PREPAYMENT CONVEXITY AND DURATION

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The bond market sell-off during March and April 1994 marked the beginning of a new debate on mortgage-backed security prepayments. Prior to the sell-off, 95% of MBS traded at premium prices, and the market's main worry was that prepayments might soar to record highs. By April, 75% of MBS were trading at discount prices, and extension risk was the talk of the market. Some investors believed historical prepayment data on discount MBS would provide a useful guide to future prepayments, others that MBS would extend more than indicated by the historical evidence.

The market initially sided with those believing in greater extension, and MBS spreads widened 25 to 30 bps during March and early April. Since then, the market has gradually revised its expectation of future prepayments upward, and MBS spreads have tightened significantly.

Such volatility in the MBS market is just the latest example of how changes in market prepayment assumptions can impact MBS returns. Prepayment revisions have always been a primary determinant of volatility in the MBS market, and prudent MBS investment strategies require knowing how prices will behave if prepayment expectations suddenly change.

Standard mortgage valuation analysis is useful for anticipating the way prepayment revisions associated with interest rate changes will impact MBS prices, but such analysis assumes that forecasted prepayments will equal actual prepayments for any given interest rate scenario. In reality, prepayments do not always behave as forecasted, so realized MBS returns tend to be more volatile than traditional valuation techniques indicate.

Prepayment forecast error cannot be eliminated,

EXHIBIT 1 ■ Prepayments on Seasoned FNMA/FHLMC Discount MBS

Year	Mortgage Rate (%)	WAC-Mortgage Rate (%)	12-Month PSA
1982	15.1	−5.5	48
1983	13.3	−3.5	102
1984	14.0	−4.0	118
1985	12.3	−2.9	132
1986	10.0	−1.3	170
1987	10.2	−1.4	180
1988	10.4	−1.3	147
1989	10.5	−1.0	130
1990	10.3	−0.8	123
1991	9.3	−0.6	137
1982-1991	11.5	−2.1	129

but it can be better understood and quantified. In our analysis, we show how the concepts of prepayment duration and convexity can be used to offset the negative effects of prepayment forecast error. We also discuss the way we incorporate forecast error into our mortgage valuation model.

Our analysis indicates that traditional mortgage valuation techniques systematically misstate the value of securities having significant prepayment convexities. Certain types of PAC IOs and extended PAC IIs are among the securities appearing more attractive once prepayment forecast error is taken into account.

I. HISTORICAL PERSPECTIVE

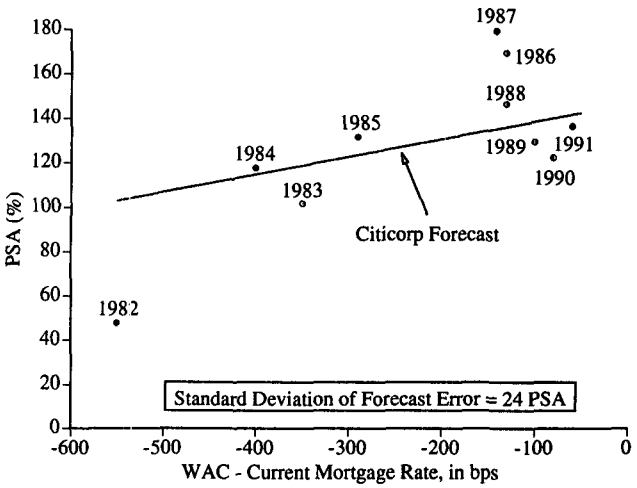
Prepayments on seasoned discount FNMA/FHLMC MBS varied considerably between 1982 and 1991. As shown in Exhibit 1, prepayments on these pools ranged from a low of 48 PSA to a high of 180 PSA over this period.

Exhibit 2 shows average historical prepayments

EXHIBIT 2 ■ Average Prepayments by Relative Coupon ■ FNMA/FHLMC Seasoned Discount MBS 1982-1991

WAC-Mortgage Rate (%)	PSA
−1 to 0	143
−2 to −1	135
−3 to −2	128
−4 to −3	117
Below −4	95

EXHIBIT 3 ■ Actual Prepayments versus Citicorp Forecast*



*For seasoned FNMA/FHLMC discount MBS.

according to the difference between prevailing mortgage rates and the pools' gross weighted-average coupons. As prevailing mortgage rates rise above the rates on existing mortgages, prepayments seem to decrease. Pools with WACs 0 to 100 bps below prevailing rates, for example, have prepaid at an average speed of 143 PSA, while pools with WACs 100 to 200 bps below prevailing rates have prepaid at an average speed of 135 PSA.

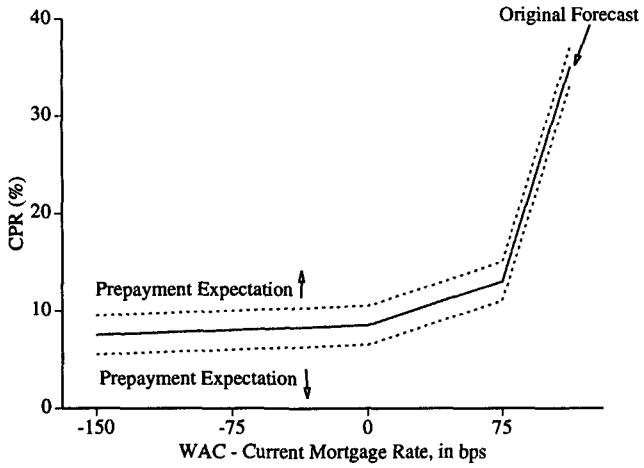
Our model projects prepayments on the basis of the characteristics of the mortgages underlying an MBS pool and an assumption about the future course of interest rates. We use our model to forecast discount MBS prepayments between 1982-1991 using the mortgage rates that actually prevailed during those years.

Exhibit 3 plots year-by-year historical prepayments against our forecasts. The model has generally fit the historical data fairly well, but there have been periods when prepayments deviated significantly from forecasted levels. Against a background of soaring mortgage rates and an economy in the midst of a major recession, prepayments fell significantly below forecasted prepayments for the year 1982. In contrast, a strong housing market caused prepayments to rise significantly above forecasted levels for the year 1987.

II. METHODOLOGY

How would MBS prices behave if market pre-

EXHIBIT 4 ■ Change in Prepayment Expectation



payment assumptions change without any movement in interest rates?

We address this issue by assuming that prepayment expectations change in such a way that the entire prepayment curve shifts by a constant amount from its previous level (see Exhibit 4). Using this new prepayment curve, we then compute the security's price under the assumption that the security's new option-adjusted spread equals its previously computed OAS. Computation of the security's new price uses the same term structure of interest rates as in the original OAS computation.

It should be emphasized that the way we shift the prepayment curve is intended to be an approximation for modeling forecast error resulting from unexpected changes in residential mobility. Modeling forecast error resulting from inability to predict mortgage refinancing behavior is beyond the scope of the discussion here.

Exhibit 5 shows the predicted relationship between prepayment revision and price for the current coupon GOLD 8.5. If prepayment expectations are revised upward, the GOLD 8.5 shortens on a positively sloped Treasury yield curve, and its predicted price rises by 16/32nds. Its price falls if prepayment expectations are revised downward and the security extends.

III. PREPAYMENT DURATION AND CONVEXITY

Just as interest rate duration and convexity are useful measures for examining a security's interest rate sensitivity, so prepayment duration and convexity are useful measures for examining a security's sensitivity to prepayments.

We define prepayment duration as the percentage change in price for a 1-percentage point change in the market's conditional prepayment rate (CPR) expectations. A principal-only MBS (PO) is an example of a security with positive prepayment duration, while an interest-only security (IO) has negative prepayment duration (see Exhibit 6).

Securities with large prepayment durations will tend to have more volatile returns than securities with smaller prepayment durations. They are also more difficult to hedge with non-mortgage instruments such as Treasury notes. Support tranches, IOs, and POs usually

EXHIBIT 5 ■ Impact of Prepayment Revision on Price

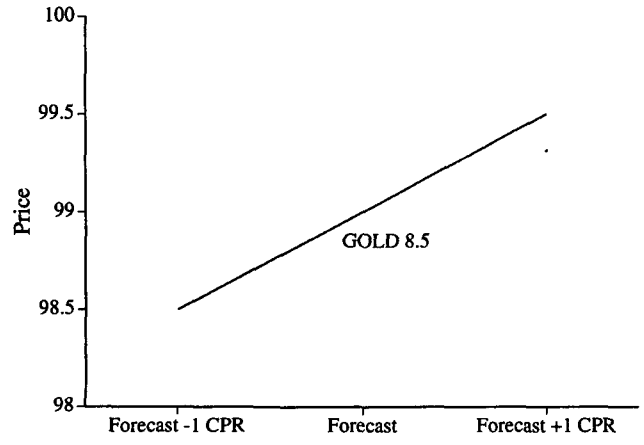


EXHIBIT 6 ■ Impact of Prepayment Revision on Price

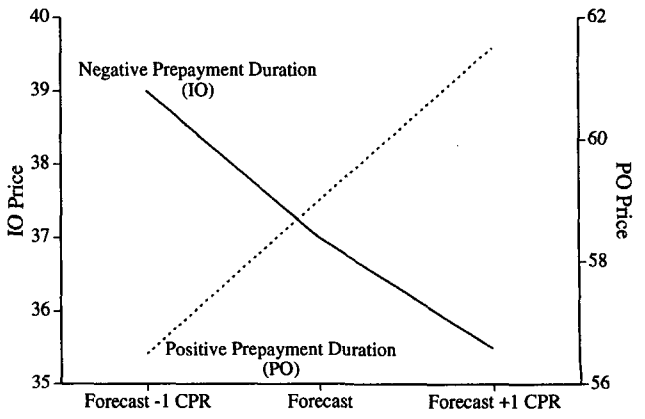
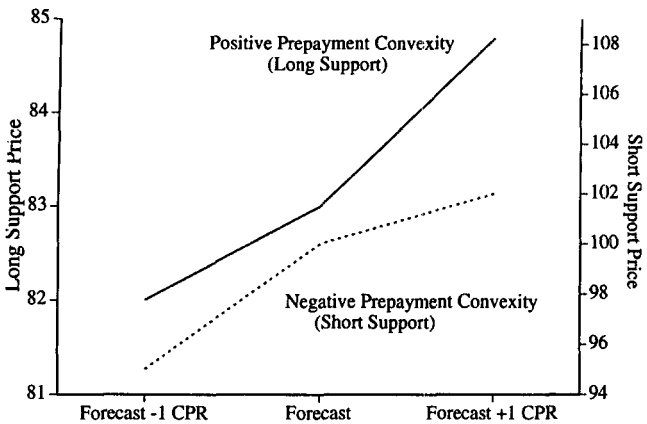


EXHIBIT 7 ■ Impact of Prepayment Revision on Price



have large prepayment durations, while PAC I tranches generally have very small prepayment durations.

Prepayment convexity measures the "curviness" of the relationship between price and prepayment assumption. Intuitively, an MBS exhibiting prepayment convexity is one in which prepayment forecast error has an asymmetric effect. In other words, a downward revision in prepayment expectations has a much different effect on price from an upward revision.

Securities with positive and negative prepayment convexities are shown in Exhibit 7. A deeply discounted long support tranche with little extension risk but with the possibility to shorten significantly is an example of a security with positive prepayment convexity, while a par-priced short support tranche is an example of a security with negative prepayment convexity.

IV. IMPACT ON VALUATION

The presence of prepayment convexity means that traditional OAS analysis is incomplete. More specifically, standard OAS analysis assumes that actual prepayments will equal forecasted prepayments for a given path of interest rates. If this assumption is relaxed, certain classes of MBS may have significantly different OAS.

To examine this further, we alter our OAS model to incorporate the effect of prepayment forecast error. We first assume that our prepayment model is correct on average, but that actual prepayments will sometimes diverge from our model's forecast. We then assume the standard deviation of forecast error to be 1.5 CPR. These assumptions are consistent with observed differences between actual prepayments and our forecasted levels for seasoned discount FNMA/FHLMC MBS over the 1982-1991 period.

Exhibit 8 shows the results for seven different securities. Incorporating prepayment forecast error into our valuation model barely impacts the OAS on the pass-through, collateral IO, or the ten-year PAC I. The OAS on the PAC IO and the PAC IIs, on the other hand, are significantly impacted.

The OAS on the PAC IO rises 167 bps once prepayment forecast error is taken into account. The reason for this pattern is that the PAC IO's current prepayment forecast is close to its lower PAC band. If prepayments slow unexpectedly, the PAC IO will extend, and its price will appreciate significantly. If prepayments rise unexpectedly, the PAC IO has enough PAC band protection so that it is unlikely to shorten and therefore incurs only a small decline in price. As a result, the PAC IO has

EXHIBIT 8 ■ Impact of Forecast Error on OAS

Security	Tranche Type	Price	Collateral	Prepayment Duration	Prepayment Convexity	OAS With Forecast Error	OAS Without Forecast Error	Difference in OAS
FHG 35 PL	4-yr. PAC IO	25-11	GNMA 8s	−6.1	557	423	256	167
FH 1629 MB	Extended PAC II	81-20	GOLD 6s	5.1	392	79	21	58
FH 1714 LB	Extended PAC II	87-00	GOLD 7s	2.3	151	73	46	27
FN Trust 254	Collateral IO	39-23	FNMA 7.5s	−5.0	41	82	79	3
FNMA 7.5	Pass-Through	93-14	FNMA 7.5s	0.6	−7	40	42	−2
FN 93-160 PK	10-yr. PAC I	83-30	FNMA 6.5s	0.2	−13	51	54	−3
FN 94-56L	3-yr. PAC II	90-07	FNMA 6.5s	6.7	−209	15	36	−21

Analysis as of October 27, 1994.

greater value than traditional analysis would indicate.

In contrast to the other securities, the three-year PAC II appears much less attractive once forecast error is considered. This tranche can extend if prepayments slow down unexpectedly, but is unlikely to shorten if prepayments are unexpectedly high. Given its discount dollar price and the likelihood of extension, its OAS is lower than traditional analysis would indicate.

The general message of our analysis is that securities with positive prepayment convexity generally appear more attractive once prepayment forecast error is taken into account. Traditional OAS analysis will understate the OAS on securities with positive prepayment convexities, and will overstate the OAS on securities with negative prepayment convexities.

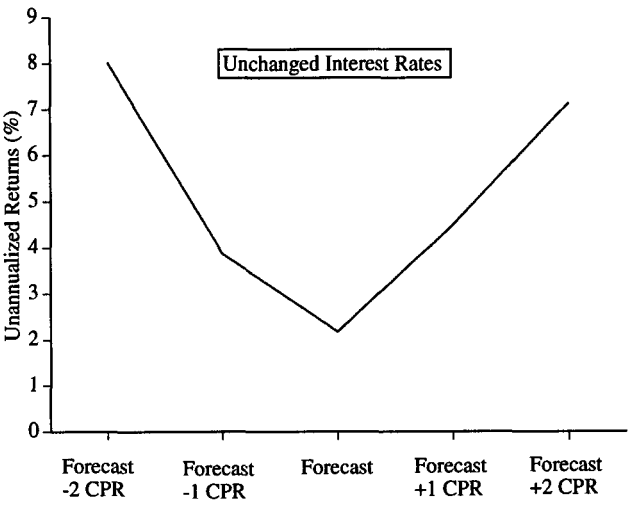
V. ENHANCING RETURNS

The concepts of prepayment duration and convexity can be used to construct portfolios with highly attractive prepayment characteristics. To show an example, we construct a portfolio using an extended PAC II (FH 1629 MB) and a PAC IO (FHG 35 PL). These securities have positive prepayment convexities, but the PAC IO has a negative prepayment duration; the PAC II has a positive prepayment duration. We assign a market weight of 46% to the PAC IO and a market weight of 54% to the PAC II. Constructed in this way, the portfolio is prepayment duration-neutral and has a 4.9 interest rate duration.

For simplicity, we assume that the prepayment forecast error on the GNMA 8s underlying the PAC IO is perfectly correlated with the forecast error on the FHLMC 6s underlying the PAC II. Relaxing this assumption does not alter our basic results, as long as the correlation of forecast error between the different collateral types is sufficiently high. We believe this is a reasonable assumption to the extent that changes in the propensity of the population to move are likely to have similar prepayment effects on different types of MBS collateral.

For the case of an unchanged interest rate scenario, we estimate that the portfolio of the PAC IO and PAC II produces a 2.17% return over a three-month holding period if market prepayment assumptions do not change (see Exhibit 9). The portfolio's performance improves considerably if prepayment expectations change over the holding period. We estimate that the portfolio returns almost 4% in the case of a one-CPR increase or decrease in market prepayment expectations. Even greater returns are realized when prepayment expectations change by more extreme amounts.

EXHIBIT 9 ■ Impact of Changes in Prepayment Expectations on Three-Month Portfolio Returns*



*Portfolio weights: 54% FH1629 MB/46% FHG35 PL.

In other words, by combining securities with positive prepayment convexities but offsetting prepayment durations, we construct a portfolio that generally performs better when market prepayment assumptions change than when assumptions remain unchanged.

VI. CONCLUSION

We have illustrated how revisions in market prepayment expectations can impact MBS prices and how incorporating prepayment forecast error into MBS valuation can systematically alter the valuation of certain classes of securities. As a beginning effort, our work focuses on the way forecast error impacts discount MBS. The assumptions we have made may be subject to debate, but we feel the basic results are realistic.

We believe that pricing in the current CMO market does not yet fully reflect the presence of prepayment forecast error, and we advise opportunistic investors to seek out MBS with positive prepayment convexity.

ENDNOTE

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