

presents the

REFERENCE IMPLEMENTATION

also known as the PURPLE PAPER

of the remarkable

DAI CREDIT SYSTEM

issuing a diversely collateralized stablecoin

formulated by

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Contents

1	Introduction							
	1.1	Naming	6					
	1.2	Motivation	6					
	1.3	Limitations	7					
	1.4	Verification	8					
Ι	Imp	plementation	9					
2	Prea	amble	10					
3	Тур	es	11					
	3.1	Numeric types	11					
	3.2	Identifiers and addresses	12					
	3.3	Jar — collateral vault	12					
		gem — collateral token	12					
		tag — market price of token	12					
		zzz — expiration time of token price feed	12					
	3.4	Gem — token model	13					
	3.5	Ilk — CDP type	13					
		jar — collateral token vault	13					
		mat — liquidation ratio	13					
		axe — liquidation penalty	13					
		hat — debt ceiling	13					
		tax — stability fee	13					
		lax — price feed limbo duration	13					
		rho - time of debt unit adjustment	13					
		rum — total outstanding debt units	13					
		chi — value of debt unit in DAI	13					
	3.6	Urn — collateralized debt position (CDP)	14					

		cat — address of liquidation initiator	14
		lad — CDP owner	14
		ilk — CDP type	14
		art — debt denominated in debt unit	14
		ink — collateral denominated in debt unit	14
	3.7	Vox — feedback mechanism data	14
		wut — market price of DAI denominated in SDR	14
		par — target price of DAI denominated in SDR	14
		how — sensitivity parameter	14
		way — rate of target price change	14
		tau — time of latest feedback cycle	14
	3.8	Vat — CDP engine aggregate	15
	3.9	System model	15
		era — current time	15
	3.10	Default data	16
4	Acts		18
	4.1	Assessment	19
		feel — identify CDP risk stage	19
	4.2	Lending	21
		open — create CDP	21
		give — transfer CDP account	21
		lock — deposit collateral	21
		free — withdraw collateral	22
		draw — issue dai as debt	22
		wipe — repay debt and burn dai	23
		shut — wipe, free, and delete CDP	23
	4.3	Adjustment	25
		prod — adjust target price and target rate	25
		drip — update debt unit and unprocessed fee revenue	26
	4.4	Price feed input	27
		$\mathtt{mark}-\mathtt{update}$ market price of collateral token	27
		tell — update market price of dai	27
	4.5	Liquidation	27
		bite — mark for liquidation	27
		grab — take tokens for liquidation	28
		plop — finish liquidation returning profit	28
		loot — take unprocessed stability fees	29
	4.6	Auctioning	29
		flip — put collateral up for auction	29
		flan — put fee revenue up for auction	20

В	Fixed point numbers with rounding							
A	Prelu	ıde	35					
	5.2	Asserting	34					
	5.1	The Maker monad	33					
5	Act framework							
		mend — burn dai and debt token	32					
		lend — mint dai and debt token	32					
		burn — deflate token	32					
		$\verb mint-inflate token $	32					
	4.10	Token manipulation	31					
		push — transfer tokens from vault	31					
		pull — transfer tokens to vault	31					
	4.9	Vaults	31					
		crop — set stability fee	31					
		cuff — set liquidation ratio	30					
		calm — set limbo duration	30					
		cork — set debt ceiling	30					
		chop — set liquidation penalty	30					
		frob — set the sensitivity parameter	30					
	1.0	form — create a new CDP type	30					
	4.8	Governance	30					
		kick — flap, flop, and whatnot	30					
	4./	tidy — burn equal quantities of DAI and SIN	30					
	4.7	Settlement	30					
		flop — put MKR up for auction	29					

Chapter 1

Introduction

The dai credit system, henceforth also "Maker," is a network of Ethereum contracts designed to issue the dai currency token and automatically adjust credit incentives in order to keep its market value stable relative to SDR¹ in the short and medium term.

New dai enters the money supply when a borrower locks an excess of collateral in the system and takes out a loan. The debt and collateral amounts are recorded in a *collateralized debt position*, or CDP. Thus all outstanding dai represents some CDP owner's claim on their collateral—until risk provokes a liquidation.

Off-chain *price feeds* give Maker knowledge of the market values of dai and the various tokens used as collateral, enabling the system to assess credit risk. If the value of a CDP's collateral drops below a certain multiple of its debt, a decentralized auction is triggered which liquidates the collateral for dai in order to settle the debt.

The system issues a separate token with symbol MKR. Since collateral auctions may fail to recover the full value of liquidated debt, the MKR token can be diluted to back emergency debt. The value of MKR, though volatile by design, is backed by the revenue from *stability fees* imposed on all dai loans. The DAI raised from stability fees is used to buy MKR tokens from the market and destroy them.

For more details on the economics of the system, as well as descriptions of governance, off-chain mechanisms that provide efficiency, and so on, see the whitepaper.

This document is an executable technical specification of the of the Maker smart contracts. It is a draft; be aware that the contents will certainly change before launch.

¹ "Special Drawing Rights" (ticker symbol xdr), the international reserve asset created by the International Monetary Fund, whose value is derived from a weighted basket of world currencies.

1.1 Naming

The implementation is formulated in terms of a parallel vocabulary whose concise words can seem meaningless at first glance (e.g., Urn, par, ink). These words are in fact carefully selected for metaphoric resonance and evocative qualities. Definitions of the words along with mnemonic reminders can be found in the glossary.

We have found that though it requires some initial indoctrination, the Maker jargon is good for development and helps when thinking and talking about the structure and mechanics of the system. Here are some of the reasons:

- The parallel jargon lets us sidestep terminological debates; for example, whether to say "rate of target price change" or "target rate."
- With decoupled financial and technical vocabularies, we can more flexibly improve one without affecting the other.
- The ability to discuss the system formally, with the financial interpretation partly suspended, has suggested insights that would have been harder to think of inside the normal language.
- The precise and distinctive language makes the structure and logic of the implementation more apparent and easier to formalize.

Some readers may perceive the Maker terminology as unnecessarily obscure despite our apologetics. In that case, we recommend a contrasting look at the Ethereum "yellow paper," after which this document should appear highly legible.

1.2 Motivation

The version of this system that will be deployed on the blockchain is written in Solidity, which is a workable smart contract implementation language. This reference implementation is a model of the behavior of those contracts, written as a "literate" Haskell program. The motivations for such a reference implementation include:

- 1. **Comparison**. Checking two free-standing implementations against each other is a well-known way of ensuring that they both behave as intended.
- 2. **Testing**. Haskell lets us use powerful testing tools such as QuickCheck and SmallCheck for comprehensively verifying key properties as a middle ground between unit testing and formal verification.

- 3. **Explicitness**. Coding the contract behavior in Haskell, a purely functional language, enforces explicit description of aspects which Solidity leaves implicit. For example, a Solidity program can read previously unwritten storage and get back a zero value, whereas in Haskell we must give explicit defaults. The state rollback behavior of failed actions is also explicit in the type of the execution function, which may return an error.
- 4. **Typing**. While Solidity does have a static type system, it is not expressive enough to encode the distinctions made by our system. In particular, the two different decimal fixed point number types that we use are typed in Solidity with one and the same uint128 type. In Haskell we can make this distinction explicit.
- 5. Formality. The work of translating a Solidity program into a purely functional program opens up opportunities for certain types of formal verification. In particular, this document will be useful for modelling aspects of the system in a proof assistant like Agda, Idris, Coq, or Isabelle. We can also use logical tools for Haskell, such as Liquid Haskell (which provides compile time logical property checking) and sbv (a toolkit for model checking and symbolic execution).
- 6. Clarity. An implementation not intended to be deployed on the blockchain is free from concerns about optimizing for gas cost and other factors that make the Solidity implementation less ideal as an understandable specification.
- 7. **Simulation**. Solidity is highly specific to the Ethereum blockchain environment and as such does not have facilities for interfacing with files or other computer programs. This makes the Solidity implementation of the system less useful for doing simulations of the system's economic, game-theoretic, or statistical aspects.

1.3 Limitations

This model is limited in that it has

- 1. a simplified version of authorization for governance;
- 2. a simplified version of ERC20 token semantics;
- 3. no implementation of the decentralized auction contracts; and
- 4. no 256-bit word limits.

These limitations will be addressed in future revisions.

1.4 Verification

Separately from this document, we are developing automatic test suites that generate many, large, and diverse action sequences for property verification. One such property is that the reference implementation exactly matches the on-chain implementation; this is verified through the generation of Solidity test cases with assertions covering the entire state. Other key properties include

- that the target price changes only according to the target rate;
- that the total dai supply is fully accounted for by CDP debts;
- that CDP acts are restricted with respect to risk stage;

along with similar invariants and conditions. A future revision of this document will include formal statements of these properties.

Part I Implementation

Chapter 2

Preamble

This is a Haskell program, and as such makes reference to a background of symbols defined in libraries, as a mathematical paper depends on preestablished theories.

Context should allow the reader to understand most symbols without further reading, but Appendix A lists and briefly explains each imported type and function.

We replace the default prelude module with our own.

```
module Maker where
import Prelude () Import nothing from Prelude
import Maker.Prelude Import everything from Maker Prelude
```

We also import our definition of decimal fixed point numbers, listed in Appendix B.

```
import Maker.Decimal
```

Now we proceed to define the specifics of the Maker system.

Chapter 3

Types

This chapter defines the data types used by Maker: numeric types, identifiers, on-chain records, and test model data.

Haskell syntax note: **newtype** defines a type synonym with distinct type identity; **data** creates a record type; and **deriving** creates automatic instances of common functionality.

3.1 Numeric types

The system uses two different precisions of decimal fixed point numbers, which we call *wads* and *rays*, having respectively 18 digits of precision (used for token quantities) and 36 digits (used for precise rates and ratios). See Appendix B for details on decimal fixed point numbers and rounding.

```
Define the distinct type of currency quantities

newtype Wad = Wad (Decimal E18)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)

Define the distinct type of rates and ratios

newtype Ray = Ray (Decimal E36)

deriving (Ord, Eq, Num, Real, Fractional, RealFrac)
```

We also define a type for time durations in whole seconds, as this is the maximum precision allowed by the Ethereum virtual machine.

```
newtype Sec = Sec Int
deriving (Eq, Ord, Enum, Num, Real, Integral)
```

Haskell number types are not automatically converted, so we convert explicitly with a *cast* function.

```
Convert via fractional n/m form. cast :: (Real \ a, Fractional \ b) \Rightarrow a \rightarrow b cast = fromRational. toRational
```

3.2 Identifiers and addresses

There are several kinds of identifiers used in the system, and we use types to distinguish them. The type parameter a creates distinct types; e.g., Id Foo and Id Bar are incompatible.

```
newtype Id a = \text{Id String deriving (Eq, Ord, Show)}
```

We define another type for representing Ethereum account addresses.

```
newtype Address = Address String deriving (Eq. Ord, Show)
```

We also have predefined entity identifiers.

```
The DAI token identifier id_{\mathrm{DAI}} = \mathrm{Id} "DAI" The internal debt token identifier id_{\mathrm{SIN}} = \mathrm{Id} "SIN" The MKR token identifier id_{\mathrm{MKR}} = \mathrm{Id} "MKR" A test account with ultimate authority id_{\mathrm{god}} = \mathrm{Address} "GOD"
```

3.3 Jar — collateral vault

The data received from price feeds is categorized by token and stored in Jar records. Our model also has the token balances embedded in these records; in reality¹, the balances are in separate ERC20 contracts.

¹We use "reality" to denote the actual state of the consensus Ethereum blockchain.

$3.4 \quad \text{Gem} - \text{token model}$

In reality, token semantics can differ, despite nominally following the ERC20 interface. Governance therefore involves reviewing the behaviors of collateral tokens. In our model, tokens behave in the same simple way. We also omit the notion of "allowance."

We define a Gem record as a map tracking the token quantity held by each holding entity.

```
data Gem = Gem { · balanceOf :: Map Entity Wad}
deriving (Eq, Show)
```

For clarity, we use a data type to distinguish the different entities that can hold a token balance.

```
      data Entity = Account Address
      External account or contract

      | Vault (Id Jar)
      Vault for collateral token

      | Joy
      Spawning account for dai

      | Woe
      Spawning account for debt

      | Ice
      Holding account for debt

      | Vow
      Settler

      | Toy
      Test driver

      deriving (Eq, Ord, Show)
```

3.5 Ilk — CDP type

Each CDP belongs to a CDP type, specified by an Ilk record. Five parameters, mat, axe, hat, tax and lax, are set by governance and are known as the *risk parameters*. The rest of the values are used by the system to keep track of the current state. The meaning of each ilk parameter is defined by its interactions in the act definitions of Chapter 4; see the whitepaper for an overview.

```
data Ilk = Ilk {
   · jar :: Id Jar, Collateral token identifier
   · tax :: Sec,
                      Grace period after price feed becomes unavailable
   · mat :: Ray,
                      Collateral-to-debt ratio at which liquidation can be triggered
   · axe :: Ray,
                      Penalty on liquidation as fraction of debt
   · hat :: Wad,
                      Limit on total dai debt for CDP type ("debt ceiling")

    tax :: Ray,

                      Stability fee as per-second fraction of debt value
   · chi::Ray,
                      Value of internal debt unit in dai
   · rho :: Sec,
                      Time of latest debt unit adjustment
   · rum :: Wad
                      Total debt in debt units
   } deriving (Eq, Show)
```

3.6 Urn — collateralized debt position (CDP)

For each CDP we maintain an Urn record identifying its type and specifying ownership, quantities of debt and collateral denominated in the CDP type's debt unit, along with who triggered liquidation (if applicable).

3.7 Vox — feedback mechanism data

The *feedback mechanism* is the aspect of the CDP engine that adjusts the target price of dai based on market price, and its data is kept in a singleton record called Vox.

```
data Vox = Vox {wut :: Wad, Market price of dai denominated in SDRpar :: Wad, Target price of dai denominated in SDR
```

```
way:: Ray, Current per-second change in target price
how:: Ray, Sensitivity parameter set by governance
tau:: Sec Time of latest feedback cycle
deriving (Eq, Show)
```

Keeping the feedback data separate allows us to more easily upgrade the mechanism in the future.

3.8 Vat — CDP engine aggregate

The Vat record aggregates the records of tokens, CDPs, CDP types, and price feeds, along with the data of the feedback mechanism.

3.9 System model

Finally we define a record with no direct counterpart in the Solidity contracts, which has the Vat record along with model state.

```
data System = System {
  · vat
               :: Vat,
                             Root Maker entity
  · era
               :: Sec,
                             Current time stamp
  · sender :: Entity,
                             Sender of current act
  · accounts :: [Address], For test suites
               :: Mode
  \_mode
                             Vow operation mode
  } deriving (Eq, Show)
data Mode = Dummy
  deriving (Eq, Show)
```

3.10 Default data

```
defaultIlk :: Id Jar \rightarrow Ilk
defaultIlk id_{jar} = Ilk  {
  · jar = id_{jar},
   · axe = Ray 1,
   · mat = Ray 1,
   · tax = Ray 1,
   · hat = Wad 0,
   · tax = Sec 0,
   · chi = Ray 1,
  · rum = Wad 0,
   \cdot rho = Sec 0
}
emptyUrn :: Id Ilk \rightarrow Entity \rightarrow Urn
emptyUrn id_{ilk} id_{lad} = Urn  {
  \cdot cat = Nothing,
   · lad = id_{lad},
   · ilk = id_{ilk},
   · art = Wad 0,
   \cdot \  \, \texttt{ink} = \texttt{Wad} \; 0
}
initial \mathcal{J}ar :: Id Jar \rightarrow Jar
initial \mathcal{J}ar id_{jar} = Jar 
   \cdot \text{ gem} = \text{Gem} \{ \cdot \text{ balanceOf} = singleton (Vault } id_{jar}) 0 \},
   · tag = Wad 0,
   \cdot zzz = 0
}
initialVat :: Ray \rightarrow Vat
initial Vat how_0 = Vat  {
   \cdot vox = Vox {
      · tau = 0,
      · wut = Wad 1,
      · par = Wad 1,
      · how = how<sub>0</sub>,
      · way = Ray 1
```

```
},
     · ilks = \emptyset,
     · urns = \emptyset,
     \cdot jars = fromList [
          (\mathit{id}_{\scriptscriptstyle \mathrm{DAI}}, \mathit{initial} \mathcal{J} \mathit{ar} \, \mathit{id}_{\scriptscriptstyle \mathrm{DAI}}),
          (\mathit{id}_{\scriptscriptstyle{\mathsf{SIN}}}, \mathit{initial} \mathcal{J} \mathit{ar} \; \mathit{id}_{\scriptscriptstyle{\mathsf{SIN}}}),
          (id_{MKR}, initial Far\ id_{MKR})
}
\mathit{initialSystem} :: \mathtt{Ray} \to \mathsf{System}
initialSystem\ how_0 = System\ \{
                                = initial Vat how_0,
     \cdot vat
                                = 0,
     · era
                                = Account id_{god},
     · sender
     \cdot accounts = mempty
```

Chapter 4

Acts

The *acts* are the basic state transitions of the system.

Unless specified as *internal*, acts are accessible as public functions on the blockchain.

The auth modifier marks acts which can only be invoked from addresses to which the system has granted authority.

For details on the underlying "Maker monad," which specifies how the act definitions behave with regard to state and rollback, see chapter 5.

4.1 Assessment

In order to prohibit CDP acts based on risk situation, we define five stages of risk.

```
data Stage = Pride | Anger | Worry | Panic | Grief | Dread
deriving (Eq, Show)
```

We define the function analyze that determines the risk stage of a CDP.

```
analyze \operatorname{era}_0 \operatorname{par}_0 \operatorname{urn}_0 \operatorname{ilk}_0 \operatorname{jar}_0 =
   if | view cat \mathtt{urn}_0 \not\equiv \mathtt{Nothing} \land \mathtt{view} \ \mathtt{ink} \ \mathtt{urn}_0 \equiv 0
          CDP liquidation triggered and started

ightarrow Dread
      | view cat urn<sub>0</sub> \not\equiv Nothing
          CDP liquidation triggered

ightarrow Grief
      | pro < min
          CDP's collateralization below liquidation ratio

ightarrow Panic
      | view zzz jar_0 + view lax ilk_0 < era_0
          CDP type's price limbo exceeded limit
             \to \mathtt{Panic}
      | view zzz jar_0 < era_0
          CDP type's price feed in limbo

ightarrow Worry
      | cap > view hat ilk_0
          CDP type's debt ceiling exceeded

ightarrow Anger
      otherwise
          No problems

ightarrow Pride
   where
    CDP's collateral value in SDR:
      \mathtt{pro} = \mathit{view} \; \mathtt{ink} \; \mathtt{urn}_0 * \mathit{view} \; \mathtt{tag} \; \mathtt{jar}_0
    CDP type's total debt in DAI:
       cap = view rum ilk_0 * cast (view chi ilk_0)
    CDP's debt in SDR:
      con = view art urn_0 * cast (view chi ilk_0) * par_0
    Required collateral as per liquidation ratio:
      min = con * cast (view mat ilk_0)
```

Table 4.1: CDP acts in the five stages of risk

	give	shut	lock	wipe	free	draw	bite	grab	plop
Pride	\mathcal{B}	E	&	₹	\oplus	\oplus	_	_	_
Anger	₾	F	S	E	⊕	_	_	_	_
Worry	₽		E	E	_	_	_	_	_
Panic	₽	E	E	E	_	_	(1)	_	_
Grief	₽	_	_	_	_	_	_	\bigcirc	_
Dread	₽	_	_	_	_	_	_	_	\bigcirc
		decrease risk		isk	increa	se risk	unwind risk		isk
allowed for anyone allowed for owner unconditionally allowed for owner if able to repay allowed for owner if collateralized allowed for settler contract									

Now we define the internal act feel which returns the value of *analyze* after ensuring that the system state is updated.

```
feel id_{\text{urn}} = \mathbf{do}

Adjust target price and target rate prod

Update debt unit and unprocessed fee revenue id_{\text{ilk}} \leftarrow look \; (\text{vat.urns.} \; ix \; id_{\text{urn.}} \; ilk)

drip id_{\text{ilk}}

Read parameters for risk analysis era<sub>0</sub> \leftarrow use era par<sub>0</sub> \leftarrow use (vat.vox.par) urn<sub>0</sub> \leftarrow look (vat.urns.ix id_{urn.}) ilk<sub>0</sub> \leftarrow look (vat.ilks.ix (view ilk urn<sub>0</sub>)) jar<sub>0</sub> \leftarrow look (vat.jars.ix (view jar ilk<sub>0</sub>))

Return risk stage of CDP return (analyze era<sub>0</sub> par<sub>0</sub> urn<sub>0</sub> ilk<sub>0</sub> jar<sub>0</sub>)
```

Acts on CDPs use feel to prohibit increasing risk when already risky, and to freeze debt and collateral during liquidation; see Table 4.1.

4.2 Lending

Any user can open one or more accounts with the system using open, specifying a self-chosen account identifier and a CDP type.

```
open id_{	ext{urn}} \ id_{	ext{ilk}} = 	ext{do}

Fail if account identifier is taken none (vat . urns . ix \ id_{	ext{urn}})

Create a CDP record with the sender as owner id_{	ext{lad}} \leftarrow use \ sender
initialize \ (vat . urns . \ at \ id_{	ext{urn}}) \ (emptyUrn \ id_{	ext{ilk}} \ id_{	ext{lad}})
```

The owner of a CDP can transfer its ownership at any time using give.

```
give id_{	ext{urn}} \ id_{	ext{lad}} = 	ext{do}

Fail if sender is not the CDP owner id_{sender} \leftarrow use \ sender

owns \ id_{	ext{urn}} \ id_{sender}

Transfer ownership

vat . urns . ix \ id_{	ext{urn}} . lad := id_{	ext{lad}}
```

Unless liquidation has been triggered for a CDP, its owner can use lock to deposit more collateral.

```
lock id_{\text{urn}} wad_{\text{gem}} = \mathbf{do}

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender owns id_{\text{urn}} \ id_{\text{lad}}

Fail if liquidation triggered or initiated want (feel id_{\text{urn}}) (\notin [Grief, Dread]) Identify collateral type id_{\text{ilk}} \leftarrow look (vat.urns.ix \ id_{\text{urn}}.ilk) id_{\text{jar}} \leftarrow look (vat.ilks.ix \ id_{\text{ilk}}.jar)

Transfer tokens from owner to collateral vault pull id_{\text{jar}} \ id_{\text{lad}} wad<sub>gem</sub>

Record an increase in collateral increase (vat.urns.ix \ id_{\text{urn}}.ink) wad<sub>gem</sub>
```

When a CDP has no risk problems (except that its CDP type's debt ceiling may be exceeded), its owner can use free to withdraw some amount of collateral, as long as the withdrawal would not reduce collateralization below the liquidation ratio.

```
free id_{\text{urn}} wad_{\text{gem}} = \mathbf{do}

Fail if sender is not the CDP owner id_{\text{lad}} \leftarrow use \ sender owns id_{\text{urn}} \ id_{\text{lad}}

Record a decrease in collateral decrease (vat.urns.ix \ id_{\text{urn}}.ink) wad_{\text{gem}}

Roll back on any risk problem except debt ceiling excess want (feel id_{\text{urn}}) (\in [Pride, Anger])

Transfer tokens from collateral vault to owner id_{\text{ilk}} \leftarrow look (vat.urns.ix \ id_{\text{urn}}.ilk) id_{\text{jar}} \leftarrow look (vat.ilks.ix \ id_{\text{ilk}}.jar) push id_{\text{jar}} \ id_{\text{lad}} wad_{\text{gem}}
```

When a CDP has no risk problems, its owner can can use draw to take out a loan of newly minted dai, as long as the CDP type's debt ceiling is not reached and the loan would not result in undercollateralization.

```
draw id_{urn} wad_{DAI} = do
 Fail if sender is not the CDP owner
   id_{\mathtt{lad}} \leftarrow use \ sender
   owns id<sub>urn</sub> id<sub>lad</sub>
 Update debt unit and unprocessed fee revenue
   id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
   chi_1 \leftarrow drip id_{ilk}
 Denominate loan in debt unit
   let wad_{chi} = wad_{DAI} / cast chi_1
 Increase CDP debt
   increase (vat . urns . ix idurn . art) wadchi
 Increase total debt of CDP type
   increase (vat.ilks.ix id_{ilk}.rum) wad<sub>chi</sub>
 Roll back on any risk problem
   want (feel id_{urn}) (\equiv Pride)
 Mint both dai and debt tokens
   lend wadDAI
 Transfer dai to CDP owner
```

```
transfer\ id_{\mathrm{DAI}}\ \mathtt{Wad}_{\mathrm{DAI}}\ \mathtt{Joy}\ id_{\mathtt{lad}} Transfer sin\ into\ debt\ vault transfer\ id_{\mathtt{SIN}}\ \mathtt{Wad}_{\mathrm{DAI}}\ \mathtt{Woe}\ \mathtt{Ice}
```

A CDP owner who has previously loaned dai can use wipe to repay part of their debt as long as liquidation has not been triggered.

```
wipe id_{\tt urn} \; {\tt wad}_{\tt DAI} = {f do}
 Fail if sender is not the CDP owner
    id_{1ad} \leftarrow use sender
   owns idurn idlad
 Fail if liquidation triggered or initiated
    want (feel id_{urn}) (\notin [Grief, Dread])
 Update debt unit and unprocessed fee revenue
    id_{\mathtt{ilk}} \leftarrow look \, (\mathtt{vat.urns.} \, ix \, id_{\mathtt{urn}} \, . \, \mathtt{ilk})
   chi_1 \leftarrow drip id_{ilk}
 Denominate dai amount in debt unit
   let wad_{chi} = wad_{DAI} / cast chi_1
 Decrease CDP debt
   decrease (vat . urns . ix idurn . art) wadchi
 Decrease total CDP type debt
   decrease (vat.ilks.ix id_{ilk}.rum) wad<sub>chi</sub>
 Transfer dai from CDP owner to dai vault
   pull id_{DAI} id_{lad} wad_{DAI}
 Destroy dai and corresponding debt tokens
   mend wad_{DAI}
```

A CDP owner can use shut to close their account—repaying all debt and reclaiming all collateral—if the price feed is up to date and liquidation has not been initiated.

```
\begin{array}{l} \text{shut } id_{\text{urn}} = \mathbf{do} \\ \\ \text{Update debt unit and unprocessed fee revenue} \\ id_{\text{ilk}} \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn.}} \text{ilk} \right) \\ \text{chi}_1 \leftarrow \text{drip } id_{\text{ilk}} \\ \\ \text{Reclaim all outstanding dai} \\ \\ \text{art}_0 \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn.}} \text{art} \right) \\ \\ \text{wipe } id_{\text{urn.}} \left( \text{art}_0 * cast \text{chi}_1 \right) \\ \\ \text{Reclaim all collateral} \\ \\ \text{ink}_0 \leftarrow look \left( \text{vat.urns.} ix \ id_{\text{urn.}} \text{ink} \right) \end{array}
```

free $id_{\mathtt{urn}}$ ink $_0$ Nullify CDP record $\mathtt{vat.urns.} \ at \ id_{\mathtt{urn}} := \mathbf{Nothing}$

4.3 Adjustment

The feedback mechanism is updated through prod, which can be invoked at any time by keepers, but is also invoked as a side effect of any CDP act that uses feel to assess the CDP risk.

```
prod = do
 Read all parameters relevant for feedback mechanism
   era_0 \leftarrow use era
   tau_0 \leftarrow use (vat.vox.tau)
   \mathtt{wut}_0 \leftarrow \mathit{use} \; (\mathtt{vat} \; . \; \mathtt{vox} \; . \; \mathtt{wut})
   par_0 \leftarrow use (vat.vox.par)
   how_0 \leftarrow use (vat.vox.how)
   way_0 \leftarrow use (vat.vox.way)
   let
    Time difference in seconds
      age = era_0 - tau_0
    Current target rate applied to target price
      par_1 = par_0 * cast (way_0 \uparrow \uparrow age)
    Sensitivity parameter applied over time
      wag = how_0 * fromIntegral age
    Target rate scaled up or down
      way_1 = inj (prj way_0 +
                     if wut_0 < par_0 then wag else - wag)
 Update target price
   vat.vox.par := par_1
 Update rate of price change
   vat.vox.way := way_1
 Record time of update
   vat.vox.tau := era_0
   where
    Convert between multiplicative and additive form
      prj \ x = if \ x \geqslant 1 \ then \ x - 1 \ else \ 1 - 1 / x
      inj x = if x \ge 0 then x + 1 else 1 / (1 - x)
```

The stability fee of a CDP type can change through governance. Due to the constraint that acts should run in constant time, the system cannot iterate over CDP records to effect such changes. Instead each CDP type has a single "debt unit" which accumulates the stability fee. The drip act updates this unit. It can be called at any time by keepers, but is also called as a side effect of every act that uses feel to assess CDP risk.

```
drip id_{ilk} = do
 Time stamp of previous drip
   \mathsf{rho}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathsf{rho})
 Current stability fee
   tax_0 \leftarrow look (vat.ilks.ix id_{ilk}.tax)
 Current debt unit value
   \mathsf{chi}_0 \leftarrow look \, (\mathtt{vat.ilks.} \, ix \, id_{\mathtt{ilk}} \, . \, \mathtt{chi})
 Current total debt in debt unit
   rum_0 \leftarrow look (vat.ilks.ix id_{ilk}.rum)
 Current time stamp
   era_0 \leftarrow use era
   let
     Time difference in seconds
       age = era_0 - rho_0
     Value of debt unit increased according to stability fee
       chi_1 = chi_0 * tax_0 \uparrow \uparrow age
     Stability fee revenue denominated in new unit
       dew = (cast (chi_1 - chi_0) :: Wad) * rum_0
 Mint dai and internal debt tokens for marginal stability fee
   lend dew
 Record time of update
   vat.ilks.ixid_{ilk}.rho:=era_0
 Record new debt unit
   \mathtt{vat.ilks.} ix id_{\mathtt{ilk}}.\mathtt{chi} := \mathtt{chi}_1
 Return the new debt unit
   return chi<sub>1</sub>
```

4.4 Price feed input

The mark act records a new market price of a collateral token along with the expiration date of this price.

```
mark id_{jar} tag<sub>1</sub> zzz<sub>1</sub> = auth $ do vat.jars.ix id_{jar}.tag := tag<sub>1</sub> vat.jars.ix id_{jar}.zzz := zzz<sub>1</sub>
```

The tell act records a new market price of the DAI token along with the expiration date of this price.

```
\mathtt{tell} \ \mathtt{wad}_{\mathtt{gem}} = \mathtt{auth} \ \$ \ \mathbf{do} \ \mathtt{vat} \ . \ \mathtt{vox} \ . \ \mathtt{wut} := \mathtt{wad}_{\mathtt{gem}}
```

4.5 Liquidation

When a CDP's risk stage marks it as in need of liquidation, any account can invoke the bite act to trigger the liquidation process. This enables the settler contract to grab the collateral for auctioning and take over the debt tokens representing "bad debt."

```
bite id_{	ext{urn}} = 	ext{do}

Fail if CDP is not in the appropriate risk stage want 	ext{ (feel } id_{	ext{urn}}) 	ext{ (} \equiv 	ext{Panic})

Record the sender as the liquidation initiator id_{	ext{cat}} \leftarrow use \ sender

vat . urns . ix \ id_{	ext{urn}} . cat := Just id_{	ext{cat}}

Apply liquidation penalty to debt

id_{	ext{ilk}} \leftarrow look 	ext{ (vat . urns . } ix \ id_{	ext{urn}} 	ext{ . ilk})

axe_0 \leftarrow look 	ext{ (vat . ilks . } ix \ id_{	ext{ilk}} 	ext{ . axe})

art_0 \leftarrow look 	ext{ (vat . urns . } ix \ id_{	ext{urn}} 	ext{ . art})

let \text{art}_1 = \text{art}_0 * cast \ \text{axe}_0

Update debt

vat . urns . ix \ id_{	ext{urn}} 	ext{ . art} := \text{art}_1
```

After liquidation has been triggered, the designated settler contract invokes grab to receive both the CDP's collateral tokens and the internal debt tokens corresponding to the CDP's debt.

```
grab id_{urn} = auth \$ do
     Fail if CDP is not marked for liquidation
        want (feel id_{urn}) (\equiv Grief)
     Transfer the collateral to the settler
        id_{vow} \leftarrow use sender
        id_{\texttt{ilk}} \leftarrow look (\texttt{vat.urns.} ix id_{\texttt{urn.}} \texttt{ilk})
        id_{\texttt{jar}} \leftarrow look (\texttt{vat.ilks.} ix id_{\texttt{ilk}}.\texttt{jar})
        ink_0 \leftarrow look (vat.urns.ix id_{urn}.ink)
       push id_{\text{jar}} id_{\text{vow}} ink_0
     Update the debt unit and stability fee
        chi_1 \leftarrow drip id_{ilk}
     Denominate the debt in dai
        \operatorname{art}_0 \leftarrow look (\operatorname{vat} . \operatorname{urns} . ix id_{\operatorname{urn}} . \operatorname{art})
       let con = art_0 * cast chi_1
     Transfer the debt tokens to the settler
       push id_{DAI} id_{vow} con
```

When the settler has finished the process of liquidating a CDP's collateral, it invokes plop on the CDP to give back any excess collateral gains.

```
plop id_{	ext{urn}} wad_{	ext{DAI}} = 	ext{auth } \$ do

Fail unless CDP is in liquidation want (feel id_{	ext{urn}}) (\equiv Dread)

Forget the CDP's requester of liquidation vat . urns . ix id_{	ext{urn}} . cat := Nothing

Return some amount of excess auction gains id_{	ext{vow}} \leftarrow use sender id_{	ext{ilk}} \leftarrow look (vat . urns . ix id_{	ext{urn}} . ilk) id_{	ext{jar}} \leftarrow look (vat . ilks . ix id_{	ext{ilk}} . jar) pull id_{	ext{jar}} id_{	ext{vow}} wad_{	ext{DAI}}

Record the gains as the CDP's collateral vat . urns . ix id_{	ext{urn}} . ink := wad_{	ext{DAI}}
```

The settler can invoke loot at any time to claim all uncollected stability fee revenue (for use in the MKR buy and burn auction).

```
loot = auth $ do

The dai vault's balance is the uncollected stability fee revenue wad_{DAI} \leftarrow look \ (balance \ id_{DAI} \ (Vault \ id_{DAI}))

Transfer the entire dai vault balance to sender id_{vow} \leftarrow use \ sender

transfer \ id_{DAI} \ wad_{DAI} \ (Vault \ id_{DAI}) Vow
```

4.6 Auctioning

```
\begin{array}{l} \texttt{flip } \textit{id}_{\texttt{gem}} \textit{ wad\_jam } \textit{wad\_tab } \textit{id}_{\texttt{urn}} = \textbf{do} \\ \textit{vow} \leftarrow \textit{look } \textit{mode} \\ \textit{case } \textit{vow } \textit{of} \\ \textit{Dummy} \rightarrow \textit{return } () \\ \\ \\ \texttt{flap} = \textbf{do} \\ \textit{vow} \leftarrow \textit{look } \textit{mode} \\ \textit{case } \textit{vow } \textit{of} \\ \textit{Dummy} \rightarrow \textit{return } () \\ \\ \\ \\ \texttt{flop} = \textbf{do} \\ \textit{vow} \leftarrow \textit{look } \textit{mode} \\ \textit{case } \textit{vow } \textit{of} \\ \textit{Dummy} \rightarrow \textit{return } () \\ \end{array}
```

4.7 Settlement

```
tidy = \mathbf{do}

Find the dai and sin balances of the sender who \leftarrow use \ sender awe \leftarrow look \ (balance \ id_{\mathrm{DAI}} \ who) woe \leftarrow look \ (balance \ id_{\mathrm{SIN}} \ who)

We can burn at most the minimum of the two balances \mathbf{let} \ x = \min \ awe \ woe

Burn the dai and sin after moving them to the vow account transfer \ id_{\mathrm{DAI}} \ x \ who \ \mathsf{Vow} transfer \ id_{\mathrm{SIN}} \ x \ who \ \mathsf{Vow} burn id_{\mathrm{DAI}} \ x \ \mathsf{Vow} burn id_{\mathrm{SIN}} \ x \ \mathsf{Vow}
```

4.8 Governance

Governance uses form to create a new CDP type. Since the new type is initialized with a zero debt ceiling, a separate transaction can safely set the risk parameters before any lending occurs.

Governance uses frob to alter the sensitivity factor, which is the only mutable parameter of the feedback mechanism.

```
frob how_1 = auth \$ do vat.vox.how := how_1
```

Governance can alter the five risk parameters of a CDP type using cuff for the liquidation ratio; chop for the liquidation penalty; cork for the debt ceiling; calm for the duration of price limbo; and crop for the stability fee.

```
cuff id_{ilk} \max_1 = \text{auth} \ \$ \ \mathbf{do} \ \text{vat.ilks.} \ ix \ id_{ilk} \ . \max := \max_1 \text{chop} \ id_{ilk} \ \text{axe}_1 = \text{auth} \ \$ \ \mathbf{do} \ \text{vat.ilks.} \ ix \ id_{ilk} \ . \text{axe} := \text{axe}_1 \text{cork} \ id_{ilk} \ \text{hat}_1 = \text{auth} \ \$ \ \mathbf{do} \ \text{vat.ilks.} \ ix \ id_{ilk} \ . \text{hat} := \text{hat}_1 \text{calm} \ id_{ilk} \ lax1 = \text{auth} \ \$ \ \mathbf{do} \ \text{vat.ilks.} \ ix \ id_{ilk} \ . \ lax := lax1
```

When altering the stability fee with crop, we ensure that the previous stability fee has been accounted for in the internal debt unit.

```
{
m crop}\;id_{
m ilk}\;{
m tax}_1= \ {
m auth}\;\$\;{
m do} \ {
m Apply}\;{
m the}\;{
m current}\;{
m stability}\;{
m fee}\;{
m to}\;{
m the}\;{
m internal}\;{
m debt}\;{
m unit} \ {
m drip}\;id_{
m ilk} \ {
m Change}\;{
m the}\;{
m stability}\;{
m fee} \ {
m vat}\;{
m .}\;{
m ilks}\;{
m .}\;ix\;id_{
m ilk}\;{
m .}\;{
m tax}:={
m tax}_1
```

4.9 Vaults

The internal act pull transfers tokens into a vault. It is used by lock to acquire collateral from a CDP owner; by wipe to acquire dai from a CDP owner; and by plop to acquire collateral from the settler contract.

```
pull id_{jar} src wad<sub>gem</sub> = 
transfer id_{jar} wad<sub>gem</sub> src (Vault id_{jar})
```

The internal act push transfers tokens out from a collateral vault. It is used by draw to send dai to a CDP owner; by free to send collateral to a CDP owner; and by grab to send collateral to the settler contract.

```
push id_{jar} dst wad_{gem} = transfer id_{jar} wad_{gem} (Vault id_{jar}) dst
```

4.10 Token manipulation

We model the ERC20 transfer function in simplified form (omitting the concept of "allowance").

```
\begin{aligned} & transfer \ id_{\texttt{jar}} \ \texttt{wad} \ src \ dst = \\ & \texttt{Operate in the token's balance table} \\ & \textit{zoom} \ (\texttt{vat. jars.} \ ix \ id_{\texttt{jar}} \ . \ \texttt{gem.} \ balanceOf) \ \$ \ \textbf{do} \end{aligned} & \texttt{Fail if source balance insufficient} \\ & \textit{balance} \leftarrow look \ (ix \ src) \\ & \texttt{aver} \ (balance \geqslant \texttt{wad}) \end{aligned}
```

```
Update balances

decrease (ix src) wad

initialize (at dst) 0

increase (ix dst) wad
```

The internal act mint inflates the supply of a token. It is used by lend to create new dat and debt tokens, and by the settler to create new MKR.

```
\label{eq:mint_id_jar} \begin{array}{l} \texttt{mint} \ id_{\texttt{jar}} \ \texttt{wad}_0 \ \textit{dst} = \\ \textit{zoom} \ (\texttt{vat.jars.} \ ix \ id_{\texttt{jar}} \ . \ \texttt{gem}) \ \$ \ \mathbf{do} \\ \textit{increase} \ (\textit{balanceOf.} \ ix \ \textit{dst}) \ \texttt{wad}_0 \end{array}
```

The internal act burn deflates the supply of a token. It is used by mend to destroy DAI and debt tokens, and by the settler to destroy MKR.

```
burn id_{jar} wad_0 src = zoom (vat . jars . ix id_{jar} . gem) \$ do decrease (balanceOf . ix src) wad_0
```

The internal act lend mints identical amounts of both dai and the internal debt token. It is used by draw to issue dai to a borrower; it is also used by drip to issue dai representing revenue from stability fees, which stays in the dai vault until collected.

```
	ext{lend wad}_{	ext{DAI}} = 	ext{do} 	ext{mint } id_{	ext{DAI}} 	ext{ wad}_{	ext{DAI}} 	ext{ Joy} 	ext{mint } id_{	ext{SIN}} 	ext{ wad}_{	ext{DAI}} 	ext{ Woe}
```

The internal act mend destroys identical amounts of both dai and the internal debt token. Its use via wipe is how the dai supply is reduced.

Chapter 5

Act framework

The reader does not need any abstract understanding of monads to understand the code. They give us a nice syntax—the **do** block notation—for expressing exceptions and state in a way that is still purely functional. Each line of such a block is interpreted by the monad to provide the semantics we want.

5.1 The Maker monad

This defines the Maker monad as a simple composition of a state monad and an error monad:

```
type Maker a = \text{StateT System (Except Error) } a
```

We divide act failure modes into general assertion failures and authentication failures.

```
data Error = AssertError Act | AuthError
deriving (Show, Eq)
```

An act can be executed on a given initial system state using *exec*. The result is either an error or a new state. The *exec* function can also accept a sequence of acts, which will be interpreted as a single transaction.

```
exec :: System \rightarrow Maker () \rightarrow Either Error System

exec sys m = runExcept (execStateT m sys)
```

5.2 Asserting

We now define a set of functions that fail unless some condition holds.

```
General assertion aver x = unless\ x\ (throwError\ (AssertError\ ?act)) Assert that an indexed value is not present none\ x = preuse\ x \gg \lambda case Nothing \to return\ () Just _- \to throwError\ (AssertError\ ?act) Assert that an indexed value is present look\ f = preuse\ f \gg \lambda case Nothing \to throwError\ (AssertError\ ?act) Just x \to return\ x Execute an act and assert a condition on its result want\ m\ p = m \gg (aver\ .\ p)
```

We define owns id_{urn} id_{lad} as an assertion that the given CDP is owned by the given account.

```
owns id_{\tt urn} \ id_{\tt lad} = {\tt do}

want \ (look \ (\tt vat . urns . ix \ id_{\tt urn} . lad)) \ (\equiv id_{\tt lad})
```

We define auth k as an act modifier that executes k only if the sender is authorized.

```
\begin{array}{l} \text{auth } continue = \textbf{do} \\ s \leftarrow \textit{use sender} \\ \textit{unless} \ (s \equiv \text{Account } \textit{id}_{god}) \ (\textit{throwError} \ \text{AuthError}) \\ \textit{continue} \end{array}
```

Appendix A

Prelude

This module reexports symbols from other packages and exports a few new symbols of its own.

```
module Maker.Prelude (module Maker.Prelude, module X) where
import Prelude as X (
 Conversions to and from strings
  Read (..), Show (..),
 Comparisons
  Eq(...), Ord(...),
 Core abstractions
               (fmap),
  Functor
  Applicative (),
  Monad
               (return, (\gg)),
 Numeric classes
  Num (..), Integral (), Enum (),
 Numeric conversions
  Real (...), Fractional (...),
  RealFrac (..),
  fromIntegral,
 Simple types
  Integer, Int, String,
 Algebraic types
         (True, False),
  Bool
  Maybe (Just, Nothing),
  Either (Right, Left),
```

```
Functional operators (.),(\$), Numeric operators (+),(-),(*),(/),(\uparrow),(\uparrow\uparrow),div, Utilities all,\lnot,elem,(\land), Constants mempty,\bot,otherwise)
```

We use a typical composition of monad transformers from the mtl library to structure stateful actions. See section 5.1 (*The Maker monad*).

```
import Control.Monad.State as X (
  StateT,
                  Type constructor that adds state to a monad type
   execStateT,
                  Runs a state monad with given initial state
                  Gets the state in a do block
  get,
  put)
                  Sets the state in a do block
import Control.Monad.Writer as X (
  WriterT,
                  Type constructor that adds logging to a monad type
  Writer.
                  Type constructor of logging monads
   runWriterT,
                  Runs a writer monad transformer
   execWriterT,
                  Runs a writer monad transformer keeping only logs
   execWriter)
                  Runs a writer monad keeping only logs
import Control.Monad.Except as X (
  MonadError, Type class of monads that fail
  Except,
                  Type constructor of failing monads
   throwError,
                  Short-circuits the monadic computation
  runExcept)
                  Runs a failing monad
```

Our numeric types use decimal fixed-point arithmetic.

```
\begin{array}{ll} \textbf{import Data.Fixed } \textit{as X (} \\ \textbf{Fixed (..)}, & \textbf{Type constructor for numbers of given precision} \\ \textbf{HasResolution (..)}) & \textbf{Type class for specifying precisions} \end{array}
```

We rely on the lens library for accessing nested values. There is no need to understand the theory behind lenses to understand this program. The notation $a \cdot b \cdot c$ denotes a nested accessor much like a.b.c in C-style languages; for more details, consult lens documentation¹.

¹Gabriel Gonzalez's 2013 article *Program imperatively using Haskell* is a good introduction.

```
import Control.Lens as X (
  Lens', lens,
  makeLenses, Defines lenses for record fields
  makeFields.
                  Defines lenses for record fields
  set,
                  Writes a lens
  use, preuse,
                  Reads a lens from a state value
  view,
                  Reads a lens from a value
  ix,
                  Lens for map retrieval and updating
  at,
                  Lens for map insertion
 Operators for partial state updates in do blocks:
  (:=),
                  Replace
  (-=), (+=), Update arithmetically
  (\% =),
                  Update according to function
  (?=))
                  Insert into map
import Control.Lens.Zoom as X (zoom)
```

Where the Solidity code uses mapping, we use Haskell's regular tree-based map type².

```
import Data.Map as X (
Map, Type constructor for mappings
Ø, Polymorphic empty mapping
singleton, Creates a mapping with a single key-value pair
fromList) Creates a mapping with several key-value pairs
```

Finally we define some of our own convenience functions.

```
decrease a \ x = a -= x

increase a \ x = a += x

initialize a \ x = a \% = (\lambda \mathbf{case} \ \text{Nothing} \to \text{Just} \ x; \ y \to y)

prepend a \ x = a \% = (x:)

x \notin xs = \neg \ (elem \ x \ xs)
```

²We assume the axiom that Keccak hash collisions are impossible.

Appendix B

Fixed point numbers with rounding

This somewhat arcane-looking code implements a wrapper around the base library's decimal fixed point type, only with x * y and x / y operations that do rounding instead of truncation of their intermediate results.

```
module Maker.Decimal (Decimal, E18, E36, Epsilon (...)) where import Data.Fixed newtype HasResolution e \Rightarrow Decimal e = D (Fixed e) deriving (Ord, Eq, Real, RealFrac)
```

We want the printed representations of these numbers to look like "0.01" and not "R 0.01".

```
instance HasResolution e \Rightarrow \text{Read} (Decimal e) where readsPrec n s = fmap (\lambda(x, y) \rightarrow (D \ x, y)) (readsPrec n s) instance HasResolution e \Rightarrow \text{Show} (Decimal e) where show (D x) = show x
```

In the Num instance, we delegate everything except multiplication.

```
instance HasResolution e \Rightarrow \text{Num (Decimal } e) where x \otimes (D \text{ (MkFixed } a)) * D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a * b + div (resolution x) 2) (resolution x)))}
D \text{ } a + D \text{ } b = D \text{ } (a + b)
D \text{ } a - D \text{ } b = D \text{ } (a - b)
negate \text{ } (D \text{ } a) = D \text{ } (negate \text{ } a)
abs \text{ } (D \text{ } a) = D \text{ } (abs \text{ } a)
```

```
signum (D a) = D (signum a)

fromInteger i = D (fromInteger i)
```

In the Fractional instance, we delegate everything except division.

```
instance HasResolution e \Rightarrow Fractional (Decimal e) where x@(D \text{ (MkFixed } a)) / D \text{ (MkFixed } b) = D \text{ (MkFixed } (div (a * resolution } x + div b 2) b))
recip (D a) = D (recip a)
fromRational r = D (fromRational r)
```

We define the E18 and E36 symbols and their fixed point multipliers.

```
data E18; data E36 instance HasResolution E18 where resolution \_=10\uparrow(18::Integer) instance HasResolution E36 where resolution \_=10\uparrow(36::Integer)
```

The fixed point number types have well-defined smallest increments (denoted ϵ). This becomes useful when verifying equivalences.

```
class Epsilon t where \epsilon :: t instance HasResolution a \Rightarrow Epsilon (Decimal a) where The use of \bot is safe since resolution ignores the value. \epsilon = 1 \ / \ from Integral \ (resolution \ (\bot :: Fixed \ a))
```