Pigeons, facebook and the birthday problem

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Abstract The unexpectedness of the birthday problem has long been used by teachers of statistics in discussing basic probability calculation. An activity is described that engages students in understanding probability and sampling using the popular Facebook social networking site.

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Introduction

The birthday problem as popularized by Feller (1968) often surprises people when the solution is revealed. By way of reminder, this problem asks what is the smallest gathering of people so that the chance of at least two people sharing the same birthday is one-half or more. At first consideration, when the number of people is small, because the number of possible birthdays is large (365 of them) the probability of two people sharing the same birthday would seem to be quite small. Most observers are surprised when they learn that just 23 people are needed to have the chance of a birthday match reach one-half or more.

We now remind the reader of the solution. Assume, for our gathering of individuals, that their birthdates are independent of each other (so, for example, no twins are present). Also, for simplicity, forget leap years and suppose that birthdays are uniformly distributed across the calendar year. Many times in computing probabilities, rather than computing the probability of an event, it is often easier to compute the probability of its complement. So first, the probability of at least one match is equal to one minus the probability of no match. Now, the probability of two individuals not sharing the same birthday is 364/365. Consider a third person: that individual's probability of not matching either of the first two selected individuals is 363/365. Consequently, (364/365)(363/365) is the chance of no birthday match among three people and one minus this is the chance of at least one birthday match among three people.

Continuing this for a total of 22 people, then, it follows that

$$\begin{aligned} &P(\text{At least one birthday match}) \\ &= 1 - \left(\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{344}{365}\right) \\ &= 0.476 \end{aligned}$$

while for 23 people we have

$$\begin{aligned} &P(\text{At least one birthday match}) \\ &= 1 - \left(\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{343}{365}\right) \\ &= 0.507 \end{aligned}$$

which verifies that 23 people is the solution to the birthday problem.

Using the so-called "pigeonhole principle", by the way, gives P(At least one birthday match) = 1 when the number of people gathered exceeds 365. The same logic used above can be applied to calculating the probability of at least one person out of 23 having a *particular* birthday (say, for example, 6 October). Specifically,

$$\begin{aligned} &P(\text{At least one person has a particular birthday}) \\ &= 1 - \left(\frac{364}{365}\right)^{23} = 0.061 \end{aligned}$$

So in a room of 23 people there is about a 51% chance that at least two people share a birthday, and about a 6% chance that someone has a particular birthday.

Facebook

With the widespread use of the social networking website Facebook, a list of birthdays to use to illustrate the birthday problem is available for your students' use. Even more fun, it's a list of birthdays of their own friends! As an aside, some Facebook users may choose not to share the month and day of their birth with their friends (part of what users select with their privacy settings). Not to worry: because the average Facebook user has 130 friends (Facebook 2011), plenty of birthdays are likely available to answer some interesting questions related to the birthday problem.

Imagine that all of your Facebook friends are "in the same room" – a virtual room so to speak. You can view your friends' birthdays by going to Events→Birthdays from the Facebook homepage. By computation like that shown above, if you have 50 Facebook friends, there is a 0.97 probability that two of your friends will share the same birthday, and there is a 0.13 probability that one of your friends will share the same birthday as you. If you have 200 Facebook friends, there is a greater than 0.99 probability that two of your friends will share the same birthday, and there is a 0.42 probability that one of your friends will share the same birthday as you (figure 1).

Activities

Try the birthday problem in class. How many people are in the class? What is the probability of at least two people sharing the same birthday? Do, in fact, any students share the same birthday? What is the probability of at least one person in the class having a particular birthday?

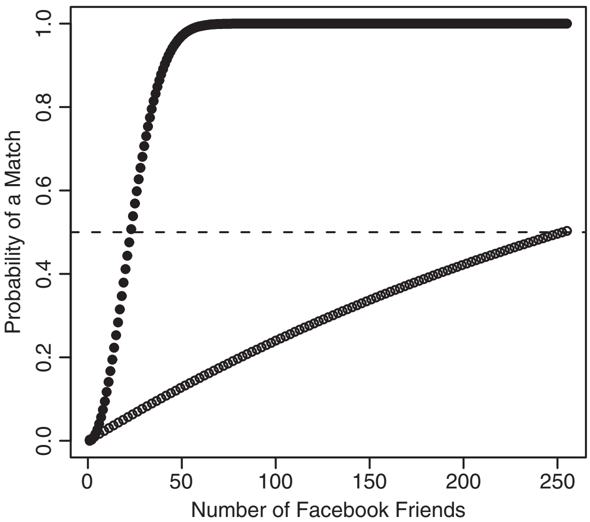

Fig. 1. The probability of a birthday match between any two people (solid dots) and the probability of a match with your birthday (open dots)

Also have your students use their Facebook accounts to gather data from their friends' birthdays to use in the following activities. First, ask students to calculate the probability of at least one of their friends "in the same room" having their birthday. For the case of the average Facebook user with 130 friends, this would equate to $1-(364/365)^{130} = 0.30$. Does the student actually share a birthday with one of their friends? Students will begin to understand the influence of the sample size on the results: the higher the number of friends, the more likely the chance that they will encounter a friend with the same birthday.

To more fully check student understanding of the birthday problem, students could be asked for the chance of a match over a different time period – a month, for example, may be used. To this end, ask the students to only look at the birthdays of their friends in the month they were born. Have them compile the list of birthdays in a table similar to table 1 or in a calendar – using pigeons, say, to represent birthdates as shown in figure 2.

For the example provided for the month of October involving a total of eleven people, P(At least one birthday match) $= 1 - (30/31 \times 29/31 \times \ldots \times 21/31) = 0.87$. Similarly, with ten other friends having an October birthday, P(Someone has the same birthday as you) would be computed as $1 - (30/31)^{10} = 0.28$. To do this calculation on their own, students will need to use different values depending on the number of days in their birth month and the number of Facebook friends they have with a birthday in that month. Indeed for the October example, Ben and Jeremy share a birthday, but no one shares the birthday of the student (6 October).

Table 1. Dates of October birthdays

Facebook friend	Birthday
Graham	Oct 5
Mine	**Oct 6**
Ben	Oct 10
Jeremy	Oct 10
Stephen	Oct 15
Emily	Oct 17
Cody	Oct 18
Rebekah	Oct 20
Aimee	Oct 22
John	Oct 27
Chris	Oct 31

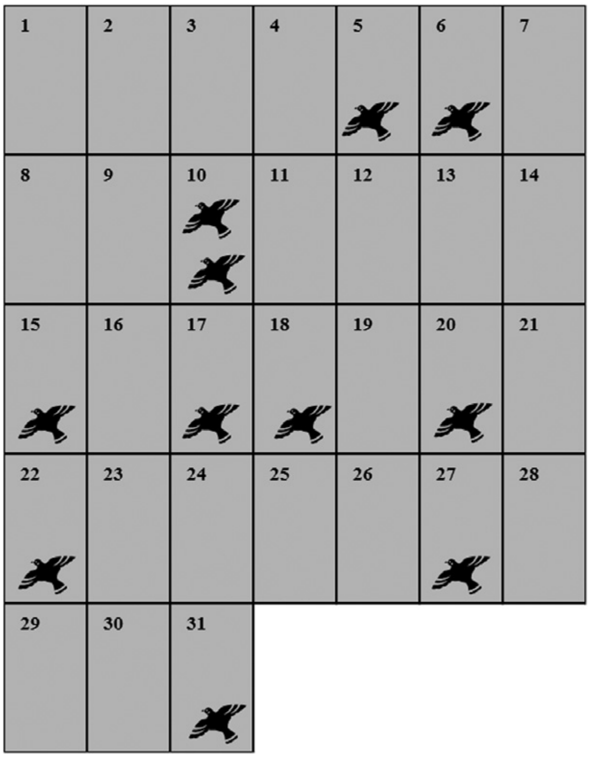

Fig. 2. Birthdays, indicated by pigeons, occupying a calendar month

Conclusion

These activities are fitting for advanced high school students or introductory college statistics course students. Indeed, investigation of the birthday problem can help students understand elements of probability and sampling. Students will be engaged in the activity by using their own friends' birthdays. After all, how many other class exercises require you to log into Facebook to work an assignment?

References

Facebook. (2011). Statistics. Available online at http://www.facebook.com/press/info.php?statistics, accessed 29 January 2012.

Feller, W. (1968). *An Introduction to Probability Theory and its Applications* (Vol. 1). New York: Wiley.