Mortgage Option Deltas

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In this article, we discuss options on TBA (to-be-announced) forward mortgage-backed security (MBS) trades. TBAs are forwards on pools of agency, fixed-rate mortgages, with a contract for each agency, coupon, loan term, and month. For example, there is a single contract for Fannie Mae pools paying a 3.5 coupon backed by 30-year mortgages to be settled in June. Another example is a TBA contract on Ginnie I pool paying a 3.0 coupon backed by 15-year mortgages to be settled in July. For simplicity, throughout this article, the reader may assume that we're talking about Fannie Mae 30-year, fixed-rate collateral. TBA contracts for such pools trade with monthly settlements for coupons in half-point increments. The methodology discussed in this article can be applied to other types of TBAs.

TBA options are European options (calls and puts) on TBA contracts. Each TBA option is on a specific TBA contract for a particular coupon with typical expiry one week before the settlement date for the contract. Mortgage options are used by mortgage originators to hedge pipeline risk and are also traded by various entities, including hedge funds, for largely speculative strategies.

Options on the most-liquid TBA contracts are themselves the most liquid. For our investigation in this note, TBA settlements are up to three months.

It is common to model TBA options with the S-curve framework proposed in Prendergast [2003]. This model takes as inputs a distribution of swap rates, as inferred from swaption prices, and DV01s for TBAs.[1] It uses the (integrated) DV01s to translate the distribution of swap rates to a distribution of TBA prices. From this price-distribution option prices may be calculated as discounted expected values. The model assumes some consistency between swaption prices, TBA DV01s, and mortgage option prices. This relationship is meaningful but may break down at times. In particular, there are somewhat frequent episodes during which mortgage option prices are significantly higher than such a model predicts, indicating a discrepancy between a rates swaption market and mortgage option market in terms of their respective supplies and demands.

There are various ways of dealing with this discrepancy, including applying a scaling factor to swaption volatilities (vols) used in calibration, adjusting the TBA DV01 curve, or adding rate-independent variance to TBA prices. As discussed in some more detail later, the first of these options seems to be common, but we have not seen discussion of the implications of this choice for risk management. This is crucial for delta hedging because the different modifications of the base S-curve framework may lead to the same option prices but different delta risks for hedging.

One can compare the deltas from the various modifications of the base S-curve model, and it is desirable to have a benchmark for them. We find that mortgage option market data are consistent with a simple local volatility framework. We estimate deltas using local volatility and compare the deltas resulting from different modifications of the base S-curve framework to these "model-free" estimates.

The choice of modification also has some implications for skew. Substantial market data on skew are difficult to come by, so it's difficult to assess models based on it. Given the local vol framework, however, we discuss the implications of model adjustment choice on skew.

THE S-CURVE MODEL

A very common framework for modeling TBA options is the S-curve model as in Prendergast [2003] and, later, Liu [2008]. The framework translates a distribution of mortgage rates to a distribution of TBA prices through a deterministic mapping function. In practice, the mortgage rate is substituted with a proxy swap rate, and the swap rate distribution is calibrated to available market swaption prices. The mapping function is usually derived from the DV01 curve of TBAs. A common parametric form for DV01s (as in Prendergast [2003]) is given by

$$\frac{-dP}{dr} = DV01(r, C_p) = d + \frac{b}{1 + e^{-c(r - C_p - a)}} \tag{1}$$

where P is TBA price, r is the representative swap rate, C_p is the coupon on the TBA contract, and a, b, c, and d are parameters to be fit. The parameters are fit to the DV01s on TBAs from some other source, for example, from an option-adjusted spread (OAS) model, as described in Hayre [2001].

In order to directly translate a representative swap rate to a TBA price the DV01 form in (1) is integrated:

$$P(r, C_p, S_t) = P_H(C_p, S_t) - d \cdot (r - C_p) - \frac{b}{c}\log(1 + e^{c(r - C_p - a)}) \tag{2}$$

where P_H is an integration constant and S_t is the settlement date. The integration constant can be set so that we price the TBA contract as an expected value:

$$E(DF \cdot P(r, C_p, S_t)) = DF_0 \cdot P_0 \tag{3}$$

where DF is a stochastic discount factor, DF_0 is the discount factor known from today's yield curve, and P_0 is the market TBA price for the given coupon and settlement date.[2]

The European option price is based on the TBA price distribution on the expiry date. Given the swap rate distribution and the rate–price mapping function, the TBA price distribution is obtained. The swap rate distribution is derived from the swap rate dynamics, which is calibrated to the implied swaption vols of the market. Note that the TBA option market trades separately from swaptions with their respective flows, therefore, the swaption vol level may not be a good indicator of TBA vol level in general. As a matter of fact, market TBA option prices are often at a significant premium over that which the S-curve model predicts, indicating that the TBA price variance is above that simply predicted from a fixed S-curve and the market swaption vol. We note on this point, too, that this fact does not seem to depend on one's OAS model: Within any reasonable range of TBA DV01s, TBA options can at times still be at a significant premium to model price. This premium was at an especially high level in the summer of 2011, which we include in our time series for the analysis to follow.

From industry publications, for example, Velayudham [2010] and Lesniewski [2008], it appears to be common to think of mortgage vol relative to swaption vol by using the S-curve framework along with a scaling factor. The scaling factor is applied to widen the swap-rate distribution. So a factor of, say, 1.2 will increase the level of (normal) swaption vol by 20%. Given this scaling, keeping the S-curve the same, one can compute a wider distribution of TBA prices and the resulting increase in the option price to the market level. Factors of up to 1.6 have been quoted in industry literature. Factors this large are seen by multiple practitioners so are not too dependent on variations in DV01s. We observed such large factors in the summer of 2011.

Two alternative ways can be adopted to match the market option prices in the S-curve framework. One way is to modify the S-curve parameters. The most obvious way to increase option values in this way is to shift the DV01 curve up by increasing d, but the other three parameters may be adjusted as well. We'll explain later why increasing only *d* may not be the best choice. A second alternative is to add an extra variance to the TBA price distribution in addition to the variance generated from the swap rate distribution through S-curve. On this point, the standard S-curve framework is based on the dynamics specified by the following SDE.

$$dP = DV01 \cdot dr \tag{4}$$

In the spirit that mortgage prices are not perfectly correlated with rates, we could also use

$$dP = DV01 \cdot dr + dS \tag{5}$$

where S is a stochastic process. In general, S can be loosely correlated to the rate, r. For simplicity, we will consider a rate-independent process in our current investigation. A suitable choice for S is a scaled Brownian motion. This amounts to adding an independent, normally distributed random variable to the S-curve-derived prices.

In summary, we have three ways to match market TBA option prices, vol scaling, S-curve modification, and rate-independent TBA price variance. In order to make distinctions among the schemes we look at the option deltas with respect to the underlying TBA contracts. To set a benchmark for deltas we use a local-vol calculation of deltas. Note that because of the potentially large-magnitude negative convexity of TBAs, the calculation of option deltas is somewhat delicate. This will be discussed in more detail in the following.

OPTION DELTA CALCULATION FROM LOCAL VOL

From market quotes of at-the-money (ATM) options across coupons (see Exhibit 1), one sees a clear inverse relationship between TBA price and Black–Scholes implied (price) vol for TBA options, implying an inverse dependence of the underlying model vol on the TBA price. A local-vol model provides a framework to investigate the option price and delta with regard to the underlying TBA price and the vol (Derman [1994], Derman [1995], Dupire [1994], and Rubinstein [1994]). From this perspective, we can see the deltas in terms of changes in the level of price volatility instead of through the negative convexity of TBAs. When the underlying price is bumped up, the local-vol model decreases the vol level according to the price level and, in turn, leads to a smaller value and also delta for the call option than those from a direct application of Black–Scholes by solely altering underlying price with the vol not dependent on price. In particular, it drives the deltas of ATM calls down below 1/2. How much below 1/2 depends on the local vol function, more specifically, on the steepness of the curve giving vol as a function of price.

Next, we outline some details on calculating deltas with a local vol model. For a local vol model for the evolution of an asset price, we have

$$dP = P \cdot r \cdot dt + P \cdot \sigma(P) \cdot dW \tag{6}$$

where P is the asset price, r is the risk-free rate, W is a Brownian motion, and $\sigma(P)$ gives the local volatility as a function of price. The local volatility may more generally depend also on time explicitly as $\sigma(t,P)$, but this simplified version $\sigma(P)$ suits our current investigation needs. Note that local vol $\sigma(P)$ is different from (Black–Scholes) implied vol, $\sigma_{BS}(P,K)$, where the implied vol depends on the strike of the option and the price of the underlying in addition to option type, expiry, and the risk-free rate.[3] In the following discussion, let us assume that we have a call option with a fixed expiry and a fixed risk-free rate.

Our main concern is to look at small perturbations in price in order to calculate deltas. Thus, we use a linear approximation for the local vol function with a steepness β ($\beta>0$) for the inverse dependence of the local vol $\sigma(P)$ on the price P:

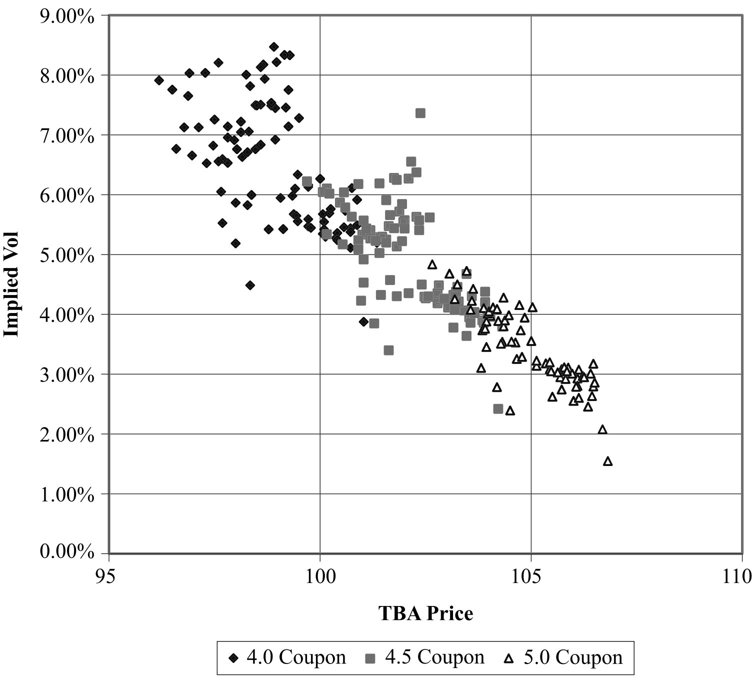
$$\sigma(P) = \alpha - \beta \cdot P \tag{7}$$

In passing, we note that, based on empirical data as shown in Exhibit 2, it appears to us that the local vol function is actually well fit globally by a linear function.

EXHIBIT 1
Market Quotes of ATM Options Across Coupons

Expiry	Coupon	TBA Price	Bid Price	Ask Price	Bid Vol	Ask Vol
Oct. 2011	4.0	104–00	0–17+	0–22+	5.67	7.29
	4.5	105–31	0–11+	0–16+	3.66	5.25
	5.0	107–25	0–06+	0–11+	2.03	3.6
Nov. 2011	4.0	103–23	0–28+	1–01+	5.74	6.75
	4.5	105–23	0–19+	0–24+	3.86	4.84
	5.0	107–17	0–11+	0–16+	2.26	3.23
Dec. 2011	4.0	103–13	1–04	1–09	5.87	6.68
	4.5	105–15	0–25+	0–30+	4.07	4.87
	5.0	107–10	0–17	0–22	2.67	3.45

EXHIBIT 2
Implied Volatility vs. Price



Given the linear local vol function, we use two key facts:

1. ATM vol is very close to local vol. This is very easy to test via Monte Carlo, and results are shown in Exhibit 3. For calculation of deltas we are primarily concerned with the slopes of the ATM implied vol and local vol curves, and less so with the absolute levels. These slopes are especially close.
2. The steepness of the implied-vol skew with regard to strike is about half the steepness of the local vol curve with regard to the price.[4] We find that the skew steepness is just slightly under $\beta/2$.

These two facts together lead to the following approximation formula to the implied vol:

$$\sigma_{BS}(P,K) \approx \sigma(P) + \frac{\beta}{2}\cdot(K-P) \tag{8}$$

That is, for a given expiry, the implied vol is roughly approximated as the sum of the local vol function and an adjustment term of strike-price difference with a $\beta/2$ slope. Setting $K = P$ we see that it follows from Formula (8) that the ATM implied vol and local vol are the same:

$$\sigma_{BS}(P,P) = \sigma(P)$$

From this it results that the derivative of the ATM backbone curve with respect to price is the same as the derivative of the local vol with respect to price:

$$\frac{\partial\sigma_{BS}(P,P)}{\partial P} = \frac{\partial\sigma(P)}{\partial P} = -\beta$$

This is important because it allows us to infer the slope of the local vol curve from ATM option (implied vol) quotes over different prices.

Given a local vol steepness β implied from the ATM backbone, we can estimate deltas as follows. Let $BS(\sigma_{BS},P,K)$ be the Black–Scholes call price with implied vol σ_{BS}, asset price P, and strike K. The delta of an ATM call is the change in option price when we increase the price of the asset. Note that when we compute the change in price of the option with a shifted asset price, the ATM option is no longer ATM, the skew needs to be taken into account. Using Formula (8) for the implied vol for strike K, the option delta can be calculated by the following first-order finite difference scheme:

$$\frac{\partial V}{\partial P} \approx \frac{BS(\sigma_{BS}(P,P)-(\beta/2)\cdot\Delta P, P+\Delta P, P) - BS(\sigma_{BS}(P,P),P,P)}{\Delta P} \tag{9}$$

Here V is the option value and ΔP is the amount we shift the price of the underlying for the computation. Alternatively, one could use the (Black–Scholes) vega of the option and the chain rule, which indicates an option delta consists of the Black–Scholes delta, and an adjustment from the Black–Scholes vega and the derivative of implied vol over the asset price (see Endnote 3).

We need values for ATM vol $\sigma_{BS}(P,P)$ and the slope β to calculate option delta in the current setup, ATM vol can be obtained directly from the market quote. We use two schemes to identify reasonable values for β.

1. Assume that the quotes for ATM TBA option and TBAs across a coupon stack in one day give us local vol as a function of price.

2. Look at historic ATM vol quotes versus TBA prices and the slope of the regression line.

Both schemes to identify values for β involve historic data. We use week-end and month-end ATM quotes for the first half of 2011. As mentioned earlier, during this period we see option prices that are especially high versus model prices. For each day, we use quotes for three expiries for each of the 4.0, 4.5, and 5.0 coupons.

For the first scheme, we use ATM option quotes across coupon stack as in Exhibit 1 to infer implied vol as a function of price. So, for example, using the numbers in Exhibit 1, we assume that the implied vol for the 4.0 coupon if it were to increase in price from 104 to 105-31 then its ATM implied bid vol would decrease from 5.67% to 3.66%, the current ATM implied vol for coupon 4.5. This assumption is consistent with the DV01s shifting over a coupon, as is commonly implemented for the S-curve framework.

For the second scheme of identifying β, we look at historic ATM implied vol for the 4.0, 4.5, and 5.0 coupons over the first six months of 2011. When we run regressions on implied vol versus price, we get very similar slopes for all three; they are −0.57, −0.62, and −0.61, respectively. The graph of volatility versus price in Exhibit 2 is fairly convincing that there is a linear relationship across coupons. When calculating deltas, we use the average over these, −0.60. The slopes inferred via Scheme 1 have considerable variation, and the slope of −0.60 turns out to be on the lower end of those.

In Exhibit 4, we display ATM call deltas coming from our two schemes of identifying a vol slope. For Scheme 1 we include output coming from the slope from the 4.0 coupon to the 4.5 coupon. For both, we include output for the expiries in the data with an implied to match the option price. Notice that the variation in the calculated slope from Scheme 1 is significant. Part of this is noise, part is inexactness from this wide market, and part is meaningful variation. One can easily identify an approximate middle range delta for each expiry.

COMPARISON OF S-CURVE DELTAS

We use the deltas from the local vol calculation as an indication of what deltas to expect from the various modifications of the S-curve model. For our tests, we use modifications of the S-curve model that all lead to the same ATM call prices and then compare the deltas that result. Even if we match prices, there can still be large differences in deltas.

As pointed out in Prendergast [2003], because of the negative convexity of the TBA,[5] we cannot simply shock the TBA price to calculate deltas. Therefore, in order to take into account the convexity of the TBA in the option delta calculation, we shift rates and look at the ratio of the change in value of the option to the change in value of the TBA:

$$\frac{dV}{dP} = \frac{dV}{dr} \Bigg/ \frac{dP}{dr} \tag{10}$$

This calculation takes in to account the negative convexity of the TBA through the change in the DV01s along the S-curve.

EXHIBIT 3
Implied ATM vs. Local Volatility at Initial Price

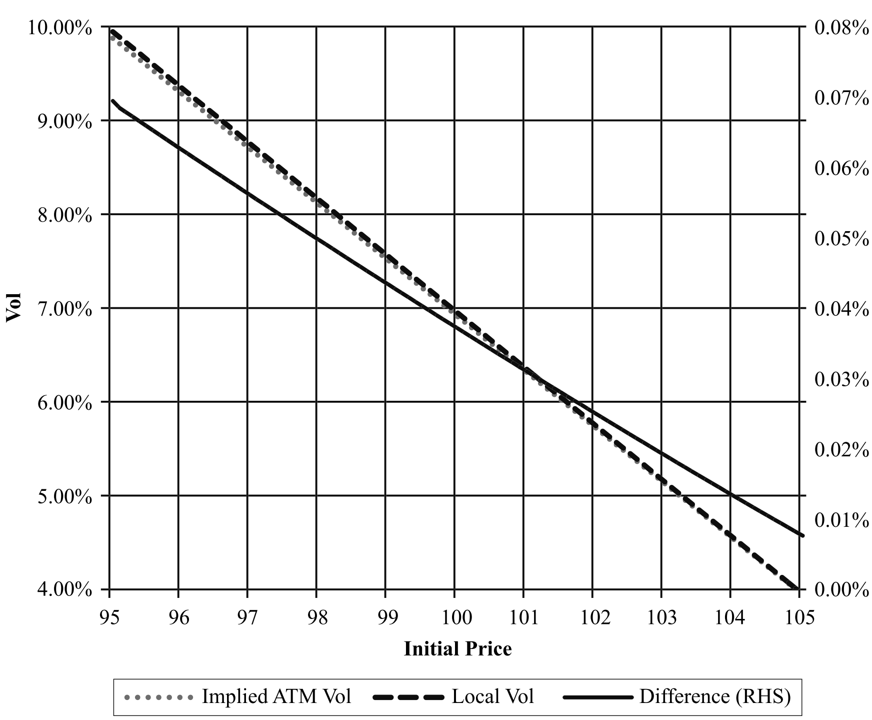
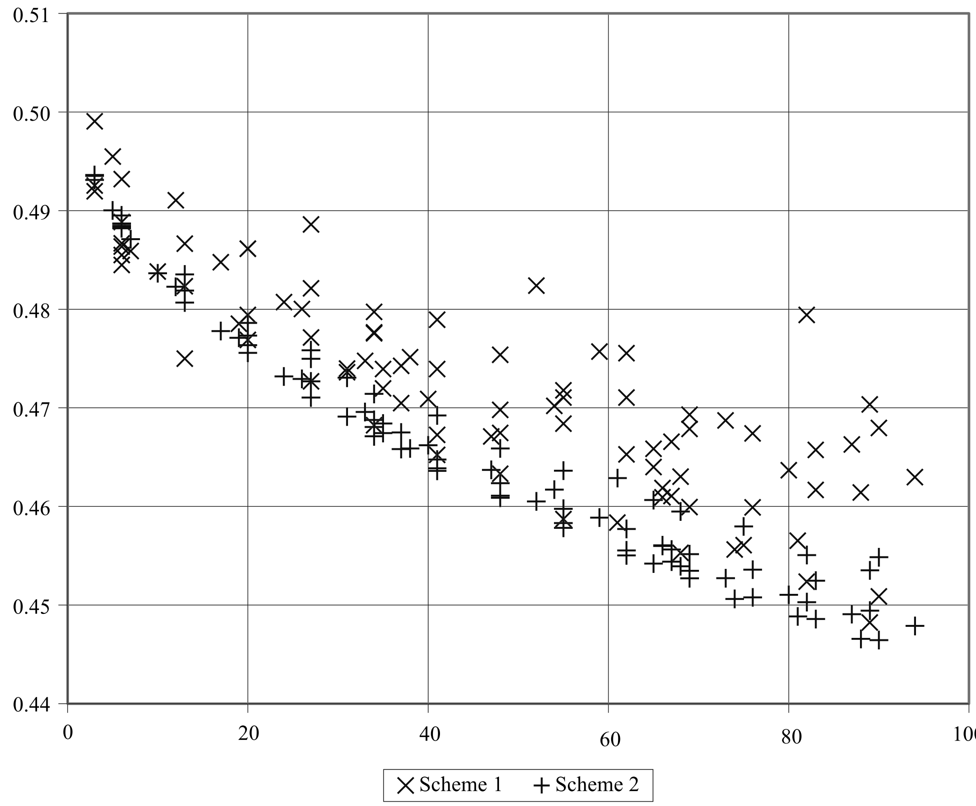


EXHIBIT 4
ATM Call Deltas Based on Local Vol Calculation



For our tests, unless explicitly stated, we will use S-curve parameters $a = -1.7$, $b = 6$, $c = 2$, and $d = 1.5$, and we assume the distribution of the representative swap rate is normal with mean 2.2% and annualized standard deviation of 103 bps. So, for an option of maturity T, we use a standard deviation of $\sqrt{T} \cdot 103$ bps. These are parameter values that are in line with the calibrations during the time period we are considering.

In our tests, besides modifying the S-curve parameters and the standard deviation of the rate distribution, we also consider the S-curve framework with rate-independent variance. Given Equation (5), we simply take the price distribution derived from using Equation (2) to transform our rate distribution and add to that an independent, normally distributed variable. Non-normal distributions may also be considered of course but would be left for future investigations.

Our goal is to make some comparisons of deltas from different model modifications that increase option prices to the same level. We start with a vol multiplier of 1.6. So, instead of using an annualized vol of 103 bps, we use 165 bps to elevate the price of ATM options. We then modify the model in other ways to match this new option price. In our investigation, we find that when we match option prices for some particular expiry (e.g., a two-month expiry) we also match very closely for all other expiries we care about, up to three months.

Three possible ways to match the 1.6 multiplier prices are shown in Exhibit 5 (as Modifications 3, 4, and 5) and the deltas of ATM calls are illustrated in

EXHIBIT 5
Three Ways to Match the 1.6 Multiplier Prices

	a	b	c	d	Vol Multiplier	Annualized Price Vol
Base S-Curve	−1.7	6	2	1.5	1	0
Modification 1	−1.7	6	2	1.5	1.6	0
Modification 2	−1.7	6	2	1.5	1	1.64
Modification 3	−1.7	6	2	4.24	1	0
Modification 4	−1.7	6	2.6	4.28	1	0

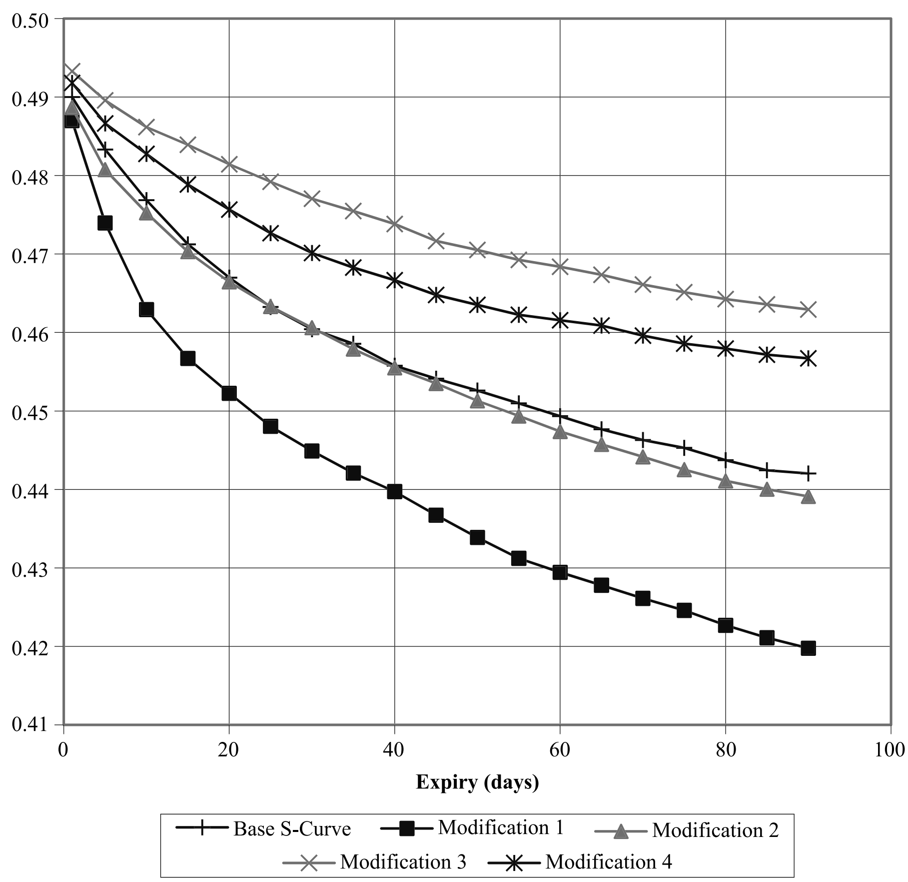
Exhibit 6. Regarding the rate-independent variance modification, we note that for a two-month option, our parameters lead to a TBA price distribution with standard deviation 4.28. For this expiry, the standard deviation of the rate-independent random variable is 0.67. For other expiries, we adjust this standard deviation to be consistent with the option expiry, meaning we use an annualized standard deviation of 1.64.

To elaborate on one subtle point about ATM call deltas: In general, the negative convexity of the TBAs is expressed in the decrease in the DV01s as rates go down. The more negatively convex, the larger the drop in the DV01s, so the steeper the S curve will be. This results in lower ATM call deltas: When rates go down, the (previously ATM) call value goes up. But the increase in option value is somewhat offset by the decrease in DV01s, because the TBA price vol is then lower. This leads to ATM calls having deltas less than 0.5. The more steep the S curve is, then the further below 0.5 the ATM call deltas will be. It is worthwhile to point out that if shifting just the P_H value, the deltas for ATM calls would not be below 0.5. This is because solely shifting P_H instead of the rate would not take into account the negative convexity of the TBA price, thus missing the effect of the vol embedded in the TBAs.

Following are our comments about the deltas for each of the four model modifications. Several of these comments deal with the steepness of the S-curve. This is closely related to the level of deltas:

1. Increasing only the TBA DV01 level parameter d leads to an increase in deltas. We interpret this to be because of the increased linearity of Equation (2) and a less steep S-curve relative to the overall level of DV01s.

EXHIBIT 6
Model ATM Call Deltas



2. With the increase in both DV01 decay speed parameter c and the level parameter d, we did not change the deltas from the base model much at all. Increasing c is increasing the steepness of the S curve. This steepness reflects an increase in the magnitude of the convexity of the TBA and thus drops the deltas. The effects of c and d may offset each other.
3. The deltas for the vol scaling are lower than the others: Scaling the vol is very similar to increasing the steepness of the S curve, and this drops the deltas.
4. Adding the rate-independent variance does not change the deltas significantly. Regardless of the level of rates, the option has some fixed minimum value because of the additional variance. The change in option value comes from just the S-curve when the underlying swap rate is bumped. Therefore, if the S-curve is fixed as in this modification, the option delta will not change much from the base case.

In addition to the absolute levels of the deltas, notice that the two modifications that deviate from the base model must also be somewhat unstable: As one increases d or the vol scaling, one is changing the deltas of ATM calls significantly. In contrast, in the Black–Scholes framework ATM deltas do not change much when implied vol changes. This seems to indicate an issue with these two S-curve modifications, and using them to determine hedge ratios may lead to unnecessary, chaotic rebalancing.

Our current investigation is based on the local vol framework to include the TBA convexity (or the equivalently the vol change) into the pricing and delta calculation of TBA options. Alternative frameworks, such as a stochastic vol framework (e.g., SABR model, Hagan et al. [2002]), could also be adopted. Different model frameworks may lead to somewhat different quantities in the resulting deltas from our current local vol framework choice, but the revelation from our current investigation, that the different schemes for market TBA option price matching qualitatively affect the hedging ratios, will persist. Also, in this article, we assume local vol is a linear function the underlying price. Thus, stochastic vol and its associated second-order smile effect on deltas (as discussed in Hagan et al. [2002]) are outside the scope of the current investigation.

SKEW

The calculation of deltas is closely related to skew. By skew, we mean implied (Black–Scholes) vol across strike for a fixed TBA price, that is, $\sigma_{BS}(P,K)$ as a function of K. Notice that this makes no explicit reference to local vol.

For a moment, we recall some behavior of a linear local vol model. For such a model there is a very direct connection between deltas and skew: If we make the local vol curve steeper, then both ATM call deltas decrease and the skew gets steeper (by half the amount that we steepened the local vol curve). We might expect the connection between deltas and skew to remain for the S-curve framework.

As one can see in Exhibit 7, this connection is not always there. For the vol multiplier modification, the relationship is clear: That modification has the lowest deltas and the steepest skew. Alternatively, the modification of adding independent price variance has relatively low deltas but the most gradual skew. One should expect this behavior of the latter modification: Unlike the other modifications, there is TBA price vol that does not depend on the level of prices. So, as TBA prices go up, there is less of a drop in price vol. This results in smaller changes in implied vol over different strikes.

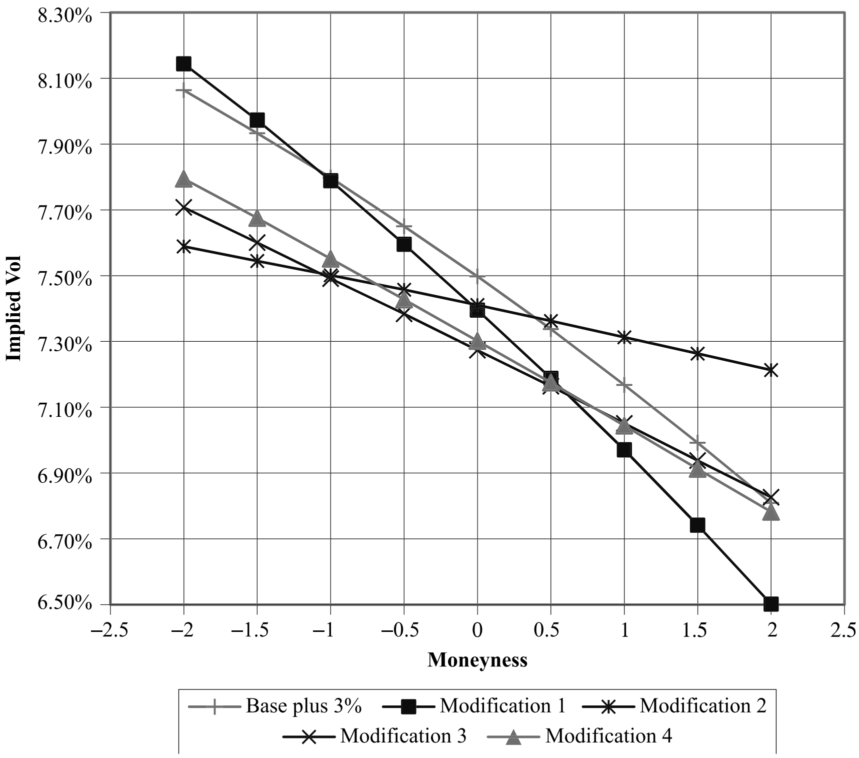
Going back to local vol, we see that the skew implied by a local vol model with $\beta = 0.6$ has a slope of -0.3. For the calibration of the S-curve that we have for the test shown here, this slope is most consistent with the base S-curve and is roughly consistent with both modified S-curves.

CONCLUSION

Mortgage options prices may be higher than the typical S-curve model suggests. Using the local vol framework, we analyzed the impact of various price-matching methodologies on the option delta. We concluded that the choice of how to deal with this significantly influences the deltas of the options with respect to the underlying TBA and thus has implications for risk management. In particular, uninformed practices in dealing with the price discrepancy may lead to erroneous deltas and hedging.

E X H I B I T 7

Implied Volatility Across Strike



ENDNOTES

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[1]DVO1 is the dollar value of a 1 basis point decrease in interest rates.

[2]The market convention for an ATM mortgage option on a specific coupon and settlement date is that the option's strike is equal to today's TBA price with the same specific coupon and settlement date. Equation (3) is a natural outcome of this convention and the assumption that an ATM mortgage option call and an ATM mortgage option put have the same present value. The drift of TBA prices derived from the swap rate dynamics and the S-curve mapping function in general may not match what is implied from the TBA and mortgage option markets. Equation (3) provides a facility to compensate the "missing" TBA price drift in order to match the market. A combination of Equations (2) and (3) calibrates P_H, which reflects this "missing" drift.

[3]The implied vol $\sigma_{BS}(P,K)$ can be considered roughly as an integration of local vol function in the range between the price level and the strike, see the appendix in Derman [1995]. The dependence of implied vol on the strike and the price introduces a correction to the option delta via the product of Black–Scholes vega and the sensitivity of implied vol w.r.t. the price, that is, $\frac{\partial V^{BS}}{\partial P} = \frac{\partial V^{BS}}{\partial P} + \frac{\partial V^{BS}}{\partial \sigma^{BS}} \frac{\partial \sigma^{BS}}{\partial P}$. In this note we tackle this effect through the local vol framework.

[4]This is "Rule of Thumb I" in Derman [1995].

[5]The convexity of TBA price with regard to the rate change is due to the embedded prepayment option that borrowers have. The option value is related to the level of rates, as accounted for in the S-curve, to general rate market vol levels (e.g., swaption vol), and to the other borrower and economic factors that affect prepayment speeds.

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