Cubic and Board Foot Volume Models for the Central States

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ABSTRACT. This paper presents tree volume models developed for major timber species in the Central States (Indiana, Illinois, Missouri, and Iowa). Models for estimating gross tree volume (either cubic foot or board foot International ¼-in. log rule) and percent cull were developed for 23 species or species groups. These models estimate volume based on observed dbh and tree site index. Nonlinear regression techniques were used to fit a Weibull-type function to estimate gross volume with a data set containing observations from more than 50,000 trees measured throughout the region. A simple linear model was used to estimate percent cull in a tree for each of several tree classes. These models are being used in the statewide inventories now underway in Missouri and Iowa and may be used by anyone desiring volume-per-tree estimates that are comparable to USDA Forest Service Forest Inventory and Analysis estimates in these areas.

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Estimating the volume of standing trees is basic to any forest inventory. Numerous volume tables and volume models have been developed for trees in the United States. These tables and models differ in many ways, including the observations required, their range of applicability, the units of measure, and the methods used to derive them. The purpose of this study was to develop a set of volume models that could consistently estimate several measures of tree volume for major timber species found in the central United States. These models will be used by the Forest Inventory and Analysis (FIA) research work unit at the North Central Forest Experiment Station to estimate the volume of sample trees when compiling volume and growth estimates for statewide inventories. The models presented here are being used in the current Missouri and Iowa forest inventories. They are also included in the computerized databases that the FIA Unit has compiled for the Central States (Hahn and Hansen 1985). These models are published here so that users of FIA data will be aware of the methods used to estimate volumes in FIA reports and can incorporate the models in their applications to ensure compatibility with FIA data.

MODEL REQUIREMENTS

FIA requires models that estimate volume based on dbh, with no observation of tree height (either total or merchantable) for several reasons. Principally, the cost of measuring plots is reduced significantly when height measurements are not required for each tree. In estimating change between inventories, local volume models are preferred because of the large measurement errors associated with height and cull measurement. When models that require height and/or cull measurements are used to estimate the various components of change (i.e., survivor growth, mortality, or removals) from remeasurement plot data, measurement errors significantly affect the results. Further, STEMS (Shifley 1987), the individual tree growth model used by FIA for much of its growth estimation work, projects diameter growth but does not project height growth. Therefore, a volume model that does not require height is necessary.

In addition to volume models that do not require height measurements, models with a broad geographic range of applicability that produce good estimates of volume in all types of naturally occurring forest stands are needed to facilitate regional analysis.

FIA statistical reports require volume models to estimate:

1. Gross cubic foot volume
2. Net cubic foot volume
3. Gross board foot volume
4. Net board foot volume

All volumes are from a 1-ft-high stump. Cubic foot volumes (1 and 2) are to a 4-in. top dob (diameter outside bark) for all trees 5 in. dbh or larger, and board foot volumes (3 and 4) are to a 9-in. top dob for hardwoods (7 in. for softwoods) for all trees 11 in. dbh (9 in. for softwoods) or larger. Board foot volumes (3 and 4) are in International ¼-in. log rule. Net cubic foot volume (2) has reductions for rotten and missing portions, and net board foot volume (4) has deductions for form defects as well as rotten and missing portions. Net cubic foot volume is the portion of the tree typically harvested in a pulp operation, and net board foot volume is the portion of the tree typically used for sawtimber.

The models developed in this study require species, dbh, and tree site index to estimate gross tree volume. Net volume estimation also requires an observation of tree class (a form classification—see the appendix for a complete definition of tree class). If tree site index is not available, it is estimated from stand site index using the models and methods presented by Carmean et al. (in prep.), or stand site index may be used directly in the models. If no estimate of site index is available, then the average site index in the area is used as an approximation. Average site index can be obtained for most areas from statistical reports published by FIA for States and Forest Survey Units.

PAST APPROACHES

Models used by FIA to estimate tree volume typically have been developed from a subset of trees measured during the inventory. On this subset of trees, detailed height, upper stem diameter, and cull volume measurements are taken for each tree, in addition to the usual data (dbh, species, and tree class). This detailed tree information is used to accurately estimate the volume of each measured tree. Using nonlinear regression methods to model these individual tree volumes, local volume models are developed and used to estimate tree volumes on plots where detailed measurements have not been taken. For many years, detailed tree measurements were taken on one-third of the plots in a statewide inventory. In more recent inventories, the proportion of detailed measurements has been reduced significantly through use of regional models.

Table 1. Various model forms used in the past to estimate tree volume in FIA inventories.

State	Reference	Model form[a]
Wisconsin	Hahn 1973	$V = \beta_1\beta_2{}^{\beta_3{}^{\beta_4 D}} + \epsilon$
Missouri	Hahn 1975	$V = \beta_1 S\beta_2{}^{\beta_3{}^{\beta_4 D}} + \epsilon$
Iowa	Hahn 1976	$V = \beta_1 S(1 - e^{\beta_2 D})^{(1/(1-\beta_3))} + \epsilon$
Minnesota	Raile 1981	$V = \beta_1 S(1 - e^{\beta_2 D})^{\beta_3} + \epsilon$
Michigan	Raile *et al.* 1982	$V = \beta_1 S^{\beta_2}(1 - e^{\beta_3 D})^{\beta_4} + \epsilon$
Indiana	Smith and Weist 1982	$V = \beta_1 S^{\beta_2}(1 - e^{\beta_3 D^{\beta_4}}) + \epsilon$

[a] D = dbh, S = site index, e = 2.71828, β_1, β_2, β_3 and β_4 = parameters, V = volume (both cubic foot and board foot), ϵ = random error.

Table 2. Number of trees used to fit gross volume and percent cull equations for the Central States.

	Gross volume equations		Percent cull equations				
			Cubic foot			Board foot	
Species group[a]	ft³	bd ft	Growing stock	Rough	Rotten	Sawtimber	Shortlog
Red-white-jack pine	814	388	86	0	0	380	8
Yellow pine	1,484	729	255	4	0	721	8
Swamp conifers	382	122	6	0	0	117	5
Baldcypress	43	42	0	0	0	42	0
Eastern redcedar	504	148	48	20	1	120	28
Select white oak	9,806	5,742	1,067	75	18	5,011	731
Other white oak	3,075	1,251	254	16	7	1,062	189
Select red oak	2,963	2,128	372	13	24	2,009	119
Other red oak	7,923	4,176	808	65	24	3,819	357
Select hickory	2,592	1,118	338	29	5	1,026	92
Other hickory	2,519	965	468	24	5	910	55
Basswood-beech-y. birch	2,342	1,465	160	50	45	1,360	105
Hard maple	2,369	1,213	409	71	41	1,114	99
Soft maple	1,747	1,029	231	59	17	906	123
Elm-hackberry	2,742	978	302	75	10	832	146
Ash	2,123	922	405	53	15	848	74
Sycamore	998	693	195	9	10	666	27
Cottonwood	933	769	99	13	5	750	19
Willow	562	284	14	5	5	260	24
Aspen	2,397	599	23	1	0	580	19
Black walnut	1,110	488	111	29	9	428	60
Yellow-poplar	1,213	872	290	15	8	857	15
Other hardwoods	3,677	1,122	554	243	62	1,010	112
Total	54,318	27,243	6,495	869	311	24,828	2,415

[a] The individual species that compose each species group are listed in Appendix Table 1.

Table 3. Diameter and site index ranges of trees used to fit equations in the Central States.

	Maximum dbh in data set used to fit the percent cull equations					Observed parameters of site index in the data set used to fit the gross cubic foot volume equations			
Species group	Growing-stock trees	Rough trees	Rotten trees	Sawtimber trees	Shortlog trees	Min.	Max.	Mean	Std dev.
Red-white-jack pine	16.9	48.8[c]	50.3[h]	28.9	31.0[l]	38	99	73.9	15.2
Yellow pine	18.4	48.8[c]	50.3[h]	20.7	31.0[l]	31	99	64.3	13 9
Swamp conifers	44.6[a]	48.8[c]	50.3[h]	27.3	31.0[l]	24	90	44.6	15 0
Baldcypress	44.6[a]	48.8[c]	50.3[h]	44.1	31.0[l]	72	90	80.7	7.5
Eastern redcedar	16.3	16.2	50.3[h]	19.2	16.7	10	99	52.9	17 4
Select white oak	39.5	44.4[d]	50.3[i]	47.0	57.9	25	99	61.9	13 6
Other white oak	27.6	44.4[d]	50.3[i]	38.1	38.5	24	99	54.5	14 7
Select red oak	38.5	48.8[e]	50.3[i]	51.2	43.9	25	99	66.5	13.8
Other red oak	41.2	48.8[e]	50.3[i]	51.7	48.8	10	99	64.8	13.6
Select hickory	29.6	25.7[f]	50.3[i]	33.1	33.0	28	99	67.5	13 4
Other hickory	30.4	25.7[f]	50.3[i]	33.7	37.0	25	99	67.1	14.7
Basswood-beech-y. birch	33.2	34.0	38.4	45.5	39.2	33	99	71.8	15 1
Hard maple	38.8	33.6	38.2	40.3	33.6	34	99	72.3	15 7
Soft maple	34.8	48.8	42.1[j]	46.3	50.3	38	99	72.1	17.2
Elm-hackberry	33.1	52.9	42.1[j]	47.2	46.5	25	99	69.2	14.5
Ash	34.7	35.2	42.1[j]	34.7	34.5	27	99	72.7	16 5
Sycamore	33.7	45.8[g]	41.5[k]	45.9	64.7[m]	35	99	79.4	14.0
Cottonwood	44.1[b]	45.8[g]	41.5[k]	61.4	64.7[m]	38	99	71.8	19.4
Willow	44.1[b]	45.8[g]	41.5[k]	32.3	64.7[m]	39	99	73.7	17.2
Aspen	44.1[b]	45.8[g]	41.5[k]	28.1	64.7[m]	35	99	67.6	12.2
Black walnut	33.3	24.4	41.5[k]	33.3	28.7	31	99	70.5	15.3
Yellow-poplar	44.6	45.8[g]	41.5[k]	44.6	64.7[m]	43	99	84.5	13 6
Other hardwoods	29.7	32.5	31.2	39.3	38.5	31	99	74.6	15.3

[a] The number of trees in this group was insufficient to fit the percent cull equation. Average percent cull from all growing-stock trees was used to estimate percent cull in this case.
[b] Data from the cottonwood, willow, and aspen groups were combined to estimate percent cull in growing-stock trees.
[c] The number of trees in this group was insufficient to fit the percent cull equation. Average percent cull from all rough trees was used to estimate percent cull in this case.
[d] Data from the select white oak and other white oak groups were combined to estimate percent cull in rough trees.
[e] Data from the select red oak and other red oak groups were combined to estimate percent cull in rough trees.
[f] Data from the select hickory and other hickory groups were combined to estimate percent cull in rough trees.
[g] Data from the sycamore, cottonwood, willow, aspen, and yellow-poplar groups were combined to estimate percent cull in rough trees.
[h] The number of trees in this group was insufficient to fit the percent cull equation. Average percent cull from all rotten trees was used to estimate percent cull in this case.
[i] Data from the oak and hickory groups were combined to estimate percent cull in rotten trees.
[j] Data from the soft maple, elm-hackberry, and ash groups were combined to estimate percent cull in rough trees.
[k] Data from the sycamore, cottonwood, willow, aspen, black walnut, and yellow-poplar groups were combined to estimate percent cull in rough trees.
[l] The number of trees in this group was insufficient to fit the percent cull equation. Average percent cull from all rotten trees was used to estimate percent cull in this case.
[m] Data from the sycamore, cottonwood, willow, aspen, and yellow-poplar groups were combined to estimate percent cull in shortlog trees

Table 4. Average taper observed in tops of sawtimber-size trees between the top of the last merchantable sawbolt[a] and the top of the last merchantable pulpstick.[b]

Species group	Number of trees[c]	Top length (ft)		Taper (in./ft)	
		Mean	Std. Dev.	Mean	Std. Dev.
Red-white-jack pine	382	13.3	6.5	0.292	0.184
Yellow pine	704	12.1	6.1	0.301	0.147
Swamp conifers	120	13.0	5.3	0.285	0.157
Baldcypress	42	18.3	6.6	0.308	0.104
Eastern redcedar	147	10.7	4.3	0.359	0.171
Select white oak	5,676	23.8	9.3	0.335	0.155
Other white oak	1,232	20.3	7.9	0.325	0.127
Select red oak	2,116	23.8	9.4	0.352	0.165
Other red oak	4,142	22.5	9.1	0.335	0.141
Select hickory	1,113	24.3	8.9	0.287	0.158
Other hickory	960	24.0	9.2	0.291	0.119
Basswood-beech-y. birch	1,435	22.5	10.4	0.356	0.244
Hard maple	1,195	26.7	10.2	0.302	0.140
Soft maple	1,021	26.9	10.6	0.305	0.169
Elm-hackbery	970	24.7	9.1	0.316	0.138
Ash	909	23.9	9.5	0.311	0.153
Sycamore	686	25.9	10.7	0.303	0.158
Cottonwood	766	26.7	10.8	0.349	0.172
Willow	273	23.8	9.7	0.250	0.106
Aspen	590	19.1	7.9	0.314	0.198
Black walnut	482	23.0	8.2	0.304	0.126
Yellow-poplar	868	25.0	9.8	0.316	0.137
Other hardwoods	1,109	24.2	9.8	0.284	0.152

[a] This is the point where dob reaches 9 in. (7 in. for softwoods) or the point above which no merchantable sawlog exists due to branching, forks, rot or other defects.

[b] This is the point where dob reaches 4 in. or the point above which no merchantable pulpwood section exists.

[c] The number of trees is slightly less than the numbers given in Table 2 because in a few trees the top of the last sawbolt occurred at the same point as the top of the last pulpstick.

Area-Specific Models

In the past, volume models were developed for fairly limited geographic areas—for example, in the 1969 Wisconsin inventory, Hahn (1973) developed separate sets of volume models for different parts of the state. This process was repeated for the 1972 Missouri inventory (Hahn 1975), the 1974 Iowa inventory (Hahn 1976), the 1977 Minnesota inventory (Raile 1980), and the 1981 Michigan inventory (Raile et al. 1982).

Producing new volume models for each inventory region caused several problems. Inventory results could not be reported until the volume models had been developed and adequately tested, adding several months to the time required to produce reports. Also, the number of trees in these areas, from which detailed measurements were taken, was not always adequate to properly fit models without lumping large numbers of species together, a procedure that could bias estimates if species with greatly different volume-to-diameter relationships were grouped.

Past volume models developed by FIA have differed in the geographic range of data used to fit the models and in the form of the model used. Since the 1968 Wisconsin inventory various nonlinear functions of dbh have been used to predict volume. The 1972 Missouri inventory models added site index as a predictor variable in an effort to add a surrogate for tree height. The various models used by FIA over the past 20 years are shown in Table 1. A general description of the models for Indiana, Iowa, Michigan, and Minnesota can be found in Yang et al. (1978).

Regional Models

The amount of work involved in producing separate volume models, the subsequent delays in publishing inventory results, and the inconsistencies in volumes per tree from one area to the next prompted development of regional volume models. Following the 1979 Michigan inventory, an adequate database was available across the Lake States (Michigan, Wisconsin, and Minnesota) to produce a single set of volume models for use in the region. These models (Hahn 1984) were used for volume estimation in the 1983 Wisconsin inventory and are based on observations from over 100,000 trees. The use of a single set of volume models for the entire Lake States has saved time and has greatly reduced the probability of error in regional analysis projects such as the 1990 RPA assessment.

A somewhat different approach to estimating volume was used in the regional Lake States models. The model form used was the common volume model, $V = \beta_1 + \beta_2 D^2 H + \epsilon$, where H is merchantable height, β_1 and β_2 are regression parameters, and ϵ is a random error. In this case, estimated height was produced by a height model that had been fit to detailed measurement tree data plus data taken on site index trees from every FIA plot in the Lake States (Ek et al. 1984). The form of the height model was a modified Weibull model that estimates H, the height to a top dob of d in., as:

$$H = 4.5 + \beta_1(1 - e^{\beta_2 D})^{\beta_3 S^{\beta_4}} (1.00001 - d/D)^{\beta_5 B^{\beta_6}} + \epsilon$$

where B is stand basal area, S is site index, β_1 through β_6 are regression parameters, and ϵ is a random error. If d is set to zero, the model estimates total tree height. The data set used to fit

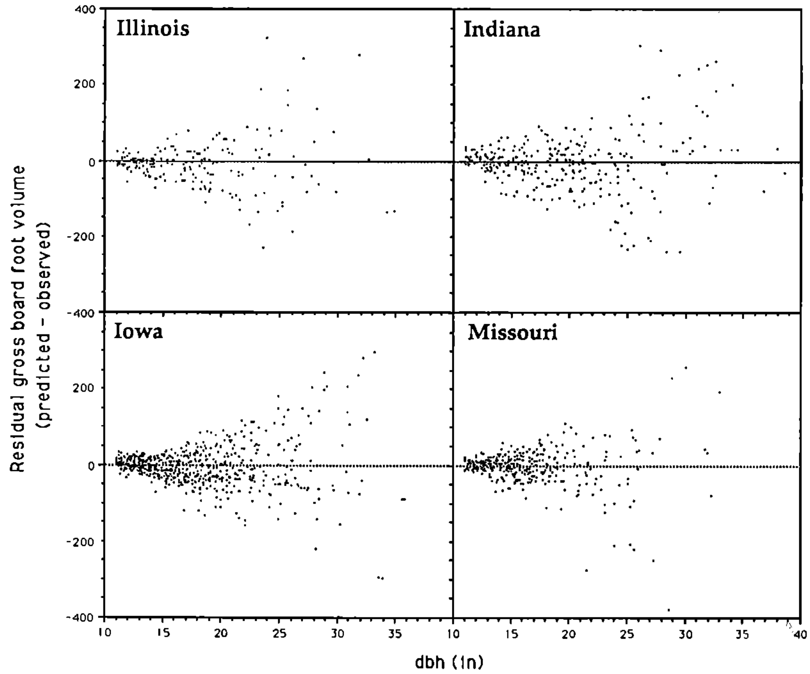

Fig. 1. Residual gross board foot volumes (predicted-observed) in select red oak for the Central States.

this model did contain some total height measurements and can be used to predict total height; however, the majority of the data contained only merchantable height measurements, making the model most useful for predicting merchantable heights.

Instead of developing independent net volume models for each tree class, models were developed for gross volume using data from all tree classes, and average percent cull volume by tree class was used to convert gross volumes to net volumes. This required fitting fewer regressions and produced consistent models that could be easily checked. A consistent model in this context refers to a reasonable relationship between estimated net volumes of trees of different tree classes. A model that produced an estimated net volume for a rotten tree that was larger than the estimated net volume for a similar growing stock tree would not be consistent. This method did; however, apply a single percentage cull to all trees of a given species within a tree class regardless of size.

CENTRAL STATES APPROACH

The approach we used for the Central States was similar to the one used for the Lake States in that we fit a single gross volume model for all tree classes. The model selected for these models was the modified Weibull model used for the 1987 Indiana net volume estimates. Percent cull was estimated with a simple linear model for each tree class rather than a single average percent cull within a tree class as done in the Lake States. Like the Lake States model, this model can be used to estimate volumes to any top diameter.

DATA

A database consisting of detailed tree measurements collected during the Missouri (1972), Iowa (1974), Illinois (1985), and Indiana (1967 and 1986) forest inventories was used to fit the volume models in this study. This database contains field data from 54,318 trees 5 in. dbh or larger. Data included are tree dbh, merchantable height, top dob, tree site index, species, and tree class. Table 2 shows the number of trees in the database by species, size, and tree class. Table 3 shows the range of diameters and site indices by species.

On each tree, dbh was measured with a diameter tape to the last one-tenth in.; merchantable height was measured to the last whole foot with a 30-ft telescopic height pole or a Suunto clinometer; and top dob was measured to the last one-tenth in. with a Wheeler pentaprism. For sawtimber-size trees, height and top dob measurements were taken at both the sawtimber and poletimber limits of merchantability. For poletimber-size trees, these measurements were taken at only the poletimber limits of merchantability. Field crews also estimated a total cull deduction (both board foot and cubic foot) for each tree following standard FIA procedures (Hahn 1984). This cull deduction was then subtracted from the gross volume to obtain the net merchantable volume for the tree. Board foot deductions were made for rotten and missing portions of the tree and for defects in form that would prevent a portion of the bole from being utilized for lumber. Cubic foot deductions were only made for rotten and missing portions of the main stem, and thus reflect the total volume of the tree available for pulp. This standard for cubic foot deduction changed between the 1974 Iowa and 1985 Illinois inventories. Because of this change in standard, only data from the 1985 Illinois and the 1986 Indiana inventories (7,675 trees) could be used for percent cubic foot cull estimation.

After the final volume models had been fit, data from an additional 10,453 detailed measurement trees became available. These trees, measured in the 1989 Missouri inventory, were used to validate the final models. The same methods were used to measure trees in both the original and validations sets.

METHODS

The gross volume of each detailed measurement tree was estimated using models that were fit to Gevorkiantz and Olsen's (1955) composite volume tables for the Lake States (Hahn 1984). These models estimate gross cubic foot volume in each tree and gross board foot volume in all sawtimber-size trees. Estimated volumes are inside bark from a 1-ft high stump to the merchantable limit based on tree measurements: dbh, merchantable height, and top dob. This estimated volume was then corrected for species differences in bark thickness using models developed for the region from felled tree measurements to arrive at adjusted gross volume (Hahn 1984). This method of estimating volume, one of ten tested

Table 5. Parameter estimates for the gross cubic foot volume and proportion cull models for the Central States.

					Percent cull[b]					
	Gross cubic foot volume[a]				Growing-stock trees		Rough trees		Rotten trees	
Species group	b_1	b_2	b_3*1000	b_4	a_1	a_2	a_1	a_2	a_1	a_2
Red-white-jack pine	122.77	0.4148	−0.02397	2.724	−0.715	0.106	18.677	0.000	47.770	0 000
Yellow pine	122.58	0.2068	−0.05767	2.772	0.211	0.000	18.677	0.000	47.770	0 000
Swamp conifers	454.13	0.1367	−0.02547	2.625	1.473	0.000	18.677	0.000	47.770	0 000
Baldcypress	337.22	0.1158	−0.00818	3.200	1.473	0.000	18.677	0.000	47.770	0 000
Eastern redcedar	112.59	0.1250	−0.10633	2.626	−0.340	0.202	4.200	0.643	47.770	0 000
Select white oak	138.51	0.1769	−0.07041	2.667	0.726	0.009	14.660	0.000	44.510	0 000
Other white oak	146.07	0.2422	−0.05269	2.632	0.436	0.071	14.660	0.000	44.510	0 000
Select red oak	167.98	0.2828	−0.04617	2.590	0.410	0.059	21.060	0.000	44.510	0 000
Other red oak	151.65	0.2597	−0.04408	2.672	0.262	0.096	21.060	0.000	44.510	0 000
Select hickory	84.15	0.2452	−0.06483	2.822	0.297	0.083	19.460	0.000	44.510	0 000
Other hickory	60.55	0.2965	−0.06537	2.874	1.637	0.000	19.460	0.000	44.510	0 000
Basswood-beech-y. birch	194.75	0.1745	−0.04508	2.732	0.506	0.129	10.437	0.459	38.440	0 000
Hard maple	118.80	0.2106	−0.07184	2.724	1.270	0.046	11.177	0.279	32.190	0 529
Soft maple	431.19	0.0194	−0.04663	2.678	−0.996	0.217	17.060	0.000	39.420	0 487
Elm-hackberry	191.45	0.1609	−0.04209	2.747	1.226	0.004	18.565	0.363	39.420	0 487
Ash	73.72	0.2283	−0.08079	2.800	1.690	0.000	21.780	0.000	39.420	0 487
Sycamore	326.30	0.1977	−0.03240	2.670	−0.686	0.100	21.469	0.180	49.990	0 000
Cottonwood	479.95	0.1104	−0.03050	2.652	1.318	0.000	21.469	0.180	49.990	0 000
Willow	39.02	0.3854	−0.08776	2.769	1.318	0.000	21.469	0.180	49.990	0 000
Aspen	61.22	0.2840	−0.11838	2.631	1.318	0.000	21.469	0.180	49.990	0 000
Black walnut	137.32	0.1399	−0.07431	2.700	2.698	0.000	11.590	0.974	49.990	0 000
Yellow-poplar	232.88	0.2093	−0.03379	2.760	0.901	0.000	21.469	0.180	49.990	0 000
Other hardwoods	94.99	0.2409	−0.04946	2.845	2.436	0.000	4.080	1.407	34.430	0 000

[a] $\hat{V}_{gross} = b_1 S^{b_2}(1 - e^{b_3 D^{b_4}})$, where D = dbh, S = tree site index, and $\hat{V}_{gross}$ = estimated gross cubic foot volume.
[b] $\hat{P} = a_1 + a_2 D$, where D = dbh, $\hat{P}$ = estimated percent cull.

Table 6. Parameter estimates for the gross board foot volume and proportion cull models for the Central States.

	Gross board foot volume[a]				Percent cull[b]			
					Sawtimber trees		Shortlog trees	
Species group	b_1	b_2	b_3*1000	b_4	a_1	a_2	a_1	a_2
Red-white-jack pine	925.5	0.2640	−0.01818	2.926	−3.217	0.505	2.172	2.823
Yellow pine	815.5	0.1327	−0.04169	2.931	2.288	0.003	2.172	2.823
Swamp conifers	915.5	0.2719	−0.03436	2.644	9.547	0.067	2.172	2.823
Baldcypress	859.7	0.2896	−0.00344	3.496	11.369	0.158	2.172	2.823
Eastern redcedar	629.3	0.1133	−0.22940	2.310	−5.533	0.812	−5.150	2.837
Select white oak	749.1	0.2214	−0.04853	2.729	−8.598	1.321	36.436	1.015
Other white oak	416.1	0.4001	−0.03862	2.750	−5.127	1.048	14.425	2.262
Select red oak	744.2	0.2627	−0.03487	2.804	−6.696	1.077	42.886	0.927
Other red oak	904.2	0.2229	−0.02808	2.861	−9.033	1.270	31.681	1.492
Select hickory	731.9	0.2265	−0.03128	2.926	−5.770	1.013	24.600	2.110
Other hickory	394.5	0.3271	−0.03303	2.999	−7.474	1.086	30.378	1.555
Basswood-beech-y. birch	1325.2	0.1748	−0.02803	2.827	−5.244	1.101	45.750	0.799
Hard maple	585.0	0.2590	−0.03684	2.925	−7.875	1.363	34.600	1.486
Soft maple	2373.9	0.0291	−0.02323	2.880	−5.115	1.251	49.050	0.512
Elm-hackberry	751.4	0.2845	−0.02692	2.819	−7.012	1.389	37.576	1.044
Ash	418.9	0.2300	−0.03323	3.090	−3.602	1.018	32.517	1.347
Sycamore	819.4	0.3402	−0.01626	2.955	−8.511	1.121	63.150	0.197
Cottonwood	916.0	0.2283	−0.02029	2.995	−1.835	0.696	63.150	0.197
Willow	103.9	0.5745	−0.03990	2.996	−9.968	1.608	63.150	0.197
Aspen	117.0	0.3714	−0.03645	3.361	−6.514	1.328	63.150	0.197
Black walnut	527.2	0.2073	−0.04310	2.918	−9.409	1.473	11.456	2.690
Yellow-poplar	569.2	0.3497	−0.01767	3.063	−8.767	0.977	33.970	1.651
Other hardwoods	648.1	0.2008	−0.02784	3.014	−6.251	1.277	33.970	1.651

[a] $\hat{V}_{gross} = b_1 S^{b_2}(1 - e^{b_3 D^{b_4}})$, where D = dbh, S = tree site index, and $\hat{V}_{gross}$ = estimated gross board foot volume.
[b] $\hat{P} = a_1 + a_2 D$, where D dbh, $\hat{P}$ = estimated percent cull.

Table 7. Summary statistics for the cubic and board foot volume and proportion cull models for the Central States.

	Gross volume				Percent cull									
					Cubic foot						Board foot			
	Cubic foot		Board foot		Growing stock		Rough		Rotten		Sawtimber		Shortlog	
Species group	$s_{y\cdot x}$[a]	R^2[b]	$s_{y\cdot x}$[a]	R^2[b]	$s_{y\cdot x}$[c]	R^2[d]	$s_{y\cdot x}$[c]	R^2[d]	$s_{y\cdot x}$[c]	R^2[d]	$s_{y\cdot x}$[c]	R^2[d]	$s_{y\cdot x}$[c]	R^2[d]
Red-white-jack pine	0.85	0.95	11.2	0.93	1.51	0.026	22.6	e	29.1	e	9.2	0.046	16.5	0.282
Yellow pine	0.75	0.94	9.7	0.91	0.91	e	22.6	e	29.1	e	6.6	0.011	16.5	0.282
Swamp conifers	0.60	0.95	8.6	0.93	4.16	e	22.6	e	29.1	e	15.3	0.013	16.5	0.282
Baldcypress	5.48	0.90	91.9	0.87	4.16	e	22.6	e	29.1	e	14.6	0.007	16.5	0.282
Eastern redcedar	0.52	0.93	9.5	0.83	1.95	0.097	20.1	0.012	29.1	e	6.2	0.074	10.0	0.107
Select white oak	1.07	0.92	25.5	0.90	2.33	0.014	19.7	e	26.3	e	12.5	0.207	12.5	0.235
Other white oak	0.79	0.94	20.5	0.90	3.10	0.009	19.7	e	26.3	e	11.3	0.083	13.1	0.245
Select red oak	1.47	0.94	27.6	0.89	3.23	0.015	19.5	e	26.3	e	12.7	0.166	12.9	0.201
Other red oak	0.96	0.94	24.0	0.90	4.01	0.016	19.5	e	26.3	e	12.7	0.185	12.9	0.336
Select hickory	0.88	0.93	24.8	0.90	3.88	0.009	19.5	e	26.3	e	11.5	0.085	10.7	0.371
Other hickory	0.80	0.94	22.3	0.92	5.99	e	19.5	e	26.3	e	10.8	0.102	13.7	0.189
Basswood-beech-y birch	1.18	0.94	29.5	0.91	5.46	0.025	19.2	0.035	20.5	e	13.5	0.131	15.1	0.118
Hard maple	0.99	0.94	27.5	0.91	3.68	0.006	17.6	0.010	25.8	0.029	12.9	0.169	11.0	0.340
Soft maple	1.24	0.95	26.5	0.94	4.82	0.083	19.5	0.126	28.7	0.019	14.4	0.244	17.1	0.079
Elm-hackberry	0.81	0.92	24.8	0.89	2.85	0.011	17.7	0.008	28.7	0.019	13.5	0.219	14.8	0.196
Ash	0.89	0.94	23.4	0.90	4.95	e	18.1	5	28.7	0.019	13.4	0.092	11.9	0.306
Sycamore	1.50	0.95	32.5	0.94	2.72	0.049	15.5	0.015	18.7	e	12.1	0.225	18.3	0.012
Cottonwood	1.95	0.90	37.1	0.91	4.04	e	15.5	0.015	18.7	e	13.7	0.138	18.3	0.012
Willow	1.16	0.90	25.1	0.85	4.04	e	15.5	0.015	18.7	e	14.6	0.115	18.3	0.012
Aspen	0.76	0.93	18.2	0.84	4.04	e	15.5	0.015	18.7	e	14.6	0.040	18.3	0.012
Black walnut	0.94	0.95	20.8	0.89	5.59	e	22.5	0.041	18.7	e	11.6	0.127	12.0	0.388
Yellow-poplar	1.47	0.94	34.1	0.91	3.17	e	15.5	0.015	18.7	e	10.8	0.152	18.3	0.012
Other hardwoods	0.77	0.95	23.8	0.92	6.50	e	29.5	0.041	42.1	0.019	14.6	0.115	17.9	0.182

[a] $s_{y\cdot x} = \sqrt{\sum_{i=1}^{N} (V_i - \hat{V}_i)^2/(N - 4)}$ where V is the observed gross volume (the adjusted gross volume of the tree based on the height and top d.o.b. measurements), $\hat{V}$ is the model predicted gross volume, and N is the number of trees used in fitting.

[b] $R^2 = 1 - \sum_{i=1}^{N} (V_i - \hat{V}_i)^2 / \sum_{i=1}^{N} (V_i - \bar{V}_i)^2$ where V, $\hat{V}$, and N are as above, and $\bar{V}$ is the average observed gross volume.

[c] $s_{y\cdot x} = \sqrt{\sum_{i=1}^{N} (P_i - \hat{P}_i)^2/(N - 2)}$ where P is the observed percent cull, $\hat{P}$ is the model predicted percent cull, and N is the number of trees used in fitting.

[d] $R^2 = 1 - \sum_{i=1}^{N} (P_i - \hat{P}_i)^2 / \sum_{i=1}^{N} (P_i - \bar{P}_i)^2$ where P, $\hat{P}$, and N are as above, and $\bar{P}$ is the average actual gross volume.

[e] The linear regression for percent cull produced an unreasonable volume that was not significantly different from zero. Mean percent cull was used to estimate percent cull in this case. The value reported for s_{yx} is:

$$s_y = \sqrt{\sum_{i=1}^{N} (P_i - \bar{P}_i)^2/(N - 1)}$$ where P, $\bar{P}$, and N are as above. No R^2 value is reported in this case.

by Martin (1984), finished first in overall ranking for bias, mean absolute difference, and standard variation in estimating log and tree volumes when true volumes were measured using water displacement techniques.

Because a portion of the data set was measured to a variable top dob rather than to the fixed top dob currently used, it was necessary to estimate the volume of the portion of the tree between the observed top dob and the fixed top dob. First, the length of the missing section was estimated based on the observed average taper from the detailed measurement trees that had upper stem diameter measurements taken at two different points (Table 4). The volume of this section was then estimated using Smalian's formula and was added to the adjusted gross volume. This estimated volume was also added to the cull deduction estimated in the field, because it would have been classed as missing. This process provided an estimate of gross tree volume to a fixed top dob for each detailed measurement tree. This volume served as the observed dependent variable in the nonlinear regression model.

The nonlinear model selected to estimate gross volume in this study was the same model used to estimate net volume in the Indiana (1987) forest inventory. The model is the modified Weibull model, $V = \beta_1 S^{\beta_2}(1 - e^{\beta_3 D^{\beta_4}}) + \epsilon$, where D = dbh, S = tree site index, V = gross volume, β_1 through β_4 are regression parameters, and ϵ is a random error. Species-specific model parameters were estimated for cubic foot and board foot (International ¼-in. log rule) volumes computed as described above for merchantable height to 4-, 7-, or 9-in. top dob, as appropriate. The values b_1 through b_4 are the estimates of β_1 through β_4 used to estimate gross volume.

Percent cull in each of the detailed measurement trees was estimated by dividing the estimated cull deduction by the gross predicted volume. After testing several models to predict percent cull, the simple linear model, $P = \alpha_1 + \alpha_2 D + \epsilon$, where D = dbh, P = percent cull, α_1 and α_2 are regression parameters, and ϵ is a random error, was selected. Different parameter sets were estimated for each tree class. Parameters were estimated for most of the species groups used by FIA. Some groupings were necessary when data were insufficient for a viable regression analysis (Table 3). As with the gross volume model, the values a_1 and a_2 are the estimates of α_1 and α_2 which are used to estimate percent cull.

We fit the gross volume models using a nonlinear regression package developed by McRoberts (in prep.).

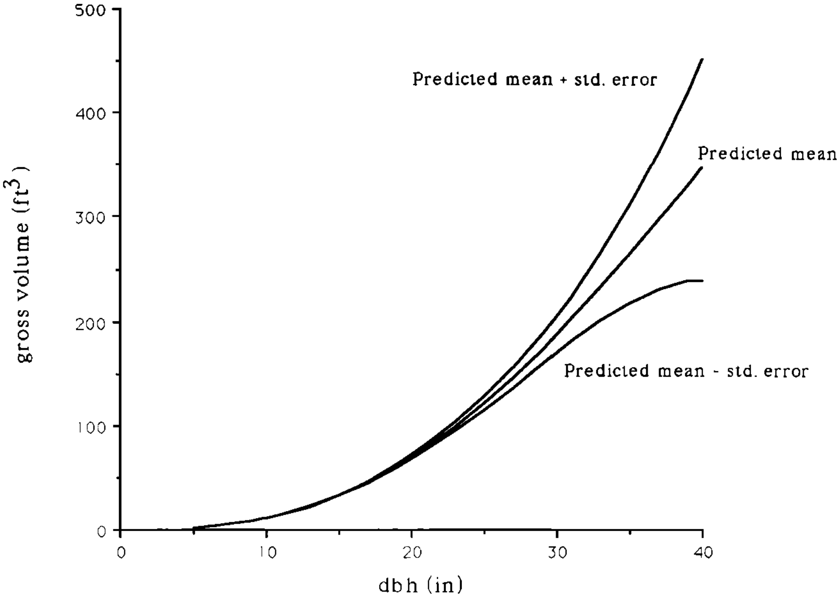

Fig. 2. Estimated gross cubic foot volume and one standard deviation confidence band from Central States model for sycamore, site index 70.

Commercial linear regression software was used to fit the percent cull models. Extensive plotting of the residual volumes against site index and species was done to detect any trends that would indicate an inappropriate model or inappropriate species groupings. Because of the typical increasing variability in the observed gross volume with larger diameters, each observation was weighted in proportion to the square of the inverse of the predicted volume when the gross volume models were fit.

Prediction errors were also plotted for each state in which the tree was measured to determine how well the model fit across the region. Figure 1 is an example of this residual analysis by state. This figure shows how well the final gross board foot volume model for one species group (select red oaks) performed across the four states. Here we have plotted the prediction residuals (predicted-observed) against dbh. Simple linear regressions were fit through these residual data sets These regressions produced parameter estimates that were not significantly different from zero. No clear tendency of the models to over or underpredict volume in any state was found.

The test data set was used to see how well the models predicted volume on a data set other than the one used in fitting. The 1989 Missouri test data set was not totally independent of the one used to fit the model because it contained some trees on remeasurement plots that were measured in 1972. The number of trees common to both data sets was not considered large enough to cause con-

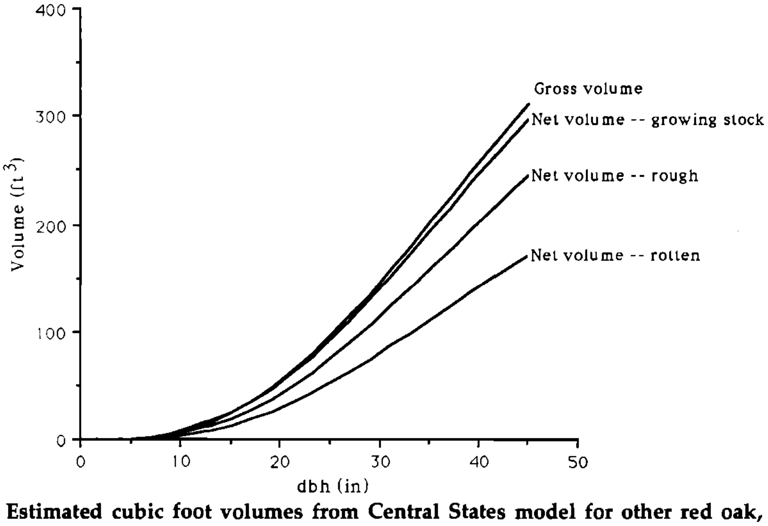

Fig. 3. Estimated cubic foot volumes from Central States model for other red oak, site index 70.

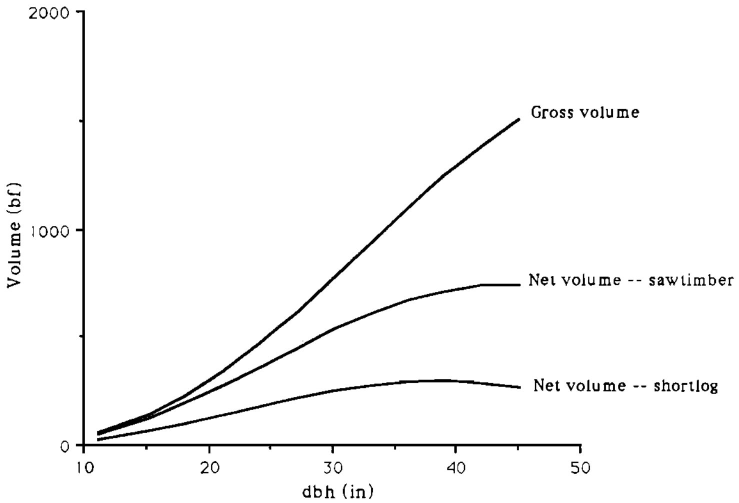

Fig. 4. Estimated board foot volumes from Central States model for select red oak, site index 70.

cern, especially because the measurement dates were 17 years apart.

Two measures of prediction error were used to evaluate how well the fitted models predicted volume. These two statistics were also used by Burk et al. (1989) to evaluate volume models used by FIA in the Northeast. Following the same notation, these two statistics are:

$$\text{ERROR} = 100 \cdot \frac{\sum_{i=1}^{N} (p_i - a_i)}{\sum_{i=1}^{N} a_i}$$

ABSOLUTE ERROR

$$= \frac{100}{N} \cdot \sum_{i=1}^{N} \left| \frac{(p_i - a_i)}{a_i} \right|,$$

where p_i is the estimated volume of the tree given by the fitted volume model, a_i is the adjusted volume of the tree based on the height and top dob measurements, and N is the number of trees. Both error statistics are expressed as percentages. ERROR is an estimate of the volume prediction error one would encounter if a large number of estimates were averaged or totaled, as is done in forest surveys. It is an estimate of the average percent bias of the model. ABSOLUTE ERROR is an estimate of the error that could be expected if one predicted the volume of an individual tree; small errors on small trees contribute as much as large errors on large trees.

RESULTS AND DISCUSSION

The parameter estimates for the gross volume and percent cull models, both cubic foot and board foot, are shown in Tables 5 and 6. The residual standard deviations ($s_{y \cdot x}$) and parameters of determination (R^2) of the regressions are shown in Table 7. The parameter estimates in Tables 5 and 6 can be used in conjunction with the gross volume and percent cull models to estimate the volume of a tree. For example, the estimated gross cubic

Table 8. Prediction errors for the cubic foot volume models observed with the 1989 Missouri test data set.

	Gross: All live trees			Net: Growing-stock trees			Net: Rough trees			Net: Rotten trees		
Species group	Number of trees	ERROR[a]	ABSOLUTE ERROR[b]	Number of trees	ERROR[a]	ABSOLUTE ERROR[b]	Number of trees	ERROR[a]	ABSOLUTE ERROR[b]	Number of trees	ERROR[a]	ABSOLUTE ERROR[b]
Red-white-jack pine	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Yellow pine	1,221	−2.4	15.6	1,190	−2.5	15.4	28	−5.7	19.9	3	25.7	53.0
Swamp conifers	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Baldcypress	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Eastern redcedar	238	2.2	14.9	178	−4.6	13.9	56	2.0	12.7	4	−25.6	27.9
Select white oak	2,348	4.0	15.2	1,641	−2.9	12.9	599	1.9	17.4	108	5.1	80.3
Other white oak	1,207	2.3	15.3	628	−8.6	13.8	457	−4.3	17.6	122	9.4	64.2
Select red oak	336	4.5	18.4	195	−2.5	13.1	85	−0.1	22.2	56	9.8	126.7
Other red oak	3,171	1.1	15.6	2,075	−5.4	13.7	852	−8.2	18.4	244	7.2	98.0
Select hickory	302	5.2	17.5	199	−2.7	15.4	92	−0.9	18.3	11	−6.7	19.9
Other hickory	453	8.7	17.6	296	1.9	14.3	123	2.5	15.1	34	17.8	109.7
Basswood-beech-y birch	12	18.4	28.9	6	−2.1	8.6	3	−23.4	18.2	3	129.0	120.6
Hard maple	70	11.9	17.8	36	−8.3	15.0	28	6.6	15.2	6	16.2	26.7
Soft maple	46	11.5	22.0	25	6.2	19.5	18	−8.7	28.2	3	12.6	55.0
Elm-hackberry	245	8.0	20.7	116	0.4	16.6	121	0.4	19.9	8	−14.3	36.1
Ash	167	7.5	15.2	80	4.8	12.8	79	−7.3	18.4	8	31.1	49.3
Sycamore	65	0.0	17.3	48	−1.3	14.2	5	−14.5	9.4	12	−14.6	32.7
Cottonwood	89	2.1	15.6	73	−0.2	14.8	13	−11.0	17.0	3	−33.2	39.5
Willow	6	27.3	34.4	3	15.5	28.0	3	0.3	18.5	0	0.0	0.0
Aspen	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Black walnut	119	−2.2	17.3	60	−7.9	15.5	54	−22.4	24.1	5	10.9	32.1
Yellow-poplar	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Other hardwoods	358	4.9	24.2	86	−8.6	28.1	219	−6.9	21.4	53	−25.0	129.2
dbh class (in.)												
5.0–9.9	4,239	−4.3	16.4	3,130	−7.9	15.4	1,024	−9.2	19.1	85	−13.5	52.7
10.0–14.9	3,937	−1.5	14.8	2,623	−6.1	12.7	1,122	−7.6	17.7	192	7.9	123.7
15.0–19.9	1,694	5.1	17.1	964	−1.1	13.4	528	0.3	17.9	202	4.9	73.4
20.0+	583	7.1	21.8	218	0.4	15.4	161	−3.0	19.4	204	4.2	86.2
Overall	10,453	2.5	16.2	6,935	−3.5	14.1	2,835	−4.0	18.4	683	4.5	88.8

[a] $\text{ERROR} = 100 \cdot \frac{\sum_{i=1}^{N} (p_i - a_i)}{\sum_{i=1}^{N} a_i}$

[b] $\text{ABSOLUTE ERROR} = \frac{100}{N} \cdot \sum_{i=1}^{N} \left| \frac{(p_i - a_i)}{a_i} \right|,$

where p_i is the estimated volume of the tree given by the volume model, a_i is the adjusted volume of the tree based on the height and top dob measurements, and N is the number of trees.

foot volume of a 15 in. dbh sycamore and having a site index of 70 would be computed as follows:

$$\begin{aligned}\hat{V}_{gross} &= b_1 S^{b_2}(1 - e^{b_3 D^{b_4}}) \\ &= 326.30 \cdot 70^{0.1977} \\ &\quad (1 - e^{-.00003240 \cdot 15^{2.670}}) \\ &= 33.1 \text{ ft}^3\end{aligned}$$

If this tree was a growing-stock tree, its estimated proportion cull would be $\hat{P} = a_1 + a_2 D = -0.0686 + 0.100 \cdot 15 = 1.43\%$. The estimated net cubic foot volume in such a tree would be

$$\begin{aligned}\hat{V}_{net} &= (\hat{V}_{gross})(1 - \hat{P}) \\ &= (33.1)(1 - .0143) \\ &= 32.8 \text{ ft}^3\end{aligned}$$

If this tree did not meet the standard of growing stock, but was classified as a rough tree, the estimated net cubic foot volume would be $33.1 \cdot (1 - (21.469 + 0.180 \cdot 15)/100) = 25.1$ ft^3. To avoid problems that might result from inappropriately using the percent cull models beyond the range of the data used to develop them, we restrict the predicted percent cull to be no greater than the predicted percent cull for the largest observed tree diameter (Table 3). Gross and net board foot volumes (International ¼ in. log rule) are estimated the same way using the parameter estimates in Table 6. Others may wish to use the models to estimate gross volumes and then apply their own estimates of cull to estimate net volume.

The computation of standard errors for these estimates is fairly complex and is discussed in the appendix. In general, the standard errors of the predicted mean gross volumes given by these models do not exceed 10% expect for diameters larger than 30 in. Standard errors also can exceed 10% for low site index values where there were few observations. Figure 2 shows the estimated gross cubic foot volume for sycamore, site index 70, across a range of diameters. This figure also shows the predicted value plus and minus one standard error. The band defined by these two lines is fairly narrow for small diameters; however, it increases in width rapidly above 35 in. dbh. The standard error of the predicted mean does not exceed 10% for diameters less than 30 in but reaches 20% at 34 in. dbh.

As with any volume model, two different standard errors exist. The commonly presented standard error is the standard error of the predicted mean This standard error is appropriate if one is using the model to estimate the average volume of all trees having a specific set of characteristics (here dbh, species, and site index). For the previous example (15 in. sycamore, site index 70) the estimated average gross volume is 33.1 ft^3. The standard error of this estimate is 0.72 ft^3 or 2.2%. If one is trying to predict the gross volume of a specific 15 in. sycamore, site index 70, the appropriate standard error to consider is the standard error of a predicted value, which is considerably higher, in this case 2.85 ft^3 or 8.6%.

Figure 3 shows the typical relationships among cubic foot volume estimates produced by these models. The net cubic foot volume of growing-stock trees is always slightly larger than that of rough trees. Growing-stock trees tend to have less rotten or

Table 9. Prediction errors for the board foot volume models observed with the 1989 Missouri test data set.

	Gross: Sawtimber and shortlog trees			Net: Sawtimber			Net: Shortlog trees		
Species group	Number of trees	Error[a]	Absolute error[b]	Number of trees	Error[a]	Absolute error[b]	Number of trees	Error[a]	Absolute error[b]
Red-white-jack pine	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Yellow pine	724	−2.7	13.5	718	−5.5	18.2	6	30.8	52.8
Swamp conifers	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Baldcypress	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Eastern redcedar	61	−4.6	12.2	51	−8.5	14.2	10	−2.0	30.8
Select white oak	953	1.8	14.4	789	3.1	26.0	164	10.9	36.7
Other white oak	298	−0.4	12.1	212	0.4	17.2	86	−3.6	29.0
Select red oak	142	2.8	14.7	119	14.9	31.3	23	20.4	218.8
Other red oak	1,230	−0.4	14.2	1,104	3.7	25.7	126	0.6	116.0
Select hickory	84	1.2	15.6	72	−0.1	17.3	12	0.6	14.1
Other hickory	114	8.8	15.8	101	11.8	24.8	13	33.1	84.6
Basswood-beech-y. birch	6	4.1	4.5	6	35.6	34.5	0	0.0	0.0
Hard maple	16	2.6	15.6	15	1.3	18.9	1	−7.3	7.3
Soft maple	8	12.6	18.3	6	38.6	45.2	2	59.2	62.1
Elm-hackberry	58	11.7	24.7	43	11.8	26.4	15	19.0	146.6
Ash	55	5.5	15.0	43	9.4	46.5	12	−8.7	44.8
Sycamore	41	−9.4	24.8	41	−2.2	26.9	0	0.0	0.0
Cottonwood	71	−7.3	16.7	66	1.4	47.6	5	1.5	21.3
Willow	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Aspen	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Black walnut	38	2.3	16.8	30	19.4	32.7	8	8.2	56.2
Yellow-poplar	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
Other hardwoods	37	−6.2	12.6	29	−9.4	18.8	8	11.0	237.5
dbh class (in.)									
9.0–14.9	2,547	−2.3	14.1	2,263	−1.9	20.8	284	0.8	51.6
15.0–19.9	1,132	1.4	14.2	964	2.4	27.3	168	15.9	108.1
20.0–24.9	186	5.5	16.4	159	12.6	42.1	27	8.3	75.2
25.0+	71	−6.0	20.3	59	7.9	55.3	12	−18.4	44.1
Overall	3,936	−0.6	14.3	3,445	2.2	24.2	491	6.0	72.1

[a] ERROR $= 100 \dfrac{\sum_{i=1}^{N} (p_i - a_i)}{\sum_{i=1}^{N} a_i}$ [b] ABSOLUTE ERROR $= \dfrac{100}{N} \sum_{i=1}^{N} \left| \dfrac{(p_i - a_i)}{a_i} \right|$,

where p_i is the estimated volume of the tree given by the volume model, a_i is the adjusted volume of the tree based on the height and top dob measurements, and N is the number of trees.

missing wood, and more of the top is typically merchantable. The estimated net volumes of rotten trees are less than those of rough trees.

The relationships among board foot volumes are shown in Figure 4. In sawtimber trees, the difference between net and gross volume is quite small for small-diameter trees; however, as the trees get larger this difference increases, reflecting the large amount of cull material typically found in large trees. For shortlog trees—those that contain a merchantable 8-ft sawlog but not a 12-ft sawlog—the difference between gross and net volume is quite large, even for small-diameter trees. The predicted volume for very large shortlog trees is lower because the merchantable section of these trees is often not the butt section, but a small-diameter log someplace in the upper portion of the bole.

The 1989 Missouri test data set was used to determine how well these models would perform on an independent data set collected in a subset of the region. Tables 8 and 9 present the results of these tests. Overall, the models overpredicted gross cubic foot volume by 2.5% and underpredicted gross board foot volume by less than 1% Applying the regional percent cull model to the estimated gross volume resulted in prediction errors of 3.5, 4.0, and 4.5% for net cubic foot volume in growing-stock, rough, and rotten trees, respectively. Net board foot volume was overpredicted by 2.2% for sawtimber trees and by 6.0% for shortlog trees. Prediction of net volume in the cull tree classes (rough, rotten, and shortlog) has always been a problem due to the large variability in percent cull in these trees and the large measurement error associated with estimating cull in standing trees. The low parameters of determination and high residual standard deviations of the regressions for percent cull in these tree classes highlight this problem. In some cases data were insufficient to fit this regression, or the slope of the fitted line was unreasonable and not significantly different from zero. In these cases, the average percent cull of all trees in a specific class was used to estimate percent cull.

The sensitivity of the gross volume models to changes in site index is reflected in the value of the estimated parameter b_2. Soft maple is a species group that had a relatively low b_2 parameter. The form of these trees is apparently not as affected by changes in site index as it is in other species groups such as the oaks and hickories. The effect that site index has on the volume models can be seen in Figure 5, where predicted gross cubic foot

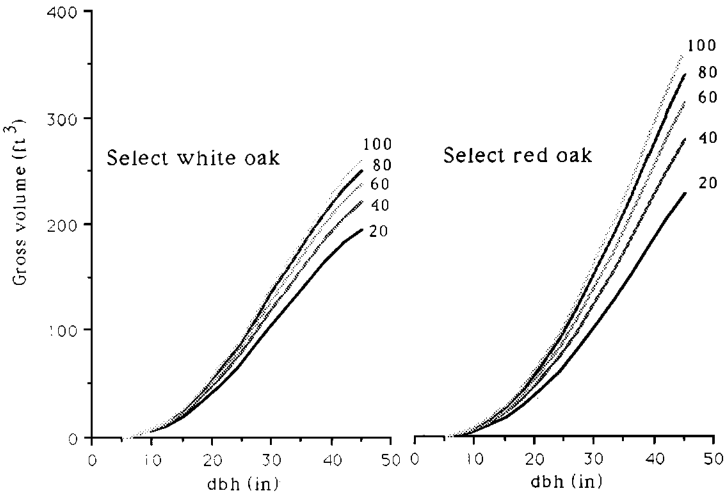

Fig. 5. Comparison of estimated gross cubic foot volumes from the Central States model for two species groups over a range of site indices.

volume of select white oak is compared to that of select red oak over a range of site index values. The volume models indicate that select red oak is more sensitive to site index than select white oak.

The gross volume models estimated in this study produce consistent tree volume estimates that are adequate for FIA needs. The percent cull models used to convert gross volume to net volume are by far the weakest part of this system, especially in the cull tree classes. The use of a simple linear model with dbh as the predictor variable to estimate percent cull was only a slight improvement over using the observed mean percent cull as the estimate for all diameters. Because growing-stock trees are of higher value and make up the majority of the population, estimating the volumes in these trees is of greater importance to FIA. Because cull trees have lower value and are less common than growing-stock trees, the impact of using less precise volume models for these trees is not as great. Additional investigation to develop better estimates of percent cull could improve net volume estimates, especially for trees in the cull classes. □

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APPENDIX

FIA tree classification

FIA classifies all live trees of commercial species into one of four tree classes. These classes are:

1. Growing stock: Trees having no serious defects that limit present or prospective use. These trees have relatively high vigor and have no visible signs of pathogens that may kill them before rotation age.
2. Rough trees: Trees that do not contain at least one merchantable 12-ft. log, now or prospectively,

Appendix Table 1. Number of trees used to fit the Central States gross cubic foot volume model by individual species.[a]

Species group name and common name	Scientific name	No. trees
Red-jack-white pine		
Red pine	*Pinus resinosa*	294
Eastern white pine	*Pinus strobus*	291
Jack pine	*Pinus banksiana*	117
Scotch pine	*Pinus sylvestris*	112
Yellow pines		
Shortleaf pine	*Pinus echinata*	1265
Virginia pine	*Pinus virginiana*	210
Loblolly pine	*Pinus taeda*	7
Swamp conifers		
Northern white-cedar	*Thuja occidentalis*	192
Tamarack	*Larix laricina*	94
Eastern hemlock	*Tsuga canadensis*	21
Balsam fir	*Abies balsamea*	36
White spruce	*Picea glauca*	22
Black spruce	*Picea mariana*	17
Baldcypress		
Baldcypress	*Taxodium distichum*	43
Eastern redcedar		
Eastern redcedar	*Juniperus virginiana*	504
Select white oak		
White oak	*Quercus alba*	8275
Bur oak	*Quercus macrocarpa*	852
Chinkapin oak	*Quercus muehlenbergii*	482
Swamp white oak	*Quercus bicolor*	182
Swamp chestnut oak	*Quercus michauxii*	15
Other white oak		
Post oak	*Quercus stellata*	2416
Chestnut oak	*Quercus prinus*	648
Overcup oak	*Quercus lyrata*	11
Select red oak		
Northern red oak	*Quercus rubra*	2891
Cherrybark oak	*Q. falcata var. pagodifolia*	43
Shumard oak	*Q. shumardii var. shumardii*	29
Other red oak		
Black oak	*Quercus velutina*	5519
Scarlet oak	*Quercus coccinea*	1018
Pin oak	*Quercus palustris*	553
Blackjack oak	*Quercus marilandica*	425
Shingle oak	*Quercus imbricaria*	232
Southern red oak	*Quercus falcata*	92
Northern pin oak	*Quercus ellipsoidalis*	72
Willow oak	*Quercus phellos*	12
Select hickory		
Shagbark hickory	*Carya ovata*	1730
Mockernut hickory	*Carya tomentosa*	653
Shellbark hickory	*Carya lacinosa*	172
Pecan	*Carya illinoensis*	37
Other hickory		
Pignut hickory	*Carya glabra*	1624
Bitternut hickory	*Carya cordiformis*	694
Black hickory	*Carya texana*	201
Basswood-beech-y. birch		
Basswood	*Tilia americana*	1434
American beech	*Fagus grandifolia*	853
Yellow birch	*Betula alleghaniensis*	55
Hard maple		
Sugar maple	*Acer saccharum*	2276
Black maple	*Acer nigrum*	93
Soft maple		
Silver maple	*Acer saccharinum*	1084
Red maple	*Acer rubrum*	663
Elm-hackberry		
American elm	*Ulmus americana*	1466
Slippery elm	*Ulmus rubra*	736
Hackberry	*Celtis occidentalis*	429
Winged elm	*Ulmus alata*	56
Rock elm	*Ulmus thomasii*	55
Ash		
White ash	*Fraxinus americana*	1558
Green ash	*Fraxinus pennsylvanica*	504
Black ash	*Fraxinus nigra*	34
Blue ash	*Fraxinus quadrangulata*	27
Sycamore		
Sycamore	*Platanus occidentalis*	998
Cottonwood		
Eastern cottonwood	*Populus deltoides*	933
Willow		
Black willow	*Salix nigra*	562
Aspen		
Quaking aspen	*Populus tremuloides*	1779
Bigtooth aspen	*Populus grandidentata*	516
Balsam poplar	*Populus balsamifera*	102
Black walnut		
Black walnut	*Juglans nigra*	1110
Yellow-poplar		
Yellow-poplar	*Liriodendron tulipifera*	1213
Other hardwoods		
Black cherry	*Prunus serotina*	590
Sassafras	*Sassafras albidum*	548
Sweetgum	*Liquidambar styraciflua*	376
Boxelder	*Acer negundo*	338
Honeylocust	*Gleditsia triacanthos*	276
River birch	*Betula nigra*	267
Black locust	*Robinia pseudoacacia*	236
Swamp tupelo	*Nyssa sylvatica var. biflora*	231
Persimmon	*Diospyros virginiana*	99
Black tupelo	*Nyssa sylvatica var. sylvatica*	96
Osage-orange	*Maclura pomifera*	90
Butternut	*Juglans cinerea*	80
Flowering dogwood	*Cornus florida*	68
Red mulberry	*Morus rubra*	57
Ohio buckeye	*Aesculus glabra*	50
Kentucky coffeetree	*Gymnocladus dioicus*	37
Eastern hophornbeam	*Ostrya virginiana*	25
Paper birch	*Betula papyrifera*	19
Eastern redbud	*Cercis canadensis*	17
Hawthorn	*Crataegus spp.*	13
Northern catalpa	*Catalpa speciosa*	13
Sugarberry	*Celtis laevigata*	11
Cucumbertree	*Magnolia acuminata*	8
White mulberry	*Morus alba*	4
American hornbeam	*Carpinus caroliniana*	4
Ailanthus	*Ailanthus altissima*	3
Apple	*Malus spp.*	1

[a] The common and scientific names are based on: Little (1979).

because of roughness or poor form.

3. Rotten trees: Rough trees in which more than 50% of the cull volume is rotten.
4. Shortlog trees: Sawtimber size trees that contain at least one merchantable 8 to 11 ft long sawlog but not a 12-ft sawlog. The shortlog tree class is a subset of the rough tree class.

Computation of approximate standard errors of estimated volumes

Computing an approximate standard error of an estimated gross volume is somewhat complicated for the nonlinear gross volume model used here. Bates and Watts (1988) present the commonly accepted method based on linear approximation. In addition to the estimated parameters of the model (b_1, b_2, b_3, and b_4) this method requires the residual mean squared error of the estimate, the derivative vector of the model evaluated at the point of interest (which can be computed), and the asymptotic variance-covariance matrix of the fitted model (which are not presented in this paper but are available from the authors on request).

The derivative vector of the model is simply the set of partial derivatives with respect to the model parameters evaluated at the parameter estimates. In this case the model is $V = \beta_1 S^{\beta_2}(1 - e^{\beta_3 D^{\beta_4}}) + \epsilon$, and the derivative vector is

$$\hat{\mathbf{V}}_o = \left[\begin{array}{c} \dfrac{\partial V}{\partial \beta_1} \\ \dfrac{\partial V}{\partial \beta_2} \\ \dfrac{\partial V}{\partial \beta_3} \\ \dfrac{\partial V}{\partial \beta_4} \end{array}\right]_{\mathbf{b}} = \left[\begin{array}{c} S^{b_2}(1 - e^{b_3 D^{b_4}}) \\ b_1 S^{b_2}(1 - e^{b_3 D^{b_4}})\ln(S) \\ -b_1 S^{b_2}(e^{b_3 D^{b_4}})D^{b_4} \\ -b_1 S^{b_2}(e^{b_3 D^{b_4}})b_3 D^{b_4}\ln(D) \end{array}\right] = \left[\begin{array}{c} 0.10135 \\ 140.33 \\ -989474 \\ 87.515 \end{array}\right]$$

when evaluated using the estimated parameters b_1, b_2, b_3, and b_4 for sycamore with $S = 70$ and $D = 15$ as in the example in the text. The estimated asymptotic variance-covariance matrix (the nonlinear equivalent of $[X'X]^{-1}$ in linear regression) of the fitted cubic foot model for sycamore is the following 4×4 matrix.

$$\hat{\mathbf{A}} = \begin{pmatrix} 0.5056E+04 & -0.1302E+01 & 0.2712E-03 & -0.6837E+00 \\ -0.1302E+01 & 0.9856E-03 & 0.4134E-08 & -0.7228E-04 \\ 0.2712E-03 & 0.4134E-08 & 0.2436E-10 & -0.4884E-07 \\ -0.6837E+00 & -0.7228E-04 & -0.4884E-07 & 0.3780E-03 \end{pmatrix}$$

The residual mean squared error of the estimated volume is also required to compute an approximate standard error of the estimated volume. In this case the residual mean squared error is dependent on the predicted gross volume $\hat{V}_{\text{gross}}$. The simple linear model

$$s^2_{\hat{V}\cdot x} = \chi_1 + \chi_2 \hat{V}_{\text{gross}} + \epsilon$$

was used to estimate residual mean squared error. For sycamore the estimated residual mean squared error is $\hat{s}^2_{\hat{V}\cdot x} = 0.8629 + 0.2013(\hat{V}_{\text{gross}})$. For the example in the text this gives an estimated residual mean squared error of $\hat{s}^2_{\hat{V}\cdot x} = 0.8629 + 0.2013(\hat{V}_{\text{gross}}) = 2.56$. The parameter estimates of χ_1 and χ_2 for other species are available from the authors on request.

Finally, given the estimated parameters, the residual mean squared error of the estimate, the derivative vector of the model evaluated at the point of interest and the asymptotic variance-covariance matrix of the fitted model, the standard error of the estimate can be approximated. The approximate standard error of the predicted mean gross cubic foot volume of 15 in. sycamores, site index 70 is

$$\sqrt{\hat{S}^2_{\hat{V}\cdot x}\, \hat{\mathbf{V}}_o^T \hat{\mathbf{A}} \hat{\mathbf{V}}_o} = 0.39 \text{ ft}^3$$

or 1.2%. The approximate standard error of the predicted gross cubic foot volume of a 15 in. sycamore, site index 7 is

$$\sqrt{1 + \hat{S}^2_{\hat{V}\cdot x}\, \hat{\mathbf{V}}_o^T \hat{\mathbf{A}} \hat{\mathbf{V}}_o} = 1.55 \text{ ft}^3$$

or 4.7%.

The Availability of Fuelwood from Vermont's Nonindustrial Private Forest Lands (NIPF)[1]

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ABSTRACT. Vermont's nonindustrial, private forest lands (NIPF) have the potential of supplying 3.3 million cords of fuelwood annually to the Northeast energy market. Annual commercial production levels during the energy crisis of the early 1980s, however, only reached 660,000 cords; 342,000 cords came from NIPF. An additional 340,000 cords were cut and consumed by NIPF owners. Low stumpage prices, landowner self-consumption, adverse esthetic impacts, and low stumpage volumes restricted the actual amount of NIPF wood sold to the region's commercial energy market. Fuelwood harvest in Vermont was associated with large parcel sizes (>100 ac), the landowner's personal use of fuelwood, and the existence of a forest management plan. Even under these conditions, the percentage of NIPF owners willing to sell to the commercial market remained small (25%). If fuelwood is to establish itself as a reliable energy resource in the Northeast, large ownerships must be protected from subdivision and fuelwood stumpage prices must increase dramatically.

North. J. Appl. For. 8(2):57–59.

Fifty percent of the woodland owners in 27 (10%) of Vermont's 276 townships were surveyed in 1984. The study towns were selected using a systematic random sampling procedure. Property tax records were used to select potential respondents, and a questionnaire was mailed to each selected individual. A second questionnaire was sent to nonrespondents 2 weeks after the initial mailing. A total of 3,024 NIPF landowners (5% of the NIPF population) were surveyed, and a 54% usable return was achieved.

NIPF HARVESTING PROFILE

Widman and Birch (1988) found that 60,600 landowners owned 3,354,400 ac of NIPF in Vermont in 1983. Thirty percent of this acreage was in parcels 100 ac or less (Table 1). An earlier study by Kingsley and Birch (1977) showed that the domestic use of wood was the third most important reason for owning Vermont NIPF, yet only

[1] This study was conducted by the School of Natural Resources at the University of Vermont as part of a regional project (NE-142) supported by the USDA Hatch and McIntire-Stennis research funds.