

# Counting Problems

## 1) Scenarios:

u used once: All five letters: 1 way

u used twice: Fill 3 spots w/4 letters: 4 ways

u used thrice: fill 2 spots w/4 letters: 6 ways

u,u,s,a,l   u,u,s,i,n   u,u,s,a,n   u,u,l,n,a

u,u,u,i,n   u,u,u,s,a   u,u,u,s,i   u,u,u,l,a   u,u,u,n,a   u,u,u,l,n

11 subsets

for # strings:

$$5! = 120$$

$$\frac{5!}{2!} \cdot 4 = 240$$

letters

$$120 + 240 + 120$$

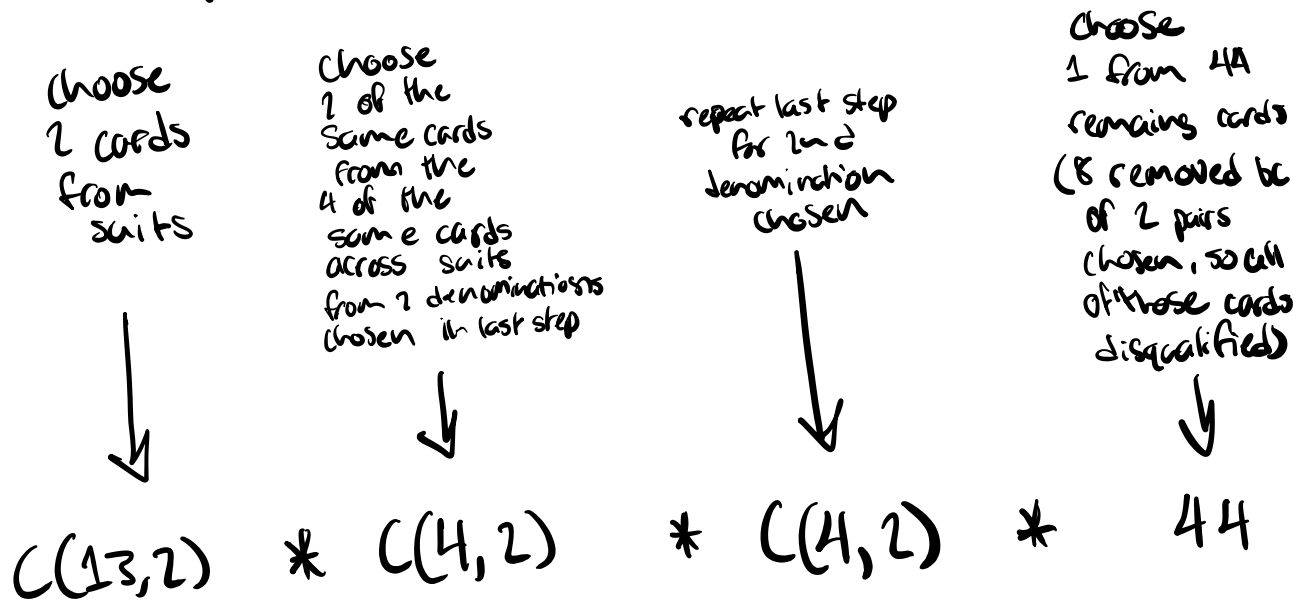
$$\frac{5!}{3!} \cdot 6 = 120$$

letters

= 480 different strings

2) Facts: 52 cards in a deck  
13 unique cards per suit

Set up:



remember  $C(n, r) = \frac{n!}{r! \times (n-r)!}$

rewrite

$$= \frac{13!}{2! \times (13-2)!} \times \frac{4!}{2! \times (4-2)!} \times \frac{4!}{2! \times (4-2)!} \times 44$$

$$= 78 \times 6 \times 6 \times 44$$

$= 123,552$

3) Set up:

Choose 1  
couple w/  
1 song

$$C(16, 1)$$

Choose to  
distribute  
6 songs to  
remaining couples

$$C(15, 6)$$

$$16$$

\*

$$\frac{15!}{6! \cdot (15-6)!}$$

$$16$$

\*

$$5,005$$

$$\boxed{= 80,080}$$

4)

remember # trees given nodes:

$$\frac{(n)!}{(n+1)! \cdot n!}$$

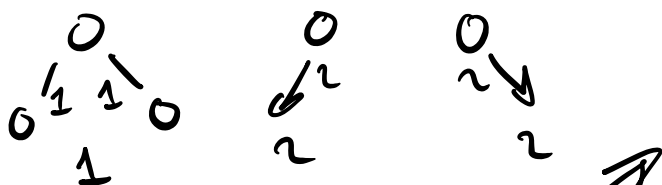
\* numbers by arrows  
denote num nodes

2 nodes:



2 options in that  
path and  
following

3 nodes

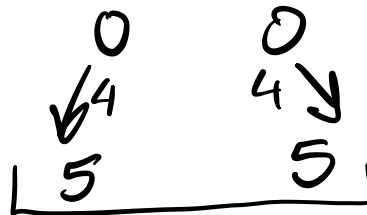
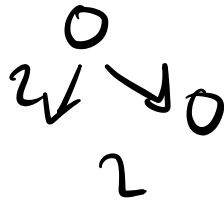
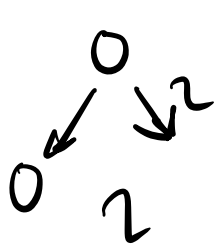


$$1 + 2 + 2 = 5$$

5 options

# is  
pathways  
resulting

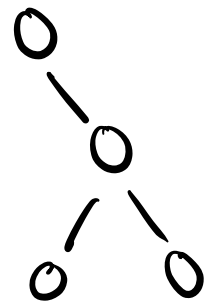
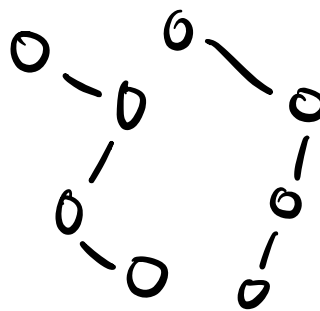
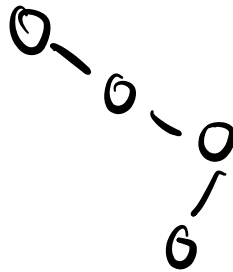
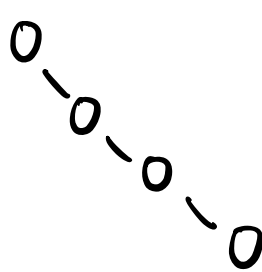
4 nodes



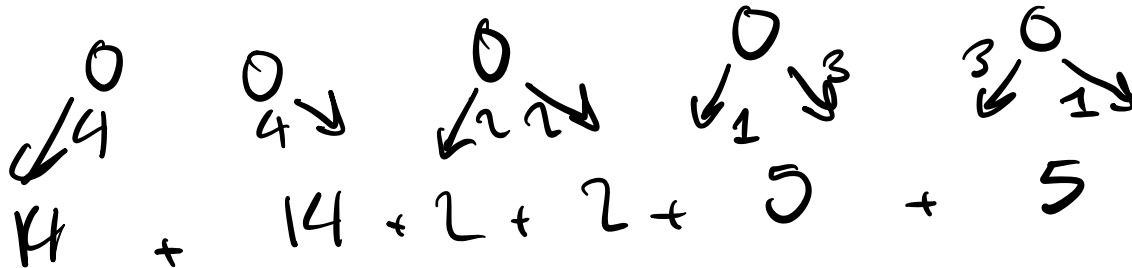
$$5+5+2+2$$

= 14 options

Ex of breakdown for 5 rightside options

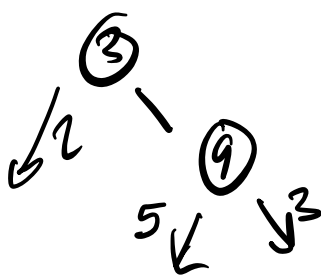


5 nodes:



= 42 options

nodes 5, 2, 3



$$42 \cdot 5 \cdot 2 = 420 \text{ ways}$$

5) 4 nurses  
Scenario

nurse			
1	2	3	4
1	1	1	7
1	1	2	6
1	2	2	5
1	2	3	4
2	2	2	4
2	2	3	3
3	3	3	1
4	4	1	1
5	3	1	1

9 ways

3 nurses  
Scenario

nurse		
1	2	3
1	1	8
1	2	7
1	3	6
1	4	5
2	2	6
2	3	5
3	3	4
4	4	2

8 ways

$$9 + 8 = \boxed{17 \text{ ways}}$$