The complex inner life of simple regression

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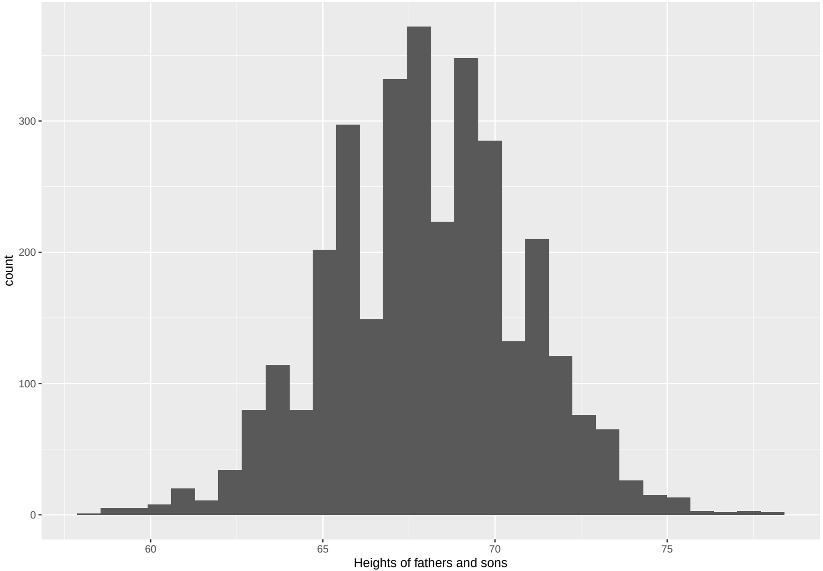
Pearson-Lee height data



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- For better predictions, use more information!

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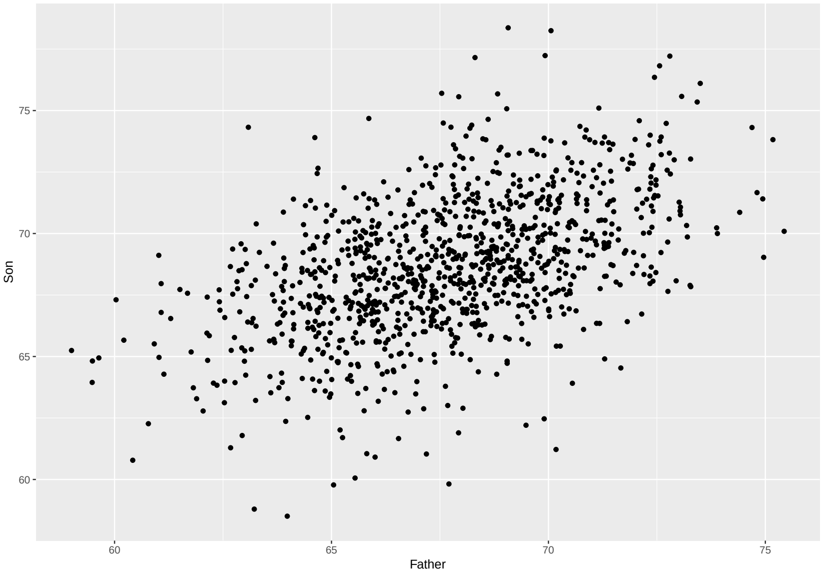

Pearson-Lee height data

Father's height	Number of sons	Average height	SD
62	15	65.59	1.78
63	36	66.51	2.91
64	60	66.70	2.31
65	101	67.22	2.53
66	139	67.66	2.35
67	134	68.14	2.24
68	157	69.09	2.76
69	142	69.44	2.30
70	115	69.77	2.49
71	77	70.54	2.31
72	50	70.68	2.33
73	28	72.05	2.76

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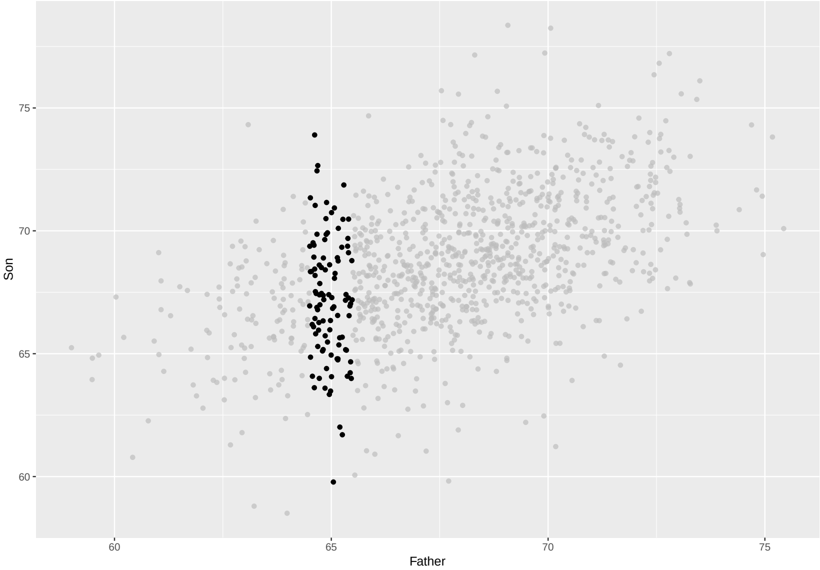

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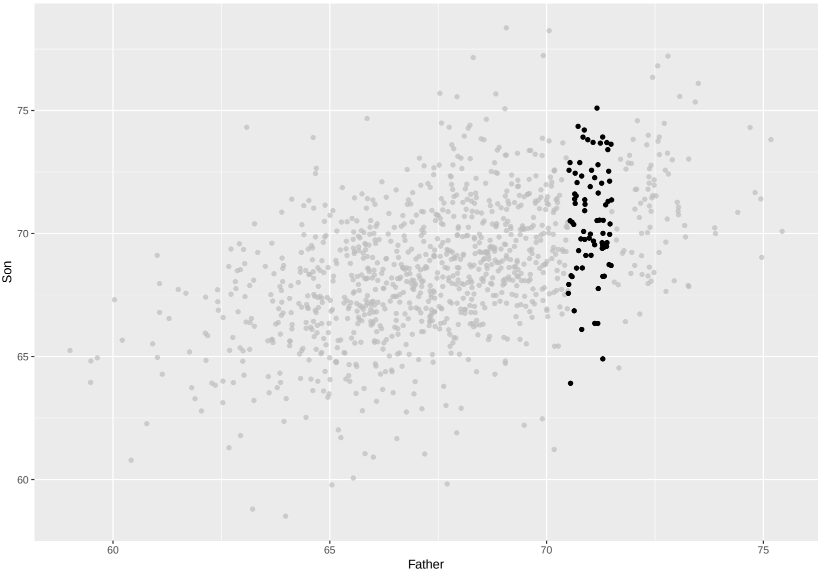

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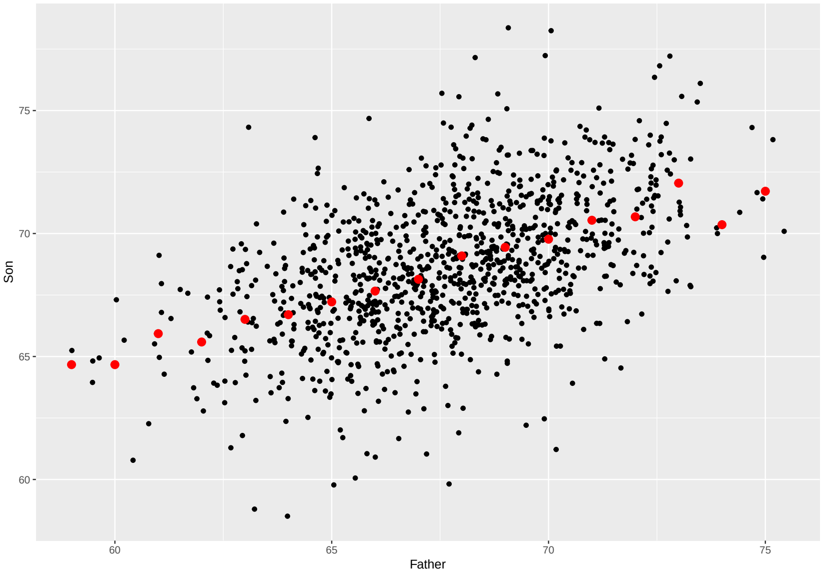

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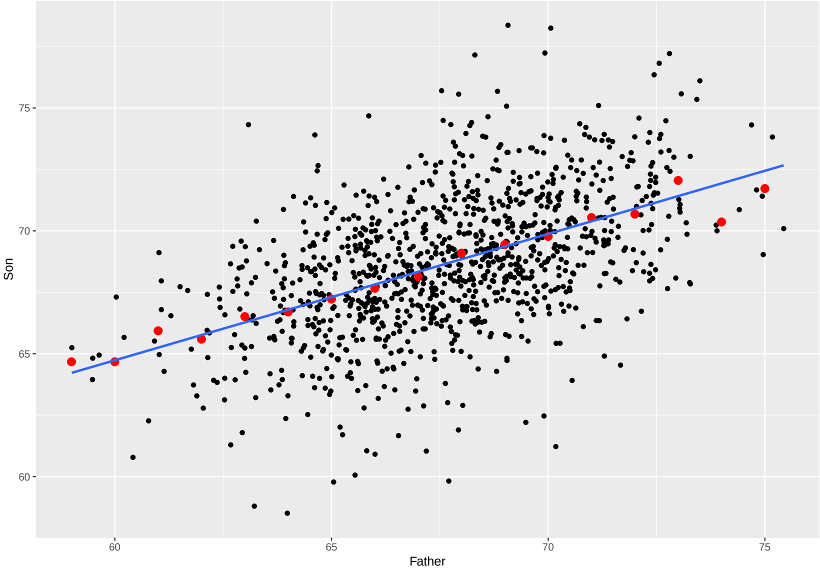

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- This is *simple linear regression*.

Pearson-Lee height data

```
##
## Call:
## lm(formula = Son ~ Father, data = heights)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.8772 -1.5144 -0.0079  1.6285  8.9685
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.88660    1.83235   18.49   <2e-16 ***
## Father       0.51409    0.02705   19.01   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
##
## Residual standard error: 2.437 on 1076 degrees of freedo
## Multiple R-squared:  0.2513, Adjusted R-squared:  0.2506
## F-statistic: 361.2 on 1 and 1076 DF,  p-value: < 2.2e-16
```

The simple linear regression model

blah β

The Gauss-Markov Theorem

you know, OLS is BLUE and whatnot

Chebyshev's Theorem

75% of observations are within 2 SDs – no matter what!