

#### presents the

## REFERENCE IMPLEMENTATION

also known as the PURPLE PAPER

of the remarkable

## **DAI CREDIT SYSTEM**

issuing a diversely collateralized stablecoin

formulated by

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## **Contents**

1	Introduction 5									
	1.1	Motivation								
	1.2	Limitations								
	1.3	Verification								
Ι	Imp	plementation 8								
2	Types 1									
	2.1	Numeric types								
	2.2	Identifiers and addresses								
	2.3	Gem — collateral price feed entry								
		jar — collateral token								
		tag — market price of token								
		zzz — expiration time of token price feed								
	2.4	ERC20 — token model								
	2.5	Ilk — CDP type								
		jar — collateral token vault								
		$mat-liquidation ratio \dots 12$								
		axe — liquidation penalty ratio								
		hat — debt ceiling								
		$tax - stability fee \dots 12$								
		lag — price feed limbo duration								
		rho — time of debt unit adjustment								
		din — total outstanding dai								
		chi — dai value of debt unit								
	2.6	Urn — collateralized debt position (CDP)								
		cat — address of liquidation initiator								
		vow — address of liquidation contract								
		lad — CDP owner								

		ilk — CDP type	13
		<pre>art — debt denominated in debt unit</pre>	13
		jam — collateral denominated in debt unit	13
	2.7	Vox — feedback mechanism data	13
		fix — market price of DAI denominated in SDR	13
		par — target price of DAI denominated in SDR	13
		how — sensitivity parameter	13
		way — rate of target price change	13
		tau — time of latest target update	13
	2.8	Vat — CDP engine data	13
		joy — unprocessed stability fee revenue	13
		sin — bad debt from liquidated CDPs	13
	2.9	System model	14
		era — current time	14
	2.10	Default data	14
3	Acts		16
	3.1	Assessment	17
		feel — identify CDP risk stage	17
	3.2	Lending	19
		open — create CDP	19
		give — transfer CDP account	19
		lock — deposit collateral	19
		free — withdraw collateral	20
		draw — issue dai as debt	20
		wipe — repay debt and burn dai	21
		shut — wipe, free, and delete CDP	21
	3.3	Adjustment	22
		prod — adjust target price and target rate	22
		drip — update debt unit and unprocessed fee revenue	23
	3.4	Price feed input	24
		mark — update market price of collateral token	24
		tell — update market price of dai	24
	3.5	Liquidation	24
		bite — mark for liquidation	24
		grab — take tokens for liquidation	25
		plop — finish liquidation returning profit	25
		heal — record bad debt as processed	25
		love — record stability fee revenue as processed	25
	3.6	Governance	26
		form — create a new CDP type	26

		frob — set the sensitivity parameter	26
		chop — set liquidation penalty	26
		cork — set debt ceiling	
		calm — set limbo duration	
		cuff — set liquidation ratio	26
		crop — set stability fee	26
	3.7	Vaults	26
		pull — transfer tokens to vault	26
		push — transfer tokens from vault	27
	3.8	Token manipulation	27
		mint — inflate token	27
		burn — deflate token	27
4	Act	framework	28
	4.1	The Maker monad	28
	4.2	Asserting	
A	Prel	ude	30
В	Fixe	d point numbers with rounding	33

## **Chapter 1**

#### Introduction

The dai credit system, henceforth also "Maker," is a network of Ethereum contracts designed to issue the dai currency token and automatically adjust credit incentives in order to keep its market value stable relative to SDR<sup>1</sup> in the short and medium term.

New dai enters the money supply when a borrower locks an excess of collateral in Maker's token vault and takes out a loan. The debt and collateral amounts are recorded in a *collateralized debt position*, or CDP. Thus all outstanding dai represents some CDP owner's claim on their collateral—until risk provokes a liquidation.

Off-chain *price feeds* give Maker knowledge of the market values of dai and the various tokens used as collateral, enabling the system to assess credit risk. If the value of a CDP's collateral drops below a certain multiple of its debt, a decentralized auction is triggered to liquidate the collateral for dai to be burned thus settling the debt.

The system issues a separate token with symbol MKR, which behaves like a "share" in Maker itself. Since collateral auctions may fail to recover the full value of liquidated debt, the MKR token can be diluted to back emergency debt. The value of MKR, though volatile by design, is backed by the revenue from a *stability fee* imposed on all dai loans and used to buy MKR for burning.

For more details on the economics of the system, as well as descriptions of governance, off-chain mechanisms that provide efficiency, and so on, see the whitepaper.

This document is an executable technical specification of the of the Maker smart contracts. It is a draft; be aware that the contents will certainly change before launch.

<sup>&</sup>lt;sup>1</sup> "Special Drawing Rights" (ticker symbol xdr), the international reserve asset created by the International Monetary Fund, whose value is derives from a weighted basket of world currencies.

#### 1.1 Motivation

The version of this system that will be deployed on the blockchain is written in Solidity, which is a workable smart contract implementation language. This reference implementation is a model of the behavior of those contracts, written as a "literate" Haskell program. The motivations for such a reference implementation include:

- 1. **Comparison**. Checking two free-standing implementations against each other is a well-known way of ensuring that they both behave as intended.
- 2. **Testing**. Haskell lets us use powerful testing tools such as QuickCheck and SmallCheck for comprehensively verifying key properties as a middle ground between unit testing and formal verification.
- 3. **Explicitness**. Coding the contract behavior in Haskell, a purely functional language, enforces explicit description of aspects which Solidity leaves implicit. For example, a Solidity program can read previously unwritten storage and get back a zero value, whereas in Haskell we must give explicit defaults. The state rollback behavior of failed actions is also explicit in the type of the execution function, which may return an error.
- 4. **Typing**. While Solidity does have a static type system, it is not expressive enough to encode the distinctions made by our system. In particular, the two different decimal fixed point number types that we use are typed in Solidity with one and the same uint128 type. In Haskell we can make this distinction explicit.
- 5. Formality. The work of translating a Solidity program into a purely functional program opens up opportunities for certain types of formal verification. In particular, this document will be useful for modelling aspects of the system in a proof assistant like Agda, Idris, Coq, or Isabelle. We can also use logical tools for Haskell, such as Liquid Haskell (which provides compile time logical property checking) and sbv (a toolkit for model checking and symbolic execution).
- 6. Clarity. An implementation not intended to be deployed on the blockchain is free from concerns about optimizing for gas cost and other factors that make the Solidity implementation less ideal as an understandable specification.
- 7. Simulation. Solidity is highly specific to the Ethereum blockchain environment and as such does not have facilities for interfacing with files or other computer programs. This makes the Solidity implementation of the system less useful for doing simulations of the system's economic, game-theoretic, or statistical aspects.

#### 1.2 Limitations

This model is limited in that it has

- 1. a simplified version of authorization for governance;
- 2. a simplified version of ERC20 token semantics;
- 3. no implementation of the decentralized auction contracts; and
- 4. no 256-bit word limits.

These limitations will be addressed in future revisions.

#### 1.3 Verification

Separately from this document, we are developing automatic test suites that generate many, large, and diverse action sequences for property verification. One such property is that the reference implementation exactly matches the on-chain implementation; this is verified through the generation of Solidity test cases with assertions covering the entire state. Other key properties include

- that the target price changes only according to the target rate;
- that the total dai supply is fully accounted for by CDP debts;
- that CDP acts are restricted with respect to risk stage;

along with similar invariants and conditions. A future revision of this document will include formal statements of these properties.

# Part I Implementation

#### **Preamble**

This is a Haskell program, and as such makes reference to a background of symbols defined in libraries, as a mathematical paper depends on preestablished theories.

Context should allow the reader to understand most symbols without further reading, but Appendix A lists and briefly explains each imported type and function.

We replace the default prelude module with our own.

```
module Maker where
```

import Prelude () Import nothing from Preludeimport Maker.Prelude Import everything from Maker Prelude

We also import our definition of decimal fixed point numbers, listed in Appendix B.

import Maker.Decimal

Now we proceed to define the specifics of the Maker system.

## Chapter 2

## **Types**

This chapter defines the data types used by Maker: numeric types, identifiers, on-chain records, and test model data.

Haskell syntax note: **newtype** defines a type synonym with distinct type identity; **data** creates a record type; and **deriving** creates automatic instances of common functionality.

#### 2.1 Numeric types

The system uses two different precisions of decimal fixed point numbers, which we call wads and rays, having respectively 18 digits of precision (used for token quantities) and 36 digits (used for precise rates and ratios). See Appendix B for details on decimal fixed point numbers and rounding.

```
Define the distinct type of currency quantities

newtype Wad = Wad (Decimal E18)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)

Define the distinct type of rates and ratios

newtype Ray = Ray (Decimal E36)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)
```

We also define a type for time durations in whole seconds, as this is the maximum precision allowed by the Ethereum virtual machine.

```
newtype Sec = Sec Int
deriving (Eq, Ord, Enum, Num, Real, Integral)
```

Haskell number types are not automatically converted, so we convert explicitly with a *cast* function.

```
Convert via fractional n/m form. cast :: (Real \ a, Fractional \ b) \Rightarrow a \rightarrow b cast = fromRational. toRational
```

#### 2.2 Identifiers and addresses

There are several kinds of identifiers used in the system, and we use types to distinguish them. The type parameter a creates distinct types; e.g., Id Foo and Id Bar are incompatible.

```
newtype Id a = \text{Id String deriving } (\text{Eq, Ord, Show})
```

We define another type for representing Ethereum account addresses.

```
newtype Address = Address String deriving (Eq, Ord, Show)
```

We also have two predefined entity identifiers.

```
The DAI token vault address id_{	ext{DAI}} = 	ext{Id} 	ext{"DAI"} A test account with ultimate authority id_{god} = 	ext{Address} 	ext{"GOD"}
```

#### 2.3 Gem - collateral price feed entry

The data received from price feeds is categorized by token and stored in Gem records. Our model also has the token balances embedded in these records; in reality<sup>1</sup>, the balances are in separate ERC20 contracts.

```
data Gem = Gem {
     erc20 :: ERC20, Token balances
     tag :: Wad, Market price denominated in SDR
     zzz :: Sec Time of price expiration
} deriving (Eq, Show)
```

<sup>&</sup>lt;sup>1</sup>We use "reality" to denote the actual state of the consensus Ethereum blockchain.

#### 2.4 ERC20 — token model

In reality, token semantics can differ, despite nominally following the ERC20 interface. Governance therefore involves reviewing the behaviors of collateral tokens. In our model, tokens behave in the same simple way. We also omit the notion of "allowance."

Tokens can be held by CDP owners, by token vaults, or by the test driver. We model this distinction with a data type.

```
data Holder = InAccount Address | InVault (Id Gem) | InToy
deriving (Eq, Ord, Show)
```

We now define an ERC20 instance as a map tracking the token quantity held by each holder.

```
data ERC20 = ERC20 { ● balanceOf :: Map Holder Wad}
deriving (Eq, Show)
```

#### 2.5 Ilk — CDP type

Each CDP belongs to a CDP type, specified by an Ilk record containing parameters for lending: the collateral token, the limit for the debt-to-collateral ratio, the penalty upon forced liquidation, and various other parameters. These are defined by governance. The meaning of each parameter is defined by its interactions in the act definitions of Chapter 3; see the whitepaper for an overview.

```
data Ilk = Ilk {
  • jar :: Id Gem, Collateral token identifier
  • lag :: Sec,
                      Maximum duration of price feed limbo
  • mat :: Ray,
                      Collateral-to-debt ratio at which liquidation can be triggered
  • axe :: Ray,
                      Penalty on liquidation as fraction of collateral value
  • hat :: Wad,
                      Limit on total debt for CDP type ("debt ceiling")
   • tax :: Ray,
                      Stability fee as per-second fraction of debt value
  • chi::Ray,
                      Dai value of internal debt unit
  • rho :: Sec,
                      Time of latest debt unit adjustment
   • rum :: Wad
                      Total debt denominated in debt unit
   } deriving (Eq, Show)
```

#### 2.6 Urn — collateralized debt position (CDP)

For each CDP we maintain an Urn record identifying its type and specifying ownership, quantities of debt and collateral denominated in the CDP type's debt unit, along with the progress of liquidation (if relevant).

#### 2.7 Vox — feedback mechanism data

The *feedback mechanism* is the aspect of the CDP engine that adjusts the target price of dai based on market price, and its data is kept in a singleton record called Vox.

```
data Vox = Vox {
      • fix :: Wad, Market price of dai denominated in SDR
      • par :: Wad, Target price of dai denominated in SDR
      • way :: Ray, Current per-second change in target price
      • how :: Ray, Sensitivity parameter set by governance
      • tau :: Sec Time of latest feedback update
    } deriving (Eq, Show)
```

Keeping the feedback data separate allows us to more easily upgrade the mechanism in the future.

#### 2.8 Vat — CDP engine data

The Vat record aggregates the records of CDPs, CDP types, and price feeds, along with the data of the feedback mechanism, and two accounting quantities.

#### 2.9 System model

Finally we define a record with no direct counterpart in the Solidity contracts, which has the Vat record along with model state.

#### 2.10 Default data

```
\begin{array}{l} \textit{defaultIlk} :: \text{Id Gem} \rightarrow \text{Ilk} \\ \textit{defaultIlk} \ id_{\text{gem}} = \text{Ilk} \ \{\\ \bullet \ \ \text{jar} = id_{\text{gem}}, \\ \bullet \ \ \text{axe} = \text{Ray 1}, \\ \bullet \ \ \text{mat} = \text{Ray 1}, \\ \bullet \ \ \text{tax} = \text{Ray 1}, \\ \bullet \ \ \text{tax} = \text{Ray 0}, \\ \bullet \ \ \text{hat} = \text{Wad 0}, \\ \bullet \ \ \text{lag} = \text{Sec 0}, \\ \bullet \ \ \text{chi} = \text{Ray 1}, \\ \bullet \ \ \text{rum} = \text{Wad 0}, \\ \bullet \ \ \text{rho} = \text{Sec 0} \\ \} \end{array}
```

```
emptyUrn :: Id Ilk \rightarrow Address \rightarrow Urn
emptyUrn\ id_{\mathtt{ilk}}\ id_{\mathtt{lad}} = \mathtt{Urn}\ \{
  • vow = Nothing,
  • cat = Nothing,
  • lad = id_{lad},
   • ilk = id_{ilk},
   • art = Wad 0,
   • jam = Wad 0
}
initialVat :: \mathtt{Ray} \rightarrow \mathtt{Vat}
initialVat how_0 = Vat  {
  • vox = Vox {
     • tau = 0,
     • fix = Wad 1,
     • par = Wad 1,
     • how = how_0,
      • way = Ray 1
   },
   • joy = Wad 0,
   \bullet sin = Wad 0,
   • ilks = \emptyset,
   • urns = \emptyset,
   ullet gems = singleton\ id_{\scriptscriptstyle \mathrm{DAI}} Gem \{
      • erc20 = ERC20 \{ \bullet \ balanceOf = \emptyset \},
     • tag
               = Wad 0,
      • ZZZ
                 = 0
}
initialSystem :: Ray \rightarrow System
initialSystem how_0 = System 
                  = initial Vat how_0,
  • vat
   • era
                  =0,
   • sender = id_{god},
   • accounts = mempty
}
```

## Chapter 3

## Acts

The *acts* are the basic state transitions of the system.

Unless specified as *internal*, acts are accessible as public functions on the blockchain.

The auth modifier marks acts which can only be invoked from addresses to which the system has granted authority.

For details on the underlying "Maker monad," which specifies how the act definitions behave with regard to state and rollback, see chapter 4.

#### 3.1 Assessment

In order to prohibit CDP acts based on risk situation, we define five stages of risk.

```
data Stage = Pride | Anger | Worry | Panic | Grief | Dread
deriving (Eq, Show)
```

We define the function analyze that determines the risk stage of a CDP.

```
analyze \operatorname{era}_0 \operatorname{par}_0 \operatorname{urn}_0 \operatorname{ilk}_0 \operatorname{jar}_0 =
   if | view vow urn<sub>0</sub> \not\equiv Nothing
          CDP liquidation in progress

ightarrow Dread
      | view cat urn<sub>0</sub> \not\equiv Nothing
          CDP liquidation triggered

ightarrow Grief
      | pro < min
          CDP's collateralization below liquidation ratio

ightarrow Panic
      | view zzz jar_0 + view lag ilk_0 < era_0
          CDP type's price limbo exceeded limit
             \to \mathtt{Panic}
      | view zzz jar_0 < era_0
          CDP type's price feed in limbo

ightarrow Worry
      | cap > view hat ilk_0
          CDP type's debt ceiling exceeded

ightarrow Anger
      otherwise
          No problems

ightarrow Pride
   where
    CDP's collateral value in SDR:
      \mathtt{pro} = \mathit{view} \; \mathtt{jam} \; \mathtt{urn}_0 * \mathit{view} \; \mathtt{tag} \; \mathtt{jar}_0
    CDP type's total debt in SDR:
      cap = view rum ilk_0 * cast (view chi ilk_0)
    CDP's debt in SDR:
      con = view art urn_0 * cast (view chi ilk_0) * par_0
    Required collateral as per liquidation ratio:
      min = con * cast (view mat ilk_0)
```

Table 3.1: CDP acts in the five stages of risk

	give	shut	lock	wipe	free	draw	bite	grab	plop
Pride	<b>(</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>₽</b>	4	_	_	_
Anger	$\mathcal{B}$	<b>&amp;</b>	<del>-</del>	<b>E</b>	$\oplus$	_	_	_	_
Worry	₾	<b>S</b>	<b>S</b>	<b>E</b>	_	_	_	_	_
Panic	₿	<b>E</b>	<b>E</b>	<b>E</b>	_	_	<b>(</b>	_	_
Grief	$\mathcal{P}$	_	_	_	_	_	_	$\bigcirc$	_
Dread	$\mathcal{P}$	_	_	_	_	_	_	_	$\bigcirc$
		decrease risk			increa	se risk	unwind risk		
allowed for anyone allowed for owner unconditionally allowed for owner if able to repay allowed for owner if collateralized									

Now we define the internal act feel which returns the value of *analyze* after ensuring that the system state is updated.

allowed for settler contract

```
feel id_{\text{urn}} = \mathbf{do}

Adjust target price and target rate prod

Update debt unit and unprocessed fee revenue id_{\text{ilk}} \leftarrow look \; (\text{vat.urns.} ix \; id_{\text{urn.}} \text{ilk})

drip id_{\text{ilk}}

Read parameters for risk analysis era<sub>0</sub> \leftarrow use era par<sub>0</sub> \leftarrow use (vat.vox.par) urn<sub>0</sub> \leftarrow look (vat.urns.ix id_{\text{urn.}})

ilk<sub>0</sub> \leftarrow look (vat.ilks.ix (view ilk urn<sub>0</sub>)) jar<sub>0</sub> \leftarrow look (vat.gems.ix (view jar ilk<sub>0</sub>))

Return risk stage of CDP return (analyze era<sub>0</sub> par<sub>0</sub> urn<sub>0</sub> ilk<sub>0</sub> jar<sub>0</sub>)
```

Acts on CDPs use feel to prohibit increasing risk when already risky, and to freeze debt and collateral during liquidation; see Table 3.1.

#### 3.2 Lending

Any user can open one or more accounts with the system using open, specifying a self-chosen account identifier and a CDP type.

```
open id_{	ext{urn}} \ id_{	ext{ilk}} = 	ext{do}

Fail if account identifier is taken none (vat . urns . ix \ id_{	ext{urn}})

Create a CDP record with the sender as owner id_{	ext{lad}} \leftarrow use \ sender
initialize \ (vat . urns . \ at \ id_{	ext{urn}}) \ (emptyUrn \ id_{	ext{ilk}} \ id_{	ext{lad}})
```

The owner of a CDP can transfer its ownership at any time using give.

```
give id_{	ext{urn}} \ id_{	ext{lad}} = 	ext{do}

Fail if sender is not the CDP owner id_{sender} \leftarrow use \ sender

owns \ id_{	ext{urn}} \ id_{sender}

Transfer ownership

vat . urns . ix \ id_{	ext{urn}} . lad := id_{	ext{lad}}
```

Unless liquidation has been triggered for a CDP, its owner can use lock to deposit more collateral.

```
lock id_{\text{urn}} wad_{\text{gem}} = \mathbf{do}

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender owns id_{\text{urn}} \ id_{\text{lad}}

Fail if liquidation triggered or initiated want (feel id_{\text{urn}}) (\notin [Grief, Dread])

Identify collateral type id_{\text{ilk}} \leftarrow look \ (\text{vat.urns.} \ ix \ id_{\text{urn}} \ . \ ilk) id_{\text{gem}} \leftarrow look \ (\text{vat.ilks.} \ ix \ id_{\text{ilk}} \ . \ jar)

Transfer tokens from owner to collateral vault pull id_{\text{gem}} \ id_{\text{lad}} \ \text{wad}_{\text{gem}}

Record an increase in collateral increase \ (\text{vat.urns.} \ ix \ id_{\text{urn.}} \ . \ jam) \ \text{wad}_{\text{gem}}
```

When a CDP has no risk problems (except that its CDP type's debt ceiling may be exceeded), its owner can use free to withdraw some amount of collateral, as long as the withdrawal would not reduce collateralization below the liquidation ratio.

```
free id_{\text{urn}} wad_{\text{gem}} = \mathbf{do}

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender owns id_{\text{urn}} \ id_{\text{lad}}

Record a decrease in collateral decrease (vat . urns . ix \ id_{\text{urn}} . jam) wad_{\text{gem}}

Roll back on any risk problem except debt ceiling excess want (feel id_{\text{urn}}) (\in [Pride, Anger])

Transfer tokens from collateral vault to owner id_{\text{ilk}} \leftarrow look (vat . urns . ix \ id_{\text{urn}} . ilk) id_{\text{gem}} \leftarrow look (vat . ilks . ix \ id_{\text{ilk}} . jar) push id_{\text{gem}} \ id_{\text{lad}} wad_{\text{gem}}
```

When a CDP has no risk problems, its owner can use draw to take out a loan of newly minted dai, as long as the CDP type's debt ceiling is not reached and the loan would not result in undercollateralization.

```
draw id_{urn} wad_{DAI} = do
 Fail if sender is not the CDP owner
   id_{\mathtt{lad}} \leftarrow use \ sender
   owns idurn idlad
  Update debt unit and unprocessed fee revenue
   id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
   \mathtt{chi}_1 \leftarrow \mathtt{drip}\ id_{\mathtt{ilk}}
 Denominate loan in debt unit
   let wad_{chi} = wad_{DAI} / cast chi_1
 Increase CDP debt
   increase (vat . urns . ix idurn . art) wadchi
 Increase total debt of CDP type
   increase (vat . ilks . ix id<sub>ilk</sub> . rum) wad<sub>chi</sub>
 Roll back on any risk problem
   want (feel id_{urn}) (\equiv Pride)
 Mint dai and transfer to CDP owner
   mint id_{DAI} wad_{DAI}
   push id_{DAI} id_{lad} wad_{DAI}
```

A CDP owner who has previously loaned dai can use wipe to repay part of their debt as long as liquidation has not been triggered.

```
wipe id_{\tt urn} \; {\tt wad}_{\tt DAI} = {f do}
 Fail if sender is not the CDP owner
    id_{1ad} \leftarrow use sender
   owns id_{\tt urn} id_{\tt lad}
 Fail if liquidation triggered or initiated
    want (feel id_{urn}) (\notin [Grief, Dread])
 Update debt unit and unprocessed fee revenue
    id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
   \mathsf{chi}_1 \leftarrow \mathsf{drip}\ id_{\mathsf{ilk}}
 Denominate dai amount in debt unit
   let wad_{chi} = wad_{DAI} / cast chi_1
 Decrease CDP debt
   decrease (vat . urns . ix idurn . art) wadchi
 Decrease total CDP type debt
   decrease (vat.ilks.ix id_{ilk}.rum) wad<sub>chi</sub>
 Transfer dai from CDP owner to dai vault
   pull id_{DAI} id_{lad} wad_{DAI}
 Destroy reclaimed dai
   burn id_{DAI} wad_{DAI}
```

A CDP owner can use shut to close their account—repaying all debt and reclaiming all collateral—if the price feed is up to date and liquidation has not been initiated.

```
shut id_{	ext{urn}} = 	ext{do}

Update debt unit and unprocessed fee revenue id_{	ext{ilk}} \leftarrow look \ (	ext{vat.urns.} ix \ id_{	ext{urn.}} . 	ext{ilk})

chi_1 \leftarrow drip \ id_{	ext{ilk}}

Reclaim all outstanding dai art_0 \leftarrow look \ (	ext{vat.urns.} ix \ id_{	ext{urn.}} . 	ext{art})

wipe id_{	ext{urn.}} \ (art_0 * cast \ chi_1)

Reclaim all collateral jam_0 \leftarrow look \ (	ext{vat.urns.} ix \ id_{	ext{urn.}} . 	ext{jam})

free id_{	ext{urn.}} \ jam_0

Nullify cdp record 	ext{vat.urns.} \ at \ id_{	ext{urn.}} := 	ext{Nothing}
```

#### 3.3 Adjustment

The feedback mechanism is updated through prod, which can be invoked at any time by keepers, but is also invoked as a side effect of any CDP act that uses feel to assess the CDP risk.

```
prod = do
 Read all parameters relevant for feedback mechanism
   era_0 \leftarrow use era
  tau_0 \leftarrow use (vat.vox.tau)
  fix_0 \leftarrow use (vat.vox.fix)
  par_0 \leftarrow use (vat.vox.par)
  how_0 \leftarrow use (vat.vox.how)
  way_0 \leftarrow use (vat.vox.way)
  let
    Time difference in seconds
      age = era_0 - tau_0
    Current target rate applied to target price
     par_1 = par_0 * cast (way_0 \uparrow \uparrow age)
    Sensitivity parameter applied over time
      wag = how_0 * fromIntegral age
    Target rate scaled up or down
     way_1 = inj (prj way_0 +
                   if fix_0 < par_0 then wag else -wag)
 Update target price
  vat.vox.par := par_1
 Update rate of price change
  vat.vox.way := way_1
 Record time of update
   vat.vox.tau := era_0
   where
    Convert between multiplicative and additive form
     prj \ x = if \ x \geqslant 1 \ then \ x - 1 \ else \ 1 - 1 / x
     inj \ x = if \ x \ge 0  then x + 1 else 1 / (1 - x)
```

The stability fee of a CDP type can change through governance. Due to the constraint that acts should run in constant time, the system cannot iterate over CDP records to effect such changes. Instead each CDP type has a single "debt unit" which accumulates the stability fee. The drip act updates this unit. It can be called at any time by keepers, but is also called as a side effect of every act that uses feel to assess CDP risk.

```
drip id_{ilk} = do
 Time stamp of previous drip
   \mathsf{rho}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathsf{rho})
 Current stability fee
   \mathsf{tax}_0 \leftarrow look \, (\mathsf{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathsf{tax})
 Current debt unit value
    \mathsf{chi}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathtt{chi})
 Current total debt in debt unit
   rum_0 \leftarrow look (vat.ilks.ix id_{ilk}.rum)
 Current unprocessed stability fee revenue
    joy_0 \leftarrow look (vat.joy)
 Current time stamp
    era_0 \leftarrow use era
   let
     Time difference in seconds
       age = era_0 - rho_0
     Value of debt unit increased according to stability fee
       chi_1 = chi_0 * tax_0 \uparrow \uparrow age
     Denominate stability fee revenue in new unit
        \mathtt{joy}_1 = \mathtt{joy}_0 + (\mathit{cast}\,(\mathtt{chi}_1 - \mathtt{chi}_0) :: \mathtt{Wad}) * \mathtt{rum}_0
 Record time of update
   \mathtt{vat.ilks.}\ ix\ id_{\mathtt{ilk}}\ .\ \mathtt{rho} := \mathtt{era}_0
 Record new debt unit
   \mathtt{vat.ilks.} ix\ id_{\mathtt{ilk}}.\mathtt{chi} := \mathtt{chi}_1
 Record fee revenue denominated in new debt unit
   vat.joy := joy_1
 Return the new debt unit
    return chi1
```

#### 3.4 Price feed input

The mark act records a new market price of a collateral token along with the expiration date of this price.

```
\begin{aligned} & \text{mark } id_{\text{gem}} \; \text{tag}_1 \; \text{zzz}_1 = \text{auth } \; \$ \; \mathbf{do} \\ & \text{vat.gems.} \; ix \; id_{\text{gem}} \; \text{.tag} := \text{tag}_1 \\ & \text{vat.gems.} \; ix \; id_{\text{gem}} \; \text{.zzz} := \text{zzz}_1 \end{aligned}
```

The tell act records a new market price of the DAI token along with the expiration date of this price.

```
\texttt{tell wad}_{\texttt{gem}} = \texttt{auth } \$ \ \mathbf{do} \ \texttt{vat.vox.fix} := \texttt{wad}_{\texttt{gem}}
```

#### 3.5 Liquidation

When a CDP's risk stage marks it as in need of liquidation, any account can invoke the bite act to trigger the liquidation process. This records the CDP's debt as bad debt in the CDP engine and enables the settler contract to later grab the collateral and begin auctioning.

```
bite id_{\tt urn} = {f do}
      Fail if CDP is not in the appropriate risk stage
        want (feel id_{urn}) (\equiv Panic)
      Record the sender as the liquidation initiator
        id_{\mathtt{cat}} \leftarrow use sender
        \mathtt{vat} . \mathtt{urns} . ix\ id_{\mathtt{urn}} . \mathtt{cat} := \mathtt{Just}\ id_{\mathtt{cat}}
      Update debt unit
        id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
        \mathsf{chi}_1 \leftarrow \mathsf{drip}\ id_{\mathtt{ilk}}
      Apply liquidation penalty to debt
        \mathtt{art}_0 \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{art})
        axe_0 \leftarrow look (vat.ilks.ix id_{ilk}.axe)
        let art_1 = art_0 * cast axe_0
      Update CDP debt
        vat . urns . ix id_{urn} . art := art<sub>1</sub>
      Record as bad debt
        increase (vat . sin) (art<sub>1</sub> * cast chi<sub>1</sub>)
```

After liquidation has been triggered, the designated settler contract invokes grab to receive the collateral tokens.

```
\begin{array}{l} \operatorname{grab}\ id_{\operatorname{urn}} = \operatorname{auth}\ \$\ \operatorname{\mathbf{do}} \\ &\operatorname{Fail}\ \operatorname{if}\ \operatorname{cdp}\ \operatorname{is}\ \operatorname{not}\ \operatorname{marked}\ \operatorname{for}\ \operatorname{liquidation} \\ & want\ (\operatorname{feel}\ id_{\operatorname{urn}})\ (\equiv\operatorname{Grief}) \\ &\operatorname{Record}\ \operatorname{the}\ \operatorname{sender}\ \operatorname{as}\ \operatorname{the}\ \operatorname{cdp's}\ \operatorname{settler} \\ & id_{\operatorname{vow}}\leftarrow use\ \operatorname{sender} \\ &\operatorname{vat}\ .\ \operatorname{urns}\ .\ ix\ id_{\operatorname{urn}}\ .\ \operatorname{vow}:=\operatorname{Just}\ id_{\operatorname{vow}} \\ &\operatorname{Forget}\ \operatorname{the}\ \operatorname{cdp's}\ \operatorname{requester}\ \operatorname{of}\ \operatorname{liquidation} \\ &\operatorname{vat}\ .\ \operatorname{urns}\ .\ ix\ id_{\operatorname{urn}}\ .\ \operatorname{cat}:=\operatorname{Nothing} \\ &\operatorname{Transfer}\ \operatorname{the}\ \operatorname{tokens}\ \operatorname{to}\ \operatorname{the}\ \operatorname{settler} \\ &\operatorname{jam}_0\leftarrow look\ (\operatorname{vat}\ .\ \operatorname{urns}\ .\ ix\ id_{\operatorname{urn}}\ .\ \operatorname{jam}) \\ &\operatorname{id}_{\operatorname{ilk}}\leftarrow look\ (\operatorname{vat}\ .\ \operatorname{urns}\ .\ ix\ id_{\operatorname{ulk}}\ .\ \operatorname{jar}) \\ &\operatorname{id}_{\operatorname{gem}}\leftarrow look\ (\operatorname{vat}\ .\ \operatorname{ilks}\ .\ ix\ id_{\operatorname{ilk}}\ .\ \operatorname{jar}) \\ &\operatorname{push}\ id_{\operatorname{gem}}\ id_{\operatorname{yow}}\ \operatorname{jam}_0 \\ \end{array}
```

When the settler has finished the process of liquidating a CDP's collateral, it invokes plop on the CDP to give back any excess collateral gains.

```
\begin{aligned} & \text{plop } \textit{id}_{\text{urn}} \; \text{wad}_{\text{DAI}} = \text{auth } \$ \; \textbf{do} \\ & \text{Fail unless cdp is in liquidation} \\ & \textit{want} \; (\text{feel } \textit{id}_{\text{urn}}) \; (\equiv \text{Dread}) \\ & \text{Forget the cdp's settler} \\ & \text{vat.urns.} \; \textit{ix} \; \textit{id}_{\text{urn.}} \; \text{vow} := \text{Nothing} \\ & \text{Return some amount of excess auction gains} \\ & \textit{id}_{\text{vow}} \; \leftarrow \; \textit{use sender} \\ & \textit{id}_{\text{ilk}} \; \leftarrow \; \textit{look} \; (\text{vat.urns.} \; \textit{ix} \; \textit{id}_{\text{urn.}} \; \text{ilk}) \\ & \textit{id}_{\text{gem}} \; \leftarrow \; \textit{look} \; (\text{vat.ilks.} \; \textit{ix} \; \textit{id}_{\text{ilk.}} \; \text{jar}) \\ & \text{pull} \; \textit{id}_{\text{gem}} \; \textit{id}_{\text{vow}} \; \text{wad}_{\text{DAI}} \\ & \text{Record the gains as the cdp's collateral} \\ & \text{vat.urns.} \; \textit{ix} \; \textit{id}_{\text{urn.}} \; \text{jam.} := \; \text{wad}_{\text{DAI}} \end{aligned}
```

When the settler has processed all the bad debt due to CDP liquidation, it invokes heal to reset the bad debt quantity.

```
heal wad_{DAI} = auth \$ do vat.sin := 0
```

When the settler has processed all the accrued stability fee revenue, it invokes love to reset the unprocessed stability fee quantity.

```
love wad_{DAI} = auth \$ do vat.joy := 0
```

#### 3.6 Governance

Governance uses form to create a new CDP type. Since the new type is initialized with a zero debt ceiling, a separate transaction can safely set the risk parameters before any lending occurs.

```
form id_{ilk} id_{gem} = auth \$ do

initialize (vat.ilks.atid_{ilk}) (defaultIlkid_{gem})
```

Governance uses frob to alter the sensitivity factor, which is the only mutable parameter of the feedback mechanism.

```
frob how_1 = auth \$ do vat.vox.how := how_1
```

Governance can alter the five risk parameters of a CDP type using cuff for the liquidation ratio; chop for the liquidation penalty; cork for the debt ceiling; calm for the duration of price limbo; and crop for the stability fee.

```
\begin{array}{l} \mathrm{cuff}\ id_{\mathtt{ilk}}\ \mathtt{mat}_1 = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat}\ .\ \mathtt{ilks}\ .\ ix\ id_{\mathtt{ilk}}\ .\ \mathtt{mat} := \mathtt{mat}_1\\ \mathtt{chop}\ id_{\mathtt{ilk}}\ \mathtt{axe}_1 = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat}\ .\ \mathtt{ilks}\ .\ ix\ id_{\mathtt{ilk}}\ .\ \mathtt{axe} := \mathtt{axe}_1\\ \mathtt{cork}\ id_{\mathtt{ilk}}\ \mathtt{hat}_1 = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat}\ .\ \mathtt{ilks}\ .\ ix\ id_{\mathtt{ilk}}\ .\ \mathtt{hat} := \mathtt{hat}_1\\ \mathtt{calm}\ id_{\mathtt{ilk}}\ \mathtt{lag}_1 = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat}\ .\ \mathtt{ilks}\ .\ ix\ id_{\mathtt{ilk}}\ .\ \mathtt{lag} := \mathtt{lag}_1 \end{array}
```

When altering the stability fee with crop, we ensure that the previous stability fee has been accounted for in the internal debt unit.

```
\begin{array}{l} \texttt{crop } id_{\mathtt{ilk}} \ \mathtt{tax}_1 = \\ \mathtt{auth} \ \$ \ \mathbf{do} \\ \\ \texttt{Apply the current stability fee to the internal debt unit} \\ \mathtt{drip } id_{\mathtt{ilk}} \\ \\ \texttt{Change the stability fee} \\ \\ \mathtt{vat.ilks.} \ ix \ id_{\mathtt{ilk}} \ . \ \mathtt{tax} := \mathtt{tax}_1 \end{array}
```

#### 3.7 Vaults

The internal act pull transfers tokens into a vault. It is used by lock to acquire collateral from a CDP owner; by wipe to acquire dai from a CDP owner; and by plop to acquire collateral from the settler contract.

```
\begin{aligned} & \text{pull } id_{\text{gem}} \ id_{\text{lad}} \ \text{wad}_{\text{gem}} = \\ & \textit{transfer } id_{\text{gem}} \ \text{wad}_{\text{gem}} \ (\text{InAccount } id_{\text{lad}}) \ (\text{InVault } id_{\text{gem}}) \end{aligned}
```

The internal act push transfers tokens out from a collateral vault. It is used by draw to send dai to a CDP owner; by free to send collateral to a CDP owner; and by grab to send collateral to the settler contract.

```
push id_{\text{gem}} id_{\text{lad}} \text{ wad}_{\text{gem}} = transfer id_{\text{gem}} \text{ wad}_{\text{gem}} (\text{InVault } id_{\text{gem}}) (\text{InAccount } id_{\text{lad}})
```

#### 3.8 Token manipulation

We model the ERC20 transfer function in simplified form (omitting the concept of "allowance").

```
transfer id_{\text{gem}} wad src\ dst =

Operate in the token's balance table zoom\ (\text{vat.gems.} ix\ id_{\text{gem}}\ .\ \text{ERC20.} balanceOf)\ \$\ \mathbf{do}

Fail if source balance insufficient balance \leftarrow look\ (ix\ src)

aver (balance \geqslant \text{wad})

Update balances decrease\ (ix\ src)\ \text{wad}
initialize\ (at\ dst)\ 0
increase\ (ix\ dst)\ \text{wad}
```

The internal act mint inflates the supply of a token. It is used by draw to create new DAI, and by settlers to create new MKR.

```
\begin{split} & \texttt{mint} \ id_{\texttt{gem}} \ \texttt{wad}_0 = \\ & \textit{zoom} \ (\texttt{vat} \ . \ \texttt{gems} \ . \ ix \ id_{\texttt{gem}} \ . \ \texttt{ERC20}) \ \$ \ \mathbf{do} \\ & \textit{increase} \ (\textit{balanceOf} \ . \ ix \ (\texttt{InVault} \ id_{\texttt{gem}})) \ \texttt{wad}_0 \end{split}
```

The internal act burn deflates the supply of a token. It is used by wipe to destroy DAI, and by settlers to destroy MKR.

```
\begin{array}{l} \mathtt{burn} \ id_{\mathtt{gem}} \ \mathtt{wad}_0 = \\ zoom \ (\mathtt{vat} \ . \ \mathtt{gems} \ . \ ix \ id_{\mathtt{gem}} \ . \ \mathtt{ERC20}) \ \$ \ \mathbf{do} \\ decrease \ (balanceOf \ . \ ix \ (\mathtt{InVault} \ id_{\mathtt{gem}})) \ \mathtt{wad}_0 \end{array}
```

## Chapter 4

## Act framework

The reader does not need any abstract understanding of monads to understand the code. They give us a nice syntax—the **do** block notation—for expressing exceptions and state in a way that is still purely functional. Each line of such a block is interpreted by the monad to provide the semantics we want.

#### 4.1 The Maker monad

This defines the Maker monad as a simple composition of a state monad and an error monad:

```
type Maker a = \text{StateT System (Except Error) } a
```

We divide act failure modes into general assertion failures and authentication failures.

```
data Error = AssertError Act | AuthError
deriving (Show, Eq)
```

An act can be executed on a given initial system state using *exec*. The result is either an error or a new state. The *exec* function can also accept a sequence of acts, which will be interpreted as a single transaction.

```
exec :: System \rightarrow Maker () \rightarrow Either Error System

exec sys m = runExcept (execStateT m sys)
```

#### 4.2 Asserting

We now define a set of functions that fail unless some condition holds.

```
General assertion aver x = unless\ x\ (throwError\ (AssertError\ ?act)) Assert that an indexed value is not present none x = preuse\ x \gg \lambda case Nothing \rightarrow return\ () Just _- \rightarrow throwError\ (AssertError\ ?act) Assert that an indexed value is present look f = preuse\ f \gg \lambda case Nothing \rightarrow throwError\ (AssertError\ ?act) Just x \rightarrow return\ x Execute an act and assert a condition on its result want m\ p = m \gg (aver\ .\ p)
```

We define owns  $id_{urn}$   $id_{lad}$  as an assertion that the given CDP is owned by the given account.

```
owns id_{\tt urn} \ id_{\tt lad} = {\bf do}

want \ (look \ (\tt vat . urns . ix \ id_{\tt urn} . \ lad)) \ (\equiv id_{\tt lad})
```

We define auth k as an act modifier that executes k only if the sender is authorized.

```
\begin{array}{l} \text{auth } continue = \mathbf{do} \\ s \leftarrow \textit{use sender} \\ \textit{unless} \ (s \equiv \textit{id}_{\textit{god}}) \ (\textit{throwError} \ \text{AuthError}) \\ \textit{continue} \end{array}
```

## Appendix A

## **Prelude**

This module reexports symbols from other packages and exports a few new symbols of its own.

```
module Maker.Prelude (module Maker.Prelude, module X) where
import Prelude as X (
 Conversions to and from strings
  Read (..), Show (..),
 Comparisons
  Eq(...), Ord(...),
 Core abstractions
  Functor
               (fmap),
  Applicative (),
  Monad
               (return, (\gg)),
 Numeric classes
  Num (..), Integral (), Enum (),
 Numeric conversions
  Real (...), Fractional (...),
  RealFrac (..),
  fromIntegral,
 Simple types
  Integer, Int, String,
 Algebraic types
  Bool (True, False),
  Maybe (Just, Nothing),
  Either (Right, Left),
```

```
Functional operators (.), (\$), Numeric operators (+), (-), (*), (/), (\uparrow), (\uparrow\uparrow), div, Utilities all, \neg, elem, Constants mempty, \bot, otherwise)
```

**import** Control.Monad.State as X (

We use a typical composition of monad transformers from the mtl library to structure stateful actions. See section 4.1 (*The Maker monad*).

```
Type constructor that adds state to a monad type
  StateT,
  execStateT,
                  Runs a state monad with given initial state
                  Gets the state in a do block
  get,
  put)
                  Sets the state in a do block
import Control.Monad.Writer as X (
  WriterT,
                  Type constructor that adds logging to a monad type
  Writer.
                  Type constructor of logging monads
  run WriterT.
                  Runs a writer monad transformer
   execWriterT, Runs a writer monad transformer keeping only logs
  execWriter)
                  Runs a writer monad keeping only logs
import Control.Monad.Except as X (
```

MonadError, Type class of monads that fail
Except, Type constructor of failing monads

throwError, Short-circuits the monadic computation

runExcept) Runs a failing monad

Our numeric types use decimal fixed-point arithmetic.

```
\begin{array}{ll} \textbf{import Data.Fixed} \ \textit{as} \ X \ (\\ \textbf{Fixed} \ (..), & \textbf{Type constructor for numbers of given precision} \\ \textbf{HasResolution} \ (..)) \ \textbf{Type class for specifying precisions} \end{array}
```

We rely on the lens library for accessing nested values. There is no need to understand the theory behind lenses to understand this program. The notation  $a \cdot b \cdot c$  denotes a nested accessor much like a.b.c in C-style languages; for more details, consult lens documentation<sup>1</sup>.

<sup>&</sup>lt;sup>1</sup>Gabriel Gonzalez's 2013 article *Program imperatively using Haskell* is a good introduction.

```
import Control.Lens as X (
  Lens', lens,
   makeLenses, Defines lenses for record fields
   makeFields.
                Defines lenses for record fields
   set,
                  Writes a lens
   use, preuse, Reads a lens from a state value
   view,
                  Reads a lens from a value
   ix,
                  Lens for map retrieval and updating
   at,
                  Lens for map insertion
 Operators for partial state updates in do blocks:
   (:=),
                  Replace
   (-=), (+=), Update arithmetically
   (\% =),
                  Update according to function
  (?=))
                  Insert into map
import Control.Lens.Zoom as X (zoom)
```

Where the Solidity code uses mapping, we use Haskell's regular tree-based map type<sup>2</sup>.

```
import Data.Map as X (
Map, Type constructor for mappings
Ø, Polymorphic empty mapping
singleton) Creates a mapping with a single key-value pair
```

Finally we define some of our own convenience functions.

```
decrease a \ x = a -= x
increase a \ x = a += x
initialize a \ x = a \% = (\lambda \mathbf{case} \ \text{Nothing} \to \text{Just} \ x; \ y \to y)
prepend a \ x = a \% = (x:)
x \notin xs = \neg \ (elem \ x \ xs)
```

<sup>&</sup>lt;sup>2</sup>We assume the axiom that Keccak hash collisions are impossible.

## Appendix B

## Fixed point numbers with rounding

This somewhat arcane-looking code implements a wrapper around the base library's decimal fixed point type, only with x \* y and x / y operations that do rounding instead of truncation of their intermediate results.

```
module Maker.Decimal (Decimal, E18, E36, Epsilon (...)) where import Data.Fixed newtype HasResolution e \Rightarrow Decimal e = D (Fixed e) deriving (Ord, Eq, Real, RealFrac)
```

We want the printed representations of these numbers to look like "0.01" and not "R 0.01".

```
instance HasResolution e \Rightarrow \text{Read} (Decimal e) where readsPrec\ n\ s = fmap\ (\lambda(x,y) \to (D\ x,y))\ (readsPrec\ n\ s) instance HasResolution e \Rightarrow \text{Show} (Decimal e) where show\ (D\ x) = show\ x
```

In the Num instance, we delegate everything except multiplication.

```
instance HasResolution e \Rightarrow \text{Num (Decimal } e) where x@(D \text{ (MkFixed } a)) * D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a*b+div (resolution x) 2) (resolution x)))}
D a + D b = D (a + b)
D a - D b = D (a - b)
negate (D a) = D (negate a)
abs (D a) = D (abs a)
```

```
signum (D a) = D (signum a)

fromInteger i = D (fromInteger i)
```

In the Fractional instance, we delegate everything except division.

```
instance HasResolution e \Rightarrow Fractional (Decimal e) where x@(D \text{ (MkFixed } a)) / D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a * resolution } x + div b 2) b))
recip (D a) = D (recip a)
fromRational \ r = D \ (fromRational \ r)
```

We define the E18 and E36 symbols and their fixed point multipliers.

```
data E18; data E36 instance HasResolution E18 where resolution \_=10\uparrow(18::Integer) instance HasResolution E36 where resolution \_=10\uparrow(36::Integer)
```

The fixed point number types have well-defined smallest increments (denoted  $\epsilon$ ). This becomes useful when verifying equivalences.

```
class Epsilon t where \epsilon :: t instance HasResolution a \Rightarrow Epsilon (Decimal a) where The use of \bot is safe since resolution ignores the value. \epsilon = 1 \ / \ from Integral \ (resolution \ (\bot :: Fixed \ a))
```