$h(k) = k \bmod m, \quad m = 2^p - 1$

$\land\ k = \sum_{i=0}^{n-1} 2^{pi} k_i$

Zz If $x$ is a permutation of $y$

$\Rightarrow h(x) = h(y)$

Let's suppose without the loss of generality that $x_a = y_b \land y_a = x_b$ but are equal on all other indices. Let $a > b$

$h(x) - h(y) =$

$$\left(\sum_{i=0}^{n-1} 2^{pi} x_i\right) \bmod 2^p - 1 - \left(\sum_{i=0}^{n-1} 2^{pi} y_i\right) \bmod 2^p - 1$$

$$= \left(\sum_{i=0}^{n-1} 2^{pi} x_i\right) - \left(\sum_{i=0}^{n-1} 2^{pi} y_i\right) \bmod 2^p - 1$$

$$= \left(2^{ap} x_a + 2^{bp} x_b\right) - \left(2^{ap} y_a + 2^{bp} y_b\right) \bmod 2^p - 1 \;\Bigg|\; \begin{matrix} x_a = y_b \\ y_a = y_b \end{matrix}$$

$$= \left(2^{ap} x_a + 2^{bp} x_b\right) - \left(2^{ap} x_b + 2^{bp} x_a\right) \bmod 2^p - 1$$

$$= 2^{ap}(x_a - x_b) + 2^{bp}(x_b - x_a) \bmod 2^p - 1$$

$$= (x_a - x_b)(2^{ap} - 2^{bp}) \bmod 2^p - 1$$

$$= (x_a - x_b)\left(2^{ap}\left(\frac{2^{bp}}{2^{bp}}\right) - 2^{bp}\right) \bmod 2^p - 1$$

$$= (x_a - x_b)\left(2^{bp}\left(2^{(a-b)p} - 1\right)\right) \bmod 2^p - 1$$

$$= (x_a - x_0)\, 2^{bp} \underbrace{\left(\frac{2^{(a-b)p} - 1}{2^p - 1}\right)}_{\text{Geometric series}} (2^p - 1) \bmod 2^p - 1$$

with $n = (a-b) - 1$

$\land\ q = 2^p$

$$\sum_{k=0}^{n} q^k = \frac{q^{n+1} - 1}{q - 1}$$

$$= (x_a - x_b)\, 2^{bp} \left(\sum_{i=0}^{a-b-1} 2^i\right)(2^p - 1) \bmod 2^p - 1$$

$$= 0$$