILE: inputfile-list-semigrouptree-7-root058.tex
```

GENERATED GRAPH:



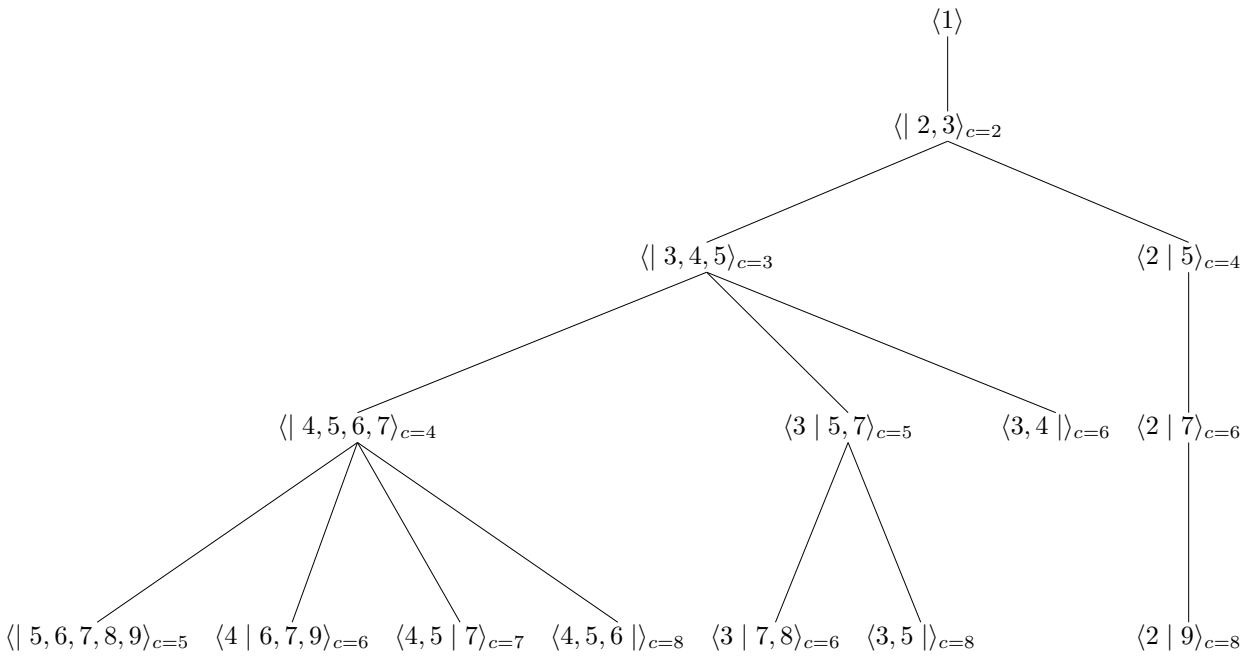
COMMAND:

```
./drawsgtree -g4 -n minimalgenerators -vertical -inputfile
```

OUTPUT:

```
[g=4] count=7 ng=7 [0 seconds]
GENERATED FILE: inputfile-minimalgenerators-semigrouptree-4.tex
```

GENERATED GRAPH:



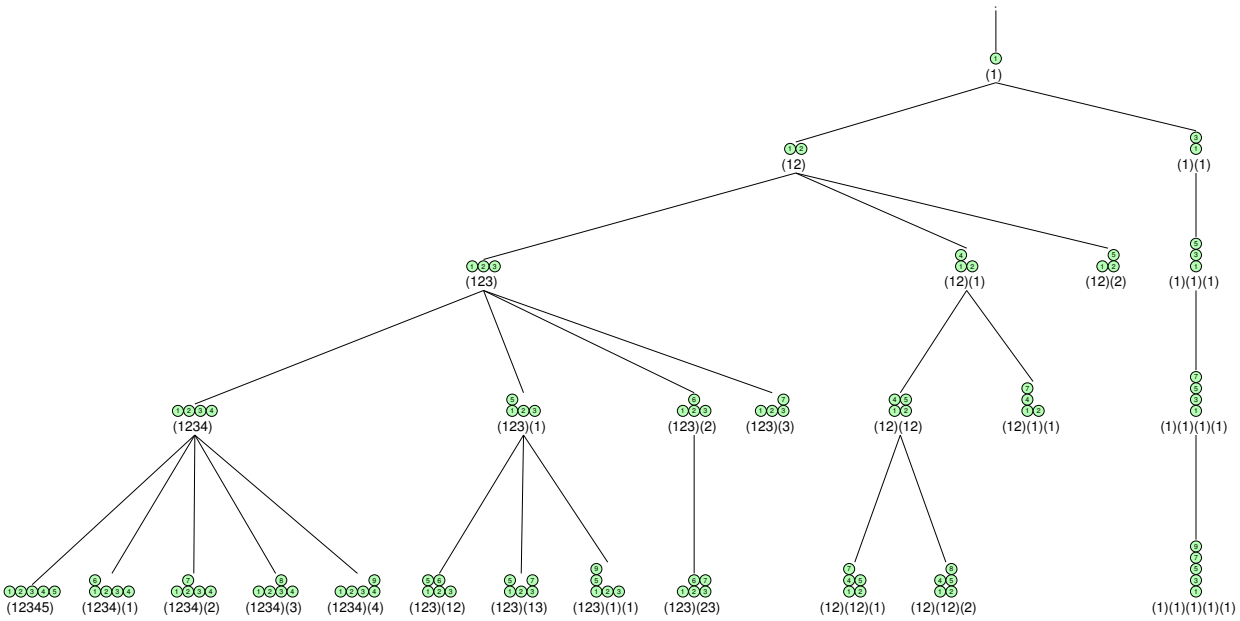
COMMAND:

```
./drawsgtree -g5 -n gapset -vertical -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
GENERATED FILE: inputfile-gapset-semigrouptree-5.tex
```

GENERATED GRAPH:



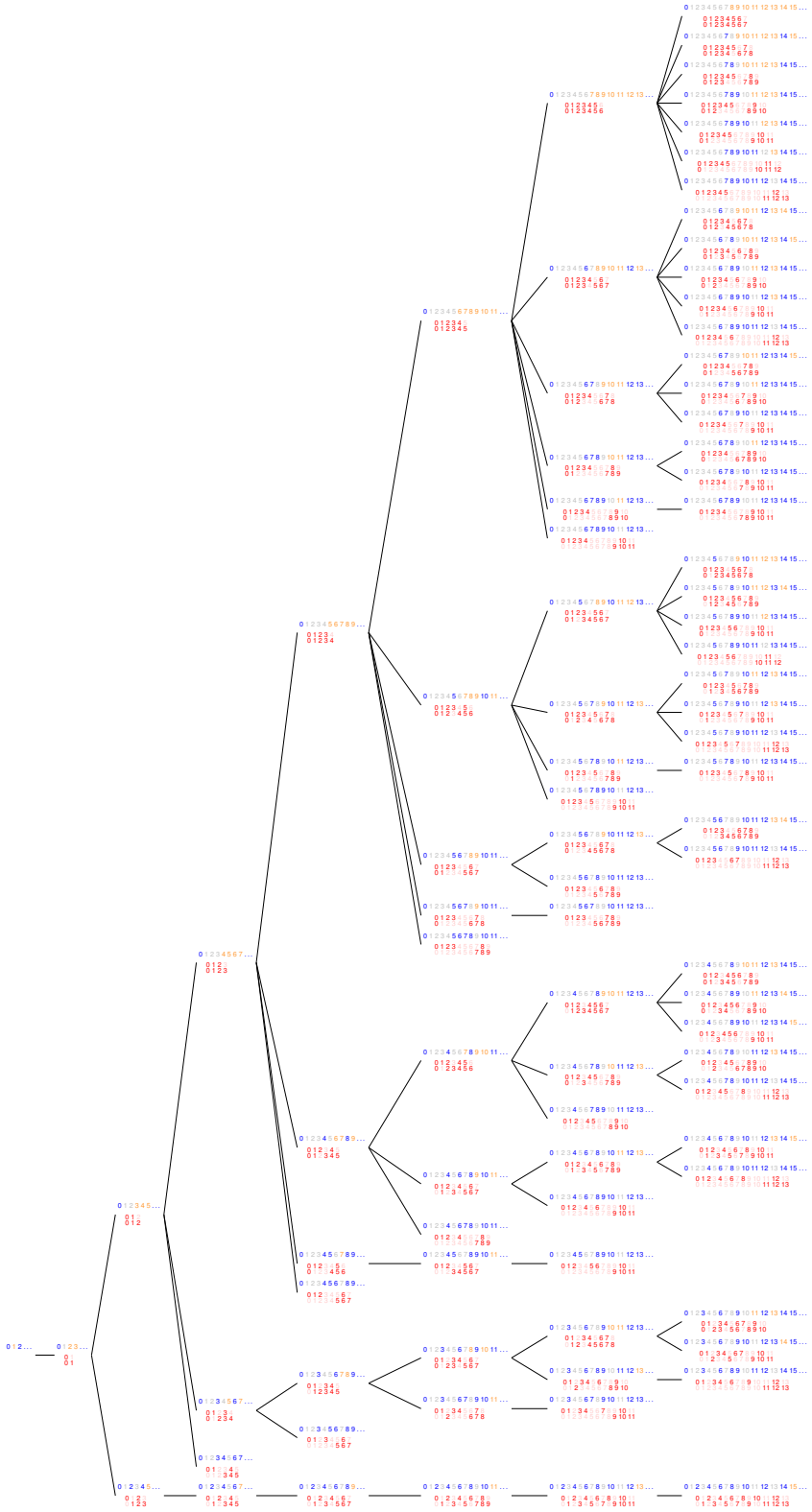
COMMAND:

```
./drawsgtree -g7 -n gapseedbitstream -n list -plain -inputfile
```

OUTPUT:

```
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: inputfile-plain-gapseedbitstream-list-semigrouptree-7.tex
```

GENERATED GRAPH:



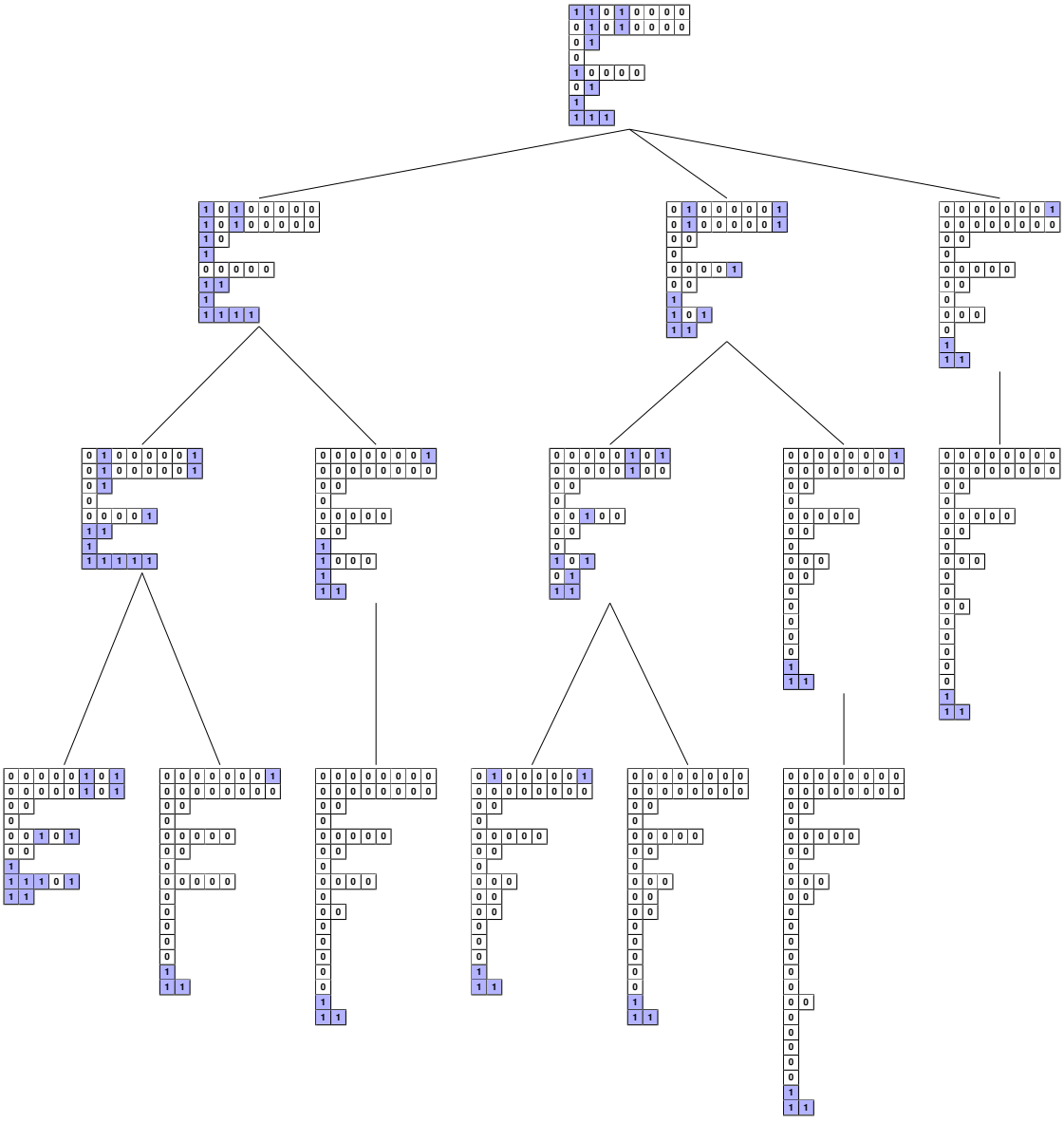
COMMAND:

```
./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27 30 -
    ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=8
N[2]=16
N[3]=18
N[4]=19
N[5]=24
N[6]=26
N[7]=27
N[8]=30
[g=25] count=6 ng=467224 [0 seconds]
GENERATED FILE: inputfile-seedstable-semigrouptree-25-root0816181924262730
    ↪ .tex
```

GENERATED GRAPH:



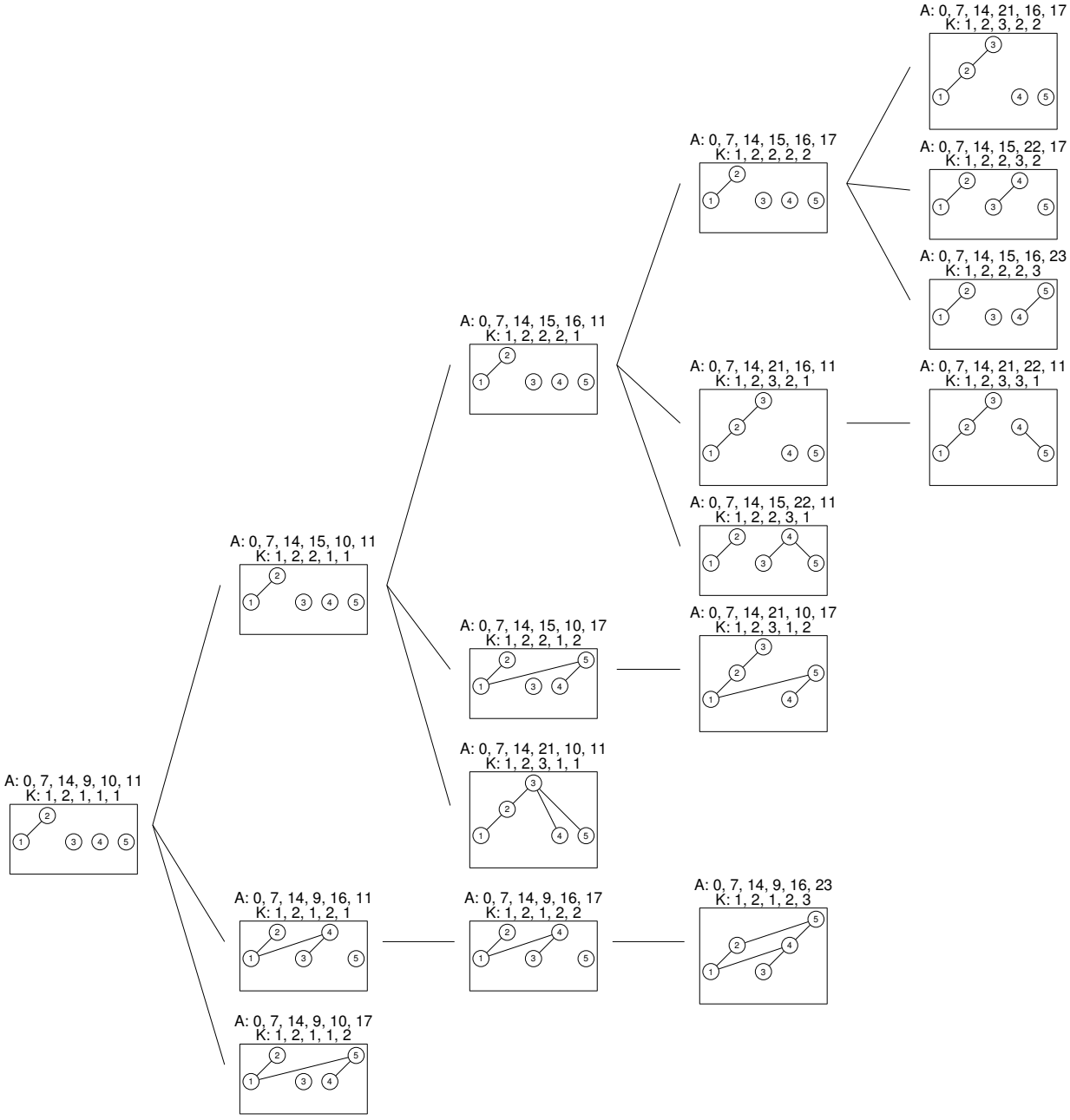
COMMAND:

```
./drawsgtree -g10 -n aperykunzposet 0 6 7 9 -inputfile
```

OUTPUT:

```
N[0]=0
N[1]=6
N[2]=7
N[3]=9
[g=10] count=4 ng=204 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-semigrouptree-10-root0679.tex
```

GENERATED GRAPH:



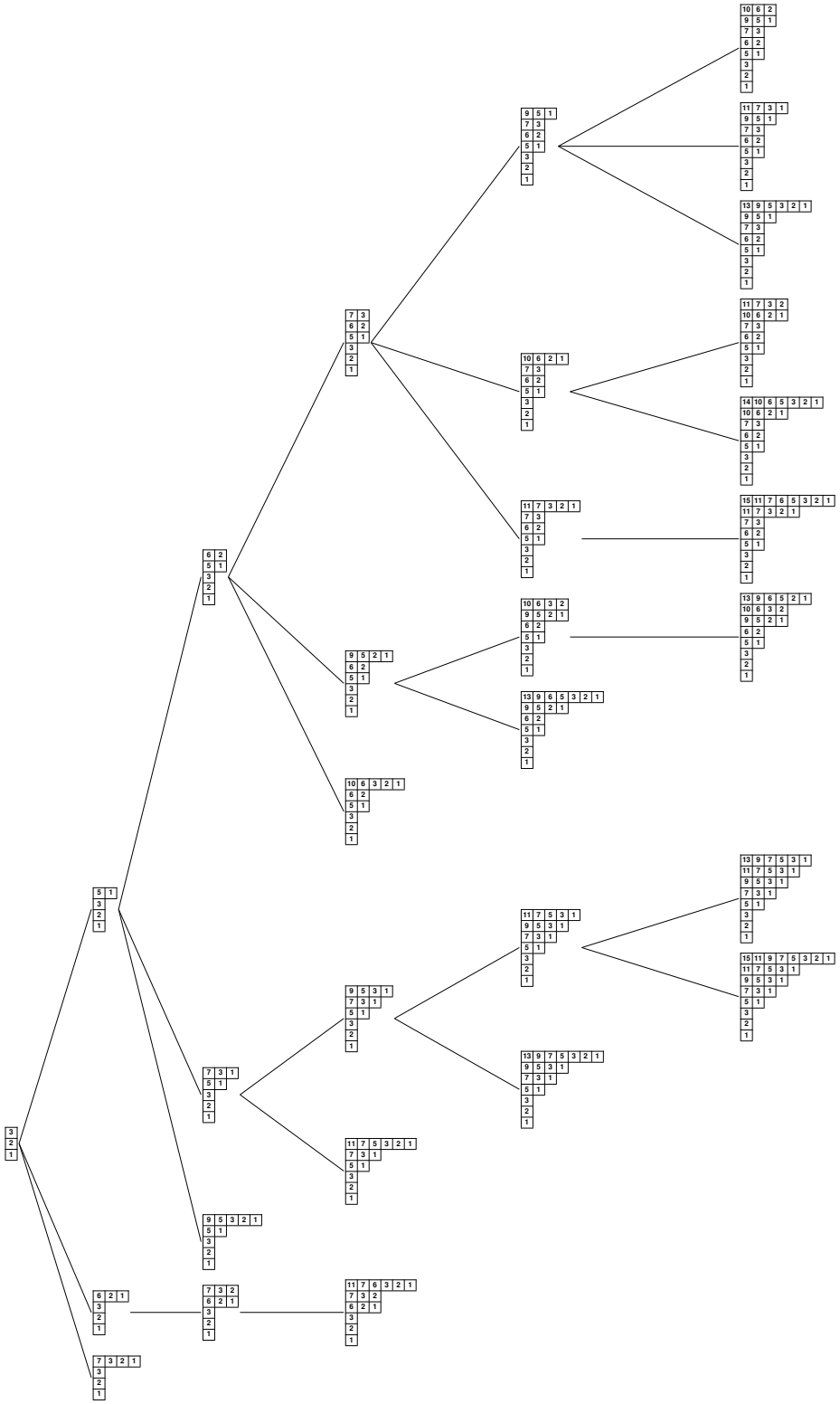
COMMAND:

```
./drawsgtree -g8 -m4 -n dyckhook -inputfile
```

OUTPUT:

```
[g=8] count=9 ng=67 [0 seconds]
GENERATED FILE: inputfile-dyckhook-semigrouptree-8-root04.tex
```

GENERATED GRAPH:



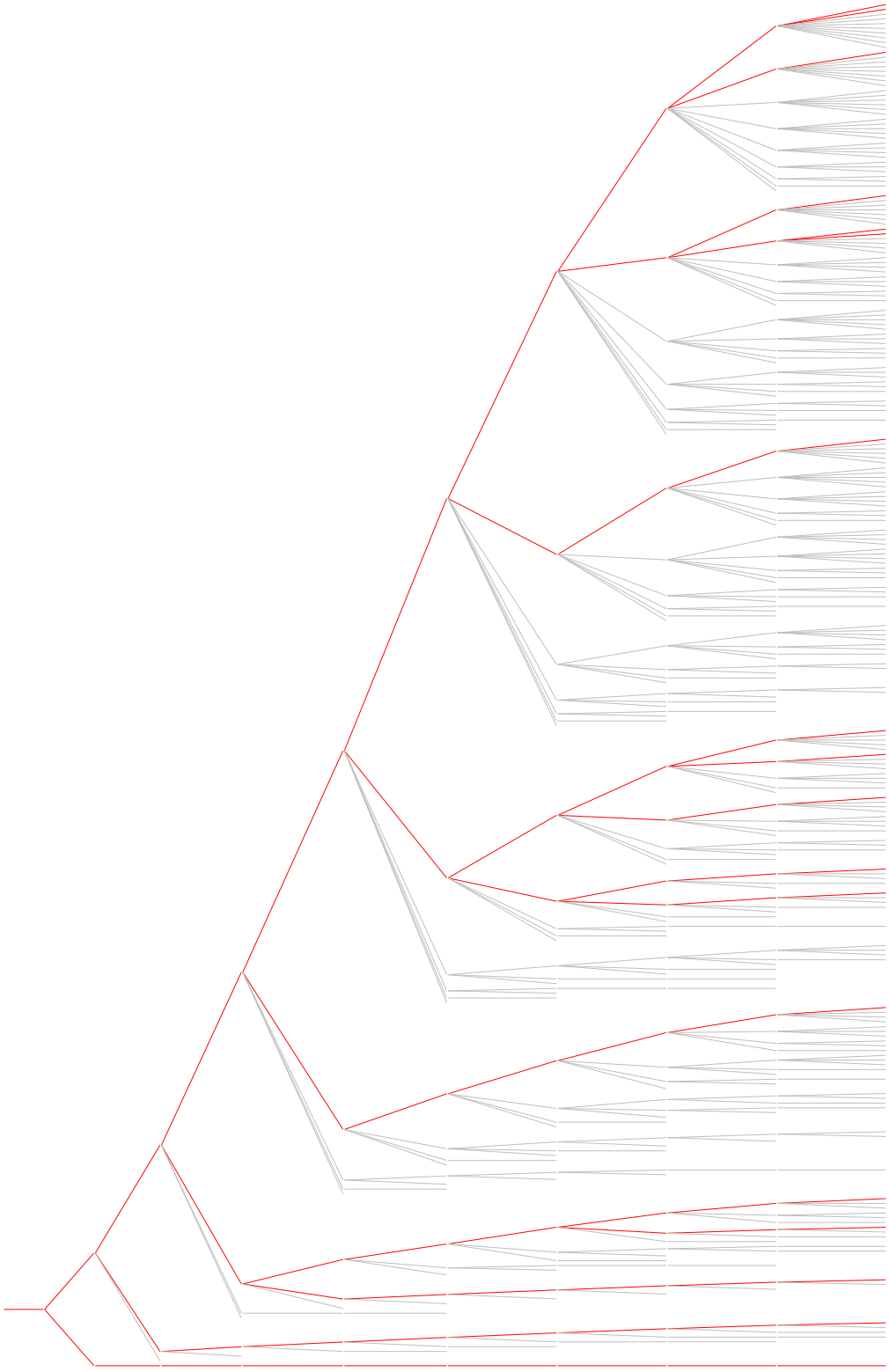
COMMAND:

```
./drawsgtree -g10 -e infinitechains -inputfile
```

OUTPUT:

```
[g=10] count=204 ng=204 [0 seconds]
GENERATED FILE: inputfile-infinitechains-semigrouptree-10.tex
```

GENERATED GRAPH:



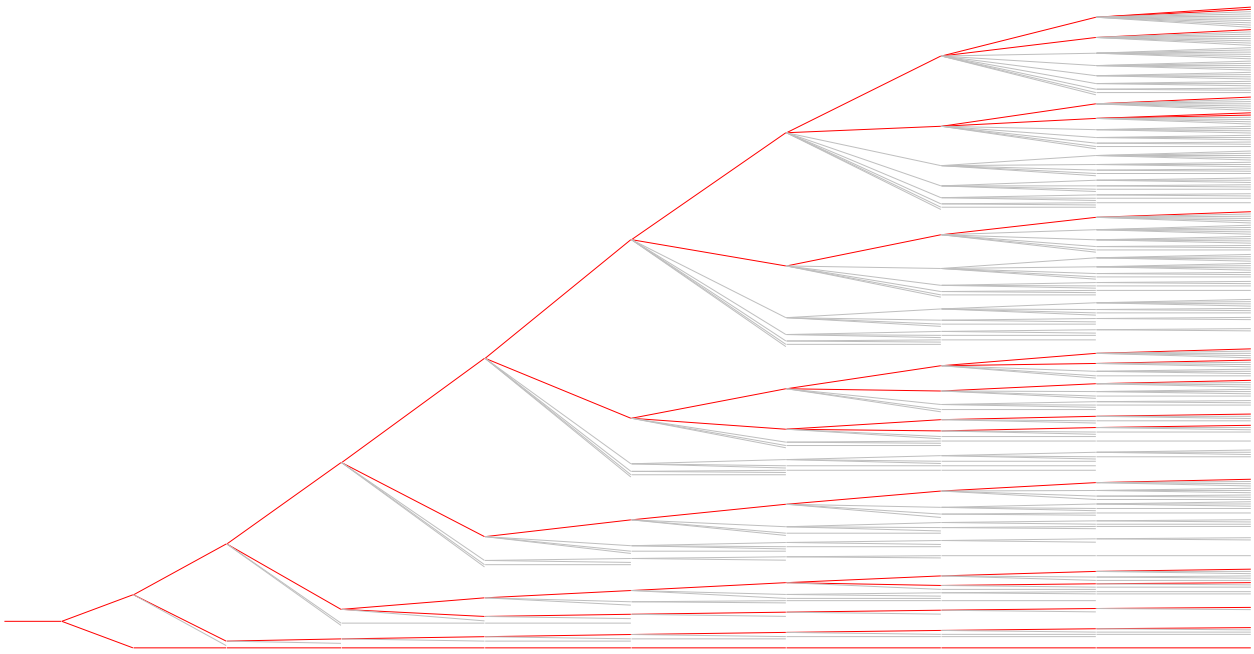
COMMAND:

```
./drawsgtree -g10 -e infinitechains -d 3. -inputfile
```

OUTPUT:

```
[g=10] count=204 ng=204 [0 seconds]
GENERATED FILE: inputfile-infinitechains-semigrouptree-10.tex
```

GENERATED GRAPH:



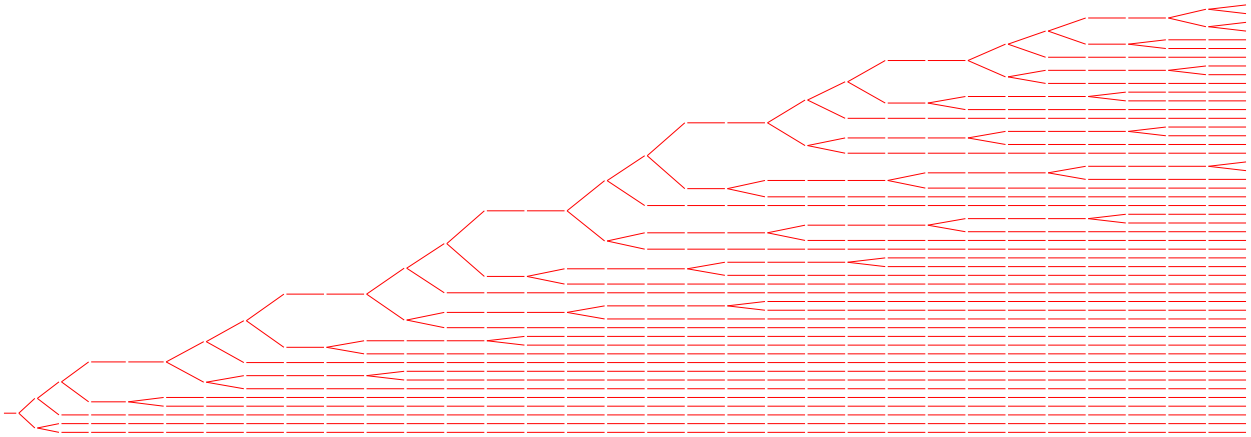
COMMAND:

```
./drawsgtree -g38 -m6 -e infinitechains -etrim -d .2 -inputfile
```

OUTPUT:

```
[g=38] count=50 ng=0 [0 seconds]
GENERATED FILE: inputfile-infinitechains-trim-semigrouptree-38-root06.tex
```

GENERATED GRAPH:



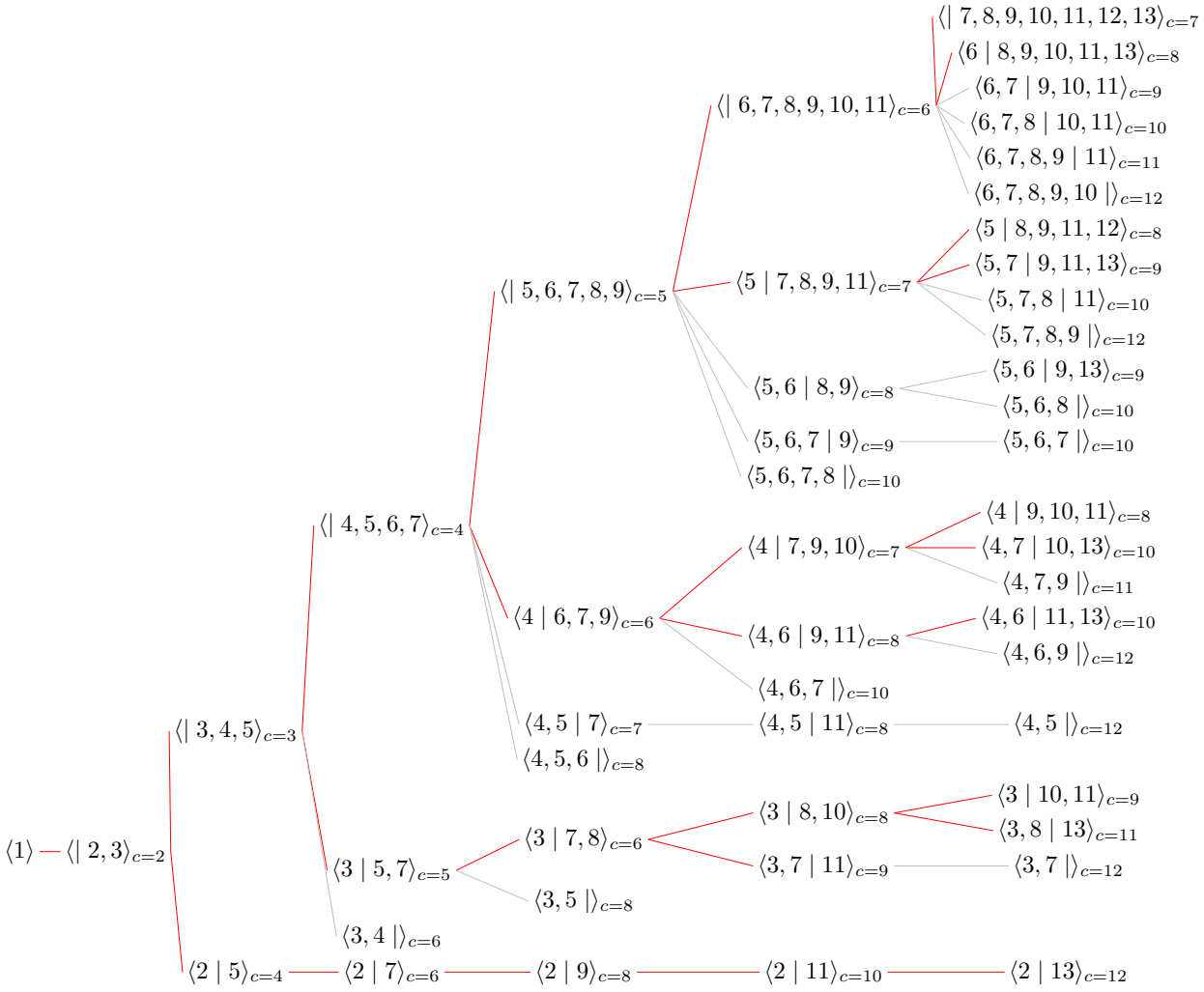
COMMAND:

```
./drawsgtree -g6 -e med -n minimalgenerators -inputfile
```

OUTPUT:

```
[g=6] count=23 ng=23 [0 seconds]
GENERATED FILE: inputfile-med-minimalgenerators-semigrouptree-6.tex
```

GENERATED GRAPH:



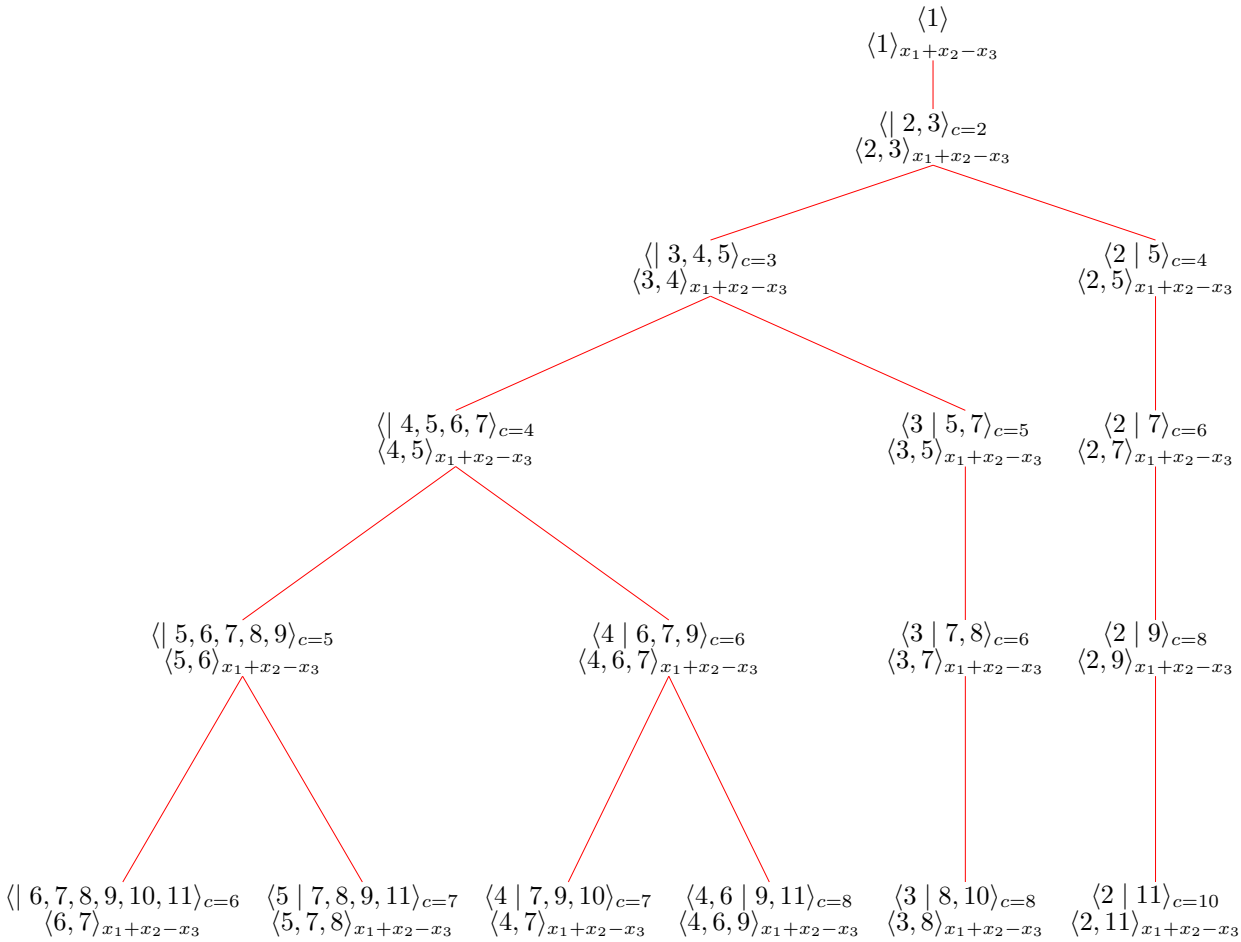
COMMAND:

```
./drawsgtree -g5 -e pattern 1+1-1 -n minimalgenerators -e trim -vertical -
    ↪ inputfile
```

OUTPUT:

```
pattern: x_{1}+x_{2}-x_{3}
[g=5] count=6 ng=12 [0 seconds]
GENERATED FILE: inputfile-pattern1+1-1-trim-minimalgenerators-
    ↪ semigrouptree-5.tex
```

GENERATED GRAPH:



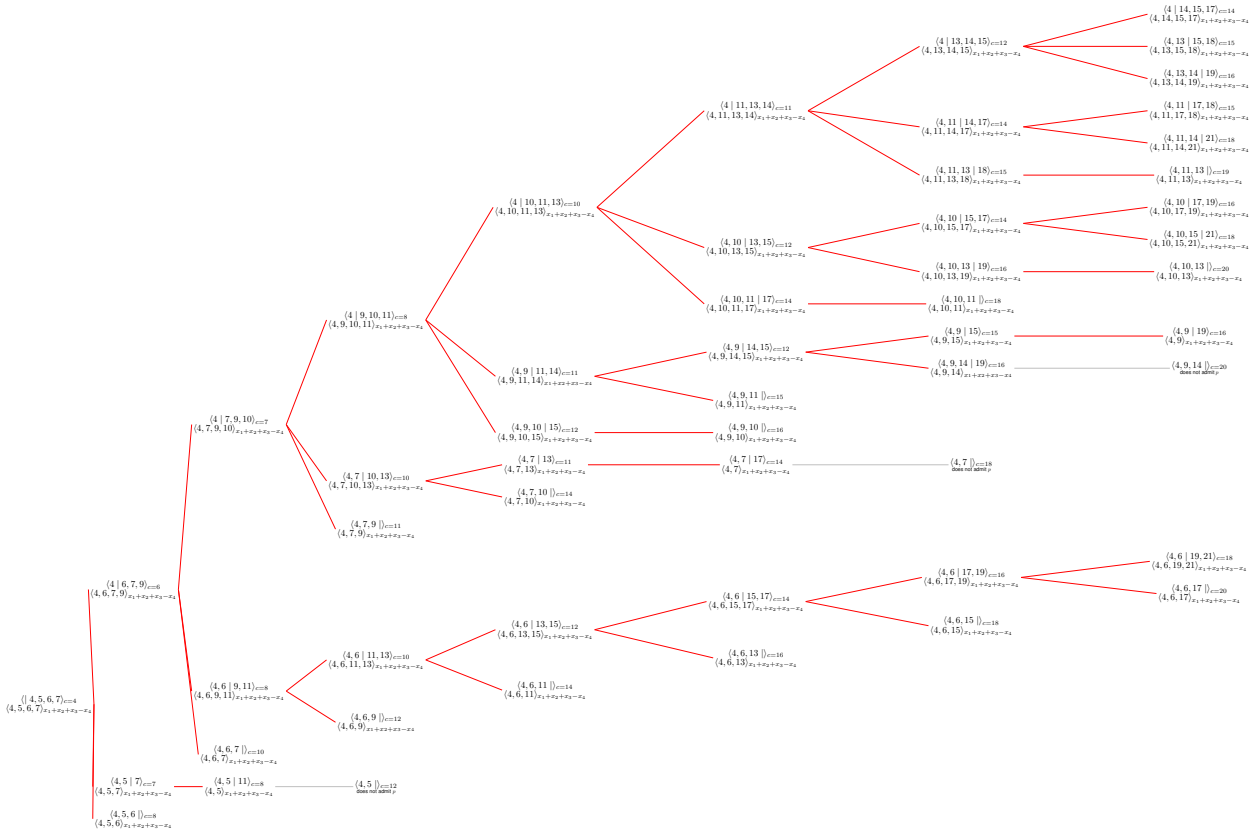
COMMAND:

```
./drawsgtree -g10 -m4 -e pattern 1+1+1-1 -n minimalgenerators -d 2.3 -s 4.
    ↪  -inputfile
```

OUTPUT:

```
pattern: x_{1}+x_{2}+x_{3}-x_{4}
[g=10] count=13 ng=204 [0 seconds]
GENERATED FILE: inputfile-pattern1+1+1-1-minimalgenerators-semigrouptree
    ↪ -10-root04.tex
```

GENERATED GRAPH:



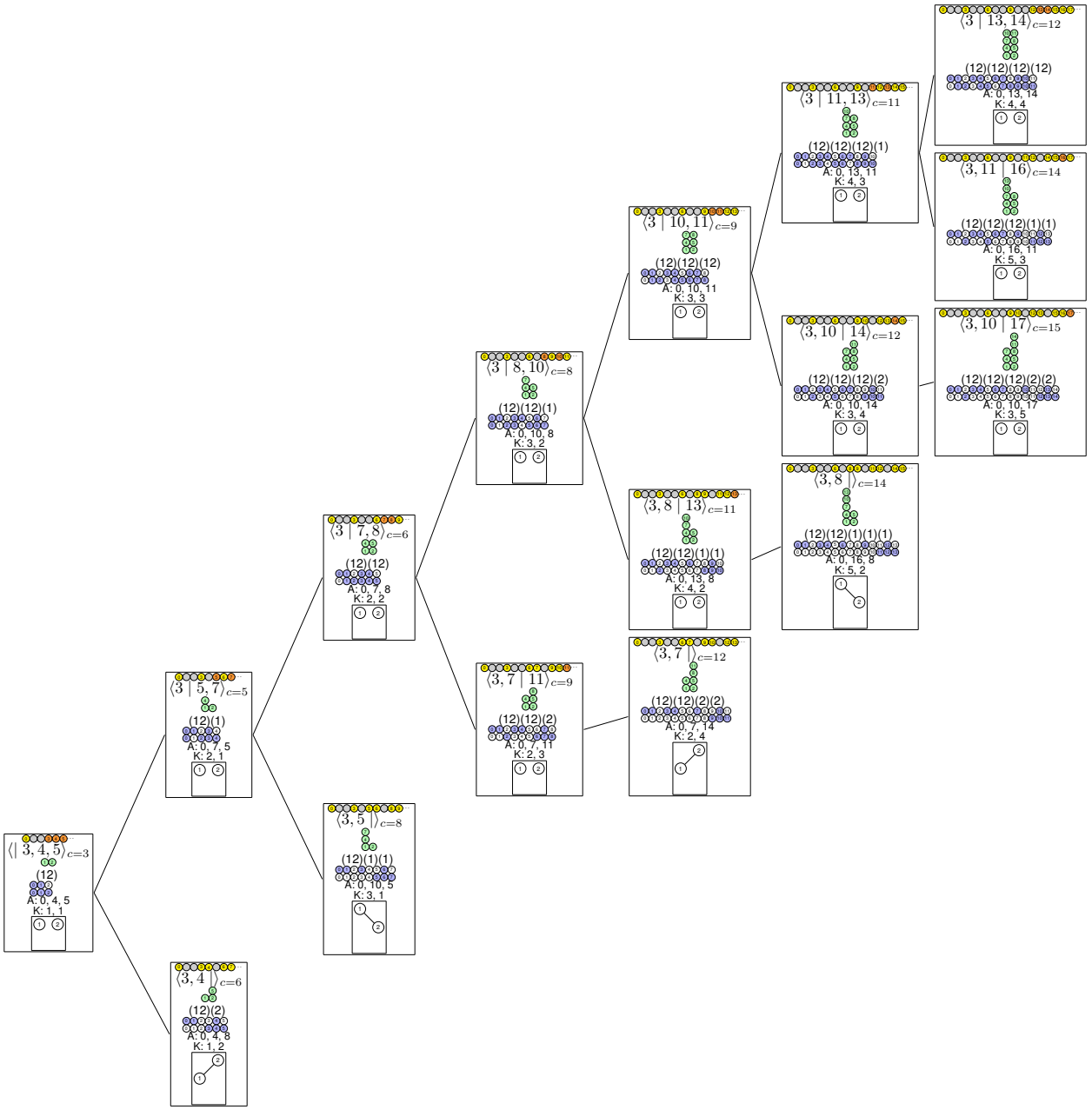
COMMAND:

```
./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
   ↪ gapseedbitstream -n aperykunzposet -framednodes -inputfile
```

OUTPUT:

```
[g=8] count=3 ng=67 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-gapseedbitstream-gapset-
   ↪ minimalgenerators-list-semigrouptree-8-root03.tex
```

GENERATED GRAPH:



COMMAND:

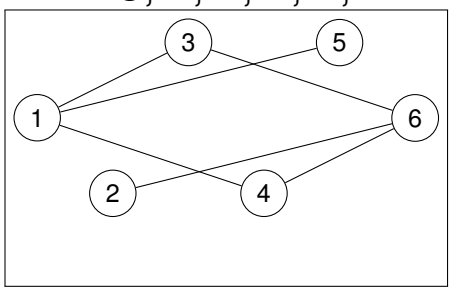
```
./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n aperykunzposet -
   ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=7
N[2]=9
N[3]=11
N[4]=14
N[5]=16
N[6]=18
N[7]=20
N[8]=21
N[9]=22
N[10]=23
N[11]=25
N[12]=27
[g=15] count=1 ng=2857 [0 seconds]
GENERATED FILE: inputfile-aperykunzposet-semigrouptree-15-
   ↪ root07911141618202122232527.tex
```

GENERATED GRAPH:

A: 0, 22, 9, 31, 11, 33, 20
K: 3, 1, 4, 1, 4, 2



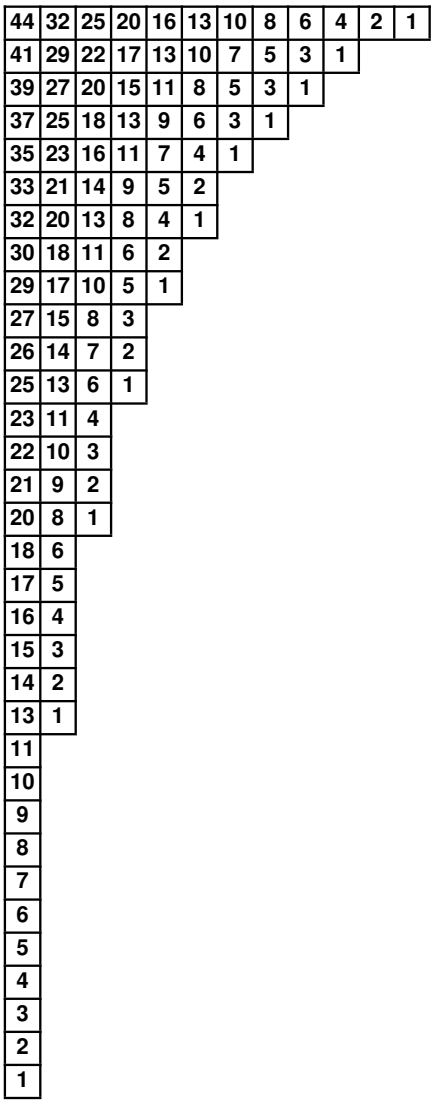
COMMAND:

```
./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n dyckhook -
   ↪ inputfile
```

OUTPUT:

```
N[0]=0
N[1]=12
N[2]=19
N[3]=24
N[4]=28
N[5]=31
N[6]=34
N[7]=36
N[8]=38
N[9]=40
N[10]=42
N[11]=43
N[12]=45
[g=33] count=1 ng=24896206 [0 seconds]
GENERATED FILE: inputfile-dyckhook-semigrouptree-33-
   ↪ root012192428313436384042 4345.tex
```

GENERATED GRAPH:



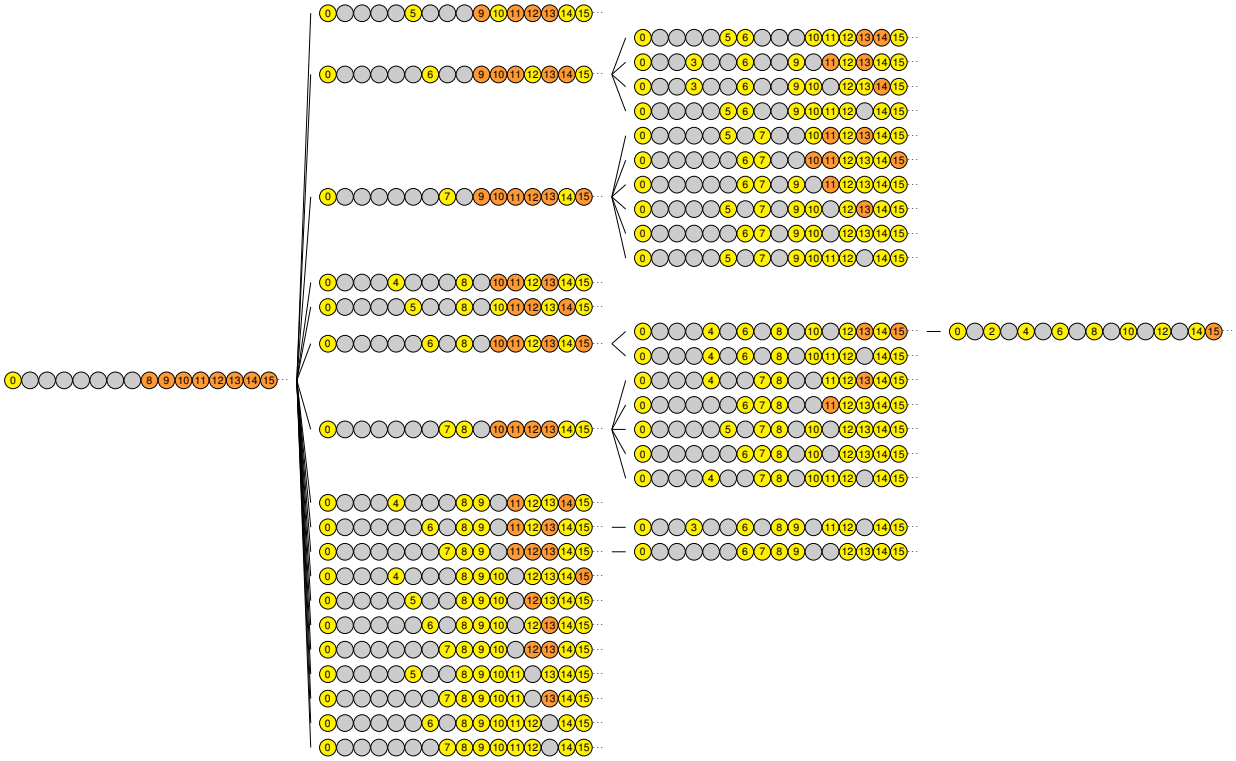
COMMAND:

```
./drawsgtree -g7 -t ordinarization -n list -inputfile
```

OUTPUT:

```
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: inputfile-list-ordinarizationtree-7.tex
```

GENERATED GRAPH:



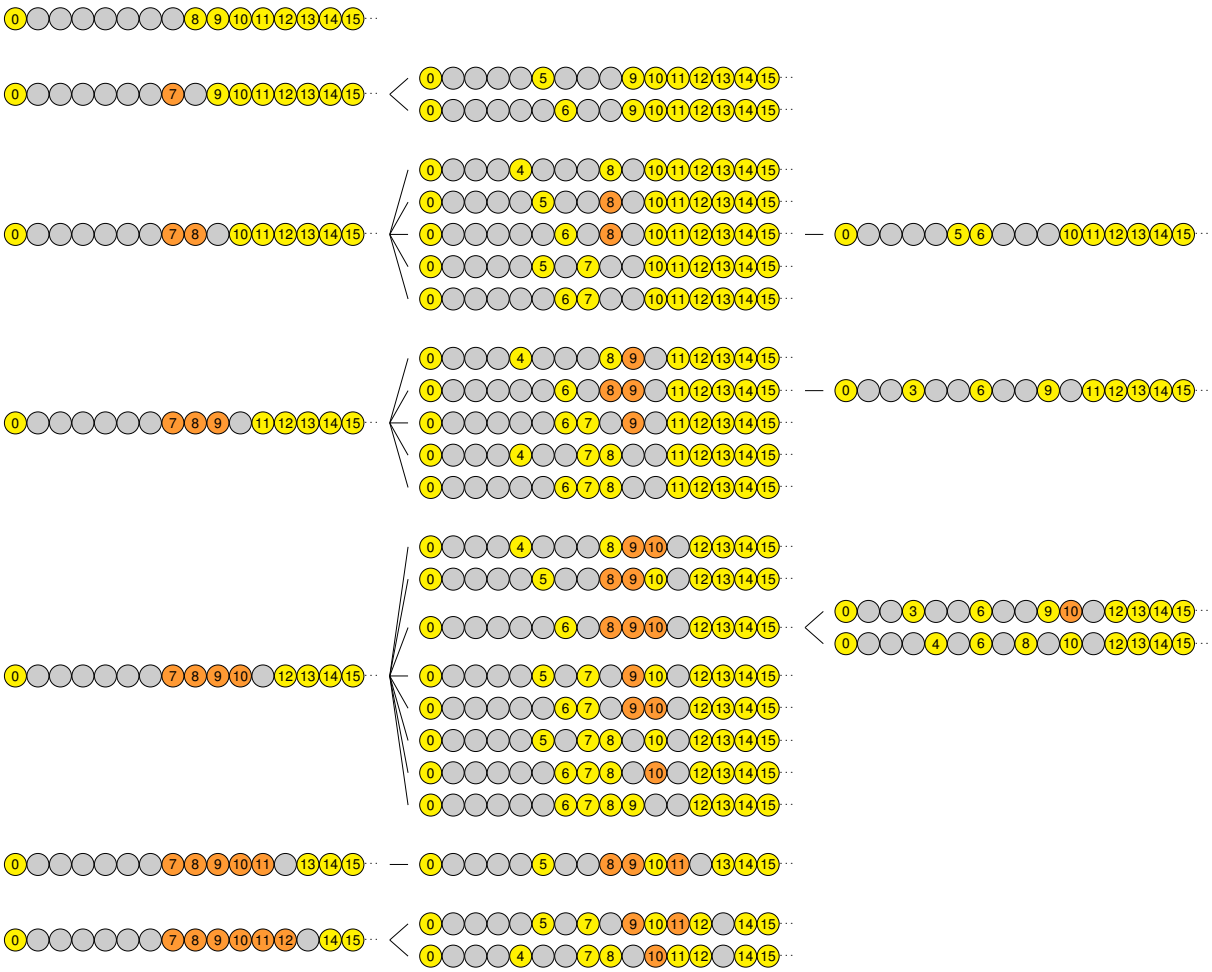
COMMAND:

```
./drawsgtree -g7 -t quasiordinarization -n list -inputfile
```

OUTPUT:

```
[g=7] count=34 ng=39 [0 seconds]
GENERATED FILE: inputfile-list-ordinarizationforest-7.tex
```

GENERATED GRAPH:



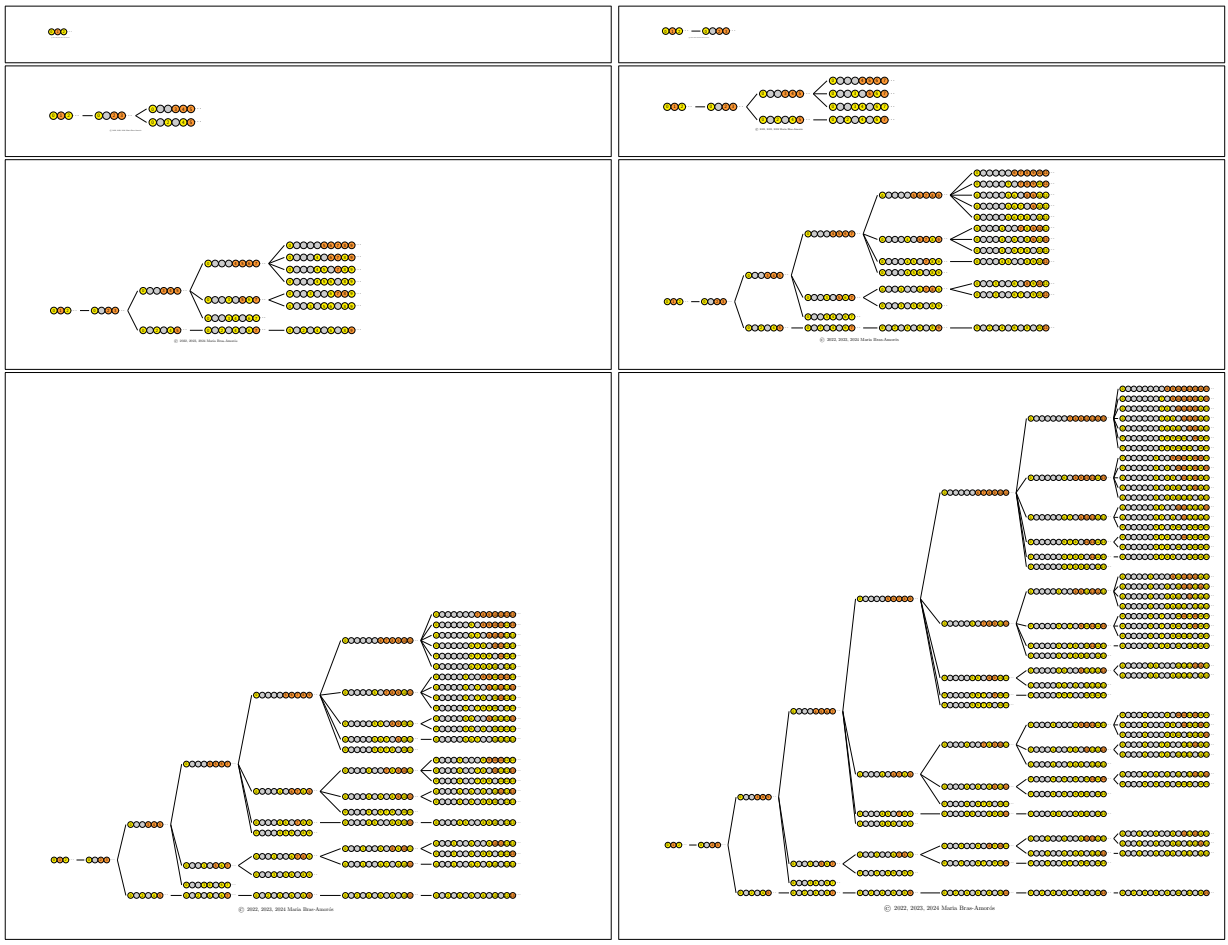
COMMAND:

```
./drawsgtree -g7 -n list -incremental
```

OUTPUT:

```
[g=0] count=1 ng=1 [0 seconds]
[g=1] count=1 ng=1 [0 seconds]
[g=2] count=2 ng=2 [0 seconds]
[g=3] count=4 ng=4 [0 seconds]
[g=4] count=7 ng=7 [0 seconds]
[g=5] count=12 ng=12 [0 seconds]
[g=6] count=23 ng=23 [0 seconds]
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: incremental-list-semigrouptree-7.tex
```

GENERATED GRAPHS:



References

[1] Maria Bras-Amorós. Drawsgtree. GitHub repository, 2022, 2023, 2024. `https://github.com/mbrasamoros/drawsgtree`.

[2] Maria Bras-Amorós. On the seeds and the great-grandchildren of a numerical semigroup. *Math. Comp.*, Accepted, 2023.

[3] Maria Bras-Amorós and Stanislav Bulygin. Towards a better understanding of the semigroup tree. *Semigroup Forum*, 79(3):561–574, 2009.

[4] Maria Bras-Amorós and Anna de Mier. Representation of numerical semigroups by Dyck paths. *Semigroup Forum*, 75(3):677–682, 2007.

[5] Maria Bras-Amorós and Julio Fernández-González. Computation of numerical semigroups by means of seeds. *Math. Comp.*, 87(313):2539–2550, 2018.

[6] Maria Bras-Amorós and Pedro A. García-Sánchez. Patterns on numerical semigroups. *Linear Algebra Appl.*, 414(2-3):652–669, 2006.

[7] Hannah Constantin, Ben Houston-Edwards, and Nathan Kaplan. Numerical sets, core partitions, and integer points in polytopes. In *Combinatorial and additive number theory. II*, volume 220 of *Springer Proc. Math. Stat.*, pages 99–127. Springer, Cham, 2017.

[8] Shalom Eliahou and Jean Fromentin. Gapsets and numerical semigroups. *J. Combin. Theory Ser. A*, 169:105129, 19, 2020.

[9] Nathan Kaplan and Christopher O'Neill. Numerical semigroups, polyhedra, and posets I: the group cone. *Comb. Theory*, 1:Paper No. 19, 23, 2021.

[10] Ernst Kunz. *Über die Klassifikation numerischer Halbgruppen*, volume 11 of *Regensburger Mathematische Schriften [Regensburg Mathematical Publications]*. Universität Regensburg, Fachbereich Mathematik, Regensburg, 1987.

[11] J. C. Rosales, P. A. García-Sánchez, J. I. García-García, and M. B. Branco. Systems of inequalities and numerical semigroups. *J. London Math. Soc. (2)*, 65(3):611–623, 2002.

[12] J. C. Rosales, P. A. García-Sánchez, J. I. García-García, and M. B. Branco. Numerical semigroups with maximal embedding dimension. *Int. J. Commut. Rings*, 2(1):47–53, 2003.

[13] Mariana Rosas-Ribeiro and Maria Bras-Amorós. Infinite chains in the tree of numerical semigroups. *Submitted*, 2023.