

presents the

REFERENCE IMPLEMENTATION

also known as the PURPLE PAPER

 $of \ the \ remarkable$

DAI CREDIT SYSTEM

issuing a diversely collateralized stablecoin

 $formulated\ by$

Daniel Brockman Mikael Brockman Nikolai Mushegian

with last update on March 16, 2017.



Contents

| 1 | Introduction | | | | | | | | | |
|---|--------------|--|--|--|--|--|--|--|--|--|
| | 1.1 | Motivation | | | | | | | | |
| | 1.2 | Limitations | | | | | | | | |
| | 1.3 | Verification | | | | | | | | |
| I | Imp | plementation | | | | | | | | |
| 2 | Types 1 | | | | | | | | | |
| _ | 2.1 | Numeric types | | | | | | | | |
| | 2.2 | Identifiers and addresses | | | | | | | | |
| | 2.3 | Gem — collateral price feed entry | | | | | | | | |
| | | jar — token vault | | | | | | | | |
| | | tag — market price of token | | | | | | | | |
| | | zzz — expiration time of token price feed | | | | | | | | |
| | 2.4 | ERC20 — token model | | | | | | | | |
| | 2.5 | Ilk — CDP type | | | | | | | | |
| | | jar — collateral token vault | | | | | | | | |
| | | mat — liquidation ratio | | | | | | | | |
| | | axe — liquidation penalty | | | | | | | | |
| | | hat — debt ceiling | | | | | | | | |
| | | tax — stability fee | | | | | | | | |
| | | lax — price feed limbo duration | | | | | | | | |
| | | rho — time of debt unit adjustment | | | | | | | | |
| | | rum — total outstanding debt units | | | | | | | | |
| | | chi — value of debt unit in DAI | | | | | | | | |
| | 2.6 | Urn — collateralized debt position (CDP) | | | | | | | | |
| | | cat — address of liquidation initiator | | | | | | | | |
| | | vow — address of liquidation contract | | | | | | | | |
| | | lad — CDP owner | | | | | | | | |
| | | ilk — CDP type | | | | | | | | |
| | | art — debt denominated in debt unit | | | | | | | | |
| | | jam — collateral denominated in debt unit | | | | | | | | |
| | 2.7 | Vox — feedback mechanism data | | | | | | | | |
| | | wut — market price of DAI denominated in SDR | | | | | | | | |
| | | par — target price of DAI denominated in SDR | | | | | | | | |
| | | how — sensitivity parameter | | | | | | | | |

| | | way — rate of target price change | 12 |
|---|-------|---|----|
| | | tau — time of latest feedback cycle | 12 |
| | 2.8 | Vat — CDP engine data | 13 |
| | | joy — unprocessed stability fee revenue | 13 |
| | | sin — bad debt from liquidated CDPS | 13 |
| | 2.9 | System model | 13 |
| | | era — current time | 13 |
| | 2.10 | Default data | 14 |
| | | | |
| 3 | Acts | | 16 |
| | 3.1 | Assessment | 17 |
| | | feel — identify CDP risk stage | 17 |
| | 3.2 | Lending | 19 |
| | 0.2 | open — create CDP | 19 |
| | | give — transfer CDP account | 19 |
| | | lock — deposit collateral | 19 |
| | | free — withdraw collateral | 20 |
| | | draw — issue dai as debt | 20 |
| | | | 20 |
| | | wipe — repay debt and burn dai | |
| | 0.0 | shut — wipe, free, and delete CDP | 21 |
| | 3.3 | Adjustment | 22 |
| | | prod — adjust target price and target rate | 22 |
| | | drip — update debt unit and unprocessed fee revenue | 23 |
| | 3.4 | Price feed input | 24 |
| | | mark — update market price of collateral token | 24 |
| | | tell — update market price of dai | 24 |
| | 3.5 | Liquidation | 24 |
| | | bite — mark for liquidation | 24 |
| | | grab — take tokens for liquidation | 24 |
| | | plop — finish liquidation returning profit | 25 |
| | | loot — take unprocessed stability fees | 25 |
| | 3.6 | Governance | 25 |
| | | form — create a new CDP type | 25 |
| | | frob — set the sensitivity parameter | 25 |
| | | chop — set liquidation penalty | 25 |
| | | cork — set debt ceiling | 25 |
| | | calm — set limbo duration | 25 |
| | | cuff — set liquidation ratio | 25 |
| | | crop — set stability fee | 26 |
| | 3.7 | Vaults | 26 |
| | 5.7 | pull — transfer tokens to vault | 26 |
| | | push — transfer tokens from vault | 26 |
| | 3.8 | Token manipulation | 26 |
| | ٥.٥ | • | |
| | | mint — inflate token | 26 |
| | | burn — deflate token | 27 |
| 1 | Act f | framework | 28 |
| | 4.1 | The Maker monad | 28 |
| | 4.1 | | 28 |
| | 4.4 | Asserting | 40 |

| A | Prelude | 30 |
|---|-----------------------------------|----|
| В | Fixed point numbers with rounding | 33 |
| | | |

Chapter 1

Introduction

The dai credit system, henceforth also "Maker," is a network of Ethereum contracts designed to issue the dai currency token and automatically adjust credit incentives in order to keep its market value stable relative to SDR¹ in the short and medium term.

New dai enters the money supply when a borrower locks an excess of collateral in the system and takes out a loan. The debt and collateral amounts are recorded in a *collateralized debt position*, or CDP. Thus all outstanding dai represents some CDP owner's claim on their collateral—until risk provokes a liquidation.

Off-chain *price feeds* give Maker knowledge of the market values of dai and the various tokens used as collateral, enabling the system to assess credit risk. If the value of a CDP's collateral drops below a certain multiple of its debt, a decentralized auction is triggered which liquidates the collateral for dai in order to settle the debt.

The system issues a separate token with symbol MKR, which behaves like a "share" in Maker itself. Since collateral auctions may fail to recover the full value of liquidated debt, the MKR token can be diluted to back emergency debt. The value of MKR, though volatile by design, is backed by the revenue from *stability fees* imposed on all dai loans. The DAI raised from stability fees is used to buy MKR tokens from the market and destroy them.

For more details on the economics of the system, as well as descriptions of governance, off-chain mechanisms that provide efficiency, and so on, see the whitepaper.

This document is an executable technical specification of the of the Maker smart contracts. It is a draft; be aware that the contents will certainly change before launch.

1.1 Motivation

The version of this system that will be deployed on the blockchain is written in Solidity, which is a workable smart contract implementation language. This reference implementation is a model of the behavior of those contracts, written as a "literate" Haskell program. The motivations for such a reference implementation include:

^{1&}quot;Special Drawing Rights" (ticker symbol XDR), the international reserve asset created by the International Monetary Fund, whose value is derived from a weighted basket of world currencies.

- 1. **Comparison**. Checking two free-standing implementations against each other is a well-known way of ensuring that they both behave as intended.
- Testing. Haskell lets us use powerful testing tools such as QuickCheck and SmallCheck for comprehensively verifying key properties as a middle ground between unit testing and formal verification.
- 3. **Explicitness**. Coding the contract behavior in Haskell, a purely functional language, enforces explicit description of aspects which Solidity leaves implicit. For example, a Solidity program can read previously unwritten storage and get back a zero value, whereas in Haskell we must give explicit defaults. The state rollback behavior of failed actions is also explicit in the type of the execution function, which may return an error.
- 4. **Typing**. While Solidity does have a static type system, it is not expressive enough to encode the distinctions made by our system. In particular, the two different decimal fixed point number types that we use are typed in Solidity with one and the same uint128 type. In Haskell we can make this distinction explicit.
- 5. Formality. The work of translating a Solidity program into a purely functional program opens up opportunities for certain types of formal verification. In particular, this document will be useful for modelling aspects of the system in a proof assistant like Agda, Idris, Coq, or Isabelle. We can also use logical tools for Haskell, such as Liquid Haskell (which provides compile time logical property checking) and sbv (a toolkit for model checking and symbolic execution).
- 6. Clarity. An implementation not intended to be deployed on the blockchain is free from concerns about optimizing for gas cost and other factors that make the Solidity implementation less ideal as an understandable specification.
- 7. Simulation. Solidity is highly specific to the Ethereum blockchain environment and as such does not have facilities for interfacing with files or other computer programs. This makes the Solidity implementation of the system less useful for doing simulations of the system's economic, gametheoretic, or statistical aspects.

1.2 Limitations

This model is limited in that it has

- 1. a simplified version of authorization for governance;
- 2. a simplified version of ERC20 token semantics;
- 3. no implementation of the decentralized auction contracts; and
- 4. no 256-bit word limits.

These limitations will be addressed in future revisions.

1.3 Verification

Separately from this document, we are developing automatic test suites that generate many, large, and diverse action sequences for property verification. One such property is that the reference implemen-

tation exactly matches the on-chain implementation; this is verified through the generation of Solidity test cases with assertions covering the entire state. Other key properties include

- that the target price changes only according to the target rate;
- that the total dai supply is fully accounted for by CDP debts;
- that CDP acts are restricted with respect to risk stage;

along with similar invariants and conditions. A future revision of this document will include formal statements of these properties.

Part I Implementation

Preamble

This is a Haskell program, and as such makes reference to a background of symbols defined in libraries, as a mathematical paper depends on preestablished theories.

Context should allow the reader to understand most symbols without further reading, but Appendix A lists and briefly explains each imported type and function.

We replace the default prelude module with our own.

```
module Maker where
import Prelude () Import nothing from Prelude
import Maker.Prelude Import everything from Maker Prelude
```

We also import our definition of decimal fixed point numbers, listed in Appendix B.

import Maker.Decimal

Now we proceed to define the specifics of the Maker system.

Chapter 2

Types

This chapter defines the data types used by Maker: numeric types, identifiers, on-chain records, and test model data.

Haskell syntax note: **newtype** defines a type synonym with distinct type identity; **data** creates a record type; and **deriving** creates automatic instances of common functionality.

2.1 Numeric types

The system uses two different precisions of decimal fixed point numbers, which we call *wads* and *rays*, having respectively 18 digits of precision (used for token quantities) and 36 digits (used for precise rates and ratios). See Appendix B for details on decimal fixed point numbers and rounding.

```
Define the distinct type of currency quantities

newtype Wad = Wad (Decimal E18)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)

Define the distinct type of rates and ratios

newtype Ray = Ray (Decimal E36)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)
```

We also define a type for time durations in whole seconds, as this is the maximum precision allowed by the Ethereum virtual machine.

```
newtype Sec = Sec Int
deriving (Eq, Ord, Enum, Num, Real, Integral)
```

Haskell number types are not automatically converted, so we convert explicitly with a *cast* function.

```
Convert via fractional n/m form. cast :: (Real \ a, Fractional \ b) \Rightarrow a \rightarrow b cast = fromRational \ . toRational
```

2.2 Identifiers and addresses

There are several kinds of identifiers used in the system, and we use types to distinguish them. The type parameter *a* creates distinct types; e.g., Id Foo and Id Bar are incompatible.

```
newtype Id a = \text{Id String deriving (Eq, Ord, Show)}
```

We define another type for representing Ethereum account addresses.

```
newtype Address = Address String deriving (Eq, Ord, Show)
```

We also have two predefined entity identifiers.

```
The DAI token vault address id_{	ext{DAI}} = 	ext{Id "DAI"} A test account with ultimate authority id_{god} = 	ext{Address "GOD"}
```

2.3 Gem — collateral price feed entry

The data received from price feeds is categorized by token and stored in Gem records. Our model also has the token balances embedded in these records; in reality¹, the balances are in separate ERC20 contracts.

2.4 ERC20 — token model

In reality, token semantics can differ, despite nominally following the ERC20 interface. Governance therefore involves reviewing the behaviors of collateral tokens. In our model, tokens behave in the same simple way. We also omit the notion of "allowance."

Tokens can be held by CDP owners, by token vaults, or by the test driver. We model this distinction with a data type.

```
\begin{aligned} \textbf{data} \; & \text{Holder} = \text{InAccount Address} \; | \; & \text{InVault (Id Gem)} \; | \; & \text{InToy} \\ & \textbf{deriving} \; & \text{(Eq, Ord, Show)} \end{aligned}
```

We now define an ERC20 instance as a map tracking the token quantity held by each holder.

```
data ERC20 = ERC20 { · balanceOf :: Map Holder Wad}
deriving (Eq, Show)
```

¹We use "reality" to denote the actual state of the consensus Ethereum blockchain.

2.5 Ilk — CDP type

Each CDP belongs to a CDP type, specified by an Ilk record. Five parameters, mat, axe, hat, tax and lax, are set by governance and are known as the Risk Parameters. The rest of the values are used by the system to keep track of the current state. The meaning of each ilk parameter is defined by its interactions in the act definitions of Chapter 3; see the whitepaper for an overview.

```
data Ilk = Ilk {
   · jar :: Id Gem, Collateral token identifier
   · tax::Sec,
                       Grace period after price feed becomes unavailable
   · mat :: Ray,
                       Collateral-to-debt ratio at which liquidation can be triggered
   · axe :: Ray,
                       Penalty on liquidation as fraction of debt
   · hat :: Wad,
                       Limit on total debt for CDP type ("debt ceiling")
   · tax::Ray,
                       Stability fee as per-second fraction of debt value
   · chi::Ray,
                       Value of internal debt unit in dai
   · rho::Sec,
                       Time of latest debt unit adjustment
   · rum :: Wad
                       Total debt in debt units
   } deriving (Eq, Show)
```

2.6 Urn — collateralized debt position (CDP)

For each CDP we maintain an Urn record identifying its type and specifying ownership, quantities of debt and collateral denominated in the CDP type's debt unit, along with the progress of liquidation (if relevant).

2.7 Vox — feedback mechanism data

The *feedback mechanism* is the aspect of the CDP engine that adjusts the target price of dai based on market price, and its data is kept in a singleton record called Vox.

```
data Vox = Vox {
wut :: Wad, Market price of dai denominated in SDR
par :: Wad, Target price of dai denominated in SDR
way :: Ray, Current per-second change in target price
```

```
    how:: Ray, Sensitivity parameter set by governance
    tau:: Sec Time of latest feedback cycle
    deriving (Eq, Show)
```

Keeping the feedback data separate allows us to more easily upgrade the mechanism in the future.

2.8 Vat — CDP engine data

The Vat record aggregates the records of CDPs, CDP types, and price feeds, along with the data of the feedback mechanism, and two accounting quantities.

Changes as of March 13 not yet incorporated into the reference implementation: The vat no longer has joy and sin directly; instead, it has a reference to a Jug, which is a two-token multivault holding dai and sin.

For accounting purposes, sin is now made into an actual ERC20 token and redefined to mean all debt, not just bad debt. The only component authorized to mint and burn either dai or sin is the Jug, and it ensures that they are always created and destroyed one-for-one.

Excess dai left in the Jug represents unprocessed stability fees and is available for the Vow to loot, whereas sin residing in the Jug represents collective CDP debt. There is no need to keep track of bad debt separately; instead, any debt found out to be bad is transferred immediately from the Jug to the Vow upon grab (along with the collateral tokens from the Jar).

2.9 System model

Finally we define a record with no direct counterpart in the Solidity contracts, which has the Vat record along with model state.

```
data System = System {
    vat :: Vat, Root Maker entity
    era :: Sec, Current time stamp
    sender :: Address, Sender of current act
    accounts :: [Address] For test suites
} deriving (Eq, Show)
```

2.10 Default data

```
defaultIlk :: Id Gem \rightarrow Ilk
\textit{defaultIlk} \ id_{\tt{gem}} = \mathtt{Ilk} \ \{
   · jar = id_{gem},
   \cdot \ \mathtt{axe} = \mathtt{Ray} \ 1,
   · mat = Ray 1,
   · tax = Ray 1,
   · hat = Wad 0,
   \cdot \  \, \texttt{tax} = \texttt{Sec} \; 0,
   · chi = Ray 1,
   · rum = Wad 0,
   · rho = Sec 0
}
emptyUrn :: Id Ilk \rightarrow Address \rightarrow Urn
emptyUrn id_{ilk} id_{lad} = Urn  {
   · vow = Nothing,
   \cdot cat = Nothing,
   · lad = id_{lad},
   · ilk = id_{ilk},
   · art = Wad 0,
     \mathtt{jam} = \mathtt{Wad}\ 0
initialVat :: \mathtt{Ray} \rightarrow \mathtt{Vat}
initialVat how_0 = Vat  {
   vatVox = Vox  {
     \cdot tau = 0,
     • wut = Wad 1,
     · par = Wad 1,
     · how = how<sub>0</sub>,
     · way = Ray 1
   },
   vat foy = Wad 0,
             = Wad 0,
   vatSin
   vatIlks = \emptyset,
   vatUrns = \emptyset,
   vatGems = singleton id_{DAI} Gem \{
     \cdot \text{ erc20} = \text{ERC20} \{ \cdot \text{ balanceOf} = \emptyset \},
     · tag = Wad 0,
      \cdot zzz = 0
}
initialSystem :: Ray \rightarrow System
initialSystem how_0 = System {
```

```
\begin{array}{lll} \textit{systemVat} & = \textit{initialVat} \ \text{how}_0, \\ \textit{systemEra} & = 0, \\ \textit{systemSender} & = \textit{id}_{god}, \\ \textit{systemAccounts} & = \textit{mempty} \\ \} \end{array}
```

Chapter 3

Acts

The *acts* are the basic state transitions of the system.

Unless specified as *internal*, acts are accessible as public functions on the blockchain.

The auth modifier marks acts which can only be invoked from addresses to which the system has granted authority.

For details on the underlying "Maker monad," which specifies how the act definitions behave with regard to state and rollback, see chapter 4.

3.1 Assessment

In order to prohibit CDP acts based on risk situation, we define five stages of risk.

```
\begin{aligned} \textbf{data Stage} &= \texttt{Pride} \mid \texttt{Anger} \mid \texttt{Worry} \mid \texttt{Panic} \mid \texttt{Grief} \mid \texttt{Dread} \\ &\quad \textbf{deriving} \; (\texttt{Eq}, \texttt{Show}) \end{aligned}
```

We define the function analyze that determines the risk stage of a CDP.

```
\mathit{analyze}\;\mathtt{era}_0\;\mathtt{par}_0\;\mathtt{urn}_0\;\mathtt{ilk}_0\;\mathtt{jar}_0 =
   if | view vow urn<sub>0</sub> \not\equiv Nothing
          CDP liquidation in progress

ightarrow Dread
      | view cat urn_0 \not\equiv Nothing
          CDP liquidation triggered
             \to \mathtt{Grief}
      | pro < min
          CDP's collateralization below liquidation ratio

ightarrow Panic
      | view zzz jar_0 + view lax ilk_0 < era_0 |
          CDP type's price limbo exceeded limit

ightarrow Panic
      | view zzz jar_0 < era_0
          CDP type's price feed in limbo

ightarrow Worry
      \mid \mathtt{cap} > \mathit{view} \, \mathtt{hat} \, \mathtt{ilk}_0
          CDP type's debt ceiling exceeded

ightarrow Anger
      otherwise
          No problems

ightarrow Pride
   where
    CDP's collateral value in SDR:
      {\tt pro} = \textit{view} \; {\tt jam} \; {\tt urn}_0 * \textit{view} \; {\tt tag} \; {\tt jar}_0
    CDP type's total debt in SDR:
       cap = view rum ilk_0 * cast (view chi ilk_0)
    CDP's debt in SDR:
      con = view art urn_0 * cast (view chi ilk_0) * par_0
     Required collateral as per liquidation ratio:
      min = con * cast (view mat ilk_0)
```

Table 3.1: CDP acts in the five stages of risk

| Pride Anger Worry Panic Grief Dread | give 爱爱爱爱 | shut PPPP PP | lock 多多罗 一 — | wipe PPPP PP | free | draw | bite | grab | plop _ _ _ _ _ _ _ |
|--|---|--------------------|-----------------------|--------------------|------|------|------|------|---|
| | decrease risk increase risk unwind risk allowed for anyone allowed for owner unconditionally allowed for owner if able to repay allowed for owner if collateralized allowed for settler contract | | | | | | | sk | |

Now we define the internal act feel which returns the value of *analyze* after ensuring that the system state is updated.

```
\begin{split} &\text{feel } id_{\text{urn}} = \mathbf{do} \\ &\text{Adjust target price and target rate} \\ &\text{prod} \\ &\text{Update debt unit and unprocessed fee revenue} \\ &id_{\text{ilk}} \leftarrow look \left( \text{vat.urns.} ix id_{\text{urn.}} \text{ilk} \right) \\ &\text{drip } id_{\text{ilk}} \\ &\text{Read parameters for risk analysis} \\ &\text{era}_0 \leftarrow use \text{ era} \\ &\text{par}_0 \leftarrow use \text{ (vat.vox.par)} \\ &\text{urn}_0 \leftarrow look \left( \text{vat.urns.} ix id_{\text{urn.}} \right) \\ &\text{ilk}_0 \leftarrow look \left( \text{vat.ilks.} ix \left( view \text{ilk urn}_0 \right) \right) \\ &\text{jar}_0 \leftarrow look \left( \text{vat.gems.} ix \left( view \text{jar.ilk}_0 \right) \right) \\ &\text{Return risk stage of CDP} \\ &\textit{return} \left( analyze \text{ era}_0 \text{ par}_0 \text{ urn}_0 \text{ ilk}_0 \text{ jar}_0 \right) \end{split}
```

Acts on CDPs use feel to prohibit increasing risk when already risky, and to freeze debt and collateral during liquidation; see Table 3.1.

3.2 Lending

Any user can open one or more accounts with the system using open, specifying a self-chosen account identifier and a CDP type.

```
open id_{	ext{urn}} \ id_{	ext{ilk}} = 	ext{do}

Fail if account identifier is taken none (vat.urns.ix id_{	ext{urn}})

Create a CDP record with the sender as owner id_{	ext{lad}} \leftarrow use \ sender

initialize \ (vat.urns.atid_{	ext{urn}}) \ (emptyUrnid_{	ext{ilk}} \ id_{	ext{lad}})
```

The owner of a CDP can transfer its ownership at any time using give.

```
give id_{	ext{urn}} \ id_{	ext{lad}} = 	ext{do}

Fail if sender is not the CDP owner id_{sender} \leftarrow use \ sender

owns \ id_{	ext{urn}} \ id_{sender}

Transfer ownership

vat . urns . ix \ id_{	ext{urn}} . lad := id_{	ext{lad}}
```

Unless liquidation has been triggered for a CDP, its owner can use lock to deposit more collateral.

```
lock id_{\text{urn}} wad_{\text{gem}} = do

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender owns id_{\text{urn}} id_{\text{lad}}

Fail if liquidation triggered or initiated want (feel id_{\text{urn}}) (\notin [Grief, Dread])

Identify collateral type id_{\text{ilk}} \leftarrow look (vat.urns.ix \ id_{\text{urn}}.ilk) id_{\text{gem}} \leftarrow look (vat.ilks.ix \ id_{\text{ilk}}.jar)

Transfer tokens from owner to collateral vault pull id_{\text{gem}} \ id_{\text{lad}} wad<sub>gem</sub>

Record an increase in collateral increase (vat.urns.ix \ id_{\text{urn}}.jam) wad<sub>gem</sub>
```

When a CDP has no risk problems (except that its CDP type's debt ceiling may be exceeded), its owner can use free to withdraw some amount of collateral, as long as the withdrawal would not reduce collateralization below the liquidation ratio.

```
\begin{aligned} & \text{free } id_{\text{urn}} \text{ wad}_{\text{gem}} = \textbf{do} \\ & \text{Fail if sender is not the CDP owner} \\ & id_{\text{lad}} \leftarrow \textit{use sender} \\ & \textit{owns } id_{\text{urn}} \ id_{\text{lad}} \\ & \text{Record a decrease in collateral} \\ & \textit{decrease} \ (\text{vat.urns.} ix \ id_{\text{urn}} \cdot \text{jam}) \ \text{wad}_{\text{gem}} \\ & \text{Roll back on any risk problem except debt ceiling excess} \\ & \textit{want} \ (\text{feel} \ id_{\text{urn}}) \ (\in [\text{Pride}, \text{Anger}]) \\ & \text{Transfer tokens from collateral vault to owner} \\ & \textit{id}_{\text{ilk}} \leftarrow \textit{look} \ (\text{vat.urns.} ix \ id_{\text{urn}} \cdot \text{ilk}) \\ & \textit{id}_{\text{gem}} \leftarrow \textit{look} \ (\text{vat.ilks.} ix \ id_{\text{ilk}} \cdot \text{jar}) \\ & \text{push} \ id_{\text{gem}} \ id_{\text{lad}} \ \text{wad}_{\text{gem}} \end{aligned}
```

When a CDP has no risk problems, its owner can use draw to take out a loan of newly minted dai, as long as the CDP type's debt ceiling is not reached and the loan would not result in undercollateralization.

```
\mathtt{draw}\;id_{\mathtt{urn}}\;\mathtt{wad}_{\mathtt{DAI}}=\mathbf{do}
  Fail if sender is not the CDP owner
    id_{lad} \leftarrow use sender
   owns \ id_{\tt urn} \ id_{\tt lad}
  Update debt unit and unprocessed fee revenue
    id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
   \mathtt{chi}_1 \leftarrow \mathtt{drip} \; \mathit{id}_{\mathtt{ilk}}
  Denominate loan in debt unit
   let wad_{chi} = wad_{DAI} / cast chi_1
  Increase CDP debt
    increase (vat . urns . ix idurn . art) wadchi
  Increase total debt of CDP type
    increase (vat . ilks . ix idilk . rum) wadchi
  Roll back on any risk problem
    want (feel id_{urn}) (\equiv Pride)
  Mint dai and transfer to CDP owner
   mint id_{DAI} wad_{DAI}
   push id_{DAI} id_{lad} wad<sub>DAI</sub>
```

Note: As of March 13, this should use the Jug instead.

A CDP owner who has previously loaned dai can use wipe to repay part of their debt as long as liquidation has not been triggered.

```
wipe id_{\text{urn}} wad<sub>DAI</sub> = \mathbf{do}

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender

owns \ id_{\text{urn}} \ id_{\text{lad}}
```

```
Fail if liquidation triggered or initiated want (feel id_{urn}) (\notin [Grief, Dread])

Update debt unit and unprocessed fee revenue id_{ilk} \leftarrow look (vat.urns.ix id_{urn}.ilk) chi_1 \leftarrow drip id_{ilk}

Denominate dai amount in debt unit let wad_{chi} = wad_{DAI} / cast chi_1

Decrease CDP debt decrease (vat.urns.ix id_{urn}.art) wad_{chi}

Decrease total CDP type debt decrease (vat.ilks.ix id_{ilk}.rum) wad_{chi}

Transfer dai from CDP owner to dai vault pull id_{DAI} id_{lad} wad_{DAI}

Destroy reclaimed dai burn id_{DAI} wad_{DAI}
```

A CDP owner can use shut to close their account—repaying all debt and reclaiming all collateral—if the price feed is up to date and liquidation has not been initiated.

```
\begin{array}{l} \text{shut } id_{\text{urn}} = \mathbf{do} \\ \text{Update debt unit and unprocessed fee revenue} \\ id_{\text{ilk}} \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn}} . \ \text{ilk} \right) \\ \text{chi}_1 \leftarrow \text{drip } id_{\text{ilk}} \\ \text{Reclaim all outstanding dai} \\ \text{art}_0 \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn}} . \ \text{art} \right) \\ \text{wipe } id_{\text{urn}} \left( \text{art}_0 * cast \ \text{chi}_1 \right) \\ \text{Reclaim all collateral} \\ \text{jam}_0 \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn}} . \ \text{jam} \right) \\ \text{free } id_{\text{urn}} \ \text{jam}_0 \\ \text{Nullify coprecord} \\ \text{vat.urns.} at \ id_{\text{urn}} := \text{Nothing} \end{array}
```

3.3 Adjustment

The feedback mechanism is updated through prod, which can be invoked at any time by keepers, but is also invoked as a side effect of any CDP act that uses feel to assess the CDP risk.

```
prod = do
 Read all parameters relevant for feedback mechanism
   era_0 \leftarrow use era
   tau_0 \leftarrow \textit{use} (vat.vox.tau)
   \mathtt{wut}_0 \leftarrow \mathit{use} \ (\mathtt{vat.vox.wut})
   par_0 \leftarrow use (vat.vox.par)
   \mathsf{how}_0 \leftarrow \mathit{use} \; (\mathsf{vat.vox.how})
   \texttt{way}_0 \leftarrow \textit{use} \; (\texttt{vat.vox.way})
   let
     Time difference in seconds
       age = era_0 - tau_0
     Current target rate applied to target price
      par_1 = par_0 * cast (way_0 \uparrow \uparrow age)
     Sensitivity parameter applied over time
       wag = how_0 * fromIntegral age
     Target rate scaled up or down
      way_1 = inj (prj way_0 +
                        if wut_0 < par_0 then wag else - wag)
 Update target price
   \mathtt{vat}.\mathtt{vox}.\mathtt{par} := \mathtt{par}_1
 Update rate of price change
   \mathtt{vat}.\mathtt{vox}.\mathtt{way} := \mathtt{way}_1
 Record time of update
   \mathtt{vat.vox.tau} := \mathtt{era}_0
   where
     Convert between multiplicative and additive form
      prj \ x = if \ x \geqslant 1 \ then \ x - 1 \ else \ 1 - 1 / x
      inj x = \text{if } x \ge 0 \text{ then } x + 1 \text{ else } 1 / (1 - x)
```

The stability fee of a CDP type can change through governance. Due to the constraint that acts should run in constant time, the system cannot iterate over CDP records to effect such changes. Instead each CDP type has a single "debt unit" which accumulates the stability fee. The drip act updates this unit. It can be called at any time by keepers, but is also called as a side effect of every act that uses feel to assess CDP risk.

```
\mathtt{drip}\ id_{\mathtt{ilk}} = \mathtt{do}
 Time stamp of previous drip
   \mathtt{rho}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathtt{rho})
 Current stability fee
   \mathtt{tax}_0 \leftarrow look \ (\mathtt{vat.ilks.} \ \mathit{ix} \ \mathit{id}_{\mathtt{ilk}} \ . \ \mathtt{tax})
 Current debt unit value
    \mathsf{chi}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathtt{chi})
 Current total debt in debt unit
   \mathtt{rum}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, \mathit{ix} \, \mathit{id}_{\mathtt{ilk}} \, . \, \mathtt{rum})
 Current unprocessed stability fee revenue
   \mathtt{joy}_0 \leftarrow look \, (\mathtt{vat.joy})
 Current time stamp
   era_0 \leftarrow use era
   let
     Time difference in seconds
        age = era_0 - rho_0
      Value of debt unit increased according to stability fee
        \mathtt{chi}_1 = \mathtt{chi}_0 * \mathtt{tax}_0 \uparrow \uparrow \mathit{age}
      Denominate stability fee revenue in new unit
        joy_1 = joy_0 + (cast (chi_1 - chi_0) :: Wad) * rum_0
  Record time of update
   \mathtt{vat.ilks.} ix id_{\mathtt{ilk}}.\mathtt{rho} := \mathtt{era}_0
  Record new debt unit
   \mathtt{vat.ilks.} ix id_{\mathtt{ilk}}.\mathtt{chi} := \mathtt{chi}_1
  Record fee revenue denominated in new debt unit
   vat.joy := joy_1
  Return the new debt unit
    return chi_1
```

3.4 Price feed input

The mark act records a new market price of a collateral token along with the expiration date of this price.

The tell act records a new market price of the DAI token along with the expiration date of this price.

```
\mathtt{tell} \ \mathtt{wad}_{\mathtt{gem}} = \mathtt{auth} \ \$ \ \mathbf{do} \ \mathtt{vat} \ . \ \mathtt{vox} \ . \ \mathtt{wut} := \mathtt{wad}_{\mathtt{gem}}
```

3.5 Liquidation

When a CDP's risk stage marks it as in need of liquidation, any account can invoke the bite act to trigger the liquidation process. This records the CDP's debt as bad debt in the CDP engine and enables the settler contract to later grab the collateral and begin auctioning.

```
bite id_{\tt urn} = {\bf do}
       Fail if CDP is not in the appropriate risk stage
         want (feel id_{urn}) (\equiv Panic)
       Record the sender as the liquidation initiator
         id_{cat} \leftarrow use sender
         \mathtt{vat}.\,\mathtt{urns}.\,\mathit{ix}\,\mathit{id}_{\mathtt{urn}}.\,\mathtt{cat} := \mathtt{Just}\,\mathit{id}_{\mathtt{cat}}
       Update debt unit
         id_{\texttt{ilk}} \leftarrow look \, (\texttt{vat.urns.} \, ix \, id_{\texttt{urn.}} \, . \, \texttt{ilk})
         \mathtt{chi}_1 \leftarrow \mathtt{drip}\ id_{\mathtt{ilk}}
       Apply liquidation penalty to debt
        \mathtt{art}_0 \leftarrow \mathit{look} \, (\mathtt{vat.urns.} \, \mathit{ix} \, \mathit{id}_{\mathtt{urn}} \, . \, \mathtt{art})
        axe_0 \leftarrow look (vat.ilks.ix id_{ilk}.axe)
        let art_1 = art_0 * cast axe_0
       Update CDP debt
        \mathtt{vat}.\mathtt{urns}.\mathit{ix}\:\mathit{id}_{\mathtt{urn}}.\mathtt{art}:=\mathtt{art}_1
       Record as bad debt
         increase (vat.sin) (art_1 * cast chi_1)
```

After liquidation has been triggered, the designated settler contract invokes grab to receive the collateral tokens.

```
\begin{array}{l} \texttt{grab} \ id_{\texttt{urn}} = \texttt{auth} \ \$ \ \textbf{do} \\ \\ \texttt{Fail} \ if \texttt{CDP} \ is \ \mathsf{not} \ \mathsf{marked} \ \mathsf{for} \ \mathsf{liquidation} \\ want \ (\texttt{feel} \ id_{\texttt{urn}}) \ (\equiv \texttt{Grief}) \\ \\ \texttt{Record} \ \mathsf{the} \ \mathsf{sender} \ \mathsf{as} \ \mathsf{the} \ \mathsf{CDP} \ \mathsf{'s} \ \mathsf{settler} \\ id_{\texttt{vow}} \leftarrow \mathit{use} \ \mathit{sender} \\ \\ \texttt{vat} \ . \ \mathsf{urns} \ . \ \mathit{ix} \ \mathit{id}_{\texttt{urn}} \ . \ \mathsf{vow} := \texttt{Just} \ \mathit{id}_{\texttt{vow}} \end{array}
```

```
Forget the cop's requester of liquidation vat . urns . ix\ id_{\text{urn}} . cat := Nothing Transfer the tokens to the settler \mathtt{jam}_0 \leftarrow look\ (\mathtt{vat}\ . \mathtt{urns}\ . ix\ id_{\mathtt{urn}}\ . \mathtt{jam}) id_{\mathtt{ilk}} \leftarrow look\ (\mathtt{vat}\ . \mathtt{urns}\ . ix\ id_{\mathtt{urn}}\ . \mathtt{ilk}) id_{\mathtt{gem}} \leftarrow look\ (\mathtt{vat}\ . \mathtt{ilks}\ . ix\ id_{\mathtt{ilk}}\ . \mathtt{jar}) push id_{\mathtt{gem}}\ id_{\mathtt{vow}}\ \mathtt{jam}_0
```

When the settler has finished the process of liquidating a CDP's collateral, it invokes plop on the CDP to give back any excess collateral gains.

```
\begin{aligned} & \text{plop } id_{\text{urn}} \text{ wad}_{\text{DAI}} = \text{auth } \$ \text{ do} \\ & \text{Fail unless CDP is in liquidation} \\ & want \text{ (feel } id_{\text{urn}} \text{ )} (\equiv \text{Dread}) \end{aligned} & \text{Forget the CDP's settler} \\ & \text{vat.urns.} ix id_{\text{urn.}} \text{ vow} := \text{Nothing} \\ & \text{Return some amount of excess auction gains} \\ & id_{\text{vow}} \leftarrow use \ sender \\ & id_{\text{ilk}} \leftarrow look \text{ (vat.urns.} ix id_{\text{urn.}} \text{ ilk)} \\ & id_{\text{gem}} \leftarrow look \text{ (vat.ilks.} ix id_{\text{ilk.}} \text{ jar)} \\ & \text{pull } id_{\text{gem}} \ id_{\text{vow}} \text{ wad}_{\text{DAI}} \end{aligned} & \text{Record the gains as the CDP's collateral} \\ & \text{vat.urns.} ix id_{\text{urn.}} \text{ jam := wad}_{\text{DAI}} \end{aligned}
```

The Vow can invoke loot at any time to have the stability fee revenue accrued so far sent to it. Note: As of March 13, this should use the Jug instead.

```
\mathtt{loot}\ \mathtt{wad}_{\mathtt{DAI}} = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat} . \mathtt{joy} := 0
```

3.6 Governance

Governance uses form to create a new CDP type. Since the new type is initialized with a zero debt ceiling, a separate transaction can safely set the risk parameters before any lending occurs.

```
form id_{ilk} id_{gem} = auth \$ do

initialize (vat.ilks.at id_{ilk}) (defaultIlk id_{gem})
```

Governance uses frob to alter the sensitivity factor, which is the only mutable parameter of the feedback mechanism.

```
\mathtt{frob}\ \mathtt{how}_1 = \mathtt{auth}\ \$\ \mathbf{do}\ \mathtt{vat}\ .\ \mathtt{vox}\ .\ \mathtt{how} := \mathtt{how}_1
```

Governance can alter the five risk parameters of a CDP type using cuff for the liquidation ratio; chop for the liquidation penalty; cork for the debt ceiling; calm for the duration of price limbo; and crop for the stability fee.

```
cuff id_{ilk} \max_1 = \text{auth } \$ \text{ do vat.ilks.} ix id_{ilk} . \max := \max_1 \text{ chop } id_{ilk} \text{ axe}_1 = \text{auth } \$ \text{ do vat.ilks.} ix id_{ilk} . \text{ axe} := \text{axe}_1
```

```
 \begin{array}{l} \texttt{cork} \ id_{\texttt{ilk}} \ \texttt{hat}_1 = \texttt{auth} \ \$ \ \mathbf{do} \ \texttt{vat} \ . \ \texttt{ilks} \ . \ ix \ id_{\texttt{ilk}} \ . \ \texttt{hat} := \texttt{hat}_1 \\ \texttt{calm} \ id_{\texttt{ilk}} \ lax1 = \texttt{auth} \ \$ \ \mathbf{do} \ \texttt{vat} \ . \ \texttt{ilks} \ . \ ix \ id_{\texttt{ilk}} \ . \ \texttt{lax} := lax1 \\ \end{array}
```

When altering the stability fee with crop, we ensure that the previous stability fee has been accounted for in the internal debt unit.

```
\begin{split} & \text{crop } id_{\mathtt{ilk}} \ \mathtt{tax}_1 = \\ & \text{auth } \$ \ \mathbf{do} \\ & \text{Apply the current stability fee to the internal debt unit} \\ & \text{drip } id_{\mathtt{ilk}} \\ & \text{Change the stability fee} \\ & \text{vat.ilks.} \ ix \ id_{\mathtt{ilk}} \ . \ \mathtt{tax} := \mathtt{tax}_1 \end{split}
```

3.7 Vaults

The internal act pull transfers tokens into a vault. It is used by lock to acquire collateral from a CDP owner; by wipe to acquire dai from a CDP owner; and by plop to acquire collateral from the settler contract.

```
\begin{aligned} & \text{pull } id_{\text{gem}} \ id_{\text{lad}} \ \text{wad}_{\text{gem}} = \\ & \textit{transfer } id_{\text{gem}} \ \text{wad}_{\text{gem}} \ (\text{InAccount } id_{\text{lad}}) \ (\text{InVault } id_{\text{gem}}) \end{aligned}
```

The internal act push transfers tokens out from a collateral vault. It is used by draw to send dai to a CDP owner; by free to send collateral to a CDP owner; and by grab to send collateral to the settler contract.

```
\begin{array}{l} {\rm push} \ id_{\rm gem} \ id_{\rm lad} \ {\rm wad}_{\rm gem} = \\ {\it transfer} \ id_{\rm gem} \ {\rm wad}_{\rm gem} \ ({\rm InVault} \ id_{\rm gem}) \ ({\rm InAccount} \ id_{\rm lad}) \end{array}
```

3.8 Token manipulation

We model the ERC20 transfer function in simplified form (omitting the concept of "allowance").

```
 \begin{array}{l} \textit{transfer } \textit{id}_{\texttt{gem}} \; \texttt{wad} \; \textit{src } \textit{dst} = \\ \\ \textit{Operate in the token's balance table} \\ \textit{zoom} \; (\texttt{vat} \; . \; \texttt{gems} \; . \; \textit{ix} \; \textit{id}_{\texttt{gem}} \; . \; \texttt{ERC20} \; . \; \textit{balanceOf}) \; \$ \; \textbf{do} \\ \textit{Fail if source balance insufficient} \\ \textit{balance} \; \leftarrow \; \textit{look} \; (\textit{ix src}) \\ \textit{aver} \; (\textit{balance} \geqslant \; \texttt{wad}) \\ \textit{Update balances} \\ \textit{decrease} \; (\textit{ix src}) \; \texttt{wad} \\ \textit{initialize} \; (\textit{at dst}) \; 0 \\ \textit{increase} \; (\textit{ix dst}) \; \texttt{wad} \\ \end{aligned}
```

The internal act mint inflates the supply of a token. It is used by draw to create new DAI, and by settlers to create new MKR.

```
\begin{aligned} & \texttt{mint} \ id_{\texttt{gem}} \ \texttt{wad}_0 = \\ & \textit{zoom} \ (\texttt{vat} \ . \ \texttt{gems} \ . \ ix \ id_{\texttt{gem}} \ . \ \texttt{ERC20}) \ \$ \ \ \mathbf{do} \\ & \textit{increase} \ (\textit{balanceOf} \ . \ ix \ (\texttt{InVault} \ id_{\texttt{gem}})) \ \texttt{wad}_0 \end{aligned}
```

The internal act burn deflates the supply of a token. It is used by wipe to destroy $dalgamma_{RR}$, and by settlers to destroy $dalgamma_{RR}$.

```
\begin{aligned} \text{burn } id_{\text{gem}} \text{ wad}_0 &= \\ zoom \text{ (vat.gems.} ix \text{ } id_{\text{gem}} \text{.ERC20)} \text{ } \$ \text{ } \mathbf{do} \\ decrease \text{ } (balanceOf.ix \text{ (InVault.} id_{\text{gem}})) \text{ wad}_0 \end{aligned}
```

Chapter 4

Act framework

The reader does not need any abstract understanding of monads to understand the code. They give us a nice syntax—the **do** block notation—for expressing exceptions and state in a way that is still purely functional. Each line of such a block is interpreted by the monad to provide the semantics we want.

4.1 The Maker monad

This defines the Maker monad as a simple composition of a state monad and an error monad:

```
type Maker a = \text{StateT System (Except Error) } a
```

We divide act failure modes into general assertion failures and authentication failures.

```
data Error = AssertError Act | AuthError
deriving (Show, Eq)
```

An act can be executed on a given initial system state using *exec*. The result is either an error or a new state. The *exec* function can also accept a sequence of acts, which will be interpreted as a single transaction.

```
exec :: System \rightarrow Maker () \rightarrow Either Error System

exec sys m = runExcept (execStateT m sys)
```

4.2 Asserting

We now define a set of functions that fail unless some condition holds.

```
General assertion aver x = unless\ x\ (throwError\ (AssertError\ ?act)) Assert that an indexed value is not present none\ x = preuse\ x >\!\!\!> \lambda case
```

```
Nothing \rightarrow return ()
 \text{Just} \_ \rightarrow \text{throwError} \text{ (AssertError ?act)} 
 \text{Assert that an indexed value is present} 
 \text{look } f = \text{preuse } f \gg \lambda \mathbf{case} 
 \text{Nothing} \rightarrow \text{throwError} \text{ (AssertError ?act)} 
 \text{Just } x \rightarrow \text{return } x 
 \text{Execute an act and assert a condition on its result} 
 \text{want } m p = m \gg \text{(aver . p)}
```

We define $\mathit{owns}\ id_{\mathtt{urn}}\ id_{\mathtt{lad}}$ as an assertion that the given CDP is owned by the given account.

```
\begin{aligned} \textit{owns } \textit{id}_{\texttt{urn}} \; \textit{id}_{\texttt{lad}} &= \mathbf{do} \\ \textit{want} \; (\textit{look} \; (\texttt{vat.urns.ix} \; \textit{id}_{\texttt{urn}} \, . \, \texttt{lad})) \; (\equiv \textit{id}_{\texttt{lad}}) \end{aligned}
```

We define auth k as an act modifier that executes k only if the sender is authorized.

```
\begin{array}{l} \text{auth } \textit{continue} = \textbf{do} \\ \textit{s} \leftarrow \textit{use sender} \\ \textit{unless} \left( \textit{s} \equiv \textit{id}_{\textit{god}} \right) \left( \textit{throwError} \, \text{AuthError} \right) \\ \textit{continue} \end{array}
```

Appendix A

Prelude

This module reexports symbols from other packages and exports a few new symbols of its own.

```
module Maker.Prelude (module Maker.Prelude, module X) where
import Prelude as X (
 Conversions to and from strings
  Read (..), Show (..),
 Comparisons
  Eq(..), Ord(..),
 Core abstractions
  Functor
                (fmap),
  Applicative (),
                (return, (\gg)),
  Monad
 Numeric classes
  Num (..), Integral (), Enum (),
 Numeric conversions
  Real (..), Fractional (..),
  RealFrac (..),
  from Integral,\\
 Simple types
  Integer, Int, String,
 Algebraic types
  Bool (True, False),
  Maybe (Just, Nothing),
  Either (Right, Left),
 Functional operators
  (.), (\$),
 Numeric operators
  (+), (-), (*), (/), (\uparrow), (\uparrow\uparrow), div,
 Utilities
  all, \neg, elem,
 Constants
```

```
mempty, \perp, otherwise)
```

We use a typical composition of monad transformers from the mtl library to structure stateful actions. See section 4.1 (*The Maker monad*).

```
import Control.Monad.State as X (
  StateT,
                  Type constructor that adds state to a monad type
  execStateT,
                  Runs a state monad with given initial state
  get,
                  Gets the state in a do block
  put)
                  Sets the state in a do block
import Control.Monad.Writer as X (
  WriterT,
                  Type constructor that adds logging to a monad type
  Writer.
                  Type constructor of logging monads
  runWriterT.
                  Runs a writer monad transformer
  execWriterT, Runs a writer monad transformer keeping only logs
  execWriter)
                  Runs a writer monad keeping only logs
import Control.Monad.Except as X (
  MonadError, Type class of monads that fail
  Except,
                  Type constructor of failing monads
  throwError,
                  Short-circuits the monadic computation
  runExcept)
                  Runs a failing monad
```

Our numeric types use decimal fixed-point arithmetic.

```
\begin{array}{ll} \textbf{import Data.Fixed } \textit{as X (} \\ \textbf{Fixed (..),} & \textbf{Type constructor for numbers of given precision} \\ \textbf{HasResolution (..))} & \textbf{Type class for specifying precisions} \end{array}
```

We rely on the lens library for accessing nested values. There is no need to understand the theory behind lenses to understand this program. The notation $a \cdot b \cdot c$ denotes a nested accessor much like a.b.c in C-style languages; for more details, consult lens documentation¹.

```
import Control.Lens as X (
  Lens', lens,
  makeLenses,
                   Defines lenses for record fields
  makeFields,
                   Defines lenses for record fields
  set,
                   Writes a lens
  use, preuse,
                   Reads a lens from a state value
  view,
                   Reads a lens from a value
  ix,
                   Lens for map retrieval and updating
  at.
                   Lens for map insertion
 Operators for partial state updates in do blocks:
  (:=),
                   Replace
  (-=), (+=), Update arithmetically
  (\% =),
                   Update according to function
  (?=))
                   Insert into map
import Control.Lens.Zoom as X (zoom)
```

¹Gabriel Gonzalez's 2013 article *Program imperatively using Haskell* is a good introduction.

Where the Solidity code uses mapping, we use Haskell's regular tree-based map type².

Finally we define some of our own convenience functions.

```
decrease a \ x = a -= x

increase a \ x = a += x

initialize a \ x = a \% = (\lambda \mathbf{case} \ \mathrm{Nothing} \to \mathrm{Just} \ x; \ y \to y)

prepend a \ x = a \% = (x:)

x \notin xs = \neg \ (\mathrm{elem} \ x \ xs)
```

²We assume the axiom that Keccak hash collisions are impossible.

Appendix B

Fixed point numbers with rounding

This somewhat arcane-looking code implements a wrapper around the base library's decimal fixed point type, only with x * y and x / y operations that do rounding instead of truncation of their intermediate results.

```
module Maker.Decimal (Decimal, E18, E36, Epsilon (...)) where import Data.Fixed newtype HasResolution e \Rightarrow Decimal e = D (Fixed e) deriving (Ord, Eq, Real, RealFrac)
```

We want the printed representations of these numbers to look like "0.01" and not "R 0.01".

```
instance HasResolution e \Rightarrow \text{Read} (\text{Decimal } e) where readsPrec \ n \ s = fmap \ (\lambda(x,y) \to (\text{D} \ x,y)) \ (readsPrec \ n \ s) instance HasResolution e \Rightarrow \text{Show} \ (\text{Decimal } e) where show \ (\text{D} \ x) = show \ x
```

In the Num instance, we delegate everything except multiplication.

```
instance HasResolution e \Rightarrow \text{Num (Decimal } e) where x@(D \text{ (MkFixed } a)) * D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a*b+div (resolution x) 2) (resolution x)))}
D \text{ } a + D \text{ } b = D \text{ } (a+b)
D \text{ } a - D \text{ } b = D \text{ } (a-b)
negate \text{ } (D \text{ } a) = D \text{ } (negate \text{ } a)
abs \text{ } (D \text{ } a) = D \text{ } (abs \text{ } a)
signum \text{ } (D \text{ } a) = D \text{ } (signum \text{ } a)
fromInteger \text{ } i = D \text{ } (fromInteger \text{ } i)
```

In the Fractional instance, we delegate everything except division.

```
instance HasResolution e \Rightarrow Fractional (Decimal e) where x@(D \text{ (MkFixed } a)) / D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a * resolution <math>x + div b 2) b))
```

```
recip (D a) = D (recip a)

fromRational r = D (fromRational r)
```

We define the E18 and E36 symbols and their fixed point multipliers.

```
data E18; data E36 instance HasResolution E18 where resolution \_=10 \uparrow (18:: Integer) instance HasResolution E36 where resolution \_=10 \uparrow (36:: Integer)
```

The fixed point number types have well-defined smallest increments (denoted ϵ). This becomes useful when verifying equivalences.

```
class Epsilon t where \epsilon::t instance HasResolution a\Rightarrow Epsilon (Decimal a) where The use of \bot is safe since resolution ignores the value. \epsilon=1 \ / \ from Integral \ (resolution \ (\bot:: Fixed \ a))
```