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A COST-EFFECTIVE APPROACH TO HEDGING MBS USING TREASURY FUTURES AND FUTURES OPTIONS

LARRY LANGOWSKI, TAE H. PARK, AND LORNE N. SWITZER

The prominence of U.S. mortgages in fixed-income markets makes the hedging of mortgage-backed securities (MBS) an issue of no little concern for mortgage investors and lenders or MBS investors. An extensive review of the issues relating to hedging MBS appears in Breeden [1994].

Three methods are currently used for hedging MBS, as documented by Arnold [1987] and Jones and Jain [1987]. The first is to hedge MBS with a short position in the corresponding over-the-counter MBS forward or options market. The second is to consider the MBS as a short put relative to Treasury yields (equivalently, as a bond and a short call with exercise price equal to the outstanding mortgage balance plus prepayment penalty), and hedge it with a long put on a Treasury instrument. The third is to hedge MBS with a short Treasury futures position, calculating the hedge ratio from the MBS price/yield relationship based on the current or projected prepayment. The hedge ratio may be continually adjusted as price and yields change.

Although the third method uses exchange-listed products, the dynamic hedging requirement may entail substantial transaction costs. In addition, hedging MBS with Treasury futures involves intermarket spread risk and yield curve shape risk.

The OTC forward market for MBS has recently become a popular venue for hedging MBS price risk and mortgage pipeline risk because of its high liquidity and readily available quotes for various settlement dates. In this article, however, we propose a new method based on estimating the cost-minimizing hedge using exchange-listed Treasury futures and futures options.

The advantages that exchange-listed contracts offer

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over OTC products are lower transaction costs, price transparency of daily positions, and elimination of counterparty risk through daily marking to market. Current emphasis by the financial industry on risk management and disclosure of derivatives positions provides a relative advantage for exchange-listed contracts over most OTC products.

Two studies of measuring the dynamic hedging performance of MBS using Treasury note futures stand out. Batlin [1987] separates the price sensitivity (or duration) of the prepayment feature of MBS from that of the fixed-income stream, and calculates the hedge ratio by assuming an inverse function of the spread between the mortgage rate and the coupon rate of the MBS for the prepayment option's duration. He estimates the price sensitivity (or duration) of MBS using the time series of historical Treasury bond yield and mortgage rates.

An article by Boudoukh, Richardson, Stanton, and Whitelaw [1995] adopts a (more theoretically sound) dynamic hedging method by differentially weighting past pairs of MBS and T-note futures returns, where the weights depend on how close current economic variables are to their historical values. The approach in Boudoukh et al. is similar in nature to the empirical duration measure, where the past premium and the discount behavior of MBS prices are taken into account in the hedge ratios just as occurs in calculation of the empirical durations.

Our approach is distinct from these MBS hedging studies in three ways. First, we estimate the duration and convexity of MBS using polynomial spline functions of MBS price (McCulloch [1975]). The structural change in the mortgage duration and convexity as the MBS price goes from discount to premium can be captured by using splines that are third-order polynomials.

Second, we use Treasury call and put options in addition to Treasury note futures as hedging instruments. The high convexity of calls and puts is expected to hedge the negative convexity of MBS more effectively than using the note futures alone.

Third, we concentrate on the total transaction costs incurred from periodic hedging as well as on reducing volatility of the portfolio. The widespread trend in the hedging literature of ignoring the total transaction costs (bid–ask spread and brokerage fees) has made certain complex hedging methods seem superior to the conventional static hedging method. Industry practice, however, has for the most part ignored the latest developments in hedging strategies, mainly because of the costs associated with dynamic hedging. It is important for hedgers to implement realistic hedging positions, taking full account of transaction costs.

As noted previously, using only futures for hedging is equivalent to duration-hedging. Duration-hedging an option-embedded security such as an MBS is risky for at least two reasons. First, duration hedges do not capture the convexity of the options. As a result, a large unexpected change in the underlying price can produce large hedge errors. Because the investor in an MBS is short the prepayment call option, the convexity in the position is negative. Any large interest rate changes, then, hurt the investor.

Second, duration hedges afford no protection against changes in implied volatility. Because the investor is short the embedded call, any increase in implied volatility increases the value of the call and reduces the value of the MBS.

At least some of these risks can be offset by including some long options in the hedge. Long options would provide a counterbalance of positive convexity (which is greater than that of futures), which would help offset the risks of large and unexpected price changes in a more cost-effective manner than using futures alone. Long options would also afford some protection against increases in implied volatilities.

The chief difficulties in using options for such volatility hedging lie in their availability. The option embedded in the MBS is long-dated, while most liquid Treasury note futures options are short-dated (expiring within three months). Since the option's sensitivity to interest rate volatility, or "vega," is proportional to time to maturity, Treasury futures options with their relatively small vegas would do little to offset the implied volatility of MBS. Thus, we concentrate only on hedging duration and convexity of MBS with Treasury futures and options.

Although the use of options in hedging MBS is widely discussed in the industry, there is so far no documented empirical evidence for the hedging effectiveness of options. Our study provides the first attempt to estimate the reliability of Treasury options for hedging MBS empirically.

The hedge ratios for Treasury futures and options are derived by minimizing the total transaction costs while offsetting the risk measures of the underlying MBS. The hedge ratios are dynamically adjusted, and the periodic (weekly) performance of the hedge portfolio is compared to that of a simple hedge using Treasury

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futures alone. Using data over the period 1988–1993 to estimate the duration and convexity of MBS, and performing an out-of-sample analysis over the period from January 1994 through September 1995, we find that this cost-effective hedging method performs considerably better (in volatility reduction and transaction costs) than the method of using Treasury futures alone.

I. EMPIRICAL DURATION AND CONVEXITY ESTIMATION

To calculate the risk measures of empirical duration and convexity used in the hedge, we rely on an estimation method that extends the approach of DeRosa, Goodman, and Zazzarino [1993]. This approach to estimating duration and convexity is different from an alternatively accepted option-adjusted risk measure that relies on an option-adjusted spread (OAS) model.[1] Since the OAS duration is dependent on a specific mortgage prepayment model, MBS dealers using different OAS models may arrive at quite different risk measures, as is shown in DeRosa, Goodman, and Zazzarino. By using the empirical duration, we avoid dependence on a specific prepayment model and make the hedge method more universally applicable.

Duration and convexity determine the variability of prices associated with changes in the interest rate r. In the case of a marginal change in the interest rate of Δr, the percentage price change of an interest rate-dependent asset can be determined by the expression:

$$\frac{\Delta P}{P} = \frac{\partial P}{\partial r}\frac{1}{P}\Delta r + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\frac{1}{P}(\Delta r)^2 + \frac{1}{3!}\frac{\partial^3 P}{\partial r^3}\frac{1}{P}(\Delta r)^3 + \ldots + \tag{1}$$

The percentage change in price relative to a change in the interest rate r can be approximated by eliminating the higher-order terms and replacing the coefficients with duration, D, and convexity, C, in (1):

$$\frac{\Delta P}{P} \equiv -D\Delta r + \frac{1}{2}C_i(\Delta r)^2 \tag{2}$$

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Thus, duration and convexity can be estimated using Equation (1) with the return of the asset.

The constant duration and convexity terms in (2) do not reflect the state-dependent duration and convexity of MBS. DeRosa, Goodman, and Zazzarino [1993] use an ad hoc version of (2), where D is a quadratic function of price, P, and ΔP and Δr are measured with daily price and interest rate changes. Such a duration measure provides a price-dependent duration that reflects the shortening of MBS duration with price and premium.

Assigning an arbitrary function of price to characterize risk, however, does not work well with convexity. Since second-order changes in the daily interest rate are often very small in magnitude, such an estimation results in very unstable convexity estimates. An alternative to avoid unstable convexity estimates is to use a step function for duration and convexity estimates where these estimates are constant for a certain range of MBS prices. This step function approach is similar to the spline function approach used by McCulloch [1975], where polynomial spline functions are used to fit the observed prices of U.S. Treasury securities.

Similar spline functions that are constant for a certain range of the price can be applied to the duration and convexity estimation for MBS as follows:

$$\frac{\Delta P_t}{P_t} = -D_i \Delta r + \frac{1}{2} C_i (\Delta r)^2$$

$$\text{for } t_{i-1} < t \leq t_i, \ i = 1, 2, \ldots, m \tag{3}$$

where $80 = t_0 < t_1 < \ldots < t_{m-1} < t_m = 110$. t_0 defines the minimum price and t_m the maximum price for the MBS price to reach during the estimation period. The coefficients, D_i and C_i, $i = 1, 2, \ldots, m$, satisfy the equation

$$-D_i \Delta r + \frac{1}{2} C_i (\Delta r)^2 = -D_{i+1} \Delta r + \frac{1}{2} C_{i+1} (\Delta r)^2$$

$$\text{for } i = 1, 2, \ldots, m-1 \tag{4}$$

Equation (3) ensures that duration and convexity of MBS meet at the "knot" points. There could be first-order conditions of Equation (4) to ensure smoothness of the spline, but they are not included because smooth-

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ness of duration and convexity are not the main objective of this empirical exercise. The more ts are used to define price ranges, the more accurately duration and convexity can be computed from Equations (3) and (4), but at the sacrifice of higher degrees of freedom.

It is important to note that parameters of the estimation are constantly updated by incorporating the current rates available to investors.

II. DYNAMIC HEDGING OF MBS

Data

The testing period is from January 1994 through September 1995. For MBS data, however, we need a longer time series to estimate the interest rate risk measures empirically. We use seven years of daily price data (from October 1988 through September 1995) on the thirty-year fixed-rate GNMA MBS, with coupons of 7.5%, 8.5%, and 9.5% (obtained from Knight-Ridder Financial). The prices represent dealer-quoted bid prices on each GNMA traded for forward delivery on a to-be-announced (TBA) basis. The representative interest rate, the ten-year Treasury note yield, is also obtained from Knight-Ridder Financial. Since the testing period was one of substantial interest rate variability (with periods of rate increases and decreases), it is expected to provide rich examples of alternative hedging scenarios.

For the Treasury futures and options data, daily closing prices of Treasury note futures and three sets (in-the-money, at-the-money, out-of-the-money) of call and put options are used for the testing period. At-the-moneyness is determined by the closeness of the strike price to the underlying futures price.

Once an at-the-money option is chosen, an in-the-money option is determined as the option with a strike price one point below (above) the at-the-money strike for call (put) options. Options with strike prices farther away from the at-the-money strike exhibit drastically different deltas and gammas, but they tend to attract less liquidity and are therefore not included.

The Treasury futures and options prices are for the nearest contracts, which are rolled over to the next contract in the expiration month. It is important to roll the options at least a couple of weeks before expiration because the options that are approaching the maturity may exhibit extremely high thetas, or time decay value, especially for those that are at the money.

The risk measures, duration and convexity, of futures are calculated according to the corresponding measures for the cheapest-to-deliver notes taken from Bloomberg. The risk measures for options are calculated using Black's model [1976], where deltas and gammas of futures options are converted to deltas and gammas with respect to Treasury yield by taking the products of the deltas and gammas, respectively, of the underlying futures.

EXHIBIT 1 ■ Duration and Convexity of GNMAs, Treasury Note Futures, and Treasury Note Futures Options

Measure	April 6, 1994	November 4, 1994
Ten-Year Treasury Yield	6.92%	8.03%
MBS Price:		
GNMA 7.5	97.69	91.50
GNMA 8.5	102.25	97.62
GNMA 9.5	105.34	103.94
Duration:		
GNMA 7.5	4.39	7.16
GNMA 8.5	3.49	6.31
GNMA 9.5	2.26	4.32
Futures	5.27	6.73
Call (in-the-money)	204.48	287.47
Call (at-the-money)	246.09	345.72
Call (out-of-the-money)	336.21	426.46
Put (out-of-the-money)	−206.28	−139.87
Put (at-the-money)	−195.63	−129.49
Put (in-the-money)	−179.44	−120.49
Convexity:		
GNMA 7.5	−126.11	918.31
GNMA 8.5	−126.08	1,005.93
GNMA 9.5	−160.91	486.32
Futures	36.67	58.31
Call (in-the-money)	272.18	268.44
Call (at-the-money)	436.49	445.84
Call (out-of-the-money)	746.51	709.83
Put (out-of-the-money)	494.42	290.99
Put (at-the-money)	413.92	230.40
Put (in-the-money)	314.71	184.40

Data Analysis

Exhibit 1 shows statistics for two representative dates in the testing period. The first date, April 6, 1994, has a relatively low interest rate (6.92%), and the GNMA 7.5 is priced at a discount, but the GNMA 8.5

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and 9.5 are priced at a premium.

The durations of GNMAs, futures, and options on this date show that the durations of GNMAs are lower than those of T-note futures. The durations of the options are very high, exhibiting the highly levered nature of options. With futures alone, one GNMA 7.5 could be reasonably hedged by shorting one futures contract, while one GNMA 9.5 could be better off with shorting one-half contract of futures.

The GNMAs have negative convexities on this date as in the case when GNMAs are traded near premium. The convexity of futures is much smaller in magnitude, and duration-based hedging with futures will not eliminate the negative convexity of GNMAs. Options, on the other hand, have very high convexities as expected, and long positions in options would offset the negative convexities of GNMAs as we noted earlier.

Given the risk measures of futures and options on April 6, it seems reasonable that an effective hedging method (both delta- and gamma-neutral, since for periods of significant variability of interest rates, delta-neutral portfolios will not hedge convexity) should involve both futures and options.

Another date, November 4, 1994, is chosen to show what happens to the risk measures when the interest rate is higher (8.03%) and GNMAs are mostly traded at discount. On this date, the GNMA 7.5 and 8.5 are priced at a discount, and their durations are much closer to that of the futures than earlier.

The convexities of GNMAs are all positive. Again on this date, short positions in futures alone would not offset the duration and convexity risk altogether. Positive convexity is harmless as long as the holding period is short enough to avoid large time decay in the portfolio value. If we want to eliminate duration risk and maintain the positive convexities of GNMAs in the hedged portfolio, futures alone could be sufficient.

A final issue of concern in comparing alternative hedging methods is the total transaction costs involved. One large component of the total transaction cost is the bid–ask spread of futures and options. Both T-note futures and (near-the-money) options usually have one-tick spreads for the nearest contracts, but one tick for T-note futures is 1/32, or $31.25, while one tick for T-note futures options is 1/64, or $15.625. Lower dollar value spreads for options can make options attractive hedging instruments.

Hedging Methodology

For cost-effective hedging, we use non-linear optimization. The objective function is the total transaction costs associated with hedging $1 million worth of MBS. Cost-effective hedging requires minimizing the total transaction costs, while eliminating the duration and convexity of the hedged portfolio.

$$\min |x_1|\, T_f + (|x_2| + |x_3| + |x_4| + |x_5| + |x_6| + |x_7|)\, T_o \tag{5}$$

subject to

$$\text{Delta}_{MBS} + x_1\text{Delta}_f + x_2\text{Delta}_{ci} + x_3\text{Delta}_{ca} + x_4\text{Delta}_{co} + x_5\text{Delta}_{pi} + x_6\text{Delta}_{pa} + x_7\text{Delta}_{po} = 0$$

$$\text{Gamma}_{MBS} + x_1\text{Gamma}_f + x_2\text{Gamma}_{ci} + x_3\text{Gamma}_{ca} + x_4\text{Gamma}_{co} + x_5\text{Gamma}_{pi} + x_6\text{Gamma}_{pa} + x_7\text{Gamma}_{po} \geq 0$$

where x_i, i = 1, ..., 7, represents the number of contracts to trade for hedging positions in futures and options (ci = in-the-money call option, ca = at-the-money call option, and so on).

The transaction costs, T_f and T_o, for futures and options are round-trip transaction costs based on a market survey of dealers. They include the bid-ask spread, or one tick size (2 × $31.25 per contract for futures and 2 × $15.625 per contract for options), clearing/exchange fee (2 × $0.57 per contract), and representative brokerage commissions (2 × $3 per contract).[2]

Deltas and gammas are the first- and second-order sensitivity to the ten-year Treasury yield for all securities. Delta_{MBS} is the dollar value change of $1 million MBS with small changes in Treasury yield, while Delta_f is the dollar value change of one Treasury note futures contract with face value of $100,000. The deltas of options are also based on $100,000 face value.[3] Similar arguments apply to gammas with the square of small change in Treasury yield. The gamma constraint has an inequality because it does no harm to have positive gammas for a short-term hedge.

If only futures are used for delta-hedging, a simple hedging strategy could be designed where the hedge ratio and the number of futures contract required are computed as the ratio of Delta_{MBS} and Delta_f:

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$$x_{fo} = \text{Delta}_{MBS} / \text{Delta}_f \qquad (6)$$

The strategy is to compare the unhedged GNMA returns, the futures-only hedged returns [Equation (7)], and cost-effective hedged returns [Equation (8)], where these returns are calculated on a weekly basis from January 1994 through September 1995. The returns are measured as after-transaction costs by dividing the total transaction costs from the hedging by the initial investment amount, which is \$1 million in this case.

$$R_{fo} = R_{MBS} - x_{fo}(1{,}000\Delta F - T_f)/\$1 \text{ million} \qquad (7)$$

$$\begin{aligned} R_{ce} = {} & w_{MBS}R_{MBS} - x_1(1{,}000\Delta F - T_f)/\$1 \text{ million} + \\ & w_2x_2R_{ci} + w_3x_3R_{ca} + w_4x_4R_{co} + \\ & w_5x_5R_{pi} + w_6x_6R_{pa} + w_7x_7\, R_{po} - \\ & (x_2 + x_3 + x_4 + x_5 + x_6 + x_7)\, T_o/\$1 \text{ million} \qquad (8) \end{aligned}$$

where R_{MBS} is the weekly percentage price change of MBS, ΔF is the weekly price change of futures, and R_{ci}, ..., R_{pi} are the rates of return for options.

In (8), weights are assigned to MBS and various option series that sum up to 1 ($w_{MBS} + w_2 + w_3 + \ldots + w_7 = 1$). These weights are the proportions invested in each security from the initial capital of \$1 million.

For instance, if \$10,000 is used in buying the net option positions, only the remaining \$990,000 is used in investment of the MBS. This approach makes the rate of returns from the two hedging strategies comparable by making the initial investment amount the same.

It is important to note that x_i, $i = 1, \ldots, 7$, and x_{fo} are integers for the optimal hedge amounts calculated from Equations (5) and (6) because they represent the actual number of contracts for futures and options. During the testing period, the optimal hedge ratios for cost-effective hedging and futures-only hedging are calculated each day, and their average rates of return and risk, calculated as the standard deviation of the return, are compared.

III. HEDGING EFFECTIVENESS

The objective function in (5) has absolute values, and thus its minimization requires non-linear programming.[4] During the test period, out-of-the-money options do not have any open interest for certain dates, and they are not included in the optimization for those dates.

Exhibit 2 provides average weekly mean returns and variabilities of the unhedged, futures-only hedged, and cost-effectively hedged returns for investing \$1 million in 7.5%, 8.5%, and 9.5% GNMA TBAs for a one-week horizon. The mean returns for the unhedged GNMAs are −0.000309, −0.00023, and −0.00019, respectively, for the 7.5s, 8.5s, and 9.5s. Because TBAs are forward contracts, these mean returns represent risk premiums for holding MBS. The low returns of GNMAs represent the bear bond market experienced in most of 1994. The high-coupon GNMA is least affected by the downturn in the bond market because it has the lowest duration of all three.

The after-transaction costs returns of the futures-only hedged portfolio are much lower than those of unhedged returns, but variabilities are significantly reduced in all three GNMAs. The futures reduce the risk of GNMA returns, but large transaction costs pull down the returns, especially for the GNMA 7.5s and 8.5s. Such poor performance of futures hedging also shows the limitations of futures when interest rates change widely as was the case in 1994.

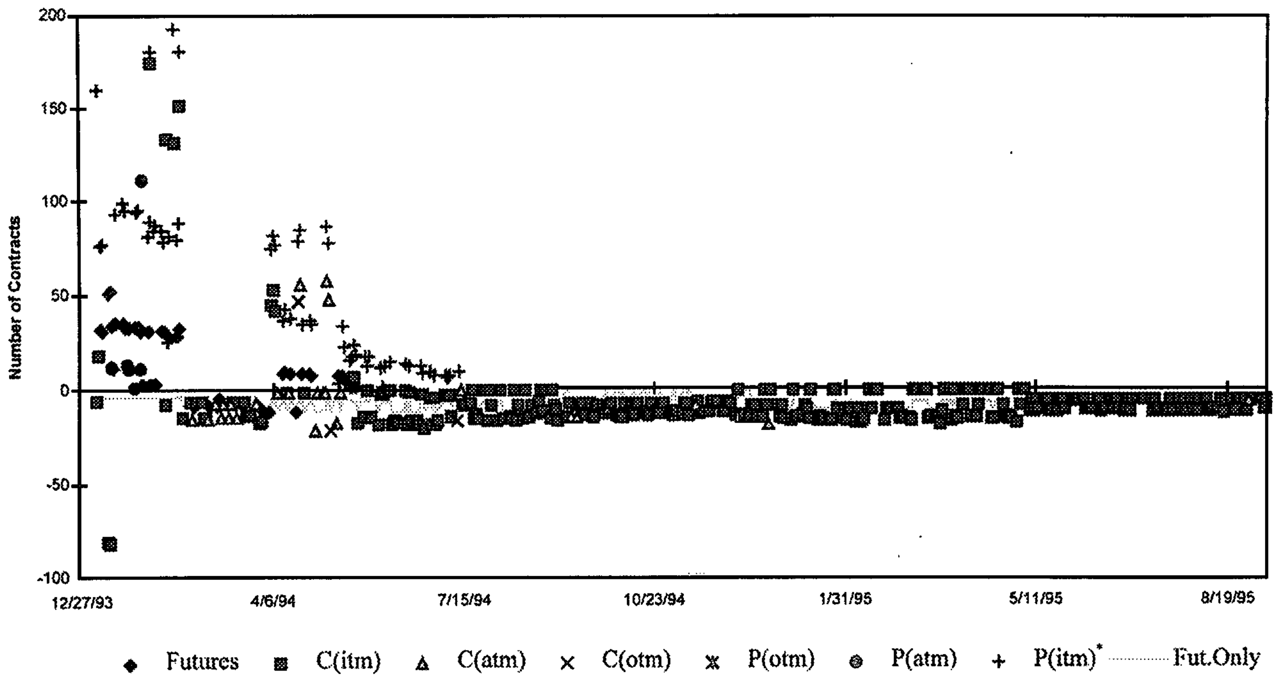
EXHIBIT 2 ■ Hedging Effectiveness: Average Weekly Mean Returns and Variabilities of Unhedged GNMA Portfolio, Futures-Only Hedged Portfolio, and Cost-Effective (Optimally) Hedged Portfolio*

Hedging Period: Weekly
Testing Period: January 4, 1994–September 21, 1995

	GNMA 7.5	GNMA 8.5	GNMA9.5
Unhedged:			
Mean	−0.00039	−0.00023	−0.00019
Standard Deviation	0.00872	0.00626	0.00392
Futures Delta Hedge:			
Mean	−0.00142	−0.00014	−0.00096
Standard Deviation	0.00359	0.00358	0.00362
Futures 1-1 Hedge:			
Mean	−0.00028	−0.00025	−0.00020
Standard Deviation	0.00800	0.00544	0.00327
Cost-Effective:			
Mean	−0.00023	0.00027	−0.00013
Standard Deviation	0.00356	0.00493	0.00352

*Mean and standard deviation of a total of 432 (overlapping) weekly returns. Standard deviations are adjusted for first-order autocorrelation of returns using $\sigma_{actual} = \sigma_{estimated}(1 + 2\rho)$, where ρ is the first-order autocorrelation coefficient.

EXHIBIT 3A ■ Futures and Options Positions for Hedging GNMA 7.5s ■ January 4, 1994 –September 21, 1995



The cost-effective or optimal hedging produces relatively favorable return and risk profiles. The returns are enhanced under this hedging method, while the variabilities of returns are still significantly reduced. The return/risk analysis shows the clear dominance of the cost-effective hedging portfolio over futures-only delta hedging for GNMA 7.5s and 9.5s, and for futures-only 1-1 hedging for GNMA 7.5s and 8.5s. The cost-effective hedging also dominates all three unhedged positions of GNMAs.

This superior performance of the cost-effective hedge is due to the rising and highly volatile interest rate environment during the testing period. In fact, the cost-effective hedging of GNMA 8.5s would have produced a rare positive return for the fixed-income market during the testing period.

Exhibit 3A shows the number of futures and options contracts to be traded for cost-effective hedging of GNMA 7.5s during the testing period. In the beginning of the period, large numbers of in-the-money puts and calls are bought in addition to long positions in futures. It is interesting to note that futures are bought in conjunction with long option positions in order to neutralize deltas and keep gammas positive.

Afterward, hedging consists mostly of short positions in in-the-money calls. It seems that writing call options is the cheapest way to hedge GNMAs during the later period. For futures-only hedging, by contrast, the number of futures contracts sold is fairly consistent throughout the period, although slightly more futures contracts are sold near the end of 1994 when the interest rate is higher.

Exhibits 3B and 3C show the numbers of futures and options contracts to be traded for GNMA 8.5s and 9.5s. For higher-coupon GNMAs, cost-effective hedging requires more positions in calls and puts, with occasional positions in futures.

The exact relationships between all the option series and futures in hedging deltas and gammas are not easily depicted. It should be noted, however, that in-the-money calls and puts are heavily used in the hedging. What is intuitively appealing is that, in the early period of 1994 when interest rates are low and GNMAs are priced at a premium, more calls and puts are bought to offset the negative convexities of GNMAs for cost-effective hedging.

IV. SUMMARY

This article shows that exchange-listed futures and options can be used to hedge the duration and convexity of mortgage-backed securities. Since such securities have

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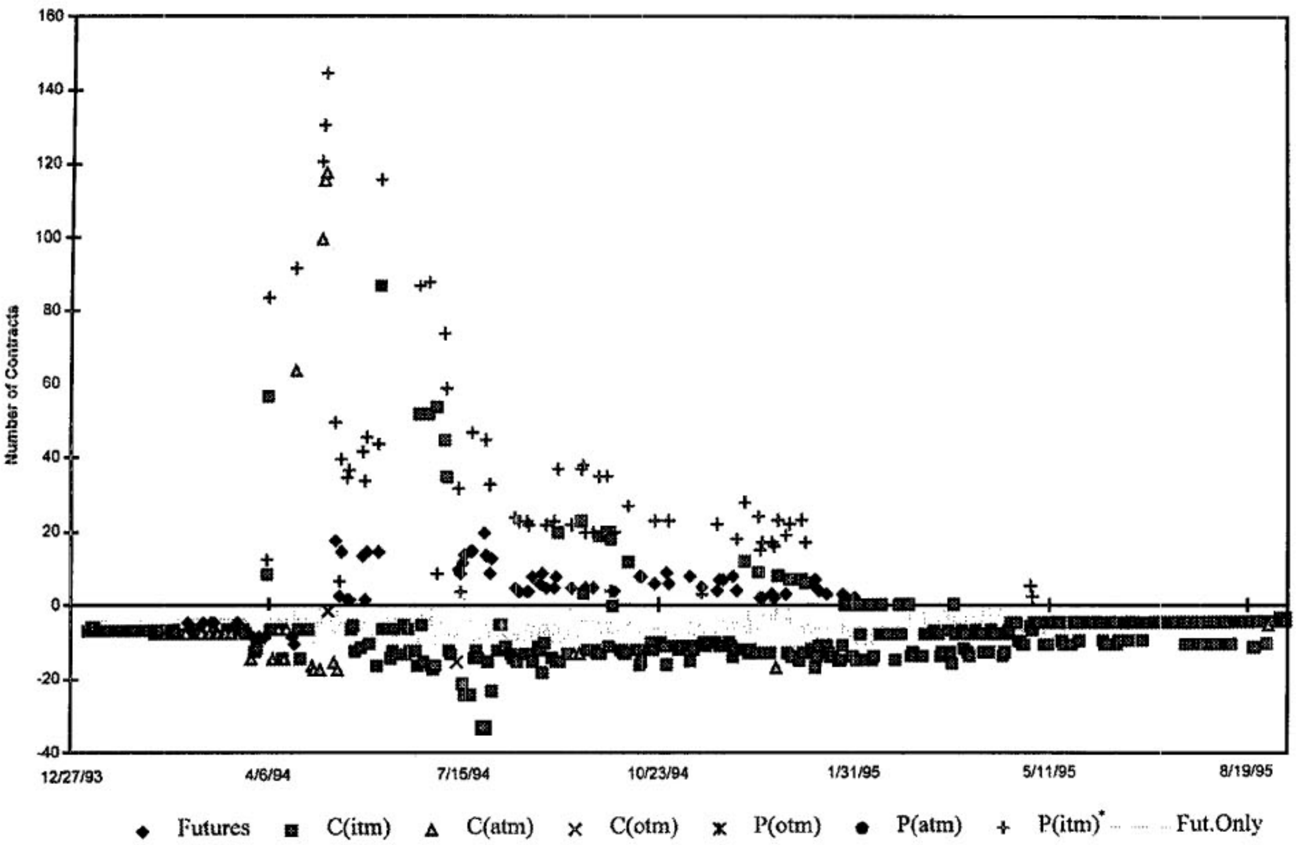
EXHIBIT 3B ■ Futures and Options Positions for Hedging GNMA 8.5s ■ January 4, 1994 –September 21, 1995



short positions of prepayment options, they tend to show negative convexity when they are trading near premium. Options that have large positive convexities can be effectively used to offset such negative convexity of MBS.

Optimization of the total transaction costs while offsetting the duration and convexity of MBS produces much more desirable risk/return characteristics of the hedged portfolio. The test results show the clear dominance of cost-effective hedging in producing higher returns with lower variabilities compared to not hedging and hedging with futures only.

During the testing period, options are consistently used to construct the hedged portfolio. In particular, when interest rates are low, large numbers of in-the-money calls and puts are used to offset the negative convexity of MBS.

These results are of obvious interest to MBS market participants, and options on Treasury note futures are likely to become important instruments in hedging MBS. Extension of these results to the use of caps or floors or other hedging instruments (such as Eurodollar futures) as alternatives in the optimizing portfolio mix deserves more research.

ENDNOTES

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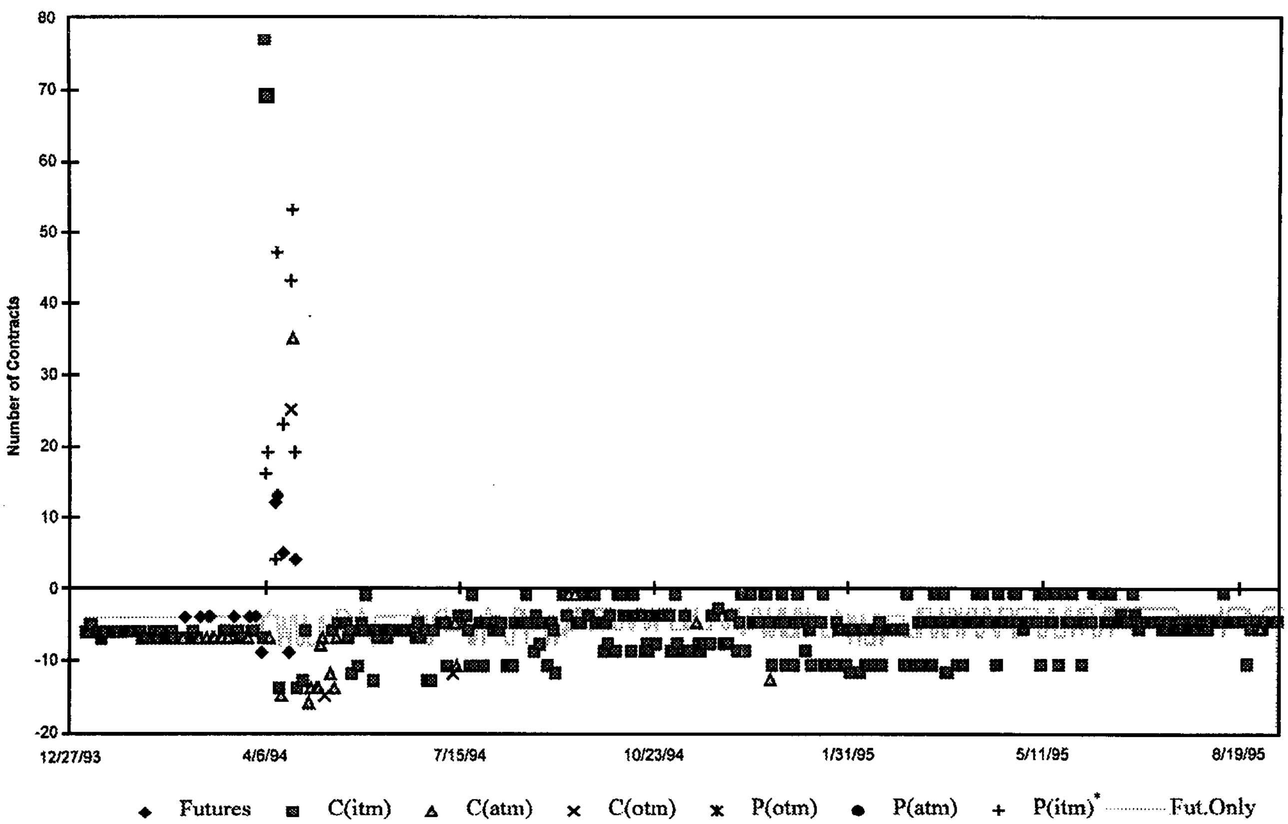
[1]The OAS method proceeds as follows. First an OAS is obtained through an interest rate tree simulation. Next, duration and convexity estimates are obtained by measuring changes in mortgage prices from raising and lowering the rate by 1 basis point. The OAS duration takes into account both the effect of the Treasury yield curve and the effect of interest rate changes on prepayment rates.

[2]Option premiums are required to be paid up-front for the options traded at the Chicago Board of Trade, so financing the purchase of options entails some financing

EXHIBIT 3C ■ Futures and Options Positions for Hedging GNMA 9.5s ■ January 4, 1994 –September 21, 1995



*itm is in-the-money; atm is at-the-money; otm is out-of-the-money. C is call option; P is put option.

costs. Opening a futures position requires similar financing cost to post initial margins, and thus the financing costs are canceled out in the objective function.

[3] $\text{Delta}_{MBS} = \text{Duration}_{MBS} \times \text{Price}_{MBS} \times 10{,}000$

$\text{Delta}_{Futures} = \text{Duration}_{Futures} \times \text{Price}_{Futures} \times 1{,}000$

$\text{Delta}_{Options} = \text{Duration}_{Options} \times \text{Price}_{Options} \times 1{,}000.$

[4] The non-linear optimization program is written using RATS.

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