# Benchmarking and Calibration of Forest Vegetation Simulator Individual Tree Attribute Predictions Across the Northeastern United States

Matthew B. Russell, Aaron R. Weiskittel, and John A. Kershaw Jr.

This study used permanent sample plot data from the USDA Forest Service's Forest Inventory and Analysis database to benchmark and calibrate three key submodels of the Forest Vegetation Simulator-Northeast variant (FVS-NE). Model predictions for total tree height (ht) and 5-year diameter ( $\Delta dbh_5$ ) and height increment ( $\Delta ht_5$ ) for the 20 most abundant species did not indicate any serious spatial trends. FVS-NE predictions for total ht performed moderately well, as mean bias averaged  $-0.9 \pm 5.2$  ft (mean  $\pm$  SD) across all species. FVS-NE  $\Delta dbh_5$  predictions fell within 15% of observed values between 8.4 and 17.3% of the time and performed best for shade-tolerant species and worst for intermediate shade intolerants. For  $\Delta ht_5$ , the number of predictions that fell within 15% of observed values averaged 7.7%. Submodel performance generally improved after calibrating FVS-NE predictions using tree size, site, and climate variables. After employing a calibrated  $\Delta dbh_5$ , 5-year basal area growth continued to be underpredicted across all ecoregions and forest types. Results indicate that (1) an assessment of overall model performance should be conducted if calibrated submodels are used and (2) alternative modeling strategies be explored to better represent the allometry and growth of the important trees species across the northeastern United States.

Keywords: Forest Inventory and Analysis (FIA), model validation, diameter increment, height increment, height-diameter

The Forest Vegetation Simulator (FVS) is a semi-distance-in-.dependent growth model that forecasts individual trees through time, which are then scaled to estimate stand-level attributes. With the principal goal of projecting future forest growth, composition, and structure, FVS has various uses in management applications ranging from weighing alternative silvicultural prescriptions for individual stands to conducting landscape and regional assessments (Crookston and Dixon 2005). Different geographic variants of FVS employ a myriad of submodels that together forecast forest growth. Specifically, the Northeast variant of FVS (FVS-NE) was originally based on the equations found in the NE-TWIGS model (Hilt and Teck 1989) and other FVS variants but has since undergone several revisions (Dixon and Keyser 2008). The diameter increment ( $\Delta$ dbh) submodel for large trees (>5.0 in. dbh) is quite similar to the original NE TWIGS implementation (with some modifications), while other FVS-NE submodels have seen more considerable revisions and updates. The current suggested geographic range for FVS-NE ranges from an eastern boundary in Maine to a western boundary of Ohio and south to West Virginia.

As the volume growth of large trees is primarily driven by  $\Delta dbh$  in FVS (Crookston and Dixon 2005), benchmarking the key submodels within FVS-NE by assessing their predictions would provide insight into the performance of the model across its suggested geographic range. Recently, Ray et al. (2009) compared predictions made by NE-TWIGS and FVS-NE and found significant differences, suggesting the need for a comprehensive assessment of

FVS-NE predictions. Assessing the baseline performance of these submodels is necessary as users employ these predictions in model extensions such as the Fires and Fuels Extension (Rebain et al. 2010) and Climate FVS (Crookston et al. 2010).

Model benchmarking of FVS variants has previously been accomplished through various techniques. For example, Lacerte et al. (2004) found inaccuracies when comparing observed and predicted stand-level basal area (BA) following a validation of the Lake States variant of FVS (FVS-LS). They conclude that improvements could be made in refining estimates of mortality (which ultimately influences stand density and, thus, BA) and site index (Lacerte et al. 2004). By assessing the  $\Delta dbh$  function of FVS-LS, Pokharel and Froese (2008) found that it overpredicted  $\Delta dbh$  by 17% on average, and Canavan and Ramm (2000) observed similar results. These studies highlight the need for a careful examination of the individual-tree submodels within the existing FVS framework, as bias at the tree level can compound to the plot and stand levels.

Researchers have observed various levels of success in updating equations and their predictions provided by the various submodels of FVS. In FVS-LS, recalibrating Holdaway's (1985)  $\Delta$ dbh AF did not lead to substantial improvements in  $\Delta$ dbh predictions for 30 species in Michigan (Pokharel and Froese 2008). Refitting submodels such as  $\Delta$ dbh and height increment ( $\Delta$ ht) did, however, lead to considerable improvements in biological consistency over FVS-LS model predictions (Lacerte et al. 2006). Similarly, predictions of plot-level BA and stem density were improved over a short time

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period (5 years) after updating the individual tree-mortality submodel in FVS's Southern variant (Radtke et al. 2012). On the other hand, Rijal et al. (2012a, 2012b) recently found refitting FVS-NE tree height and crown ratio equations to be ineffective for improving predictions when compared to utilizing a different equation form for several species in Maine. This suggests that effectiveness of model calibration may depend on the performance of each submodel, the species examined, and the geographic extent of the evaluation data sets. It should be noted that although FVS-NE has an inherent self-calibration algorithm for some submodels (Dixon and Keyser 2008); calibration here is defined as improvements that can be applied to individual tree attribute predictions without requiring direct measurements such as tree diameter or height increment. Most importantly, the performances of key FVS-NE submodels have yet to be tested for the dominant species, ecoregions, site quality classes, and forest types that are found across the 13 northeastern states.

The purpose of this analysis was to investigate the performance of key submodels within FVS-NE and to seek improvements of individual submodels. Specific objectives were to: (1) quantify the performance of the ht-dbh,  $\Delta$ dbh, and  $\Delta$ ht submodels within FVS-NE using independent data gathered across the northeastern United States (benchmarking), (2) examine predictions for individual tree attributes after making adjustments to submodel output using a modified site index or a suite of tree, stand, and climate variables (calibration), and (3) compare the accuracy of a noncalibrated and calibrated FVS-NE in predicting plot-level BA growth.

## **Methods**

#### **FVS-NE Submodels**

Total tree height (ht; ft) is estimated using tree dbh (in.) for trees where ht is missing in a tree list and is influential in estimating  $\Delta dbh$  for small trees in FVS-NE (Dixon and Keyser 2008). Model form of the ht-dbh equation depends on tree species. For the species examined in this analysis, conifer species are estimated with the Curtis (1967) form:

$$ht = 4.5 + \alpha_1 \exp(-\alpha_2 dbh^{\alpha_3})$$
 (1)

where as hardwoods are estimated with the Wykoff et al. (1982) form

$$ht = 4.5 + \exp(\beta_1 + \beta_2/(dbh + 1.0))$$
 (2)

where the regression coefficients  $\alpha_i$  and  $\beta_i$  are species specific and derive from FVS's Southern variant, while a separate set of coefficients are available for trees found in the Allegheny National Forest in Pennsylvania (Dixon and Keyser 2008).

The  $\Delta$ dbh for trees  $\geq$  5.0 in. in FVS-NE is estimated using the potential-modifier approach initially outlined in Teck and Hilt (1991) but with some modification as implemented in FVS-NE. These modifications include a different interpretation of BA in larger trees, an adjustment factor (AF) (0.7), and bounds applied to the growth-modifier component. First, potential BA growth ( $\Delta$ ba<sub>pot</sub>) is estimated using dbh and site index (SI). Second, a growth modifier is estimated using the BA in larger trees two 1-in. diameter classes below the subject tree (bal). Lastly, predicted annual BA growth ( $\Delta$ ba) is estimated by multiplying the potential and modifier components

$$\Delta ba = (\gamma_1 SI(1 - \exp(-\gamma_2 dbh)) * 0.7) * (\exp(-\gamma_3 bal))$$
 (3)

where  $\gamma_i$  are species group-specific regression coefficients from the current implementation of FVS-NE (Dixon and Keyser 2008, Table 4.7.1.1). Data used to initially parameterize the large-tree  $\Delta$ dbh equation were collected from permanent sample plots in the 1960s through the 1980s from 14 states in the northeastern United States (Teck and Hilt 1991).

Similarly, the potential-modifier approach is used in FVS-NE for predicting  $\Delta ht$ . First, growth effective age (GEA) is estimated using ht and SI with the equations of Carmean et al. (1989). Second, the tree is grown for 10 years and GEA is used to then predict an updated tree ht. The difference between these two ht values is assumed to be potential height increment (ht<sub>pot</sub>). Modified ht increment is estimated using the growth modifier from Equation 3 and the subject tree's ht divided by the 40 largest-diameter trees in the stand, termed relative ht (ht<sub>rel</sub>)

$$\Delta ht = ht_{pot} * [1 - ((1 - (exp(-\gamma_3 bal)))(1 - ht_{rel}))] * 0.8$$
 (4)

## Forest Inventory and Analysis Data

Tree and plot records were obtained from the USDA Forest Service's Forest Inventory and Analysis (FIA) program (Forest Inventory and Analysis 2011). The majority of these inventory plots were remeasured (73% of all plots), with a mean remeasurement interval of  $5.2 \pm 0.9$  (mean  $\pm$  SD) years. Data from 13 US states where the suggested FVS-NE geographic range lies were used. Measurements from 1998 onward were used, which was when the FIA's annual inventory design was implemented across the United States (McRoberts et al. 2005). Data from the FIA plots were compiled from both subplots (trees  $\geq$  5.0 in. dbh) and microplots (trees 1.0-4.9 in. dbh). Only plots with no visible disturbance since the last remeasurement (or in the last 5 years for plots that were initially measured) were used in this analysis.

Compiled data spanned seven ecoregions as defined by Bailey (1980). Ecoregions examined were (codes in parentheses): Laurentian mixed forest (212), eastern broadleaf (oceanic [221] and continental [222]), western Allegheny plateau (223), outer coastal plain mixed forest (232), Adirondack-New England mixed forest (M212), and central Appalachian broadleaf forest (M221). The primary forest type groups examined were: aspen-birch (A-B), elm-ash-cottonwood (E-A-C), maple-beech-birch (M-B-B), oak-hickory (O-H), oak-pine (O-P), spruce-fir (S-F), and white-red-jack pine (W-R-JP). The five most abundant species in the compiled data included balsam fir (*Abies balsamea* [L.] Mill.), eastern hemlock (*Tsuga canadensis* [L.] Carr), red spruce (*Picea rubens* Sarg.), and red (*Acer rubrum* L.) and sugar maple (*Acer saccharum* Marsh.; Table 1).

Assessments of model performances was limited to all trees with dbh  $\geq 5.0$  in. because (1) it is the threshold for measurement trees on FIA phase II plots, and (2) it is the threshold for the large-tree  $\Delta$ dbh and  $\Delta$ ht equations within FVS-NE. Plot-level metrics such as BA were computed for the conditions of each plot-year combination. Individual species were examined and their size and growth measurements analyzed. As most remeasurements occurred on a 4–6-year interval (84% of all plots), increments for dbh and ht were standardized for each tree that survived a remeasurement period

$$\Delta \operatorname{inc}_5 = \frac{\operatorname{size}_2 - \operatorname{size}_1}{Y_2 - Y_1} \times 5 \tag{5}$$

where  $\Delta inc_5$  is 5-year diameter ( $\Delta dbh_5$ ) or height ( $\Delta ht_5$ ) increment, size<sub>1</sub> is dbh or ht at initial measurement year  $Y_1$ , and size<sub>2</sub> is dbh or ht at remeasurement year  $Y_2$ .

Table 1. Common and scientific names with species codes and number of observations for the most abundant tree species across 13 Northeast states on 9,908 FIA plots.

			ľ	Number of observations			
Species code	Common name	Scientific name	ht	$\Delta dbh_5$	$\Delta ht_5$		
AB	American beech	Fagus grandifolia Ehrh.	6,433	8,460	191		
BC	Black cherry	Prunus serotina Ehrh.	5,177	6,503	-		
BF	Balsam fir	Abies balsamea (L.) Mill.	9,143	12,409	612		
ВО	Black oak	Quercus velutina Lam.	1,704	1,903	-		
BS	Black spruce	Picea mariana (Mill.) B.S.P.	1,846	2,164	-		
CO	Chestnut oak	Quercus prinus L.	4,475	5,306	-		
EH	Eastern hemlock	Tsuga canadensis (L.) Carr.	10,255	12,381	437		
PB	Paper birch	Betula papyrifera Marsh.	4,430	6,973	175		
QA	Quaking aspen	Populus tremuloides Michx.	2,301	2,680	_		
RM	Red maple	Acer rubrum L.	27,926	35,084	1,206		
RO	Northern red oak	Quercus rubra L.	6,671	7,834	_		
RS	Red spruce	Picea rubens Sarg.	9,234	11,835	573		
SB	Sweet birch	Betula lenta L.	4,011	4,990	_		
SM	Sugar maple	Acer saccharum Marsh.	12,768	14,728	-		
WA	White ash	Fraxinus americana L.	5,066	6,011	-		
WC	Northern white-cedar	Thuja occidentalis L.	5,406	8,925	379		
WO	White oak	Quercus alba L.	3,656	4,100	-		
WP	Eastern white pine	Pinus strobus L.	7,783	9,473	363		
YB	Yellow birch	Betula alleghaniensis Britton	5,057	6,769	196		
YP	Yellow poplar	Liriodendron tulipifera L.	1,940	2,045	_		

## **FVS-NE Benchmarking**

Predicting tree ht was accomplished after employing the appropriate ht-dbh model form suggested for the individual species and if the FIA plot was located in the Allegheny National Forest (Dixon and Keyser 2008, p. 10–12). Only heights that were directly measured and displayed positive growth were used in this analysis. Tree records that contained ocular estimates of ht or records where ht was predicted with a model were omitted.

Predicted annual BA increment from Equation 3 was first added to the current tree's BA, and then converted to dbh. This process occurred iteratively until the 5-year cycle was complete. For the increment equations, the FIA SI for the condition of the plot was used. Per their protocol, FIA estimates SI from a single tree or by averaging SI values for multiple site trees (Woudenberg et al. 2010). For plot conditions where SI was not measured, the FVS-NE default setting of 56 ft at a base age of 50 years for sugar maple was used. For all species on the FIA plot, SI was then converted to species SI using the appropriate equation for a species group (Dixon and Keyser 2008, p. 6–7). For predicting  $\Delta$ ht, ht<sub>rel</sub> was computed for each tree in predicting  $\Delta$ ht; if a tree's actual ht was recorded in the FIA tree record, it was used. If not, the tree's estimated ht from (Equation 1 or 2) was used.

For each of the ht,  $\Delta dbh_5$ , and  $\Delta ht_5$  equations for the 20 species, root mean squared error (RMSE) and mean bias (MB) were computed

$$RMSE = \sqrt{\sum_{i=1}^{n} (obs_i - pred_i)^2/n}$$
 (6)

$$MB = \sum_{i=1}^{n} (obs_i - pred_i)/n$$

where obs<sub>i</sub> and pred<sub>i</sub> are the observed and predicted values using the FVS-NE equation for the species of interest for tree i, respectively, and n is the number of observations for a species. The percentage of predictions accurate to within  $\pm$  15% of the observed value (PA<sub>15%</sub>)

was also calculated for each submodel (Rykiel 1996, Radtke et al. 2012). This level of tolerance is likely to account for measurement tolerant thresholds inherent in the FIA data collection process (e.g.,  $\pm$  0.1 in. for dbh; USDA Forest Service 2011) and provides a moderate amount of disagreement between observed and predicted values. Similarly, we conducted equivalence tests comparing observed and predicted values using two one-sided tests (Wellek 2003). Mean bias for individual FIA plots was also computed to examine the spatial pattern of the performance of the FVS-NE equations. This bias was mapped if there were at least five observations on the plot. The  $\Delta$ dbh equation from Equation 3 was used to calculate plot-level basal area (BA;  $\rm ft^2/ac)$  at the remeasurement period and corresponding plot-level BA growth ( $\Delta BA_5$ ) and then was compared to observed  $\Delta BA_5$ .

#### **FVS-NE Calibrations**

The process of calibrating the ht,  $\Delta$ dbh, and  $\Delta$ ht equations began by first examining the prediction errors (observed value—predicted value from FVS-NE) calculated for each tree record. In the first approach, alternative estimates of SI were evaluated. Since 60% of all FIA plots contained no estimate of SI, using a default SI of 56 ft for sugar maple may be questionable for use in the  $\Delta$ dbh and  $\Delta$ ht equations for the entire Northeast. For the remaining 40% of FIA plots, we calculated the average SI for each ecoregion within each of the 13 states (Table 5). After converting to species-specific SI, we computed  $\Delta$ dbh and  $\Delta$ ht predictions using this ecoregion-specific estimate of SI (SI<sub>ECO</sub>) for plots where SI was not recorded.

In the second approach, an adjustment was fitted for each submodel of each species (Holdaway 1985). Prediction errors were plotted against tree, plot, and spatial location variables such as tree size (e.g., dbh or ht), BA, and latitude and longitude (LAT and LONG; decimal degrees) of the plot where the tree was located. Thirty-year (1961–1990) average climate data were obtained by specifying LAT, LONG, and elevation of each FIA plot location to a spline surface model developed from climate station data across forests of North America (Rehfeldt 2006, USDA Forest Service

Table 2. Summaries of plot conditions (n = 9,908) for data acquired from 13 states across the northeastern United States from the Forest Inventory and Analysis database.

Attribute	Mean	SD	Min	Max
BA (ft²/ac)	110.5	63.1	3.3	513.9
Percent BA in conifers	27.8	36.1	0.0	100.0
Percent BA in hardwoods	72.2	36.1	0.0	100.0
Mean annual temperature (° C)	6.8	2.6	0.7	14.0
Growing season precipitation (in.)	23.0	1.9	17.4	34.0
Growing season degree days (>5° C)	1,670.0	444.1	693.0	3,174.0
Latitude (°)	42.9	2.3	37.2	47.4
Longitude (°)	-74.0	4.61	-84.8	-67.0

2012). Climate variables, including mean annual temperature (MAT), growing season precipitation (GSP), and growing season degree-days > 5° C (GSDD5), were similarly plotted against prediction errors.

Hence, the adjustment factor  $(AF_{jk})$  was estimated for the *j*th submodel for the *k*th species in a stepwise regression framework as

$$AF_{jk} = \theta_{1jk} + \theta_{2jk} \operatorname{size} + \theta_{3jk} \operatorname{size}^2 + \theta_{4jk} \operatorname{BA}$$

$$+ \theta_{5jk} \operatorname{LAT} + \theta_{6jk} \operatorname{LONG} + \theta_{7jk} \operatorname{MAT} + \theta_{8jk} \operatorname{GSP}$$

$$+ \theta_{9jk} \operatorname{GSDD} 5 \quad (7)$$

where size is (1) initial tree dbh when calibrating the ht or  $\Delta dbh_5$  submodel or (2) initial tree ht for calibrating the  $\Delta ht_5$  submodel. The calibrated  $\Delta dbh_5$  prediction was used to calculate  $\Delta BA_5$  and then was compared to observed growth.

## Results

## **FVS-NE Benchmarking**

The 20 most abundant species investigated (6 conifers and 14 hardwoods) comprised 86% of all  $\Delta dbh_5$  observations (Table 1). Mean plot BA was  $110.5 \pm 63.1$  ft²/ac (mean  $\pm$  SD; Table 2). Plots contained a greater degree of hardwood dominance, as percent BA in hardwoods was  $72.2 \pm 36.1$ %. Mean observed  $\Delta dbh_5$  ranged from  $0.27 \pm 0.23$  in./5 years (in./5-yr) for black spruce (*Picea mariana* [Mill.] B.S.P.) to  $1.01 \pm 0.78$  in./5-yr for yellow-poplar (*Liriodendron tulipifera* L.). Mean observed  $\Delta ht_5$  ranged from  $3.5 \pm 3.2$  ft/5 years (ft/5-yr) for northern white-cedar (*Thuja occidentalis* L.) to  $5.8 \pm 4.8$  ft/5-yr for red maple.

Performances of the FVS-NE submodels were variable across the region as measured by mean bias (Figure 1). There was general trend of overprediction of ht across northern New England states and underprediction of ht in western locales in the region. In predicting  $\Delta dbh_5$ , there was a general trend of underprediction across much of the region, except in the Allegheny subregion where overprediction of  $\Delta dbh_5$  was evident. Although there were data available from only two states (Maine and Pennsylvania), there were no indications of spatial trends in the submodel performance of  $\Delta ht_5$ .

The PA<sub>15%</sub> for total tree ht predictions for individual species predicted with the FVS-NE model ranged from 13.2% for northern white-cedar to 67.9% for quaking aspen (*Populus tremuloides* Michx.; Table 3). Mean bias for ht averaged across all species was  $-0.9\pm5.2$  ft (Figure 2). The PA<sub>15%</sub> for  $\Delta dbh_5$  ranged from 8.4% for paper birch (*Betula papyrifera* Marsh.) to 17.3% for northern red oak (*Quercus rubra* L.). Mean bias and RMSE for  $\Delta dbh_5$  averaged across all species was  $-0.04\pm0.15$  and  $0.47\pm0.13$  in./5-years, respectively. For the species examined, FVS-NE tended to perform

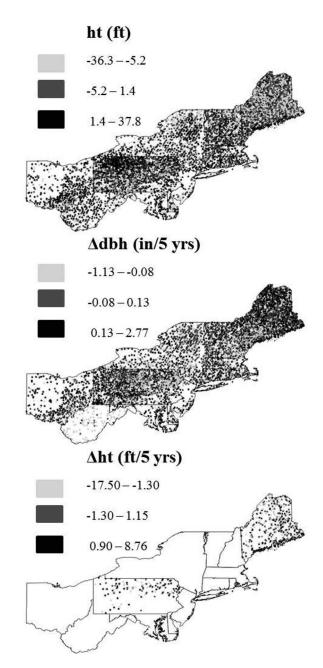


Figure 1. Approximate FIA plot locations displaying mean bias for predictions from FVS-NE variant for total height (ht), and 5-year diameter ( $\Delta dbh_5$ ), and height increment ( $\Delta ht_5$ ). Legend group cutoffs are the 33rd and 67th percentiles of the predictions.

best for predicting the  $\Delta dbh_5$  for shade-tolerant species (e.g., eastern hemlock, northern white-cedar, and red maple) and worst for intermediate shade-intolerant species (e.g., black cherry [*Prunus serotina* Ehrh.], paper birch, and quaking aspen) (Figure 2). The PA<sub>15%</sub> for  $\Delta ht_5$  averaged 7.7% and ranged from 3.4% for paper birch to 9.8% for eastern hemlock (Table 2). For 85, 40, and 0% of the species examined, equivalence tests with a null hypothesis of dissimilarity and a threshold of  $\pm$  15% were rejected for the ht,  $\Delta dbh_5$ , and  $\Delta ht_5$  submodels, respectively (Table 4).

### **FVS-NE Calibrations**

Employing  $SI_{\rm ECO}$  (Table 5) generally did little to improve the accuracy of species-specific FVS-NE growth equations. If employing  $SI_{\rm ECO}$  increased the  $PA_{15\%}$  metric for  $\Delta dbh_5$ , improvements in

Table 3. Percentage of predictions accurate to within  $\pm$  15% of observed values (PA<sub>15%</sub>) and root mean square error (RMSE) using FVS-NE variant predictions, FVS predictions using a modified site index specific to state/ecoregion (FVS<sub>SI</sub>), and calibrated FVS predictions (FVS<sub>CAL</sub>) for total height (ht; ft), and 5-year diameter ( $\Delta$ dbh<sub>5</sub>; in/5-yr) and height increment ( $\Delta$ ht<sub>5</sub>; ft/5-yr).

PA <sub>15%</sub>						RMSE										
ht		ht	$\Delta dbh_5$		$\Delta ht_5$		ht		$\Delta dbh_5$			$\Delta ht_5$				
Species code	FVS	FVS <sub>CAL</sub>	FVS	FVS <sub>SI</sub>	FVS <sub>CAL</sub>	FVS	FVS <sub>SI</sub>	FVS <sub>CAL</sub>	FVS	FVS <sub>CAL</sub>	FVS	FVS <sub>SI</sub>	FVS <sub>CAL</sub>	FVS	FVS <sub>SI</sub>	FVS <sub>CAL</sub>
AB	47.5	55.2	14.4	14.7	16.0	9.4	7.3	20.4	13.2	11.0	0.41	0.40	0.39	4.62	4.77	4.12
BC	44.6	60.4	13.0	13.4	13.5	-	-	-	15.6	11.6	0.67	0.65	0.56	-	-	-
BF	59.1	65.9	14.4	14.9	18.4	9.6	10.8	15.4	8.1	7.2	0.43	0.43	0.38	3.80	3.79	3.05
ВО	62.9	66.3	15.7	15.4	19.9	-	-	-	11.9	10.8	0.52	0.52	0.48	-	-	-
BS	50.4	68.8	14.1	14.0	16.8	-	-	-	9.2	6.8	0.24	0.25	0.23	-	-	-
CO	57.8	61.9	16.6	16.8	19.8	-	-	-	11.1	10.2	0.35	0.35	0.30	-	-	-
EH	41.8	54.9	16.3	16.8	28.4	9.8	8.7	15.3	12.6	9.7	0.41	0.40	0.38	4.72	4.62	4.02
PB	47.7	65.9	8.4	8.0	10.7	3.4	6.3	11.4	12.3	8.2	0.46	0.47	0.32	4.04	3.98	3.39
QA	67.9	71.3	14.7	15.1	16.7	-	-	-	9.6	9.0	0.66	0.63	0.58	-	-	-
RM	57.7	64.2	13.2	13.9	13.4	7.3	7.0	15.5	11.7	10.2	0.44	0.44	0.43	5.52	5.55	4.59
RO	63.8	66.2	17.3	17.4	19.6	-	-	-	11.7	11.0	0.54	0.53	0.49	-	-	-
RS	61.6	66.3	13.1	13.1	16.8	8.9	9.8	†	8.2	7.5	0.35	0.35	0.31	4.11	3.94	†
SB	60.2	63.5	16.0	16.0	28.3	-	-	-	10.7	10.2	0.38	0.38	0.37	-	-	-
SM	62.2	66.1	13.9	15.5	14.1	-	-	-	11.1	10.3	0.47	0.47	0.46	-	-	-
WA	50.6	61.0	14.5	14.5	15.8	-	-	-	14.0	11.7	0.56	0.55	0.52	-	-	-
WC	13.2	71.0	12.6	13.6	17.5	4.5	13.2	11.6	15.0	6.0	0.31	0.30	0.29	4.07	3.85	3.19
WO	58.9	65.0	15.0	15.1	18.0	-	-	-	11.7	10.2	0.38	0.38	0.32	-	-	-
WP	52.5	55.3	13.5	13.5	14.0	9.4	5.0	16.0	12.7	11.7	0.67	0.66	0.60	4.85	4.73	4.29
YB	48.8	64.7	13.4	13.6	14.4	7.1	6.1	13.3	12.4	9.2	0.45	0.44	0.43	5.40	5.32	3.84
YP	63.1	67.9	15.6	15.8	19.4	-	-	-	14.1	12.8	0.74	0.74	0.65	-	-	-

<sup>†</sup> Calibrations were not significant.

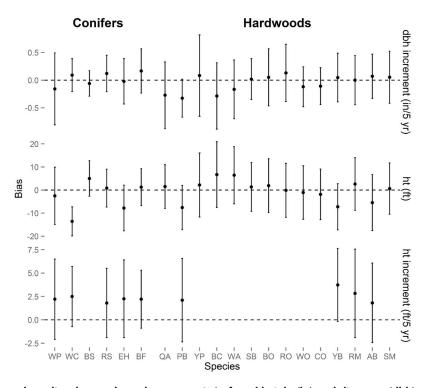


Figure 2. Mean bias (observed-predicted; error bars denote one SD) of total height (ht) and diameter (dbh) and ht increment predictions using FVS-NE variant for 20 species across the northeastern United States. Values above and below zero represent underprediction and overprediction, respectively. Species are sorted by increasing shade tolerance level from left to right within a species group.

predictions generally were less than 1%. Similar results were observed for  $\Delta dbh_5$ , although substantial improvements were observed for paper birch and northern white-cedar when using SI\_{ECO}. For 35 and 0% of the species examined, equivalence tests of dissimilarity were rejected for the  $\Delta dbh_5$  and  $\Delta ht_5$  submodels, respectively, when SI\_{ECO} was used (Table 4).

In fitting the AF (Equation 7), various independent variables helped to explain the prediction error of the FVS-NE submodel and species of interest varied (Table 6). For the 20 species examined, the AF could generally be explained by dbh, BA, and LAT for most species when investigating the ht submodel. To a lesser extent, dbh² and climate variables explained some of the variability in estimating

Table 4. Results of equivalence tests of dissimilarity using a threshold of  $\pm$  15% indicating equivalence (E) or no equivalence (NE) comparing observed values for FVS-NE variant predictions, FVS predictions using a modified site index specific to state/ecoregion (FVS $_{\rm Sl}$ ), and calibrated FVS predictions (FVS $_{\rm CAL}$ ) for total height (ht; ft), and 5-year diameter ( $\Delta$ dbh $_{5}$ ; in/5-yr) and height increment ( $\Delta$ ht $_{5}$ ; ft/5-yr).

Species		ht		Δdbh	5	$\Delta ht_{_{5}}$			
code	FVS	FVS <sub>CAL</sub>	FVS FVS <sub>SI</sub>		FVS <sub>CAL</sub>	FVS	$FVS_{SI}$	FVS <sub>CAL</sub>	
AB	E	E	E	NE	E	NE	NE	E	
BC	E	E	NE	NE	E	-	-	-	
BF	E	E	NE	NE	E	NE	NE	E	
BO	E	E	E	E	E	-	-	-	
BS	E	E	NE	NE	E	-	-	-	
CO	E	E	NE	NE	E	-	-	-	
EH	NE	E	E	E	E	NE	NE	E	
PB	NE	E	NE	NE	E	NE	NE	E	
QA	E	E	NE	NE	E	-	-	-	
RM	E	E	E	E	E	NE	NE		
RO	E	E	NE	NE	E	-	-	-	
RS	E	E	NE	NE	E	NE	NE	†	
SB	E	E	E	E	E	-	-	-	
SM	E	E	E	E	E	-	-	-	
WA	E	E	NE	NE	E	-	-	-	
WC	NE	E	NE	NE	E	NE	NE	E	
WO	E	E	NE	NE	E	-	-	-	
WP	E	E	NE	NE	E	NE	NE	E	
YB	E	E	E	E	E	NE	NE	E	
YP	E	E	E	E	E	-	-	-	

<sup>†</sup> Calibrations were not significant.

the ht prediction error. After applying calibrations to the FVS-NE predictions,  $PA_{15\%}$  increased by an average of 10%. The most sizeable gains were made in improving the accuracy of FVS-NE ht predictions for northern white-cedar and black spruce (Table 3).

The AF for  $\Delta dbh_5$  could generally be explained by dbh, dbh<sup>2</sup>, BA, LONG, and GSDD5. The PA<sub>15%</sub> improved by an average of 3% for all species after applying calibrations to FVS-NE  $\Delta dbh_5$  predictions, and substantial improvements were made in calibrating  $\Delta dbh_5$  predictions for eastern hemlock and sweet birch (*Betula lenta L.*). For seven of the nine species with  $\Delta ht_5$  data available from Maine and Pennsylvania, GSP was significant and positive in predicting the AF. Other variables contributed little to reduce the variability of the FVS-NE  $\Delta ht_5$  predictions. Despite the limited data analyzed, PA<sub>15%</sub> improved on average by 7% and RMSE was reduced by 18%, if calibrations were successful for a species (Table 3). For all of the submodels and species examined, equivalence tests of dissimilarity were rejected using calibrated FVS-NE predictions (Table 4).

The FVS-NE prediction of  $\Delta BA_5$ , derived from the baseline  $\Delta dbh_5$  equation from Equation 1, differed somewhat from the  $\Delta BA_5$  predicted using  $SI_{ECO}$  and the calibrated  $\Delta dbh_5$  (Figure 3). Across the seven ecoregions, FVS-NE predictions continued to underpredict  $\Delta BA_5$  using the three approaches, as indicated by a positive bias. Substantial reductions in  $\Delta BA_5$  bias were observed for some ecoregions. For example, bias was reduced from  $0.72 \pm 0.06$  (mean  $\pm$  standard error) to  $0.32 \pm 0.005$  ft<sup>2</sup>/ac and from  $2.77 \pm 0.37$  ft<sup>2</sup>/ac to  $1.85 \pm 0.35$  ft<sup>2</sup>/ac for the Laurentian mixed forest (code 212) and eastern broadleaf (continental; code 222) ecoregions, respectively. There was a trend of decreasing bias as species SI increased, and  $\Delta BA_5$  bias in FVS-NE predictions using SI<sub>ECO</sub> were minimal (0.17  $\pm$  0.07) for high-quality sites > 61.4 ft at 50 years. Predictions using a noncalibrated FVS-NE showed minimal bias for  $\Delta BA_5$  in oak-pine and white-red-jack pine forest types, while bias

Table 5. Mean site index (base age 50 years; SD in parentheses) by ecoregions within states for FIA plots across the northeastern United States.

State	Ecoregion	Mean (SD)		
СТ	221	62.1 (14.0)		
DE	232	65.4 (17.5)		
MA	221	57.7 (12.5)		
MA	M211	58.6 (11.8)		
MD	232	66.3 (14.2)		
MD	M221	67.8 (21.8)		
ME	211	46.4 (14.5)		
ME	221	60.0 (12.7)		
ME	M211	44.4 (11.7)		
NH	221	61.8 (12.1)		
NH	M211	52.2 (14.0)		
NJ	221	64.0 (0.9)		
NJ	232	54.2 (15.0)		
NY	211	63.9 (14.0)		
NY	221	65.7 (12.9)		
NY	222	70.1 (16.0)		
NY	M211	53.6 (10.4)		
OH	221	72.6 (14.9)		
OH	222	70.3 (14.9)		
OH	223	63.1 (20.1)		
PA	211	63.4 (13.0)		
PA	221	70.5 (16.6)		
PA	222	59.7 (3.0)		
PA	M221	64.5 (13.8)		
RI	221	63.7 (10.2)		
VT	211	64.1 (12.4)		
VT	221	55.4 (4.7)		
VT	M211	57.7 (12.5)		
WV	221	79.7 (17.8)		
WV	M221	65.7 (17.2)		

was reduced in oak-hickory and spruce-fir forest types using a calibrated  $\Delta dbh_5$  equation to predict future BA (Figure 3).

#### Discussion

Through this analysis, the performance of the static height-diameter submodel within FVS-NE was shown to perform moderately well. However, accuracy tests revealed that 5-year  $\Delta$ dbh and  $\Delta$ ht performed quite poorly. Trends in predictions across the northeastern United States did not indicate any strong spatial trends for the submodels examined. Employing FVS-NE prediction errors as the dependent variable, calibration efforts using AFs found that tree size, BA, geographic coordinates, and climate variables could help explain the prediction variability for some species when considering specific submodels. Calibrating growth equations using an ecoregion-specific SI computed from FIA plot data within states generally did not improve the accuracy of the FVS-NE  $\Delta$ dbh and  $\Delta$ ht submodels. Computing plot-level  $\Delta$ BA<sub>5</sub> using a calibrated  $\Delta dbh_5$  tended to outperform  $\Delta BA_5$  estimated using noncalibrated FVS-NE predictions, but results were dependent on ecoregion, site quality class, and forest type. This suggests that calibrations applied to predictions at the tree level should be examined closely when scaled to represent plot-level predictions. Results could also be complicated by the interactions between  $\Delta dbh_5$  and the tree mortality equation, which were not evaluated in this analysis.

#### **FVS-NE Benchmarking**

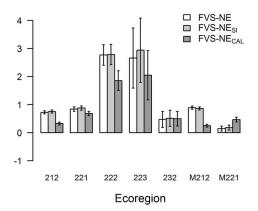
This investigation was an attempt to benchmark key submodels of FVS-NE across its suggested geographic range throughout the

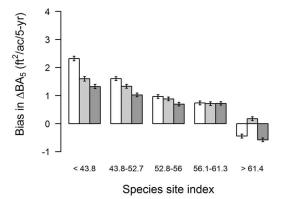
Parameters for AFs for FVS-NE variant submodels using tree, stand, and climate variables with associated fit statistics. Table 6.

\*Variables are AF; total tree height (ht; ft); diameter increment ( $\Delta dbh_5$ ; in/5-yr); height increment ( $\Delta dh_5$ ; ft/5-yr); initial tree diameter when estimating ht and  $\Delta dbh_5$  or initial tree height when estimating  $\Delta ht_5$  (size); basal area (BA; ft²/ac for trees  $\geq 1.0$  in. dbh); longitude (LONG; °); mean annual temperature (MAT; ° C); growing season precipitation (GSP; in.); growing season degree days (GSDD5; >5° C).

Model is:  $AF_{p} = \theta_{1,p}$  size  $AF_{p,p} = BA + B_{2,p}$  LAT  $AF_{p,p} = BA + B_{2,p}$  LONG  $AF_{p,p} = BA + B_{2,p}$  GSD 5.

81





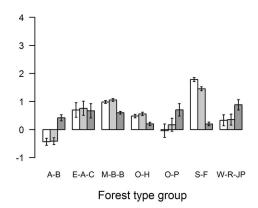


Figure 3. Bias (observed-predicted; with standard errors) of net BA growth using FVS-NE variant predictions, FVS-NE predictions using a modified site index specific to state/ecoregion (FVS-NE<sub>SI</sub>) and calibrated FVS predictions (FVS<sub>CAL</sub>) found in the northeastern United States.

northeastern United States. Other assessments of FVS-NE performance have occurred in regions that expand outside of this range (e.g., the Canadian Maritime Provinces; Rijal et al. 2012a, 2012b) or at more local levels. Specifically, Ray et al. (2009) present  $\Delta dbh$  adjustments based on long-term growth records from one site in central Maine. Outside of the Northeast, Shaw et al. (2006) presented a case study investigating how FVS can be locally calibrated by modifying key submodels such as total tree ht and  $\Delta dbh_5$ . Although local assessments and calibrations will continue to be necessary for routine forest management activities, the benchmarking of growth models is particularly needed at the geographic scale for which they are designed. As questions arise that ask the role that forests play in the global carbon cycle, assessments on much broader scales, such as regional ones, are

necessary (e.g., see Nunery and Keeton 2010). Exercises that benchmark growth model performance in their present form are also necessary prior to calibrating or reengineering them to reflect future climate scenarios (e.g., Crookston et al. 2010) or carbon storage potential (e.g., Woodall et al. 2011). The observation that FVS-NE predictions did not exhibit an overtly spatial pattern (Figure 1) could signify that the geographic region for the model is appropriate for making these kinds of regional assessments.

Results that tested the accuracy of Δdbh<sub>5</sub> predictions here generally agreed with those found for many of the same species in the Lake States implementation of FVS (Pokharel and Froese 2008): PA<sub>15%</sub> values for the 20 species investigated were nearly all under 15% for Δdbh<sub>5</sub>, indicating poor performance for the large-tree Δdbh equation of FVS-NE. Results here showed FVS-NE to overpredict Δdbh<sub>5</sub> for both quaking aspen and black spruce, but Lacerte et al. (2004) found general agreement between the observed and predicted  $\Delta$ dbh for the two species in Ontario at various projection lengths. The fact the  $\Delta dbh_5$  was underpredicted for shade-tolerant species (e.g., balsam fir, red spruce, and northern white-cedar) and overpredicted for intermediate-shade intolerants (e.g., eastern white pine [Pinus strobus L.], paper birch, and quaking aspen) agrees with the magnitude of the FVS-NE  $\Delta$ dbh multipliers suggested by Ray et al. (2009) and potentially provides justification for incorporating shade tolerance metrics into future modeling efforts.

The performance of the ht submodel was surprising, considering that equation parameters are adopted from the southern variant of FVS (Dixon and Keyser 2008) and the recent findings of Rijal et al. (2012b). Results showed that 12 of the 20 species displayed a mean bias for ht within 3 ft (Figure 2), which speaks to their relative robustness. However, as suggested by Rijal et al. (2012b), the FVS-NE ht equations can result in bias when evaluated at a more local level. Tests of the  $\Delta ht_5$  equations were much more variable because of the limited  $\Delta$ ht data available and the variability observed in the measurements. This resulted in low accuracies, as PA<sub>15%</sub> was below 10% for all species where  $\Delta$ ht data were available. Variability in predictions of  $\Delta ht_5$  are likely due to the reliance on the Carmean et al. (1989) equations that use SI computed primarily from localized studies and the requirement of FVS-NE to estimate growth effective age before predicting increment. Using a modified SI<sub>ECO</sub> appeared to do well in estimating  $\Delta BA_5$  at high-quality sites but was otherwise ineffective for the majority of ecoregions and forest types examined. As the annual FIA program continues, ht data for additional species gathered across broader geographic regions will help to further investigate the performance of the  $\Delta$ ht equations within FVS.

#### **FVS-NE Calibrations**

Building from AFs that take into account measures of tree size to calibrate model predictions (Holdaway 1985), results indicated that improvement in model predictions could be attained for some species by using tree size, BA, spatial location, and/or climate information for the FIA plots. Although they observed some variability in model residuals for  $\Delta$ dbh, Pokharel and Froese (2008) did not pursue model calibrations due to the complexities of residuals with multiple plot attributes such as SI, BA, and quadratic mean diameter. Calibrations applied here saw the most improvement for the  $\Delta$ ht<sub>5</sub>, ht, and  $\Delta$ dbh<sub>5</sub> submodels, respectively. Tree size (whether dbh or ht) accounted for much of the variability for the three submodels investigated, suggesting the Holdaway (1985) model calibration technique can be successfully applied on a regional basis.

The finding that ht predictions were improved for many species by incorporating geographic locale variables agrees with previous studies (Huang et al. 2000, Russell et al. 2010). Whether one refits submodels or employs calibration adjustments, improvements can be achieved if the focus is solely on the individual tree. The manner in which these updated predictions influence other submodels' performances and model performance at large, e.g., plot scales, should be investigated—adjusting component equations could lead to unanticipated overall model behavior (Westfall and Burkhart 2001).

Although studies have calibrated tree-level predictions of  $\Delta dbh$  (Holdaway 1985, Pokharel and Froese 2008), there has been little work examining the impact of scaling these calibrated predictions to compute plot-level attributes. The results here showed that the bias of  $\Delta BA_5$  predictions could be reduced for some ecoregions, site quality classes, and forest types by using a calibrated  $\Delta dbh_5$  equation, while predictions showed little improvement for other regions and forest types (Figure 3). As the focus here was solely on trees that survived a remeasurement interval, one factor that may improve plot-level predictions is an accurate assessment of the mortality model, similar to the approach outline by Radtke et al. (2012). The mortality equation is likely to be just as influential as  $\Delta dbh_5$  in accurately determining future BA, as suggested by the work of Lacerte et al. (2004). Similarly, estimates of tree recruitment would likely influence future BA in these multicohort, mixed-species stand types.

As national forest inventory data such as those available through the FIA program continue to be collected across expansive geographic ranges, this information can serve as an excellent clearinghouse for such exercises in benchmarking, calibrating, and refitting growth-and-yield submodels such as those found in FVS. Analyses herein indicate that reengineering model forms and refitting models, rather than calibrating existing model structures and predictions, may be warranted.

## **Conclusions**

Using national forest inventory data gathered from permanent sample plots across the northeastern United States, this analysis found that the current implementation of the FVS-NE growth-andyield model performed moderately well for reflecting the ht-dbh relationship of 20 common species but poorly for predicting shortterm Δdbh<sub>5</sub> and Δht<sub>5</sub>. Calibrating FVS-NE predictions helped to improve the accuracy of the three submodels investigated, but little improvement was observed for predicting Δdbh<sub>5</sub>. Using calibrated predictions for the Adbh<sub>5</sub> submodel led to improvements for some ecoregions and forest types when scaled to represent plot-level  $\Delta BA_5$ , however, underprediction was still present. Future work can follow by investigating the performance of the tree mortality and ingrowth submodels within FVS-NE and its ability to represent future stocking levels. Results indicate that efforts in model reengineering, as opposed to calibrating existing model predictions or refitting models of a similar form, should also be undertaken.

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