Assignment Instructions: Module 2 - The LP Model

Problem 1:

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a longterm contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Mini's can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. **Decision Variables:**

The decision variables are x_1 and x_2, the number of Collegiate and Mini units produced respectively.

b. **The Objective function** is the quantity of each type of backpack (the Collegiate and Mini) to produce each week to maximize profit.

c. **Constraints:**

1) Back Savers receives from a supplier 5000 sq. ft.. of nylon fabric each week. The Collegiate and Mini requires 3 and 2 sq. ft. of nylon to produce one of each respectively.

2) Sales forecast at most 1000 Collegiate's and 1200 Mini's being sold each week.

3) The Collegiate takes 45 minutes to produce and the Mini requires 40 minutes. Their are 35 laborers who work a combined 1400 of hours (each one works 40 hours per week).

d. Mathematical Formulation

Sales & Materials

Backpack Model	Profit per Unit	Sales Forecast (units)	Material Used (sq. ft.)	Restriction on Material (per week)
Collegiate	$32	1000	3	≤ 5000
Mini	$24	1200	2	

Labor & Production

Backpack Model	Production Labor (hours)	Restriction on Labor (hours)
Collegiate	0.75	≤ 1400
Mini	0.6667	

Maximize $Z = 32x_1 + 24x_2$

Constraints:

Material: $3x_1 + 2x_2 \leq 5000$

Sales: $x_1 \leq 1000$
$x_2 \leq 1200$

Production Hours: $\frac{3}{4}x_1 + \frac{2}{3}x_2 \leq 1400$
where $x_1, x_2 \geq 0$ and $x_1, x_2 \in \mathbb{Z}$

Problem 2:

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

The decision variables are the allocation of space for each size (L, M, S) of the product at each of factory (1, 2, and 3), which are represented by the following variables:

Factory 1 (L, M, S) = x_1, x_4, and x_7

Factory 2 (L, M, S) = x_2, x_5, and x_8

Factory 3 (L, M, S) = x_3, x_6, and x_9

b. Formulate a linear programming model for this problem.

Product Profit & Production Space (per unit)

New Product Size	Profit	Production Space. (sq. ft)	Sales Forecast
L	$420	20	900
M	$360	15	1200
S	$300	12	750

Allocation of Space for Each Product Size

Product Size	Allocation of Factory for Units of Product		
	Factory		
	1	2	3
L	x_1	x_2	x_3
M	x_4	x_5	x_6
S	x_7	x_8	x_9

Factory Unit Capacity & Production Rates

Factory	Plant Capacity (units)	Storage Space Available (sq. ft)
1	750	13,000
2	900	12,000
3	450	5,000

Maximize $Z = 420(x_1 + x_2 + x_3) + 360(x_4 + x_5 + x_6) + 300(x_7 + x_8 + x_9)$

Constraints:

Factory Space:

$20x_1 + 15x_4 + 12x_7 \leq 13{,}000$

$20x_2 + 15x_5 + 12x_8 \leq 12{,}000$

$20x_3 + 15x_6 + 12x_9 \leq 12{,}000$

Plant Unit Capactity:

$x_1 + x_4 + x_7 \leq 750$

$x_2 + x_5 + x_8 \leq 900$

$x_3 + x_6 + x_9 \leq 450$

Sales:

$x_1 + x_2 + x_3 \leq 900$

$x_4 + x_5 + x_6 \leq 1200$

$x_7 + x_8 + x_9 \leq 750$

where $x_1, x_2, \ldots, x_9 \geq 0$ and $x_1, x_2, \ldots, x_9 \in \mathbb{Z}$