How Impulse Responses Become Sound Pressure Level (SPL) Plots:

The Good, the Bad, and the Ugly

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1 Motivation: Why Frequency Response Matters

- The goal is to determine how loud a loudspeaker is at each frequency.
- A straightforward approach is to play pure tones at different frequencies, record the sound pressure level (SPL), and connect the dots to form a curve of SPL vs. frequency.
- This method is slow and affected by reflections and echoes, unless performed in an anechoic chamber.
- A more efficient and accessible alternative relies on the **impulse response** and the **Fourier transform**.
- This text is a practical and occasionally irreverent guide to turning impulse-response measurements from the time domain into meaningful SPL data in the frequency domain. It explores how an impulse response represents a loudspeaker's acoustic behaviour and how the data is processed, interpreted, and occasionally abused.

Figure 1: Measuring tone-by-tone versus using a single broadband test.

2 Time domain vs. frequency domain: the Fourier transform

Assume a sound pressure signal evolving as a function of time, $p(t)$. The signal is recorded by a microphone and digitized by an A/D converter with sampling rate F_s. As a result, we obtain N discrete samples $p_i = p(t_i)$ at times

$$t_i = (i-1)\tau, \qquad \tau = \frac{1}{F_s}, \quad i = 1, 2, \dots, N.$$

The idea of the Fourier transform is to describe each time-domain data point p_i as a combination of sine waves, each with amplitude A_j and phase φ_j, at discrete frequencies

$$f_j = \frac{F_s}{N} j, \quad j = 1, 2, \dots, M,$$

where $M = (N-1)/2$ if N is odd, or $M = N/2 - 1$ if N is even.

The discrete Fourier expansion is then written as:

$$\text{For odd } N: \quad p_i = A_0 + \sum_{j=1}^{M} A_j \sin\big(2\pi f_j t_i + \varphi_j\big), \tag{1}$$

$$\text{For even } N: \quad p_i = A_0 + \sum_{j=1}^{M} A_j \sin\big(2\pi f_j t_i + \varphi_j\big) + A_{N/2}\cos\big(\pi\, t_i/\tau\big). \tag{2}$$

Each amplitude–phase pair (A_j, φ_j) represents one Fourier component at frequency $f_j = jF_s/N$. The A_0 coefficient is the DC component of the p_i data. Note that the total number of coefficients A_j (including A_0) and φ_j together equals N. In other words, the number of data points in the frequency domain is the same as in the time domain. The above equations therefore represent a fully determined system. The coefficients (A_j, φ_j) can be uniquely obtained from the measured samples p_i by solving this system, typically using efficient numerical algorithms such as the *Fast Fourier Transform (FFT)*.

The Fourier transform is reversible. Given the frequency-domain data (A_j, φ_j), the corresponding time-domain signal p_i can be reconstructed exactly using the same equations.

The time domain describes how the signal evolves over time, whereas the frequency domain expresses the same information in terms of the amplitude and phase distribution across frequencies.

3 How Loud Is the Speaker? One Test for All Frequencies

- Imagine a test signal that contains all frequencies with equal amplitude and zero phase.
- Using the inverse Fourier transform, this frequency-domain description can be converted into the time domain.
- The result is a very short pulse—mathematically a **Dirac delta**, or in practice a broadband **impulse**.
- When such a signal drives the loudspeaker, the microphone records its acoustic output over time.
- The resulting waveform, $h(t)$, is the **impulse response**, expressed in sound pressure (Pa) as a function of time.
- Each peak, dip, and reflection in $h(t)$ corresponds to how the loudspeaker and room respond to different frequencies.

Figure 3: Ideal impulse (test signal) and measured impulse response of a loudspeaker.

4 From Impulse to Frequency — The FFT

- Applying a Fourier transform to $h(t)$ yields $H(f)$, the loudspeaker's frequency response.
- The magnitude of $H(f)$ indicates the relative sound level of each frequency; the phase describes timing and coherence.
- With microphone calibration, magnitudes can be expressed as sound pressure level (SPL) in decibels (re 20 µPa).
- The transformation from $h(t)$ to $H(f)$ turns a single broadband measurement into the familiar SPL vs. frequency curve.

Figure 4: Impulse response and its corresponding SPL frequency response.

5 The Good, the Bad, and the Ugly (Preview)

- The following sections discuss how real-world processing shapes this transformation:
  - **The Good:** gating and windowing to suppress reflections.
  - **The Bad:** excessive windowing that removes low-frequency content.
  - **The Ugly:** manipulation of data through zeroing, smoothing, or padding.
- Each step is an attempt to recover the true loudspeaker behaviour from a reflection-rich environment—but not all are equally honest.

This conceptual introduction explains how an impulse response represents a complete description of a loudspeaker's behaviour. Later sections explore how this data is processed, interpreted, and occasionally abused.