AMIA

Guía 2

G3

3) Sea $A=\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix}$

a) Demostrar q' A es def. positiva

1) A es simétrica ✓ $A^T=\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix}=A$

2) Sea $x=\begin{pmatrix} a \\ b \end{pmatrix}\in\mathbb{R}^2$

$$\Rightarrow x^T\cdot A\cdot x=(a\ b)\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=\left(a+\frac{b}{2},\frac{a}{2}+\frac{b}{3}\right)\begin{pmatrix} a \\ b \end{pmatrix}=$$

$$=a^2+\frac{ab}{2}+\frac{ab}{2}+\frac{b^2}{3}=a^2+\frac{2ab}{2}+\frac{b^2}{3}=\frac{1}{4}a^2+\frac{3}{4}a^2+ab+\frac{b^2}{3}$$

$$=\frac{a^2}{4}+\left(\frac{\sqrt{3}a}{2}\right)^2+a\cdot b+\left(\frac{b}{\sqrt{3}}\right)^2=\frac{a^2}{4}+\underbrace{\left(\frac{\sqrt{3}a}{2}+\frac{b}{\sqrt{3}}\right)^2}_{=\left(\frac{\sqrt{3}a}{2}\right)^2+\cancel{2}\cdot\frac{\cancel{\sqrt{3}}a}{\cancel{2}}\cdot\frac{b}{\cancel{\sqrt{3}}}+\left(\frac{b}{\sqrt{3}}\right)^2}>0 ✓ \quad \forall a,b\in\mathbb{R}$$

$\Rightarrow x^T\cdot A\cdot x>0\ \forall x\in\mathbb{R}^2$ ✓

$\Rightarrow$ A es definida positiva

3b) Sea $V = \mathbb{R}^{2\times1}$ con prod. int $\langle X, Y\rangle = Y^T A X$. Aplicar Gram-Schmidt.

$B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} = \{ b_1, b_2 \}$

$k_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$k_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - P_{k_1}(b_2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \dfrac{\langle k_1, b_2\rangle}{\langle k_1, k_1\rangle} \cdot k_1$

a) $\langle k_1, k_1 \rangle = (1\ 0) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1\ \ 1/2) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$

b) $\langle k_1, b_2 \rangle = (0\ 1) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1/2,\ 1/3) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1/2$

$\Rightarrow k_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \dfrac{1/2}{1} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$

$\Rightarrow \tilde{B} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} \right\} = \{ \tilde{b}_1, \tilde{b}_2 \}$

Normalizo con la norma

$\|\tilde{b}_1\| = \langle \tilde{b}_1, \tilde{b}_1 \rangle^{1/2} = \left[(1\ 0) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]^{1/2} = \sqrt{1} = 1$

$\|\tilde{b}_2\| = \langle \tilde{b}_2, \tilde{b}_2 \rangle^{1/2} = \left[(-\tfrac{1}{2}\ \ 1) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix} \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} \right]^{1/2} = \left[(0\ \ \tfrac{1}{12}) \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} \right]^{1/2} = \dfrac{1}{\sqrt{12}}$

$\Rightarrow BON = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{12}}{2} \\ \sqrt{12} \end{pmatrix} \right\}$

Chequeo

$\begin{pmatrix} -\frac{\sqrt{12}}{2} \\ \sqrt{12} \end{pmatrix}^T \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left(0\ \ \frac{\sqrt{12}}{12}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ✓

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