1) $\frac{15}{15} * \frac{14}{15} * \frac{13}{15} * \frac{12}{15} * \frac{11}{15} * \frac{10}{15} * \frac{9}{15} * \frac{8}{15}$

$= \frac{\frac{15!}{7!}}{15^8} = \frac{259459200}{15^8} \approx 0.10123 = \boxed{10.123\%}$

2)

Total options: 00000 - 99999

100,000 options

Let's think by position

0	0	0	0	0
5 options	4 options	7 options	6 options	5 options

Probability $\frac{4200}{100,000} = .042$ 8-choose 5

Use formula:

$= (0.042)^5 (1 - 0.042)^{8-5} \binom{8}{5}$

$= (0.042)^5 (1 - 0.042)^3 \left(\frac{8!}{5!(8-5)!}\right)$

$\approx 6.4347 \cdot 10^{-6}$

3) A = 2 dice 4 or above

die 1: roll 3 dice choose 2 to be 4 or above so ~~odds~~ odds on each die are 1/2

$\hookrightarrow \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$

$= \frac{3!}{2!\,(3-2)!}\left(\frac{1}{2}\right)^2 \frac{1}{2} = 3 \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{8}$

$\hookrightarrow \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$

$= 1\left(\frac{1}{8}\right)(1) = \frac{1}{8}$ $\rightarrow + = \frac{1}{2}$

$P(A) = \frac{1}{2}$

B = All three dice show same value

$\hookrightarrow$ Any for d1, 1/6 for both d2 + d3

$\hookrightarrow 1 \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(B)$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$

$P(A \cap B) = \frac{1}{2} \cdot \left(\frac{1}{6}\right)^2 = \frac{1}{72}$

Since $P(A) \cdot P(B) = P(A \cap B)$, they are $\boxed{\text{independent}}$

4) To consider:

- 13 cards in each hand
  - must choose 5 for a flush
- comparing this to probability of choosing any 5 cards from 52 card deck
- 4 different suits so increase chances by four times

write it out:

$$= 4 \cdot \frac{C(13,5)}{C(52,5)}$$

$C(13,5) \rightarrow$ P(choosing 5 1 suit)

$C(52,5) \rightarrow$ P(choosing 5 from deck)

$4 \rightarrow$ suits

$$= 4 \cdot \frac{\frac{13!}{5!\,8!}}{\frac{52!}{5!\,47!}} \approx .00198 = P(\text{flush})$$

We're trying to find how many hands it would take, so geometric (around 505)

$$\text{expected \# hands} = \frac{1}{P(\text{flush})} = \frac{1}{.00198} \approx \boxed{505.05 \text{ hands}}$$

(answer may vary slightly depending on # decimals used for this var)

5) P(Superstar plays) = .75

P(Win W/O superstar) = .50

P(Win W/ Superstar) = .70

P(Superstar plays and win 4/5)

$= \binom{5}{4} * .7^4 \cdot (1 - .7)$

$= \frac{5!}{4!} * .7^4 \cdot .3 = 5 * .7^4 * .3 = .36015$

P(superstar doesn't play + win 4/5)

$= \binom{5}{4} * .5^5 = 5 * .5^5$

$\approx .15625$

$P(\text{total}) = .36015(.75) + .15625(.25)$

$\approx .309175$

Bayes Theorem: $\rightarrow \left(P(F|E) = \frac{P(E|F) \cdot P(F)}{P(E)}\right)$

$$\frac{.36015(.75)}{.309175} \approx .87365$$

$\boxed{\text{87.365\% chance he played}}$