Continuous Time Transfer function - Lead/Lag:

$$C(s) = \frac{\alpha\tau s + 1}{\tau s + 1}$$

$\alpha > 1$ implies it is a lead controller, $\alpha < 1$ implies a lag controller.

1. Forward Euler

Substituting,

$$s = \frac{z-1}{T}$$

We get,

$$C(z) = \frac{\alpha\tau(z-1) + T}{\tau(z-1) + T} = \frac{U}{E}$$

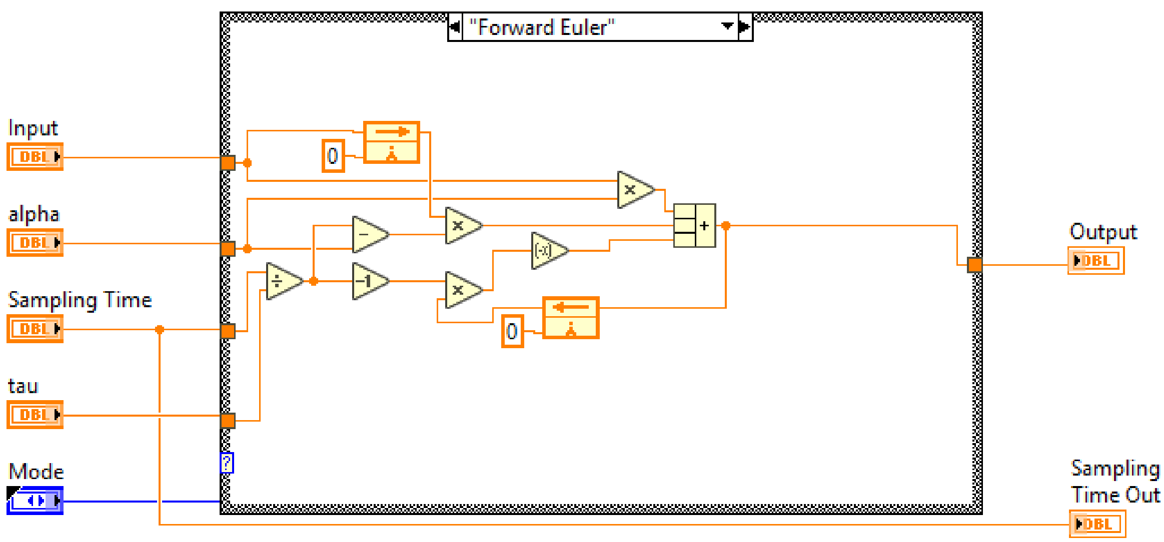
Converting to difference equations,

$$\tau . u(k+1) = \alpha\tau . e(k+1) + (T - \alpha\tau) . e(k) - (T - \tau) . u(k)$$

Using the time-shift property, taking τ to the right-hand side,

$$u(k) = \alpha . e(k) + \left(\frac{T}{\tau} - \alpha\right) . e(k-1) - \left(\frac{T}{\tau} - 1\right) . u(k-1)$$

LabVIEW implementation:



2. Backward Euler

Substituting,

$$s = \frac{z-1}{Tz}$$

We get,

$$C(z) = \frac{\alpha\tau(z-1) + Tz}{\tau(z-1) + Tz} = \frac{U}{E}$$

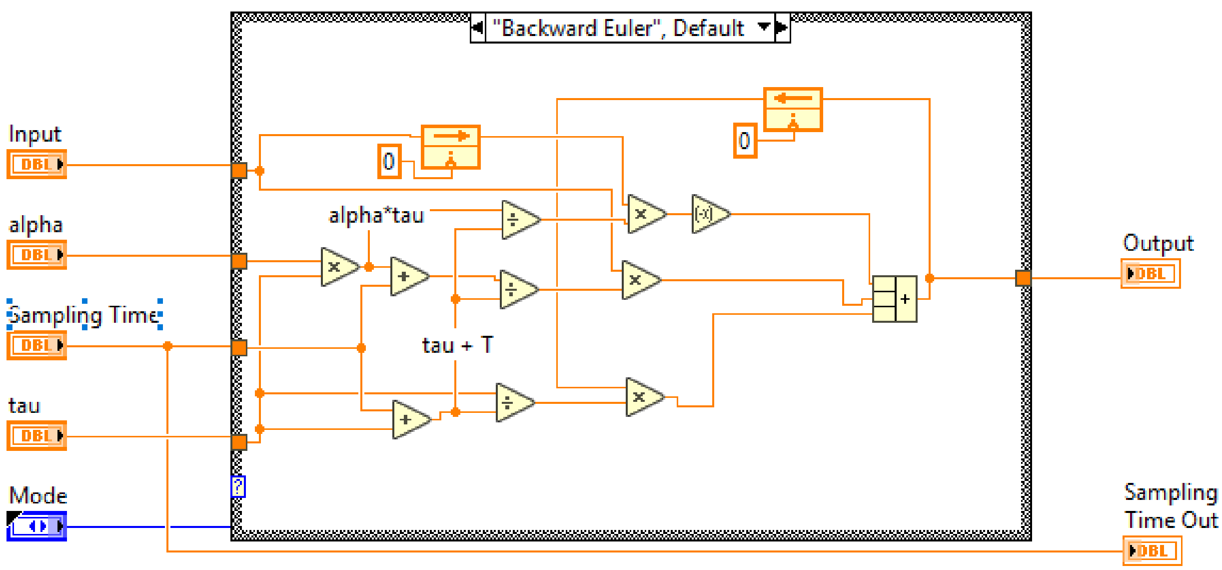
Converting to difference equations,

$$(\tau + T).u(k+1) = (\alpha\tau + T).e(k+1) - \alpha\tau.e(k) + \tau.u(k)$$

Using the time-shift property, taking $(\tau + T)$ to the right-hand side,

$$u(k) = \left(\frac{\alpha\tau + T}{\tau + T}\right).e(k) - \left(\frac{\alpha\tau}{\tau + T}\right).e(k-1) + \left(\frac{\tau}{\tau + T}\right).u(k-1)$$

LabVIEW implementation:



3. Trapezoidal / Bi-linear / 'Tustin'

Substituting,

$$s = \frac{2}{T} \cdot \frac{z-1}{z+1}$$

We get,

$$C(z) = \frac{2\alpha\tau(z-1) + T(z+1)}{2\tau(z-1) + T(z+1)} = \frac{U}{E}$$

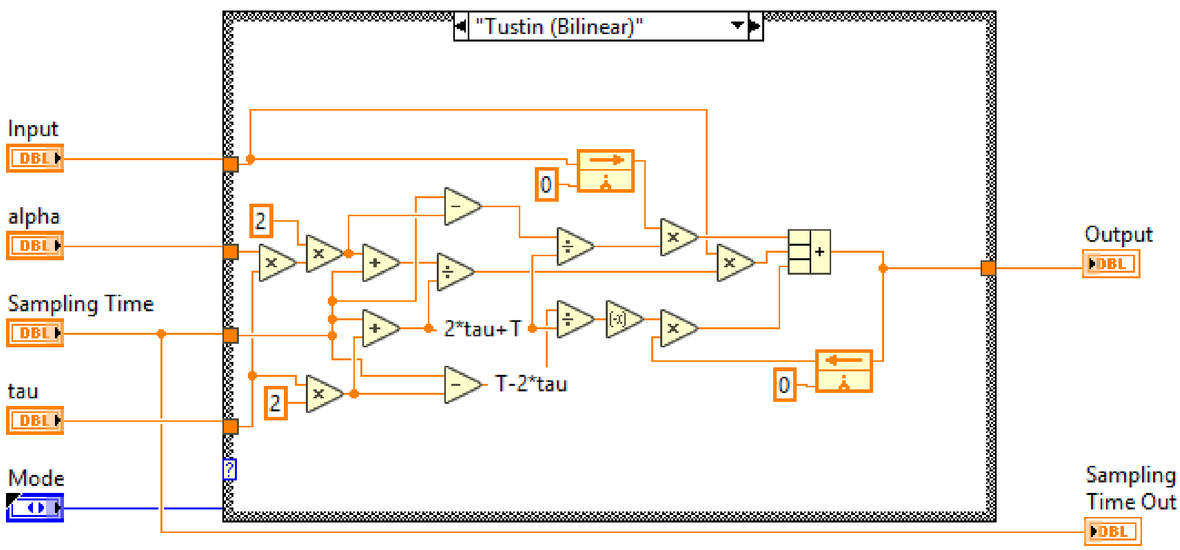
Converting to difference equations,

$$(2\tau + T).u(k+1) = (2\alpha\tau + T).e(k+1) + (T - 2\alpha\tau).e(k) - (T - 2\tau).u(k)$$

Using the time-shift property, taking $(T + 2\tau)$ to the right-hand side,

$$u(k) = \left(\frac{T + 2\alpha\tau}{T + 2\tau}\right).e(k) + \left(\frac{T - 2\alpha\tau}{T + 2\tau}\right).e(k-1) - \left(\frac{T - 2\tau}{T + 2\tau}\right).u(k-1)$$

LabVIEW implementation:



Continuous Time Transfer function - Integrator:

$$C(s) = \frac{K_I}{s} = \frac{1}{T_i s}$$

1. Forward Euler

Substituting,

$$s = \frac{z-1}{T}$$

We get,

$$C(z) = \frac{K_I T}{z-1} = \frac{U}{E}$$

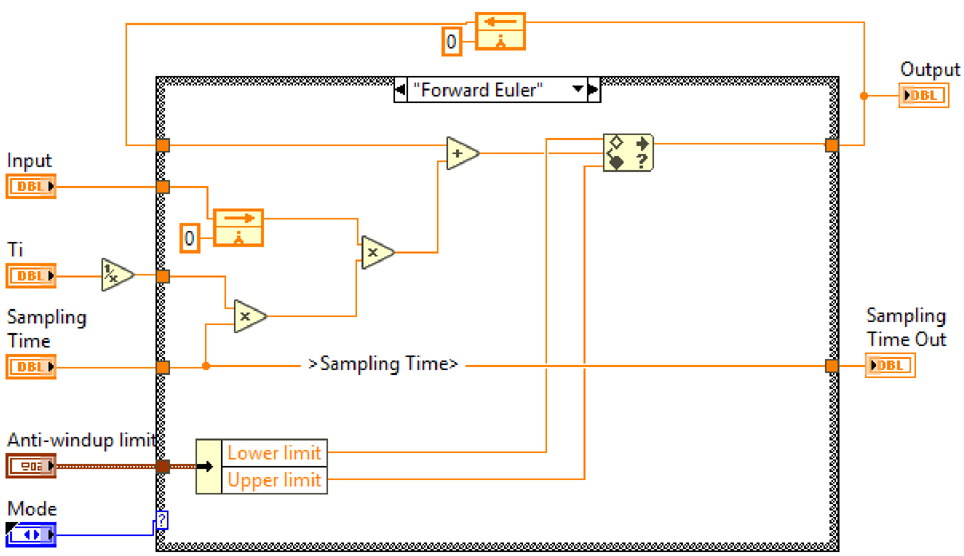
Converting to difference equations,

$$u(k+1) = K_I T.e(k) + u(k)$$

Using the time-shift property, taking τ to the right-hand side,

$$u(k) = K_I T.e(k-1) + u(k-1)$$

LabVIEW implementation (with anti-windup):



2. Backward Euler

Substituting,

$$s = \frac{z-1}{Tz}$$

We get,

$$C(z) = \frac{K_I T z}{z-1} = \frac{U}{E}$$

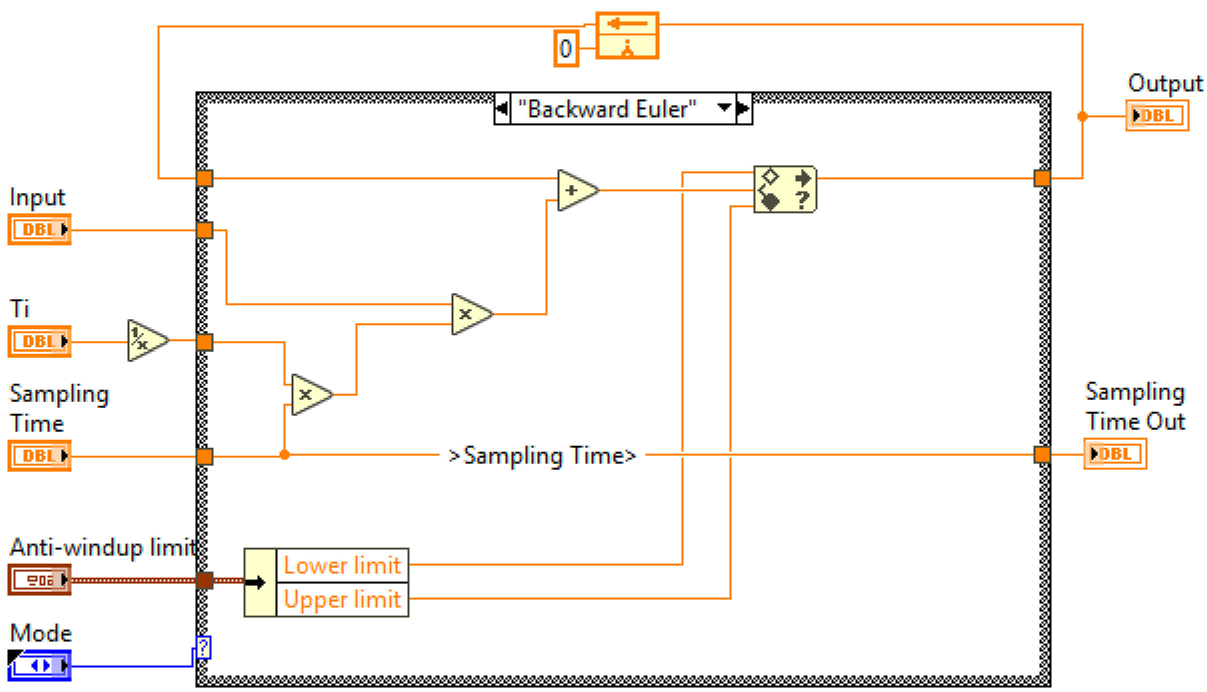
Converting to difference equations,

$$u(k+1) = K_I T.e(k+1) + u(k)$$

Using the time-shift property, taking τ to the right-hand side,

$$u(k) = K_I T.e(k) + u(k-1)$$

LabVIEW implementation (with anti-windup):



3. Trapezoidal / Bi-linear / 'Tustin'

Substituting,

$$s = \frac{2}{T} \cdot \frac{z-1}{z+1}$$

We get,

$$C(z) = \frac{K_I T}{2} . \frac{z+1}{z-1} = \frac{U}{E}$$

Converting to difference equations,

$$u(k+1) = \frac{K_I T}{2} . \{e(k) + e(k+1)\} + u(k)$$

Using the time-shift property, taking τ to the right-hand side,

$$u(k) = \frac{K_I T}{2} . \{e(k) + e(k-1)\} + u(k)$$

LabVIEW implementation (with anti-windup):