

Drawsgtree: a tool for visualizing properties in the semigroup tree

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In these pages we illustrate examples drawn by the code `drawsgtree`.
The code `drawsgtree` can be downloaded from <https://github.com/mbrasamoros/drawsgtree>.
Contact `maria.bras@urv.cat` for any comment or questions.
Please, cite as [?].

```
./drawsgtree -h
```

```

./sgroup [options] generate a latex file with the semigroup tree
-h display this help
-g <int> [mandatory option] maximum genus
-m <int> multiplicity
-n [option] node representation
  -n list list of semigroup elements
  -n minimalgenerators representation by minimal generator set
  -n gapset representation by gapsets
              (S. Eliahou, J. Fromentin: Gapsets and
                ↪ numerical semigroups, Journal of
                ↪ Combinatorial Theory, Series A, 2020)
-n gapseedbitstream representation with the gap bitstream and the
  ↪ seed bitstream
              (M. Bras-Amoros, J. Fernandez-Gonzalez:
                ↪ Computation of numerical semigroups
                ↪ by means of seeds, Math of Comput,
                ↪ 2018
                M. Bras-Amoros: On the seeds and the great
                ↪ -grandchildren of a numerical
                ↪ semigroup, Math of Comput, Accepted,
                ↪ 2023)
-n seedstable representation by seeds tables
              (M. Bras-Amoros, J. Fernandez-Gonzalez:
                ↪ Computation of numerical semigroups
                ↪ by means of seeds, Math of Comput,
                ↪ 2018
                M. Bras-Amoros: On the seeds and the great
                ↪ -grandchildren of a numerical
                ↪ semigroup, Math of Comput, Accepted,
                ↪ 2023)
-n dyckhook representation by augmented Dyck paths and Hook
  ↪ lengths
              (M. Bras-Amoros, A. de Mier: Representation
                ↪ of numerical semigroups by Dyck
                ↪ paths, Semigroup Forum, 2007)
              H. Constantin, B. Houston-Edwards, N.
                ↪ Kaplan: Numerical sets, core
                ↪ partitions, and integer points in
                ↪ polytopes, Combinatorial and
                ↪ Additive Number Theory, 2017)
-n aperykunuzposet representation by Apery sets, Kunz coordinates,
  ↪ and posets
              (E. Kunz: Uber die Klassifikation
                ↪ numerischer Halbgruppen, Regensburger
                ↪ Mathematische Schriften, 1987
                J.C. Rosales, P.A. Garcia-Sanchez, J.I.
                ↪ Garcia-Garcia, M.B. Branco: Systems

```

↪ of inequalities and numerical
 ↪ semigroups, J. Lond. Math. Soc.,
 ↪ 2002
 N. Kaplan, K. O'Neill: Numerical
 ↪ semigroups, polyhedra, and posets I:
 ↪ the group cone, Combinatorial
 ↪ Theory, 2021)

-e [option] edge distinction
 -e infinitechains distinguish the infinite chains in the
 ↪ semigroup tree
 (M. Bras-Amoros, S. Bulygin: Towards a
 ↪ better understanding of the semigroup
 ↪ tree, Semigroup Forum, 2009
 M. Rosas-Ribeiro, M. Bras-Amoros: Infinite
 ↪ chains in the tree of numerical
 ↪ semigroups. Submitted, 2023)

-e med distinguish the chains of MED semigroups
 (J.C. Rosales, P.A. Garcia-Sanchez, J.I.
 ↪ Garcia-Garcia, M.B. Branco: Numerical
 ↪ semigroups with maximal embedding
 ↪ dimension, Int. J. Commut. Rings,
 ↪ 2003)

-e pattern <sign1>a1<sign2>a2..
 distinguish the semigroups admitting the (
 ↪ strongly admissible) pattern <sign1>a1x1
 ↪ +<sign2>a2x2+...+<signn>anxn
 (M. Bras-Amoros, P.A. Garcia-Sanchez:
 ↪ Patterns on numerical semigroups,
 ↪ Linear Algebra App. 2006)

-etrim discard the non-distinguished edges together with
 ↪ all its descendants

-t [option] alternative tree
 -t ordinarization
 (M. Bras-Amoros: The ordinarization
 ↪ transform of a numerical semigroup
 ↪ and semigroups with a large number of
 ↪ intervals, J. of Pure and App.
 ↪ Algebra, 2012)

-t quasiordinarization
 (M. Bras-Amoros, H. Perez-Roses, J. M.
 ↪ Serradilla-Merino: Quasi-
 ↪ ordinarization transform of a
 ↪ numerical semigroup, Symmetry, 2021)

-incremental incremental with genus
 -inputfile input file (not compiling without a calling file)
 -vertical vertical tree growing down
 -plain plain representation of objects using less memory
 -blackandwhite graph without colors
 -framednodes frame each tree node
 -d <float> enlarge distance between generations by the
 ↪ specified factor

-s <float> enlarge distance between siblings by the specified
 ↪ factor
 -rotated rotated 90 degrees
 -o <filename> output file name
 0 N[1] N[2] ... N[k] root at the semigroup $\{0, N[1], N[2], N[k], N[k]+1, N[k]$
 ↪ $]+2, \dots\}$

examples: ./drawsgtree -g5 -n list
 ./drawsgtree -g7 -n list -incremental
 ./drawsgtree -g7 -n list 0 5 8 -s .37 -d 1.2
 ./drawsgtree -g4 -n minimalgenerators -vertical
 ./drawsgtree -g5 -n gapset -vertical
 ./drawsgtree -g7 -n gapseedbitstream -n list -plain
 ./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27
 ↪ 30
 ./drawsgtree -g10 -n aperykunzposet 0 6 7 9
 ./drawsgtree -g8 -m4 -n dyckhook
 ./drawsgtree -g10 -e infinitechains
 ./drawsgtree -g10 -e infinitechains -d 3.
 ./drawsgtree -g42 -m6 -e infinitechains -etrim -d .2
 ./drawsgtree -g6 -e med -n minimalgenerators
 ./drawsgtree -g5 -e pattern 1+1-1 -n minimalgenerators -e trim
 ↪ -vertical
 ./drawsgtree -g10 -m4 -e pattern 1+1+1-1 -n minimalgenerators -
 ↪ d 2.3 -s 4.
 ./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
 ↪ gapseedbitstream -n aperykunzposet -framednodes
 ./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n
 ↪ aperykunzposet
 ./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n
 ↪ dyckhook
 ./drawsgtree -g7 -t ordinarization -n list
 ./drawsgtree -g7 -t quasiordinarization -n list

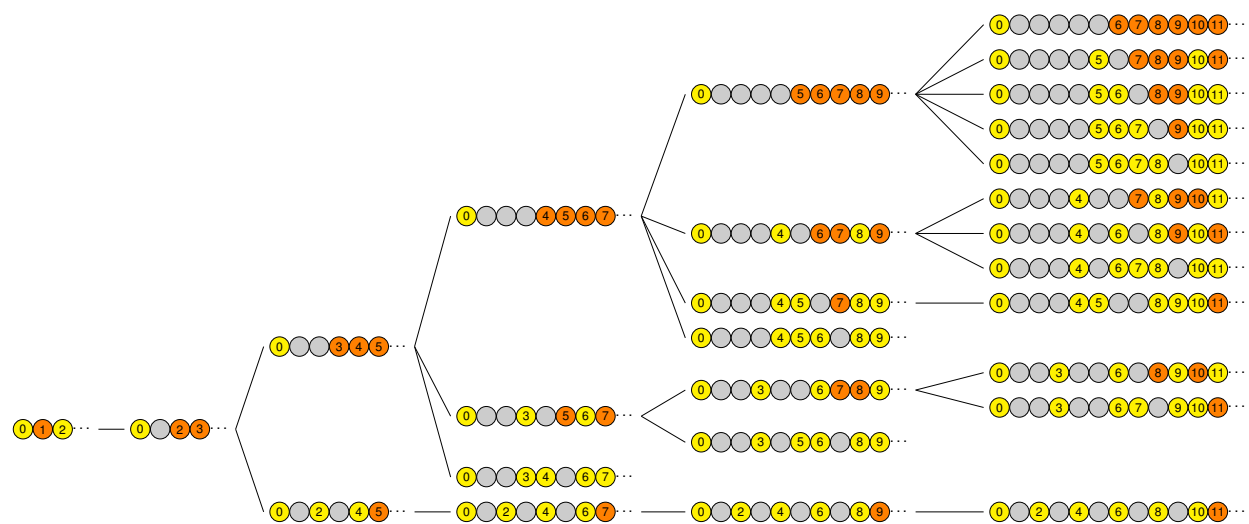
COMMAND:

```
./drawsgtree -g5 -n list -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]  
GENERATED FILE: inputfile-list-semigrouptree-5.tex
```

GENERATED GRAPH:



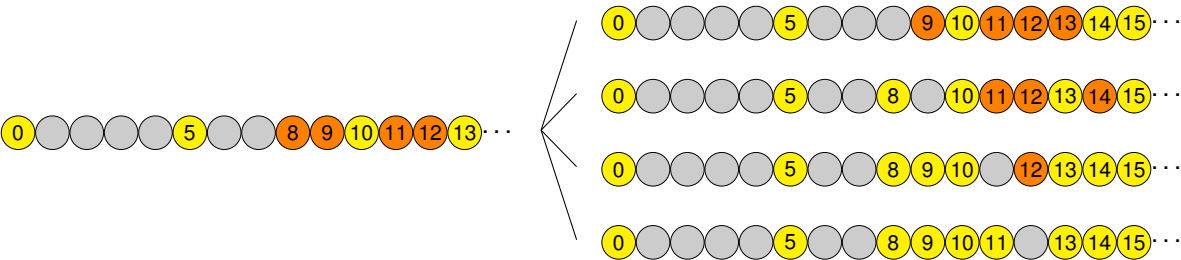
COMMAND:

```
./drawsgtree -g7 -n list 0 5 8 -s .37 -d 1.2 -inputfile
```

OUTPUT:

```
N[0]=0
N[1]=5
N[2]=8
[g=7] count=4 ng=39 [0 seconds]
GENERATED FILE: inputfile-list-semigrouptree-7-root058.tex
```

GENERATED GRAPH:



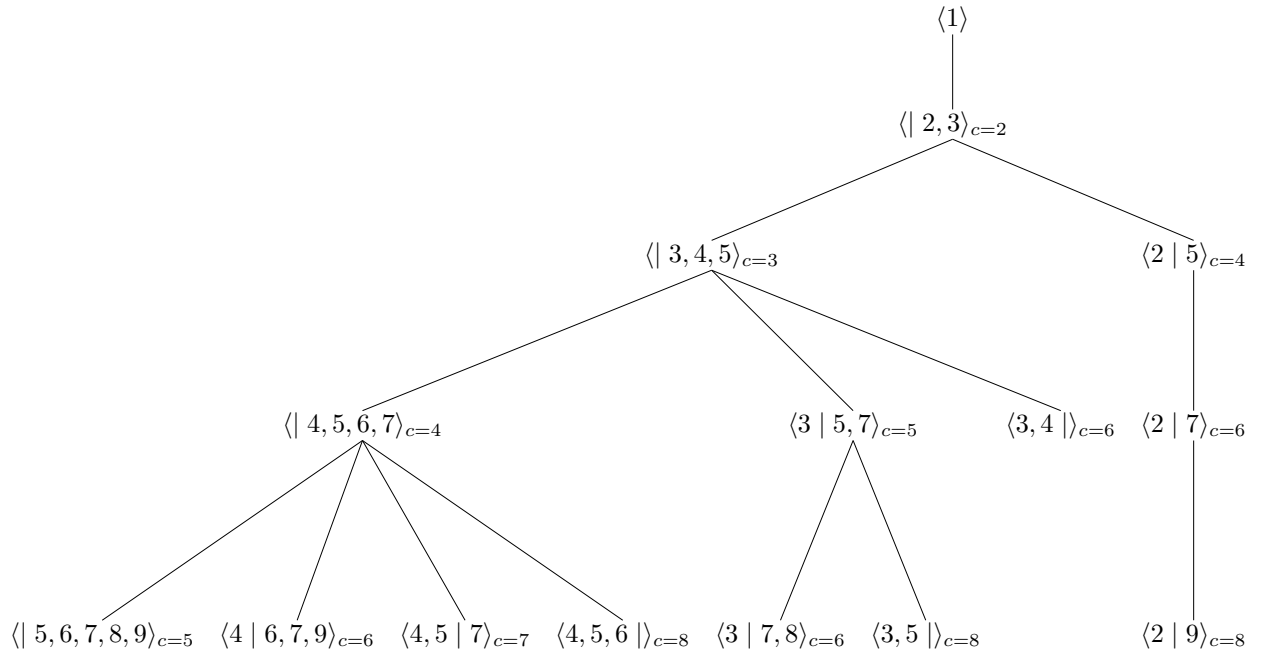
COMMAND:

```
./drawsgtree -g4 -n minimalgenerators -vertical -inputfile
```

OUTPUT:

```
[g=4] count=7 ng=7 [0 seconds]
```

```
GENERATED FILE: inputfile-minimalgenerators-semigroup-tree-4.tex
```

GENERATED GRAPH:

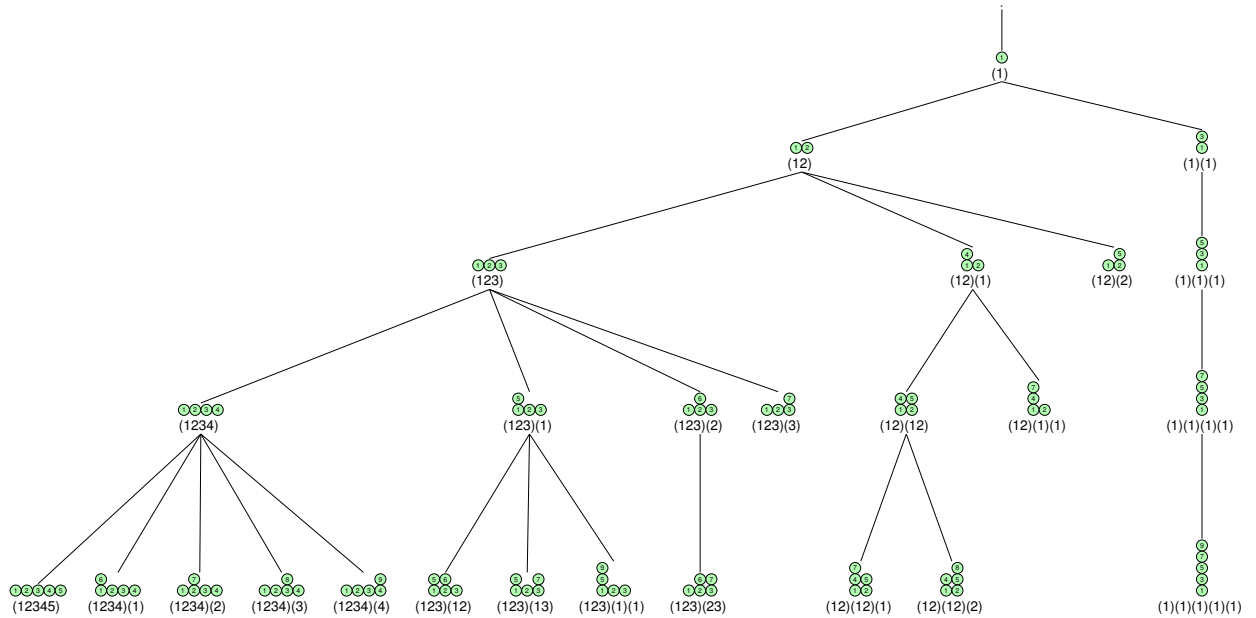
COMMAND:

```
./drawsgtree -g5 -n gapset -vertical -inputfile
```

OUTPUT:

```
[g=5] count=12 ng=12 [0 seconds]
```

```
GENERATED FILE: inputfile-gapset-semigroupptree-5.tex
```

GENERATED GRAPH:

COMMAND:

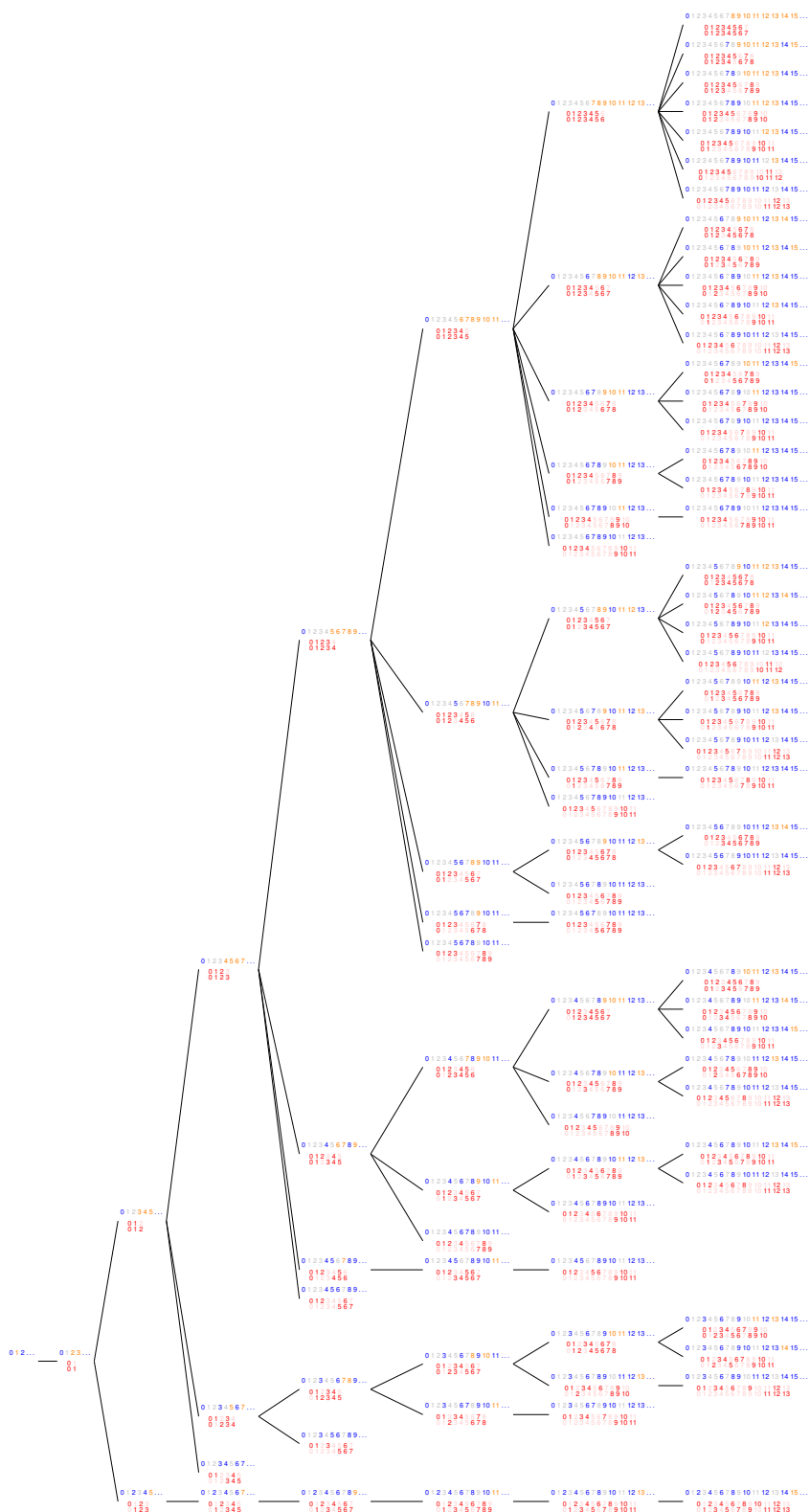
```
./drawsgtree -g7 -n gapseedbitstream -n list -plain -inputfile
```

OUTPUT:

[g=7] count=39 ng=39 [0 seconds]

GENERATED FILE: inputfile-plain-gapseedbitstream-list-semigrouptree-7.tex

GENERATED GRAPH:



COMMAND:

```
./drawsgtree -g25 -n seedstable -vertical 0 8 16 18 19 24 26 27 30 -  
↪ inputfile
```

OUTPUT:

N[0]=0

N[1]=8

N[2]=16

N[3]=18

N[4]=19

N[5]=24

N[6]=26

N[7]=27

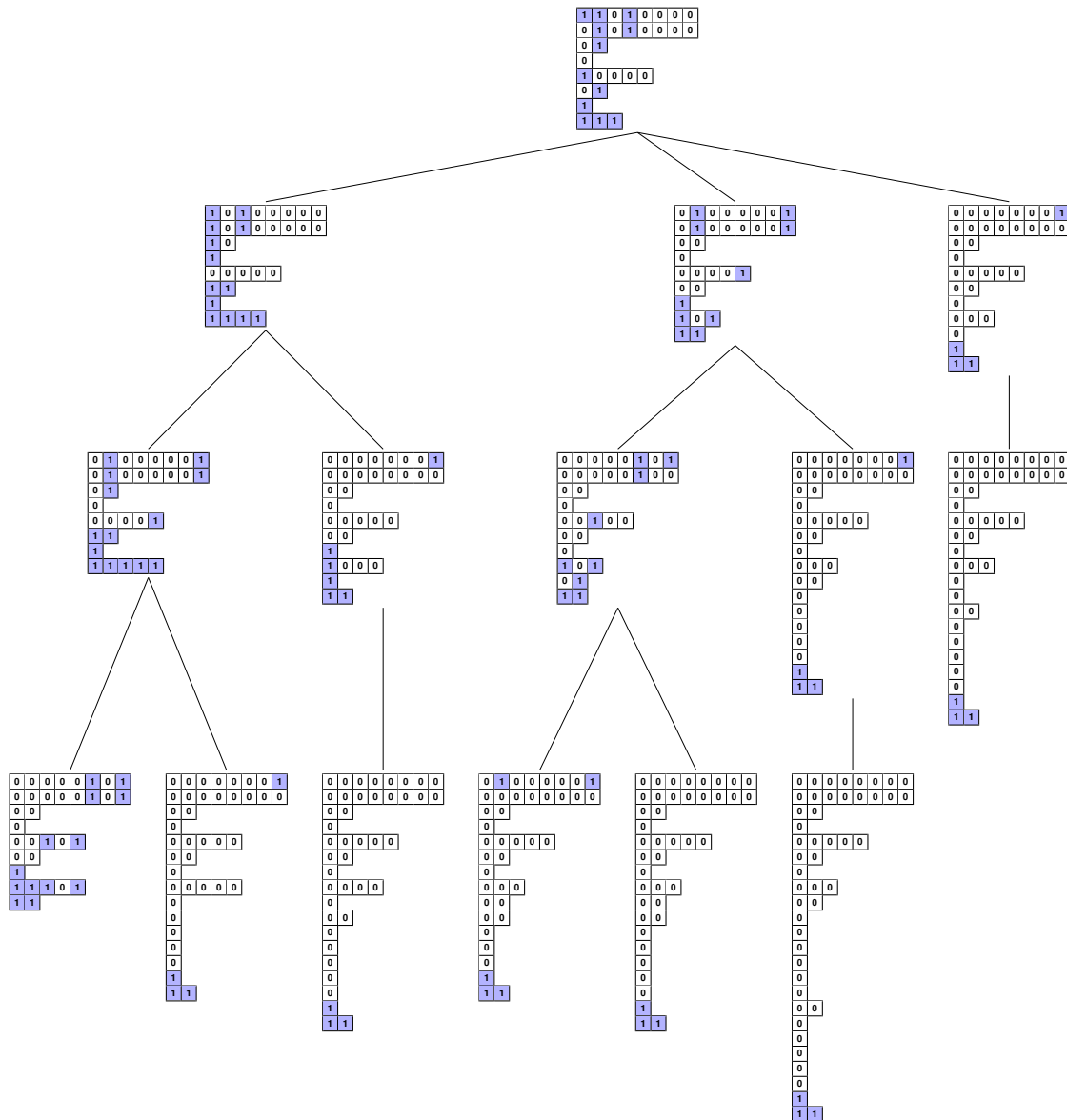
N[8]=30

[g=25] count=6 ng=467224 [0 seconds]

GENERATED FILE: inputfile-seedstable-semigrouptree-25-root0816181924262730

↪ .tex

GENERATED GRAPH:



```
./drawsgtree -g10 -n aperykunuzposet 0 6 7 9 -inputfile
```

OUTPUT:

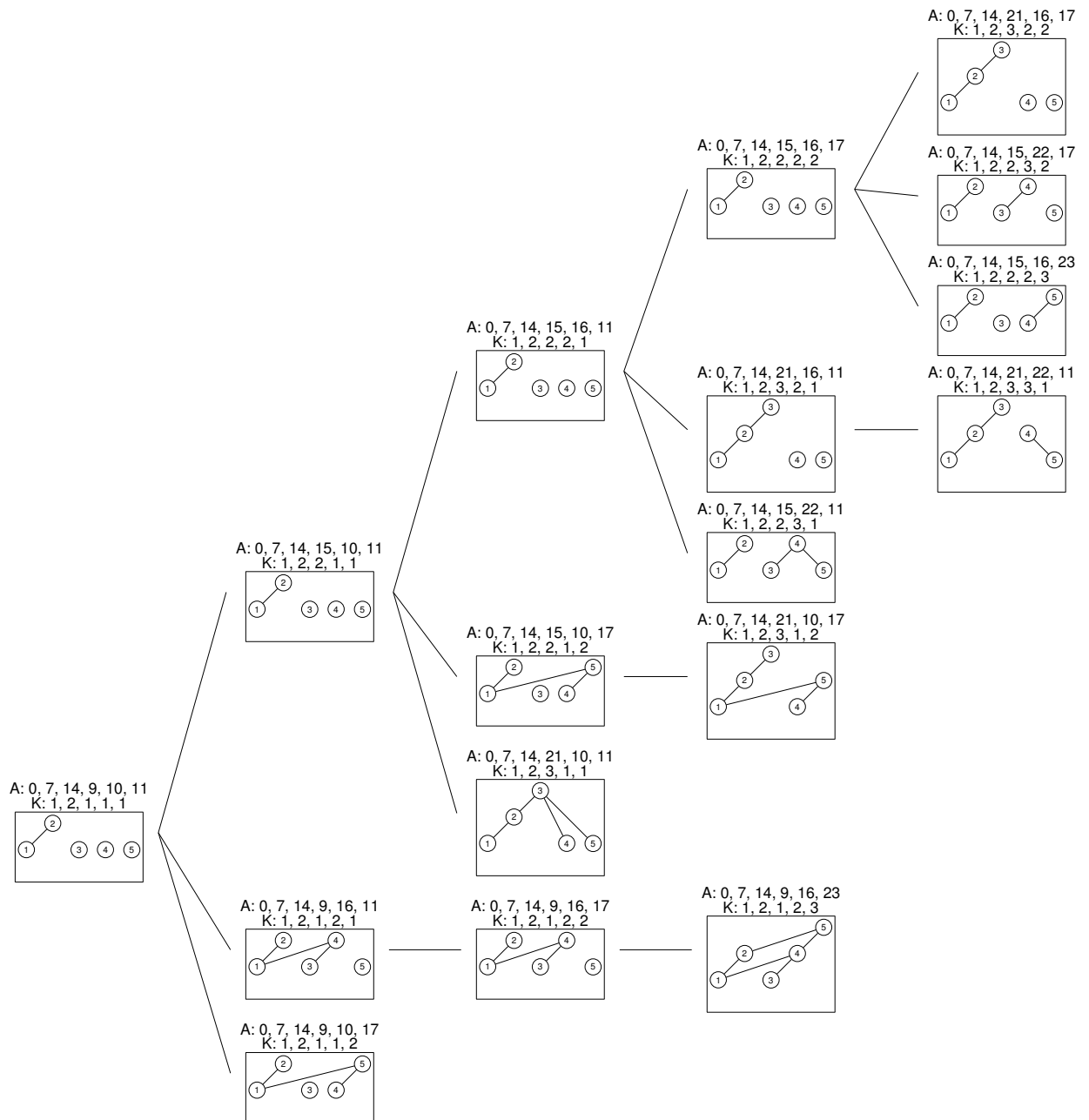
$$N[0] = 0$$
$$N[1] = 6$$
$$N[2] = 7$$

N [3] = 9

```
[g=10] count=4 ng=204 [0 seconds]
```

GENERATED FILE: inputfile-aperykunuzposet-semigrouptree-10-root0679.tex

GENERATED GRAPH:



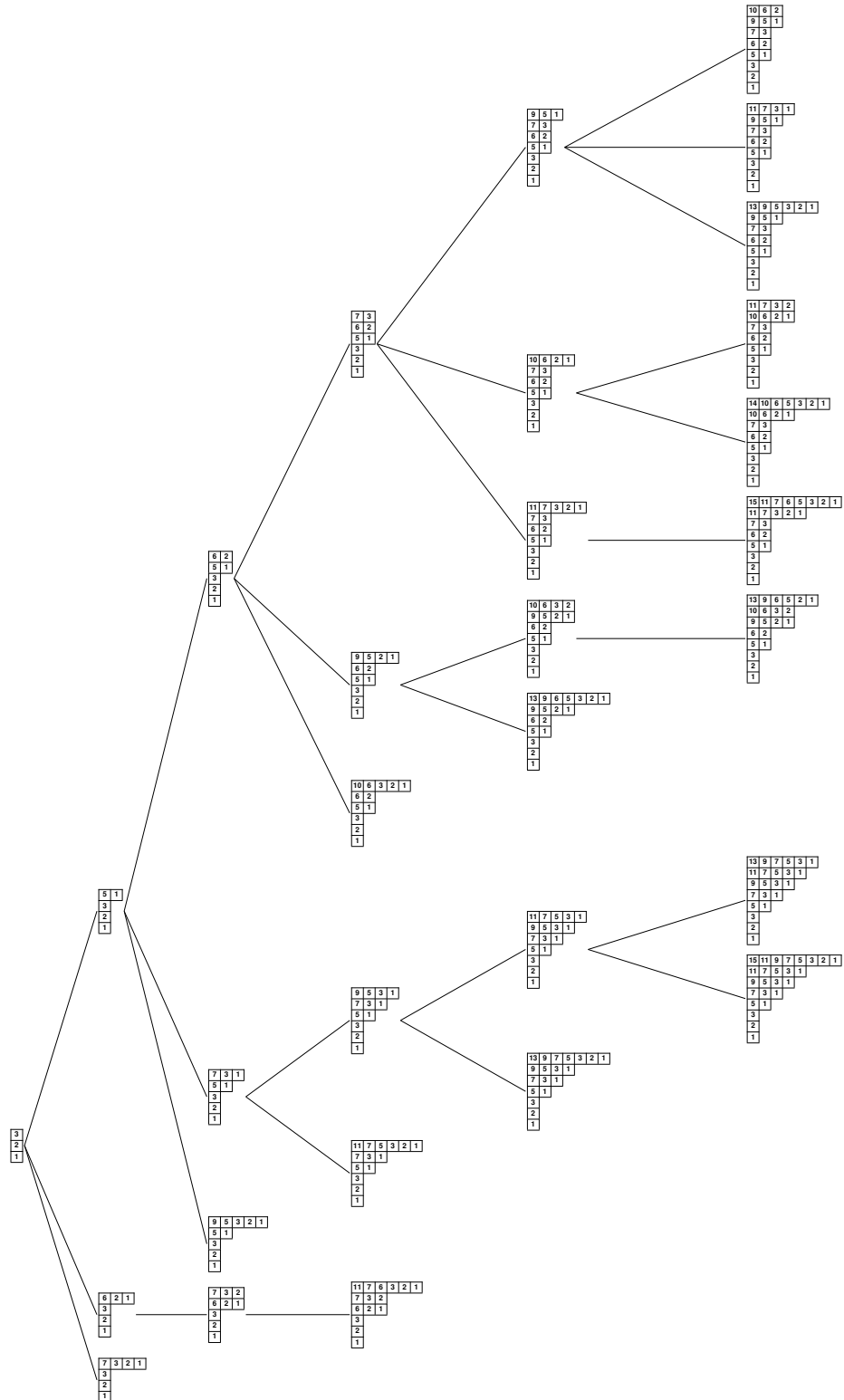
COMMAND:

```
./drawsgtree -g8 -m4 -n dyckhook -inputfile
```

OUTPUT:

```
[g=8] count=9 ng=67 [0 seconds]
```

```
GENERATED FILE: inputfile-dyckhook-semigrouptree-8-root04.tex
```

GENERATED GRAPH:

COMMAND:

```
./drawsgtree -g10 -e infinitechains -inputfile
```

OUTPUT:

```
[g=10] count=204 ng=204 [0 seconds]
```

```
GENERATED FILE: inputfile-infinitechains-semigroupptree-10.tex
```

GENERATED GRAPH:

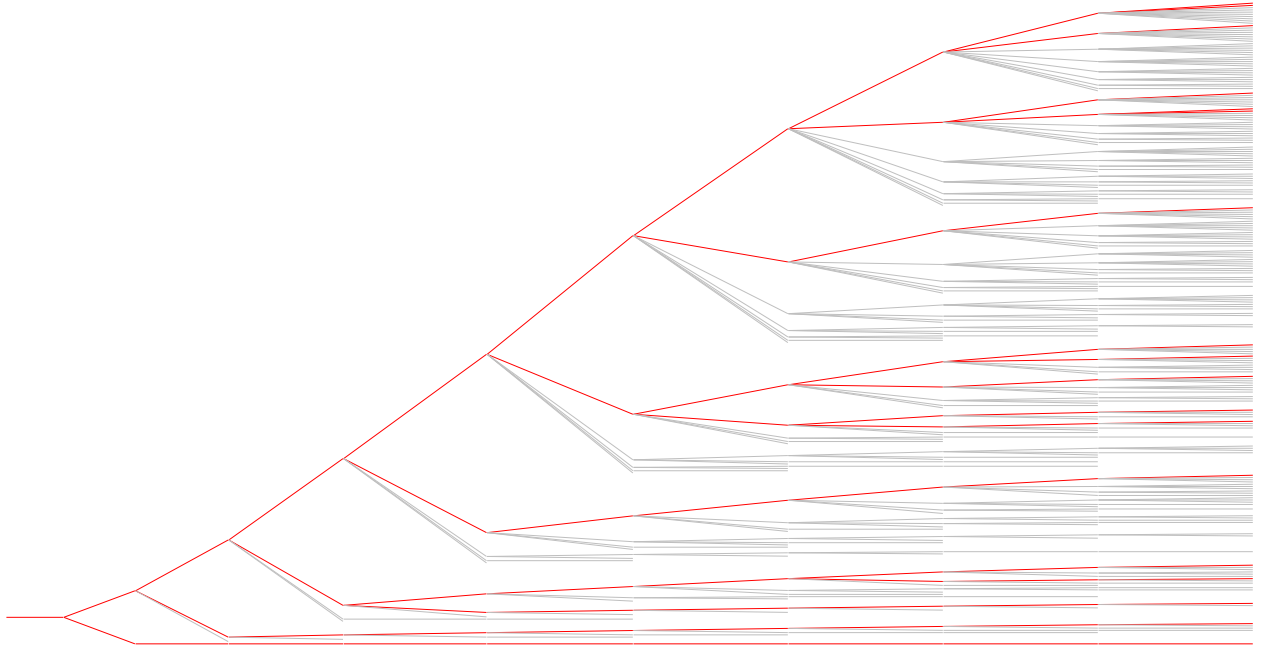
COMMAND:

```
./drawsgtree -g10 -e infinitechains -d 3. -inputfile
```

OUTPUT:

```
[g=10] count=204 ng=204 [0 seconds]
```

```
GENERATED FILE: inputfile-infinitechains-semigrouptree-10.tex
```

GENERATED GRAPH:

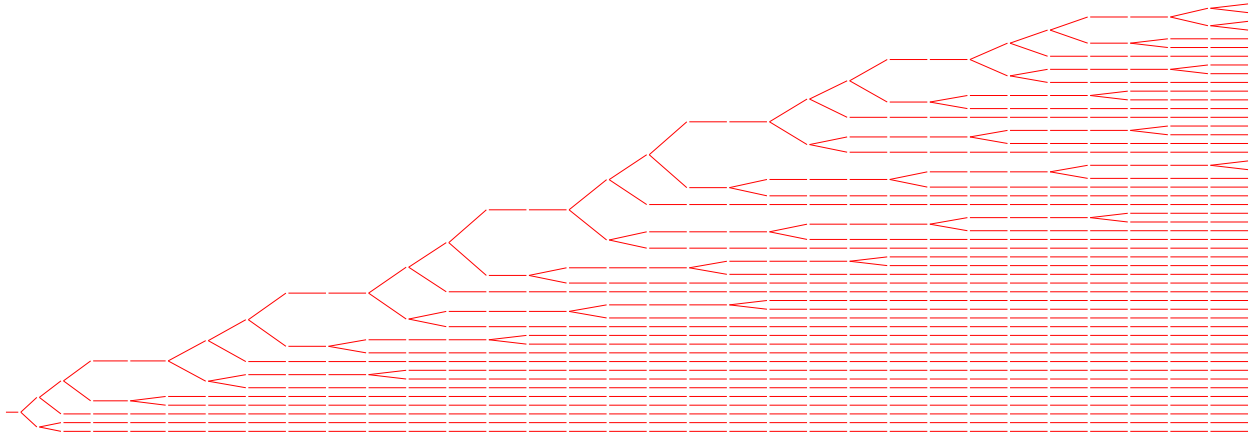
COMMAND:

```
./drawsgtree -g38 -m6 -e infinitechains -etrim -d .2 -inputfile
```

OUTPUT:

```
[g=38] count=50 ng=0 [0 seconds]
```

```
GENERATED FILE: inputfile-infinitechains-trim-semigroup-tree-38-root06.tex
```

GENERATED GRAPH:

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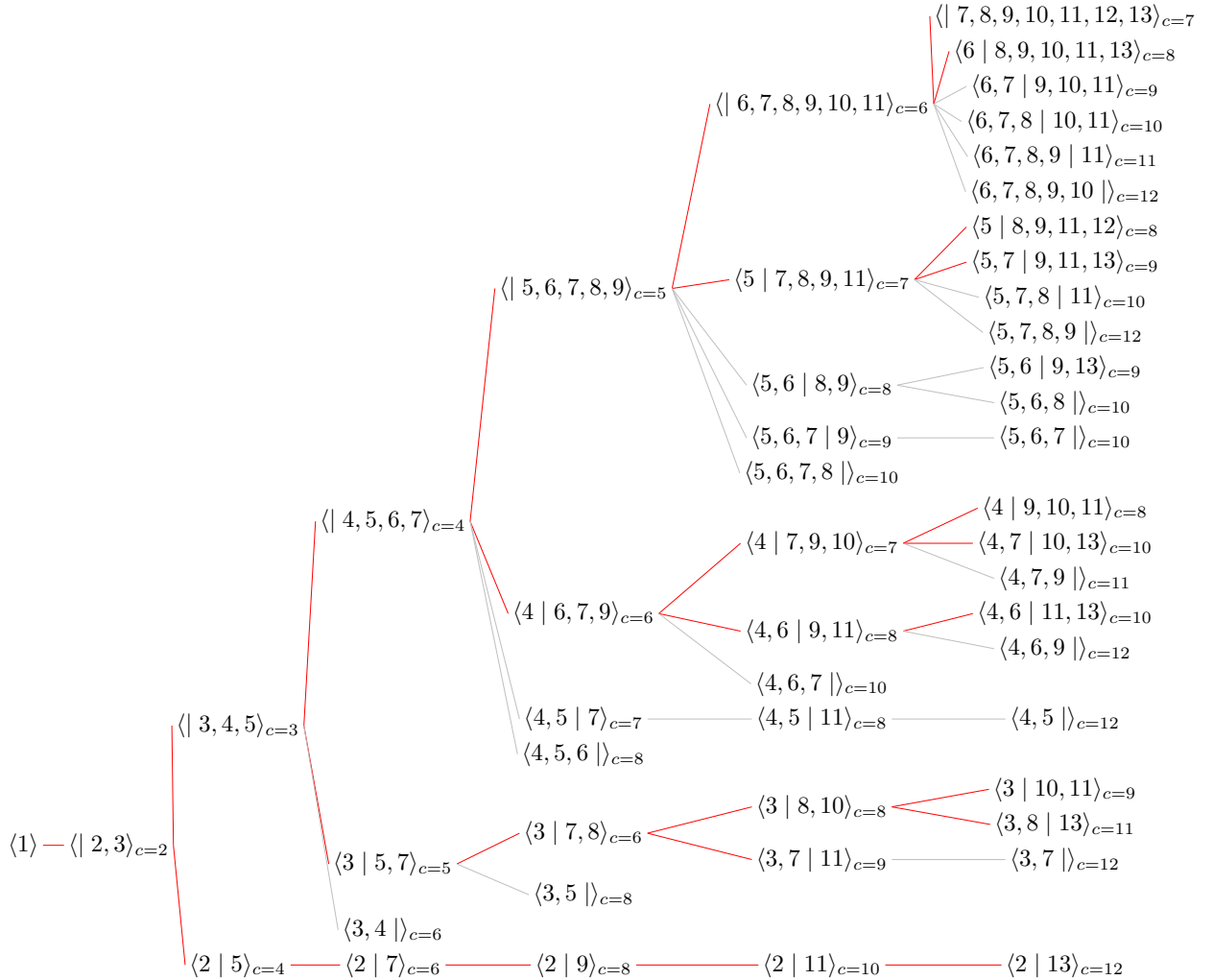
COMMAND:

```
./drawsgtree -g6 -e med -n minimalgenerators -inputfile
```

OUTPUT:

```
[g=6] count=23 ng=23 [0 seconds]
```

```
GENERATED FILE: inputfile-med-minimalgenerators-semigrouptree-6.tex
```

GENERATED GRAPH:

COMMAND:

```
./drawsgtree -g5 -e pattern 1+1-1 -n minimalgenerators -e trim -vertical -
↳ inputfile
```

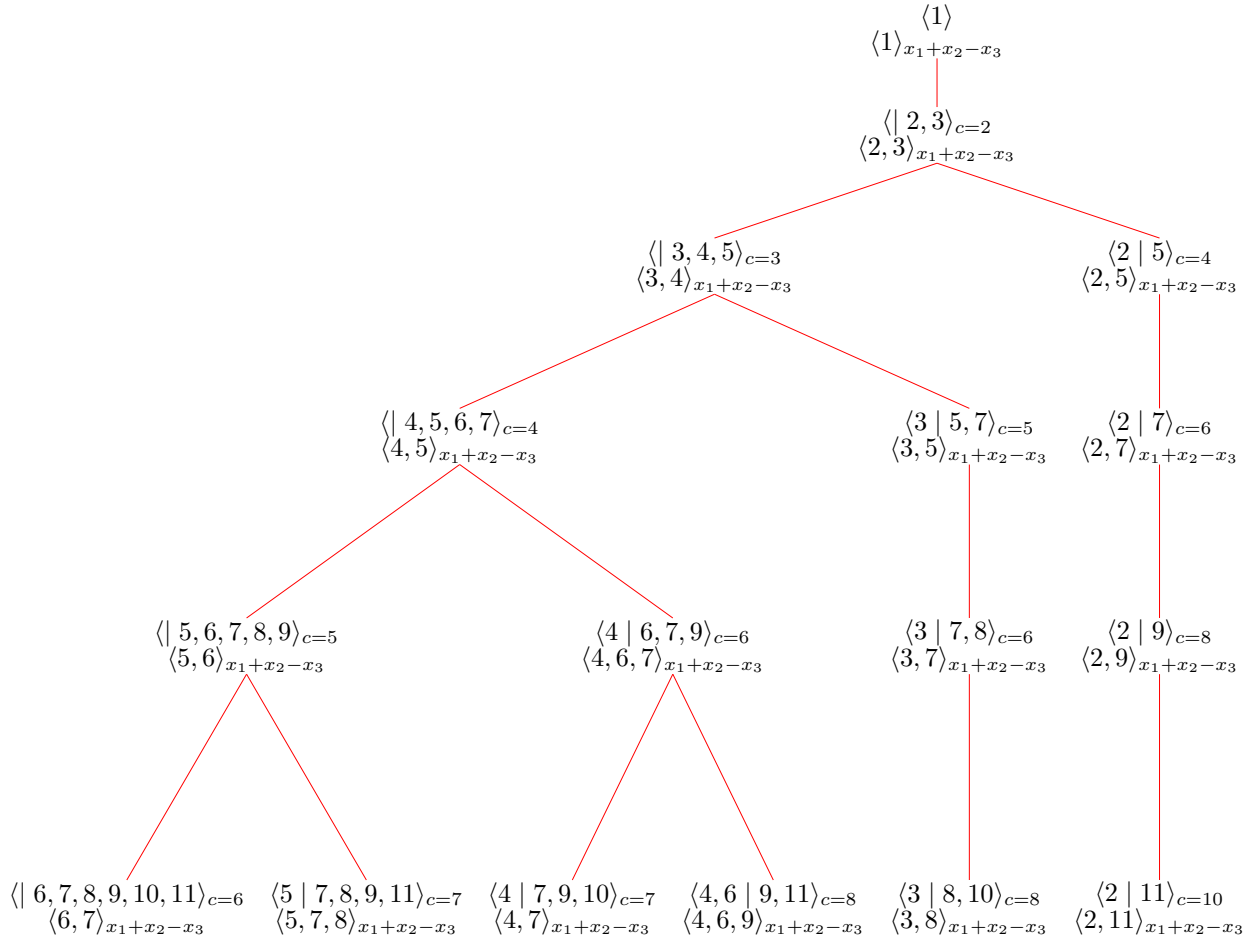
OUTPUT:

pattern: $x_{\{1\}}+x_{\{2\}}-x_{\{3\}}$

[g=5] count=6 ng=12 [0 seconds]

GENERATED FILE: inputfile-pattern1+1-1-trim-minimalgenerators-

↳ semigrouptree-5.tex

GENERATED GRAPH:

COMMAND:

```
./drawsgtree -g10 -m4 -e pattern 1+1+1-1 -n minimalgenerators -d 2.3 -s 4.  
↪ -inputfile
```

OUTPUT:

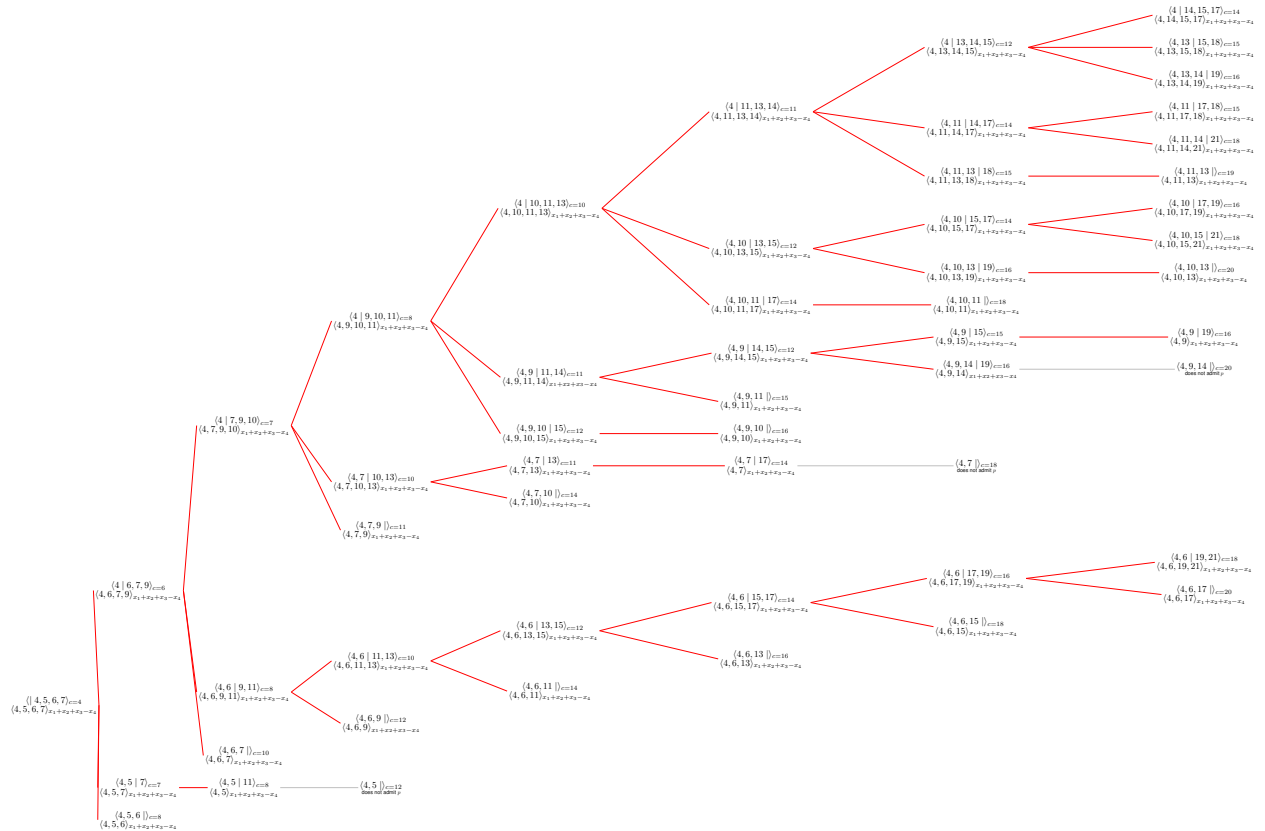
pattern: $x_{\{1\}} + x_{\{2\}} + x_{\{3\}} - x_{\{4\}}$

[g=10] count=13 ng=204 [0 seconds]

GENERATED FILE: inputfile-pattern1+1+1-1-minimalgenerators-semigrouptree

↪ -10-root04.tex

GENERATED GRAPH:



COMMAND:

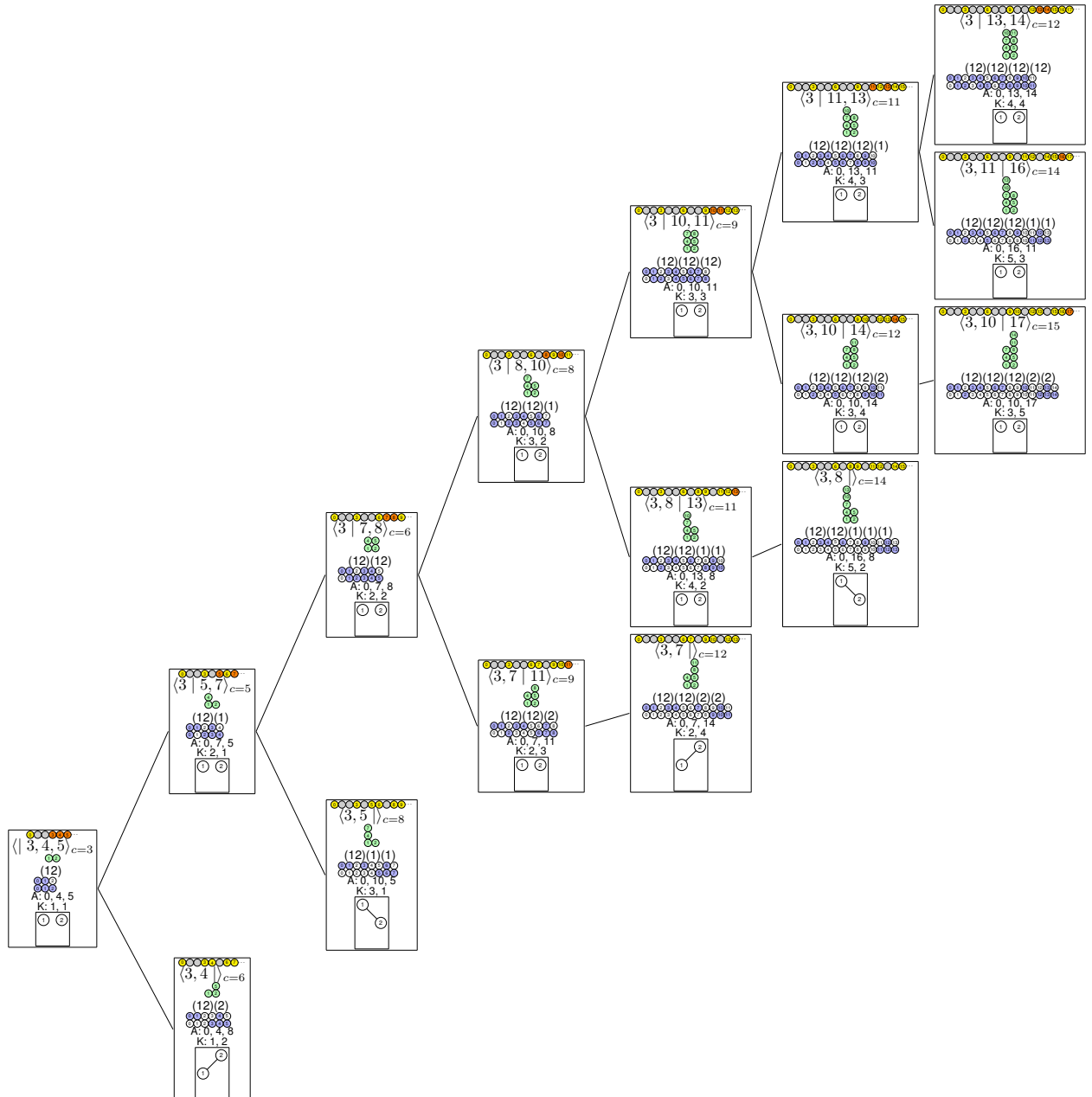
```
./drawsgtree -m3 -g8 -n list -n gapset -n minimalgenerators -n
  ↪ gapseedbitstream -n aperykunzposet -framednodes -inputfile
```

OUTPUT:

```
[g=8] count=3 ng=67 [0 seconds]
```

GENERATED FILE: inputfile-aperykunuzposet-gapseedbitstream-gapset-
 ↪ minimalgenerators-list-semigrouptree-8-root03.tex

GENERATED GRAPH:



COMMAND:

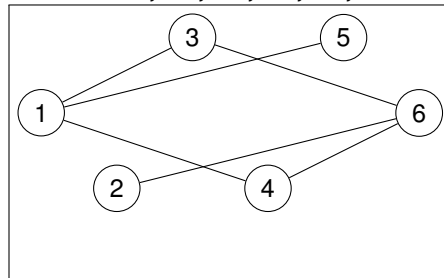
```
./drawsgtree -g15 0 7 9 11 14 16 18 20 21 22 23 25 27 -n aperykunuzposet -  
  ↪ inputfile
```

OUTPUT:

```
N[0]=0  
N[1]=7  
N[2]=9  
N[3]=11  
N[4]=14  
N[5]=16  
N[6]=18  
N[7]=20  
N[8]=21  
N[9]=22  
N[10]=23  
N[11]=25  
N[12]=27  
[g=15] count=1 ng=2857 [0 seconds]  
GENERATED FILE: inputfile-aperykunuzposet-semigroupptree-15-  
  ↪ root07911141618202122232527.tex
```

GENERATED GRAPH:

A: 0, 22, 9, 31, 11, 33, 20
K: 3, 1, 4, 1, 4, 2



COMMAND:

```
./drawsgtree -g33 0 12 19 24 28 31 34 36 38 40 42 43 45 -n dyckhook -  
  ↪ inputfile
```

OUTPUT:

```
N[0]=0  
N[1]=12  
N[2]=19  
N[3]=24  
N[4]=28  
N[5]=31  
N[6]=34  
N[7]=36  
N[8]=38  
N[9]=40  
N[10]=42  
N[11]=43  
N[12]=45  
[g=33] count=1 ng=24896206 [0 seconds]  
GENERATED FILE: inputfile-dyckhook-semigroupptree-33-  
  ↪ root0121924283134363840424345.tex
```

GENERATED GRAPH:

44	32	25	20	16	13	10	8	6	4	2	1
41	29	22	17	13	10	7	5	3	1		
39	27	20	15	11	8	5	3	1			
37	25	18	13	9	6	3	1				
35	23	16	11	7	4	1					
33	21	14	9	5	2						
32	20	13	8	4	1						
30	18	11	6	2							
29	17	10	5	1							
27	15	8	3								
26	14	7	2								
25	13	6	1								
23	11	4									
22	10	3									
21	9	2									
20	8	1									
18	6										
17	5										
16	4										
15	3										
14	2										
13	1										
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											

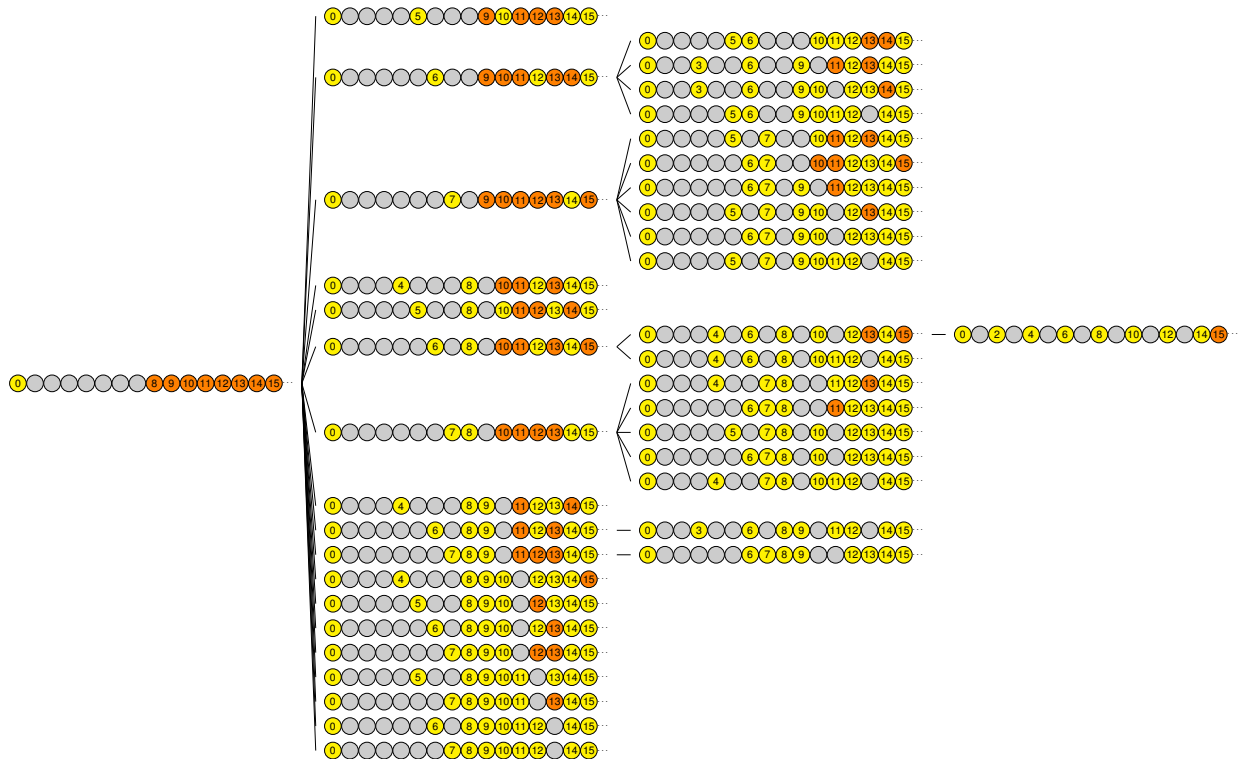
COMMAND:

```
./drawsgtree -g7 -t ordinarization -n list -inputfile
```

OUTPUT:

```
[g=7] count=39 ng=39 [0 seconds]
```

```
GENERATED FILE: inputfile-list-ordinarizationtree-7.tex
```

GENERATED GRAPH:

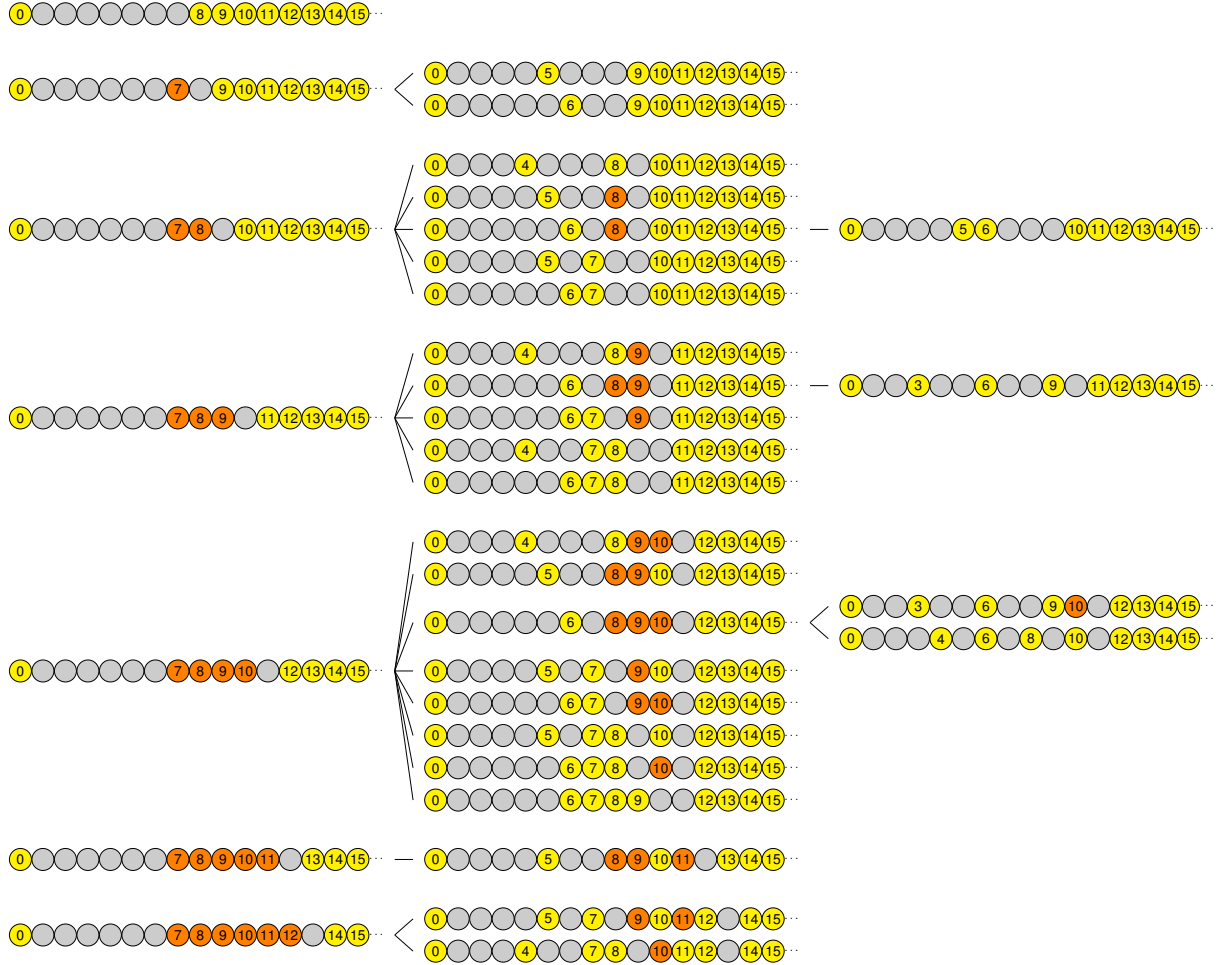
COMMAND:

```
./drawsgtree -g7 -t quasiordinarization -n list -inputfile
```

OUTPUT:

```
[g=7] count=34 ng=39 [0 seconds]
```

```
GENERATED FILE: inputfile-list-ordinarizationforest-7.tex
```

GENERATED GRAPH:

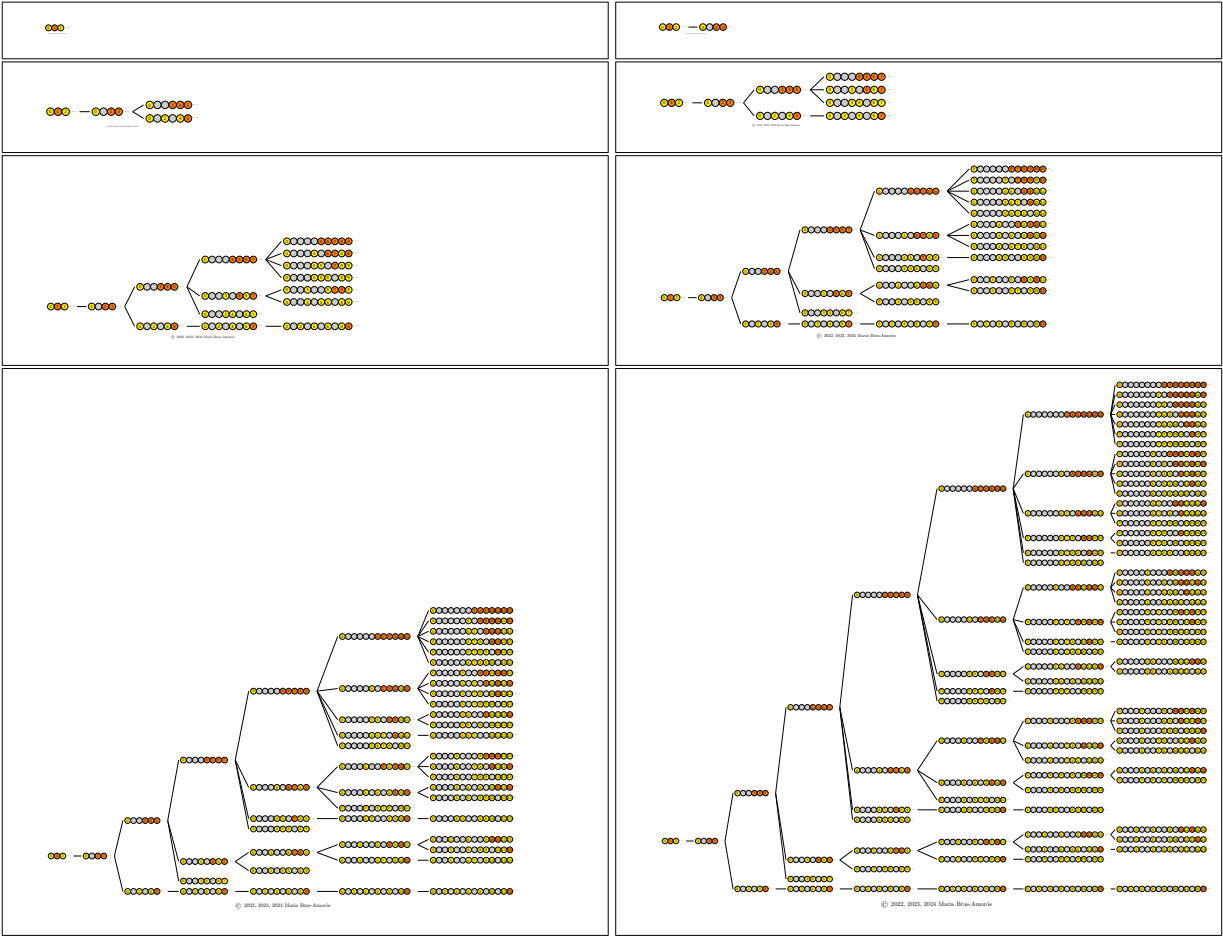
COMMAND:

```
./drawsgtree -g7 -n list -incremental
```

OUTPUT:

```
[g=0] count=1 ng=1 [0 seconds]
[g=1] count=1 ng=1 [0 seconds]
[g=2] count=2 ng=2 [0 seconds]
[g=3] count=4 ng=4 [0 seconds]
[g=4] count=7 ng=7 [0 seconds]
[g=5] count=12 ng=12 [0 seconds]
[g=6] count=23 ng=23 [0 seconds]
[g=7] count=39 ng=39 [0 seconds]
GENERATED FILE: incremental-list-semigroup-tree-7.tex
```

GENERATED GRAPHS:



References

- [1] Maria Bras-Amorós. Drawsgtree. GitHub repository, 2022, 2023, 2024. <https://github.com/mbrasamoros/drawsgtree>.
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- [3] Maria Bras-Amorós and Stanislav Bulygin. Towards a better understanding of the semigroup tree. *Semigroup Forum*, 79(3):561–574, 2009.
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- [5] Maria Bras-Amorós and Julio Fernández-González. Computation of numerical semigroups by means of seeds. *Math. Comp.*, 87(313):2539–2550, 2018.
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- [8] Shalom Eliahou and Jean Fromentin. Gapsets and numerical semigroups. *J. Combin. Theory Ser. A*, 169:105129, 19, 2020.
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- [11] J. C. Rosales, P. A. García-Sánchez, J. I. García-García, and M. B. Branco. Systems of inequalities and numerical semigroups. *J. London Math. Soc. (2)*, 65(3):611–623, 2002.
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- [13] Mariana Rosas-Ribeiro and Maria Bras-Amorós. Infinite chains in the tree of numerical semigroups. *Submitted*, 2023.