

# Prepayment Risk- and Option-Adjusted Valuation of MBS

*Opportunities for arbitrage.*

Alexander Levin and Andrew Davidson

**O**n option-adjusted spread (OAS), while a much better measure than yield or static spread, still falls short in explaining the dynamics of mortgage pricing. The standard OAS typically varies across instruments (pass-throughs, collateralized mortgage obligations, interest-only securities, principal-only securities), coupons, prepayment option moneyness, and pool seasoning stages.

Premium and discount MBS are often priced at wider OAS than the current-coupon issues. Premium MBS and IOs stripped off premium pools are considered hazardous, and their higher OAS reflect concerns of understated refinancing. Naturally, the respective POs look rich. In the discount sector, higher OAS reflects the risk associated with possible overstatement of the housing turnover rate.

Clearly, these market phenomena defeat the very purpose of a constant OAS approach. Rich-cheap judgments become inconclusive, and rate shock analysis can produce inaccurate hedge ratios.<sup>1</sup>

Like Cheyette [1996] and Cohler, Feldman, and Lancaster [1997], we attribute the OAS and its variability to the prepayment risk premium, i.e., possible non-diversifiable deviations of actual future prepayments from a best guess prepay model's forecast.

It is the market's fear of systematic bias in prepayment forecasts that leads to a risk premium. If prepayments were perfectly predictable, then an exact, even inefficient, option exercise model should produce a zero OAS to an appropriate option-free benchmark, just as options and

**ALEXANDER LEVIN** is head of Valuation Development at Andrew Davidson & Co., Inc., in New York City.  
[alex@ad-co.com](mailto:alex@ad-co.com)

**ANDREW DAVIDSON** is the president and founder of Andrew Davidson & Co., Inc.  
[andy@ad-co.com](mailto:andy@ad-co.com)

embedded option instruments with known algorithms of exercise such as swaptions, callable agency debt, and corporate bonds are all priced flat to their respective option-free rate curves.

While OAS varies widely among instruments, our new spread measure, called *prepayment risk- and option-adjusted spread* (prOAS, pronounced pro-A-S), accounts for both option and prepayment risk. We posit that, on a prOAS basis, all liquid agency MBS should be priced flat to agency debentures, eliminating the variability found in traditional OAS measures. Our method has its roots in the capital asset pricing model (CAPM) and its extension, arbitrage pricing theory (APT), where return compensation for risk and the concept of equivalent risk-neutrality play key roles.

For unstructured pass-throughs, we propose a prOAS valuation model that is armed with the power of backward induction and allows for endogenously finding risk measures and prices reflecting embedded prepay uncertainty—in the form of return spread compensation—computed for each investment period and level of interest rates. All the required prices and risk spreads can be found concurrently in the course of valuation performed backward on a probability tree or a finite-difference grid.

A price obtained in this manner will reflect the differences in prepayment uncertainty without the need to vary OAS across instruments. Values of IOs, POs, and mortgage servicing rights (MSR) can be objectively derived without knowing the OAS level for each instrument; if necessary, traditional OAS can later be calculated from the resulting prices.

We prove that this process of explicit endogenous prepay risk accounting is mathematically equivalent to risk-neutral prepayment modeling. Such a model retains the structure and the features of a physical prepay model, but operates with risk factors stressed to their undesired directions (where value deteriorates). A risk-neutral prepay model easily solves the issue of CMO valuation under price of prepay risk implicitly, without reliance on a non-feasible backward valuation. That is, we could refine the handling of CMOs by replacing the time-consuming prepayment stress tests described in Cohler, Feldman, and Lancaster [1997] with risk-neutral forecasts.

We find that two prepayment risk factors are essential: the risk of refinancing understatement, and the risk of turnover overstatement. Having calibrated prices of these two prepayment risks to a set of widely traded MBS, we can then produce prices for all other instruments exposed to the same risks. Without two independent risk

factors, it would be impossible to explain why both discounts and premiums of nearly all MBS collateral types are traded at higher OAS than non-MBS instruments. This paradox has apparently puzzled some authors; see Kupiec and Kah [1999] and Gabaix, Krishnamurthy, and Vigneron [2004]. A single-dimensional risk analysis would allow for hedging prepayment risk by combining premium MBS and discount MBS, a strategy any experienced trader knows would fail.

## PRICING PDE FOR OPTION-ADJUSTED VALUATION

We start with a hypothetical dynamic asset (mortgage) whose market price  $P(t, x)$  depends on time  $t$  and one market factor  $x$ . We treat  $x(t)$  as a random process with a (generally variable) drift rate  $\mu$  and a volatility  $\sigma$ , disturbed by a standard Brownian motion  $z(t)$ :

$$dx = \mu dt + \sigma dz \quad (1)$$

We assume further that the asset pays the continuous coupon rate  $c(t, x)$ , and its balance  $B$  is amortized at the  $\lambda(t, x)$  rate, so that  $(\partial B / \partial t) = -\lambda B$ . Then, one can prove that the price function  $P(t, x)$  should solve the partial differential equation (PDE):

$$\underbrace{r + OAS}_{\text{expected return}} = \underbrace{\frac{1}{P} \frac{\partial P}{\partial t} + \frac{1}{P}(c + \lambda) - \lambda}_{\text{time return}} + \underbrace{\frac{1}{P} \frac{\partial P}{\partial x} \mu + \frac{1}{2P} \frac{\partial^2 P}{\partial x^2} \sigma^2}_{\text{drift return diffusion return}} \quad (2)$$

A derivation of this PDE can be found in Levin [1998], but it goes back at least to Fabozzi and Fong [1994]. If our amortizing asset is an interest-only strip (IO), the pricing equation is modified by excluding the  $\lambda/P$  term in the time return expression.

A notable feature of the PDE (2) is the absence of the balance variable,  $B$ . The entire effect of possibly random prepayments is represented by the amortization rate function,  $\lambda(t, x)$ . Although the total cash flow observed for each accrual period does depend on the beginning-period balance, construction of a backward induction scheme will require the knowledge of  $\lambda(t, x)$ , not the balance.

Pricing PDE (2) can be solved on a probability tree or a finite-difference grid that has as many dimensions as the total number of factors or state variables  $r, c$ , and  $\lambda$ ,

or by Monte Carlo simulation. If the coupon rate is fixed, and the amortization rate  $\lambda$  depends only on current time (loan age) and the immediate market factor  $x$ , the entire valuation problem can be solved backward on a two-dimensional ( $x, t$ ) lattice (the lattice will require more dimension if the market factor  $x$  is a vector).

To implement this method, we would start the valuation process from maturity  $T$  when we surely know that the price is par,  $P(T, x) = 1$  (zero for an IO), whatever the value of factor  $x$ . Working backward, we deduce prices at age  $t - 1$  from prices found at age  $t$ . In doing so, we replace derivatives in PDE (2) by finite-difference approximations, or weigh branches of the lattice by explicitly computed probabilities.

Even for a simple fixed-rate mortgage pass-through, though, total amortization speed  $\lambda$  usually cannot be modeled as a function of time and the immediate market. Prepayment burnout is a strong source of path-dependence because future refinancing activity is affected by the past incentives. One can think of a mortgage pool as a heterogeneous population of participants with different refinancing propensities. Some mortgagors have higher rates or better credit or larger loans, or perhaps they face lower regional transaction costs. Once they leave the pool, future prepayment activity gradually declines. Hence,  $\lambda$  depends on historical market rates, making the valuation problem path-dependent.

Instead of considering pricing PDE (2) for the entire collateral, Levin [2001, 2004] proposes decomposing it into two components, active and passive, differing in refinanceability. Under the active-passive decomposition (APD) model, mortgage path-dependent collateral can be deemed a simple portfolio of two path-independent instruments if:

- Active and passive components prepay differently, but follow immediate market rates and loan age.
- Any migration between components is prohibited.

Alternatives and variations of this modeling structure include decomposition into several groups, as well as stratifying the entire pool explicitly by available loan characteristics (see Davidson [1987], Davidson, Herskovitz, and Van Drunen [1988], Hayre [1994, 2000], and Kalotay, Yang, and Fabozzi [2004]). With a decomposed collateral, it is possible to value mortgage-backed securities using backward induction and, as we show further, explicitly account for prepayment risk.

## VALUATION WITH PREPAYMENT RISK: BASIC CONCEPTS

Mortgage practitioners use the term *prepayment risk* loosely. Most often they simply mean prepayment variability, but this is not what we attempt to capture. Indeed, a large portion of prepayment uncertainty is associated with interest rates and is thus explained by the prepayment models that are inherent in modern option-adjusted spread analytical systems.

If prepayments were perfectly explained by a model, there would not be any prepayment risk premium. An option model coupled with an exact prepayment formula should be able to deliver the right price for an agency-backed (default-protected) MBS operating with OAS = 0. MBS would be valued flat to a known benchmark curve—similarly to swaptions or callable notes, except with a more complex exercise rule.

Savvy market participants realize that a model can tell only part of the prepayment story. Because a model cannot predict prepayments exactly, we have unexplained deviations of prepayment speeds above or below the model's forecast, often called prepayment surprises or prepayment errors. Not all prepayment surprise should require market compensation. Random oscillations of actual prepayments around the model are diversifiable over time. Prepayment errors typically seen in small pools are diversifiable in large pools.

We associate the notion of prepayment risk with *non-diversifiable uncertainty*, common for the MBS market, systematic in trend and unexplained by an otherwise best-guess prepay model. The varying level of OAS on different instruments represents compensation for this risk.

All the terms in pricing Equation (2) represent different sources of return, but none of them explicitly quantifies prepayment risk. The entire compensation for bearing this risk is hidden in the OAS term.

How would we price prepayment risk? Suppose that the prepayment rate  $\lambda(t, x, \xi)$ , depends on one uncertain variable or uncertain parameter,  $\xi$ , independent of the interest rate market. For the first conceptual illustration, we assume  $\xi(t)$  is a Wiener process with zero drift and volatility of  $\sigma_\xi$ , i.e.,  $d\xi = \sigma_\xi dz_\xi$ , and known initial value,  $\xi(0)$ . According to the CAPM/APT, the risky return should be proportional to the price volatility due to the risky factor  $\xi$ . A common multiplier,  $\pi_\xi$ , called price of risk, should apply to every asset exposed to the same risk factor  $\xi$ .

Using this notation, we therefore state that, for every

investment period, the expected return,  $r + OAS$  on the left-hand side of PDE (2), should be adjusted for risk as:

Single-Period Expected Return =

$$r + prOAS + \underbrace{\pi_\xi \sigma_\xi \frac{1}{P} \frac{\partial P}{\partial \xi}}_{\text{prepay risk spread}} \quad (3)$$

where the *prepay risk- and option-adjusted spread* (*prOAS*) is a “risk-free” OAS; in the absence of any other risk factors, it should be zero for a properly selected pricing benchmark.

For example, we may assume that all agency MBS should be valued flat to the same agency yield curve, on this prepay risk-adjusted basis ( $prOAS = 0$ ). For non-agency MBS, this should certainly account for an additional risk associated with imperfect credit, and thus becomes equal to the pure credit spread that can be derived from the S&P or Moody’s rankings for a non-agency pass-through or a particular CMO tranche.

The risky spread term, unlike the traditional OAS, is not constant—it varies with interest rates and loan age. It is also directional—it can be both positive and negative—depending on the sign of price exposure to the factor  $\xi$ . Since the market provides a return premium for bearing the risk, it must make hedge instruments rich so that a properly constructed  $\xi$ -neutral portfolio of an asset and its hedge will earn nothing but risk-free OAS, i.e., *prOAS*. This is a traditional arbitrage argument that prevents construction of a risk-free portfolio earning any excess above the risk-free return (see Hull [2000]).

## VALUATION METHOD 1: ENDOGENOUS ASSESSMENT OF RISK

Factor volatility  $\sigma_\xi$  and price of risk constant  $\pi_\xi$  are defined outside the pricing model; they are common for all instruments exposed to prepay factor  $\xi$ . Let us show how price  $P(t, x; \xi)$  and its partial derivative taken with respect to the risk factor,  $\partial P / \partial \xi$ , denoted from now on as  $P_\xi$ , can be estimated internally and simultaneously in the course of backward valuation.

Suppose we have a path-independent instrument in hand that is subject to pricing Equation (2), and the amortization rate  $\lambda$  depends on risk factor  $\xi$ . Let us first multiply both sides of pricing Equation (2) by price  $P$  and replace the traditional expected return,  $r + OAS$ , with the risk-adjusted version from (3):

$$(r + prOAS)P = \frac{\partial P}{\partial t} + c + \lambda - P\lambda + \frac{\partial P}{\partial x}\mu + \frac{1}{2} \frac{\partial^2 P}{\partial x^2} \sigma^2 - \underbrace{\pi_\xi \sigma_\xi P_\xi}_{\text{price of risk}} \quad (4)$$

Second, we take the first derivative of both sides of Equation (4) with respect to  $\xi$ :

$$(r + prOAS)P_\xi = \frac{\partial P_\xi}{\partial t} - P_\xi \lambda + \frac{\partial P_\xi}{\partial x} \mu + \frac{1}{2} \frac{\partial^2 P_\xi}{\partial x^2} \sigma^2 + (1 - P) \frac{\partial \lambda}{\partial \xi} - \underbrace{\pi_\xi \sigma_\xi P_{\xi\xi}}_{\text{term to omit or approximate}} \quad (5)$$

where for now we disregard the last term.

We have here a system of two linear partial differential Equations (4) and (5) with two unknown functions,  $P(t, x)$  and  $P_\xi(t, x)$ . It can be simultaneously solved backward starting from the terminal (maturity  $T$ ) conditions:  $P(T, x) = 1$ ,  $P_\xi(T, x) = 0$ . This backward induction is carried out on a usual  $(t, x)$ -grid or probability tree, for one value of  $\xi = \xi(0)$ , and separately for the active and passive part.

Omission or approximation of the prepayment convexity term, proportional to  $P_{\xi\xi}$ , avoids expanding the pricing grid to the  $\xi$ -dimension. If we want to account for the convexity term, we can do so by adding  $\frac{1}{2} P_{\xi\xi} \sigma_\xi^2$  to the PDE, and then differentiate both parts twice while neglecting (or approximating) the third derivative. We would end up with a system of three PDEs, to be solved on the same  $(t, x)$ -grid for three unknown functions,  $P(t, x)$ ,  $P_\xi(t, x)$ , and  $P_{\xi\xi}(t, x)$ —see Levin [2004] for details.

## VALUATION METHOD 2: EQUIVALENT RISK-NEUTRAL PREPAY MODEL

Pricing Equation (4) allows for an important financial interpretation. Suppose we still work with the traditional PDE (2), ignoring the price of risk but instead letting the risk factor  $\xi$  drift with a negative  $\pi_\xi \sigma_\xi$  rate per year:

$$d\xi = \sigma_\xi dz_\xi - \pi_\xi \sigma_\xi dt \quad (6)$$

It is easy to see that such a drift contributes a systematic return that is mathematically identical to the above marked price of risk. Indeed, the full-time derivative term,  $\partial P / \partial t$  on the right-hand side of Equation (2), will now be composed of a  $\partial P / \partial t$  term measured due to a simple pas-

sage of time (i.e., with unchanged  $\xi$ ) minus a  $\pi_\xi \sigma_\xi P_\xi$  term that comes from the Itô lemma applied to price  $P$  as a function of random variable  $\xi$  defined by Equation (6). This leads directly to the pricing Equation (4) with risk.

We essentially arrive at a powerful risk-neutrality concept for prepayment risk: Pricing with risk consideration can be replaced with pricing without it, but with the prepay risk factor  $\xi$  drifting in the undesired direction. The rate of this drift is proportional to volatility  $\sigma_\xi$ , and the coefficient of this proportionality is the price of risk,  $\pi_\xi$ . This is the same concept we use in constructing term structure models and pricing financial derivatives while taking forward rates and prices into consideration.

A prepayment model with the factor  $\xi$  set to drift at the risk-adjusted rate can be logically called a *risk-neutral prepayment model*, like all other financial models that exploit this concept. It is therefore meaningless to wonder how well such a model fits actual prepayments—it is simply not meant to do this job.

The risk-neutral drift for factor  $\xi$  can also be partly caused by a model's systematic bias. For example, the market may expect the refinancing process to become more efficient going forward than it has been. Mathematically this bias cannot be distinguished from the price of risk; it too results in a drift change for  $\xi$ . Using the risk-neutral prepay model and the prOAS should lead to the same valuation results as using an empirical prepayment model with the traditional OAS.

Mortgage market participants are accustomed to seeing differences in broker and analyst forecasts of prepayments and reports of OAS numbers for the same instruments and market conditions. The transition from an objective model, which often uses historical prepayments, to a risk-neutral one that targets known prices for known instruments may explain the disparities. Hence, risk-neutral prepay speeds should differ less across firms. Since Black and Scholes, risk-neutral modeling has been known for its ability to reduce bias from systematic model-induced errors.

Although we have shown that a risk-neutral prepay model is theoretically equivalent to the explicit risk assessment performed for each investment period, from a financial engineer's point of view there is a fundamental difference between the two techniques. The explicit risk assessment method computes the partial derivative  $P_\xi$  directly for each investment period and rate level. This task is feasible if the entire valuation is performed backward, and the MBS price and its derivatives with respect to risky factor  $\xi$  can be found for every node of the pricing grid.

For example, unstructured MBS can be priced this

way using the burnout-curing active-passive decomposition idea. CMOs, though, are heavily path-dependent beyond burnout, and are therefore not subject to backward valuation. For CMOs, the risk-neutral prepay model would be an ideal method as it does not require computing  $P_\xi$  directly. Letting the risk factor  $\xi$  drift in the undesired direction naturally explores the price's dependence on  $\xi$ , without measuring  $P_\xi$  explicitly.

We have so far assumed for simplicity that the risk factor follows a simple Wiener process. In general, the behavior of prepayment factors is more complex. For example, prepayments cannot grow more uncertain over time without bounds, suggesting that the dynamics of risk factors are mean-reverting. Yet purely diffusive behavior cannot capture uncertainty in the starting values, i.e., risk present at time zero. For example, the price of risk constant model considers risky prepayment parameters where values are constant but drawn from some distribution (see Cohler, Feldman, and Lancaster [1997]).

In general, we may extend the stochastic model for factor  $\xi(t)$  to:

$$\begin{aligned} d\xi &= a_\xi(\bar{\xi} - \xi)dt + \sigma_\xi dz_\xi \\ E\xi(0) &= \xi_0 \\ Var\xi(0) &= \sigma_{\xi_0}^2 \end{aligned} \quad (7)$$

where  $a_\xi$  is the mean reversion parameter, and  $\bar{\xi}$  is the long-term equilibrium. The starting value is now considered uncertain and drawn from  $N[\xi_0, \sigma_{\xi_0}]$ . This mean-reverting pattern will add more terms in pricing PDEs. Also, the very last (time zero) step in backward valuation will become a special one—to account for the time zero risk and convexity.

Transforming model (7) into a risk-neutral form, we again add a  $-\pi_\xi \sigma_\xi$  drift rate, and shift the initial condition by  $-\pi_\xi \sigma_{\xi_0}$ :

$$\begin{aligned} d\xi &= [a_\xi(\bar{\xi} - \xi) - \pi_\xi \sigma_\xi]dt + \sigma_\xi dz_\xi \\ E\xi(0) &= \xi_0 - \pi_\xi \sigma_{\xi_0} \\ Var\xi(0) &= \sigma_{\xi_0}^2 \end{aligned} \quad (7-RN)$$

These are the generalized dynamics of the risk factor  $\xi(t)$  in a risk-neutral prepayment model. We see that risk-neutrality lowers both the starting value and (in the presence of mean reversion) the long-term equilibrium for  $\xi(t)$ .

## A prOAS PRICING MODEL WITH REFINANCING AND TURNOVER RISK

We have noted that both premium and discount MBS are often traded at somewhat elevated option-adjusted spreads. IOs stripped from premium collateral are priced progressively cheaper, on an OAS basis, than IOs taken from discount or current-coupon collateral. We conjecture that two major prepayment sources, the refinancing process (driving the premiums) and the turnover process (vital for understanding the discounts), are perceived as risky by the mortgage market. To put this into simple practical terms, there are two distinct market fears—refinancing understatement, and turnover overstatement. Hence, the model of risk should be at least two-dimensional, which is a rather simple extension of what we have considered so far. Allowing the refinancing and the turnover processes to be randomly scaled, we can model the total prepayment speed as:

$$\text{SMM} = \rho \text{RefiSMM} + \tau \text{TurnoverSMM} \quad (8)$$

where the prepay multipliers,  $\rho$  and  $\tau$ , are considered uncertain but centered on 1, and mutually independent. Thus, instead of one hypothetical prepay risk factor  $\xi$ , we have now two,  $\rho$  and  $\tau$ .

If we use the active-passive (or other) collateral decomposition, we apply the additive rule in Equation (8) to each constituent piece. Every risk premium and convexity cost found in pricing equations now becomes a simple sum of two terms associated with the refinancing risk and the turnover risk. We assume known volatilities,  $\sigma_\rho$  and  $\sigma_\tau$  of two Wiener processes,  $\rho(t)$  and  $\tau(t)$ , as well as two prices of risk,  $\pi_\rho$  and  $\pi_\tau$ .

The conceptual risk-adjusted return formula in Equation (3) now becomes

Single-Period Expected Return =

$$r + prOAS + \underbrace{\pi_\tau \sigma_\tau \frac{P_\tau}{P} - \pi_\rho \sigma_\rho \frac{P_\rho}{P}}_{\text{prepay risk spread}} \quad (9)$$

where  $P_\tau$  and  $P_\rho$  stand for partial derivatives. Note the negative sign for the refinancing risk. Since premium fixed-rate MBS typically have  $P_\rho < 0$ , we can reward them by either assuming a negative price of risk constant,  $\neq_\rho$ , or using the negative sign in the spread formula. Discount fixed-rate MBS have  $P_\tau > 0$ , so the positive sign in (9) produces positive return compensation for bearing turnover risk.

Extension of the single risk factor is rather simple. In the fundamental pricing PDE (2), we now replace the expected return,  $r + OAS$ , with Equation (9). Thus modified risk-adjusted PDE will apparently include unknown derivatives  $P_\tau$  and  $P_\rho$ . The next step is to differentiate this equation with respect to each risk factor,  $\rho$  and  $\tau$ , thereby adding two more equations and closing mathematical construct. The total number of pricing PDEs to solve will be either three with prepay convexity cost disregarded (for  $P$ ,  $P_\rho$ , and  $P_\tau$ ), or six with prepay convexity cost included (for  $P$ ,  $P_\rho$ ,  $P_\tau$ , the second derivatives,  $P_{\rho\rho}$ ,  $P_{\tau\tau}$  and the mixed derivative  $P_{\rho\tau}$ ).<sup>2</sup>

For this two-factor prepay risk setting, a risk-neutral prepay model remains an attractive alternative method of computing prOAS. We accelerate refinancing (set  $\rho$  drifting above 1 at the rate of  $\neq_\rho \sigma_\rho$ ), and retard turnover (set  $\tau$  drifting below 1 at the rate of  $\neq_\tau \sigma_\tau$ ). For a framework that combines single (time-zero) jump and mean-reverting diffusion, these risk-neutral drifts for the refinancing and the turnover multiples are shown in Exhibit 1. We can split the risk between jump and diffusion, and alter the rate of reversion.

The dynamics shown in Exhibit 1 associate about half of refinancing risk with the refinancing uncertainty present at time zero; the other half appears gradually. We are generally more certain about the starting turnover rate, and it may take a while before future macroeconomic conditions will alter it, so a smaller portion of the total drift is present at time zero for the turnover risk component.

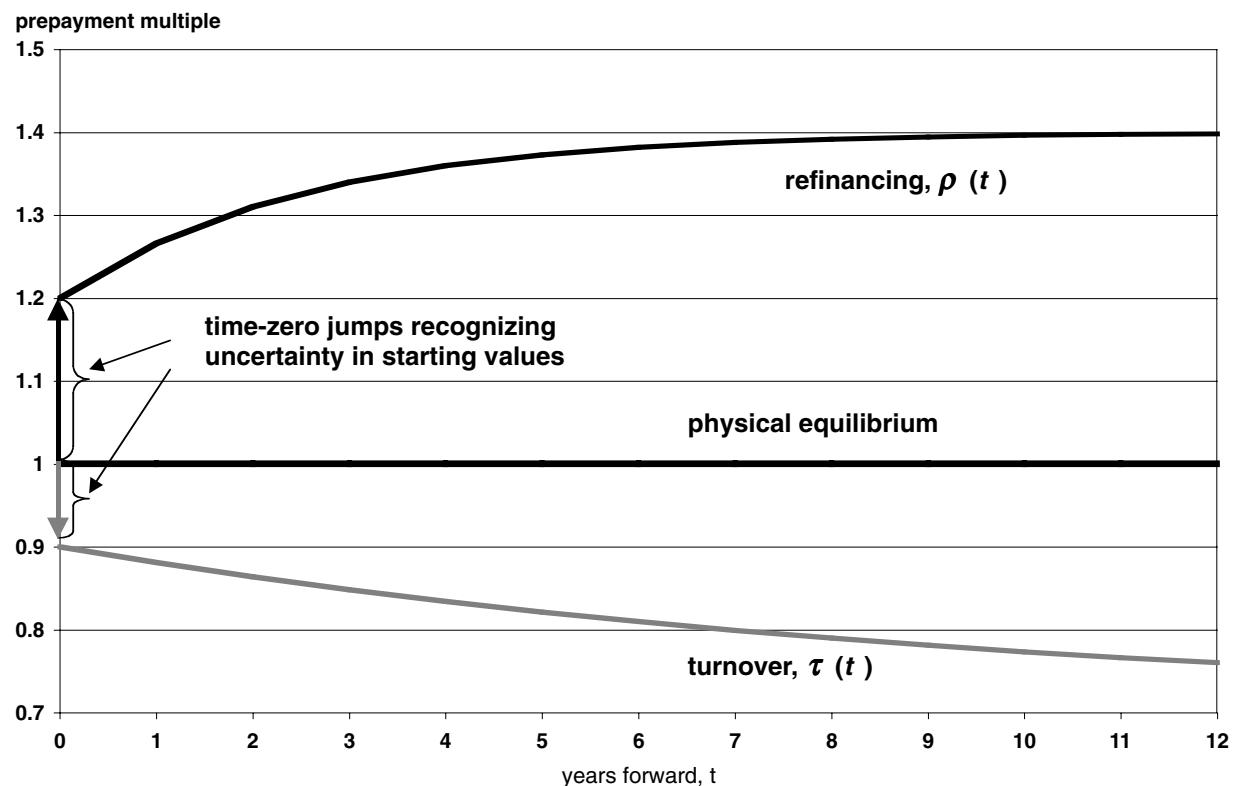
## DETERMINING PRICES OF RISK: CALIBRATION TO TBAs

Our discussion of practical steps in using the prOAS model includes the problems it may address and the valuation results it generates. Assuming we know volatility and mean-reversion parameters for the risk factors  $\rho(t)$  and  $\tau(t)$ , we need to find market levels for (calibrate) the price of risk constants,  $\neq_\rho$  and  $\neq_\tau$ . We can tune these constants to match market prices (or the OAS), for a range of actively traded securities.

To find these parameters, we first need to define the appropriate target for prOAS. The target prOAS should reflect the spread on a similar security with no prepayment uncertainty. While assuming a zero prOAS to the swap curve may be appropriate, we find that agency debentures are a better benchmark, as they have the same credit guarantee as agency MBS and entail no prepayment uncertainty.<sup>3</sup>

## EXHIBIT 1

### Risk-Neutral Prepay Multiples



An example of the calibration results is shown in Exhibit 2. We use eight TBA instruments priced on August 29, 2003, with net coupons ranging from 4.5% to 8.0%. On that day, the mortgage current coupon rate was 5.67%, so there were both premiums and discounts in our sample. First, we measure OAS numbers using the traditional valuation method, without any risk adjustment (black bars). We then use the prOAS pricing method and select the price of risk constants  $\neq_{\rho}$  and  $\neq_{\tau}$  so as to minimize the prOAS levels (grey bars). The calibration works fairly well across the range of TBAs, with a 3–4 basis point mean-squared accuracy in reaching the debenture (zero) prOAS target.

The lines drawn in Exhibit 2 show the principal components of OAS, i.e., OAS compensation due to refinancing risk and turnover risk. The direction of both lines is apparent, but some important points should be made. For one, the turnover line almost never leaves positive territory. Discounts would certainly lose value with slow turnover, but why will premiums suffer? The very steep yield curve is primarily responsible for this effect; slowing turnover pushes cash flows to longer maturities

with higher discount rates. It also slightly inflates the time value of the prepayment option.

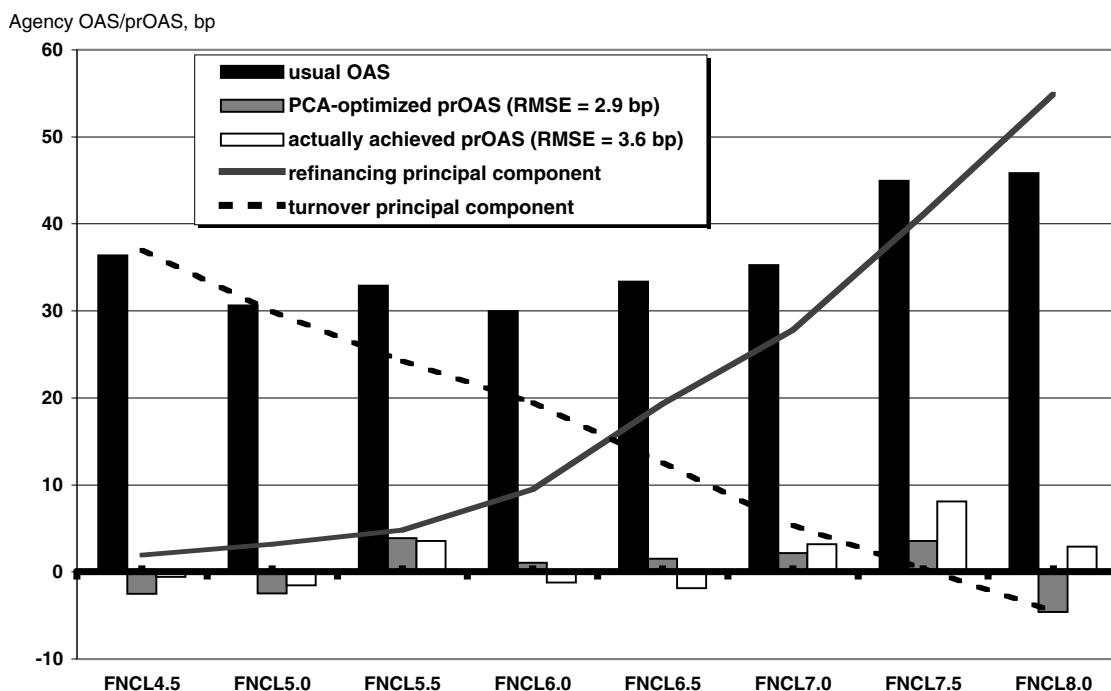
The best mix of principal components is found assuming that prOAS is linear in prices of risk. Hence, our actually achieved prOAS levels are suboptimal (white bars).

This calibration exercise clearly shows the value of the two-risk factor model for several reasons. In a single-risk factor model, prepayment risk is associated with either global acceleration or deceleration, regardless of the source. First, in seeking a single price of risk, it would be impossible to move the premium OAS levels toward the zero level without moving the discount OAS levels away from zero, and vice versa. Second, for any market condition, a single-risk factor model would allow one MBS coupon (perhaps, interpolated) to be prepay-neutral, i.e., not exposed to the overall prepay scale. Hence, the theory requires such an MBS be traded at a zero, not a positive, OAS, which is not the case for the market shown in Exhibit 2. Third, as we mentioned in the introduction, a single-factor risk model leads to an unworkable single-factor hedge strategy.

Will the parameters of the prOAS model be stable

## EXHIBIT 2

### Prices of Risk Calibrated to FNCL TBAs as of August 29, 2003



over time, or do we need to calibrate them on a daily basis? While one goal of physical models is to stay steady, the concept of risk-neutrality is linked to changing market prices for benchmark instruments, which reflect the dynamics of market preferences for risk. If the market prices for TBAs exhibit OAS tightening or widening over time, they are sending us a message of changing perceptions of prepayment risk.

This conjecture is borne out when we examine the trends in results of the calibration of the prices of risk constants,  $\neq_p$  and  $\neq_r$ , at different dates, as shown in Exhibit 3. These parameters are not constant and even show an exaggerated reaction to interest rate dynamics.

When rates dropped to their 40-year record lows (May–June 2003), refinancing fears reached panic stage. High-premiums (e.g., FNCL7.5 and FNCL8.0) did not appreciate, which meant that their agency OAS levels increased to 100 basis points and above to absorb much of the rate plunge. During that period, the calibrated price of refinancing risk surged. The calibration revealed no concern about housing turnover, as the discount sector had evaporated.

When rates moved back up through summer 2003, the refinancing wave started to cool off; large volumes of

freshly originated FNCL4.5 and FNCL5.0 became discounts. This was when the turnover concerns became apparent. The rest of the time we witnessed a general stabilization in risk prices.

In our opinion, it is dynamics of interest rates, not their levels, that induce exaggerated prices of risks. Both irrational pricing and sharp changes in the mortgage market composition may explain this phenomenon (see also Gabaix, Krishnamurthy, and Vigneron [2004]).

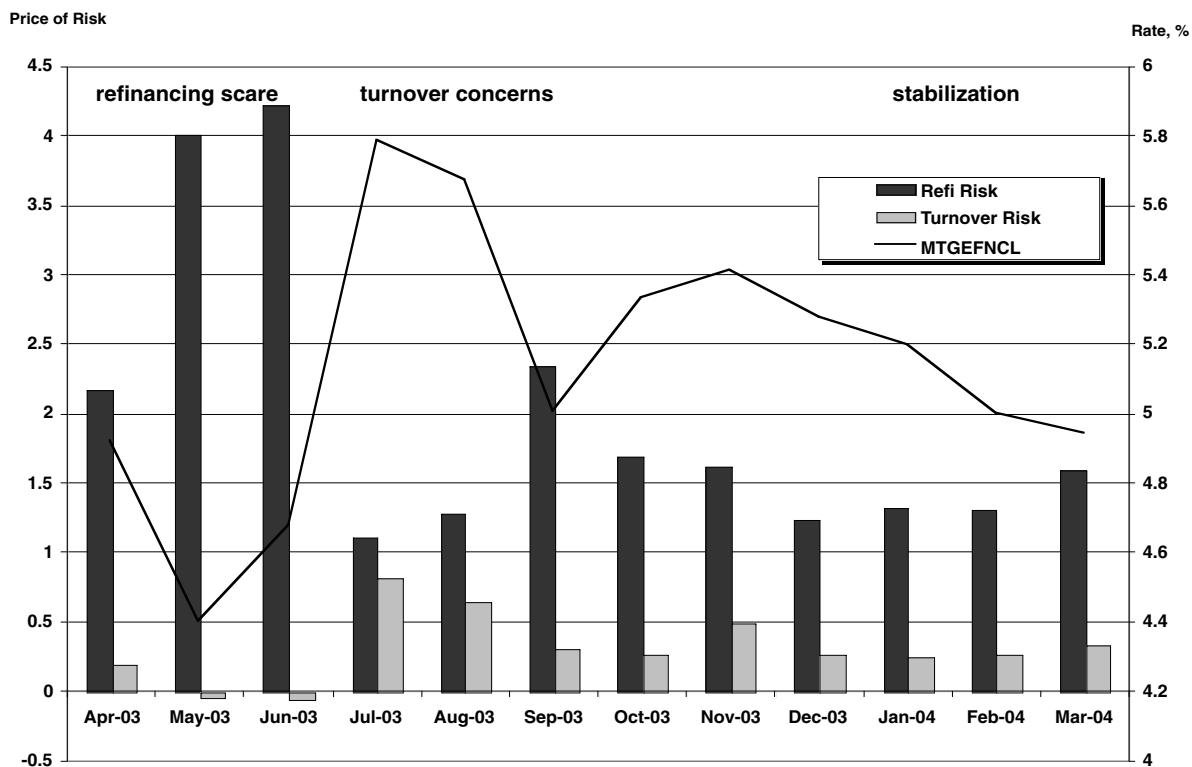
Comparing the heights of the bars in Exhibit 3, one might conclude that the MBS market is systematically dominated by refinancing fears, not turnover fears. This is likely a misperception. As Exhibit 2 shows, the principal components of OAS are of comparable size even when  $\neq_p$  is significantly greater than  $\neq_r$ . When rates rise, prices for discount MBS can suffer substantial drops that are limited by and therefore depend on the turnover speed.

## VALUATION OF MBS STRIPS WITH prOAS

In the traditional OAS valuation, either price or OAS should be given as input. Under prOAS, the role of OAS is performed by a better-defined measure. The goal of prOAS pricing is to eliminate differences in OAS

## EXHIBIT 3

### Historical Prices of Refinancing and Turnover Risk



among instruments that are exposed to prepayment risk.

As we asserted earlier, the prOAS measure should value agency MBS flat to agency debentures. Therefore, once the risk factors are given their stochastic specifications (jump sizes, diffusive volatility, and mean reversion) and the prices of risk constants are determined, we can value any agency MBS or its IO and PO strips much like swaptions, i.e., by looking at the benchmark rate and volatility structure, but without any knowledge of the traditional OAS.

Exhibit 4 shows valuation results for agency trust IOs using prices of risk constants, a high  $\neq_p$ , and a near-zero  $\neq_\tau$ , obtained from the calibration to Fannie Mae TBAs on May 30, 2003. Application of the prOAS method first leads to values that are then converted into conventional OAS measures.<sup>4</sup> The prOAS model explains IO cheapness (and therefore PO richness) naturally, and correctly predicts an OAS level of (and above) 1,000 basis points.

As POs stripped off premium pools should be looked at as hedges against refinancing risk, they have to be traded rich according to arbitrage pricing theory. Our prOAS model successfully confirms this in that virtually all OAS for trust POs are deep in negative territory (not shown in Exhibit 4). Results in Exhibit 4 also provide

some degree of confidence for managers of mortgage servicing rights (MSR) portfolios (not actively traded or frequently quoted)—they can use the prOAS measure to better assess the risk of their portfolios.

Exhibit 3 shows in a historical risk chart that on May 30, 2003, the entire risk perception was evolving out of a refinancing scare. Exhibit 4 shows that both the TBA market and the trust IO market agree with one another in incorporating this risk into pricing.

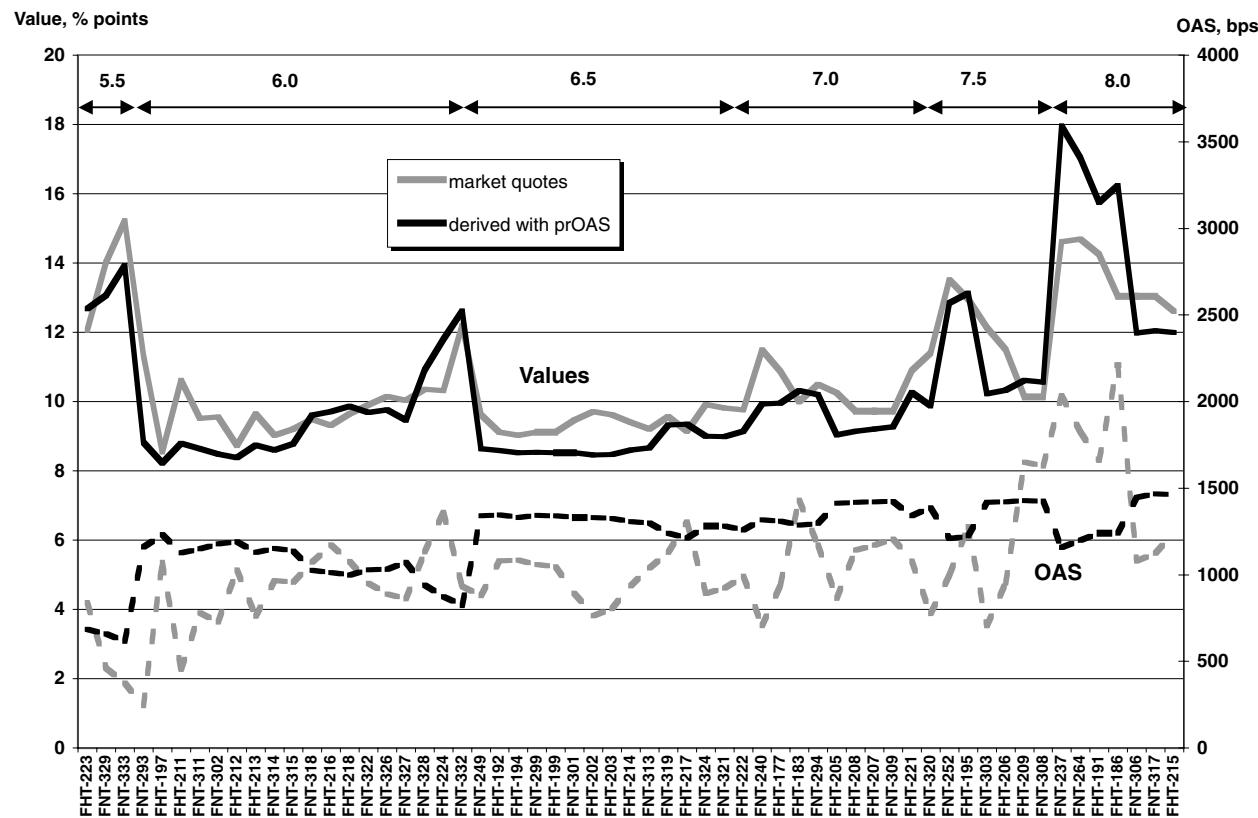
In the summer months of 2003, rates rose sharply, pushing lower-coupon MBS (4.5s and 5.0s) into discount territory. Exhibit 2 confirms that by the end of that summer the refinancing fear had dissipated, making room for turnover concerns (slower-than-modeled turnover results in a loss for a discount MBS holder). It is not surprising that the price for turnover risk, virtually non-existent in May 2003, grew considerably (Exhibit 3).

What if we apply prices of risk calibrated to the August 29, 2003, TBA market to value trust IOs? Exhibit 5 shows two stages in application of the prOAS model, valuation with refinancing risk only, and with total risk.

Comparing the market prices and related OAS (light grey lines) with the valuation results under the prOAS model with refinancing risk only (darker grey lines), we

## EXHIBIT 4

### Trust IO Valuation as of May 30, 2003



see that the single-risk factor prOAS model just slightly overstates values compared to the actual market. It shows a directionally correct OAS tendency (tighter spread for discount IOs, wider for premiums) and magnitude.

Disaster strikes when we add the true turnover risk calibrated to the TBAs (black lines). Since IOs can be used as hedges against turnover risk, theory says they should be penalized, not rewarded. An almost constant 250-basis point OAS reduction is seen as the result.

On that day, the actual IO market did not seem to appreciate this theory. Price quotes were much lower than the full two-risk factor prOAS model suggests they should have been. According to the APT, such mispricing should allow construction of a fully hedged risk-free portfolio that earns more than the risk-free rate.

After analyzing a number of trading days, we believe that the TBA-IO dislocation coincides with a sharp surge of rates when the IO market, driven predominantly by the acceleration fear, misses the hedging aspect against slowing down housing turnover. In those market conditions, there is a theoretical opportunity to create a dynamically hedged mortgage portfolio that is prepay-neutral and earns

an excess return over funding rates. Recognize, however, that the market value of this portfolio would remain exposed to the risk of further TBA-IO dislocation.

### MODERNIZED GREEKS

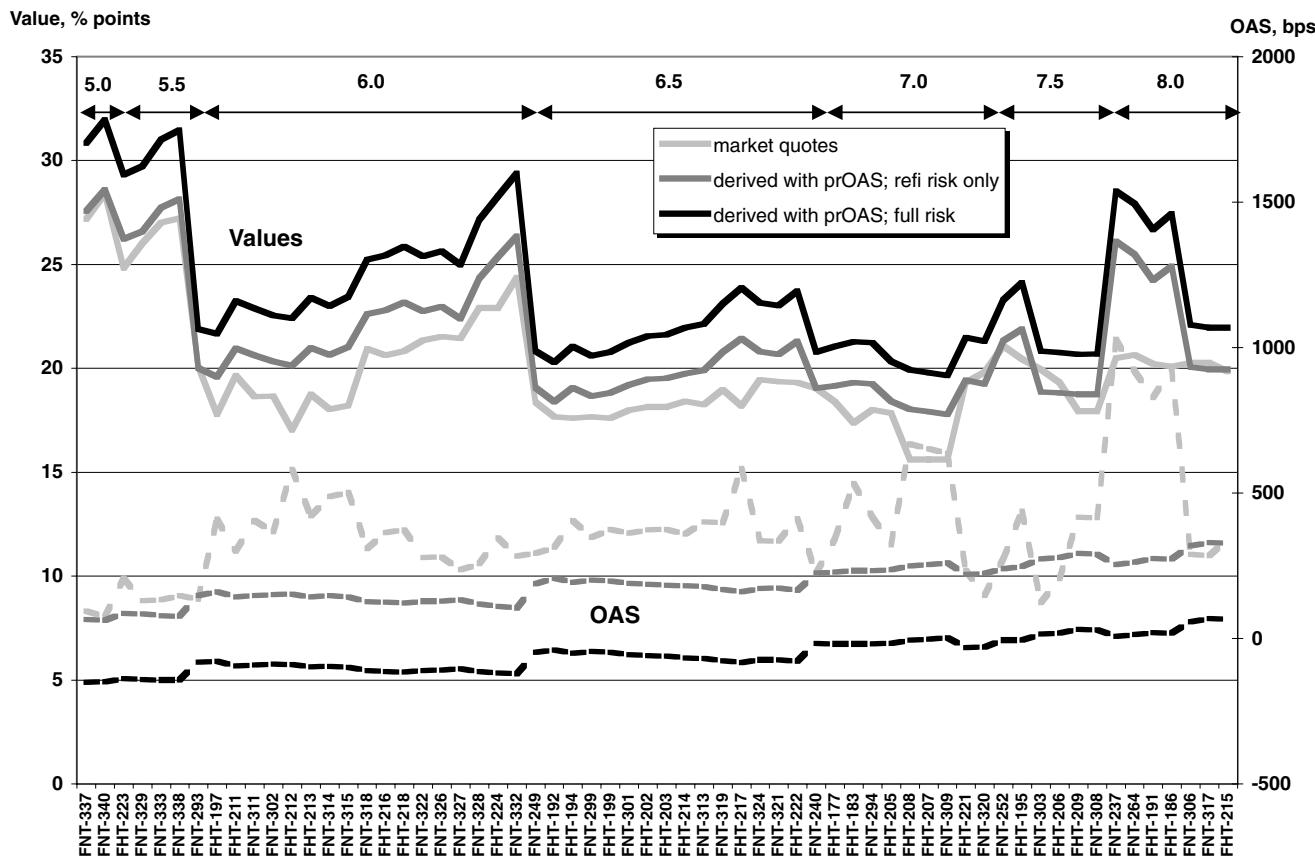
Valuation adjusted for prepayment risk leads to different rate sensitivity from the traditional approach. Intuitively, premium pass-throughs become less rate-sensitive because their risky spread absorbs interest rate moves following the prepay option moneyness. Indeed, any rate drop elevates the refinancing risk and thereby inflates the traditional OAS; any rate rise reduces the risk and compresses the OAS.

Since the discount MBS react inversely, they are more rate-sensitive under the prOAS method than under the constant OAS risk assessment. A flat OAS profile for the current-coupon to cuspy-premium TBA sector seen in Exhibit 2 suggests that the constant OAS valuation is a valid way to assess their rate sensitivity.

All these findings (confirmed in Exhibit 6) can be explained even more easily by the equivalent transition

## EXHIBIT 5

Trust IO Valuation as of August 29, 2003



from the objective to the risk-neutral prepayment model, faster for premiums and slower for discounts.

Exhibit 7 compares valuation profiles for an MSR stripped off a 6.5% near current-coupon pool. We see that a constant OAS valuation systematically understates rate sensitivity for all rate levels by as much as one-third. This implies, for example, that MSR managers would under-hedge if they used a traditional constant OAS duration.

## CONCLUDING REMARKS

The two-risk factor prOAS valuation approach that we analyze is a well-defined extension of the traditional OAS method that draws its roots from arbitrage pricing theory. It successfully explains many phenomena in the MBS market such as OAS variability among MBS coupons and instrument types, the IO-PO pricing paradox, and the divergence of practical durations from the theoretical. At the same time, the method points to some lacks in the mortgage market that reveal inefficiencies and possible arbitrage.

Two particular anomalies—missed hedging power of IOs against turnover risk, and exaggerated dynamics of risk prices—let savvy investors construct prepay risk-neutral MBS portfolios that earn excess returns and conscientiously speculate on taking a risky position.

## ENDNOTES

The authors thank Jay Delong for help in integrating the model into their valuation system; Dan Szakallas for tuning and optimizing the prepay model to historical prepay data; William Searle for model implementation; and Ilda Pozhegu for publication help. This article has benefited from the comments of Lily Chu, Frank Fabozzi, Yung Lim, Anthony Sanders, and William M. Storms.

<sup>1</sup>In addition, holding OAS constant for the purpose of computing duration and convexity for an MBS pass-through and its strip derivatives (IO and PO) is inconsistent from a simple mathematical view. When rates change, so do values of the IO and PO, and thus their weights in the pass-through change.

## EXHIBIT 6

### From OAS to prOAS—TBA Duration Difference

#### A. Market as of May 30, 2003 (MTGEFNCL = 4.397%)

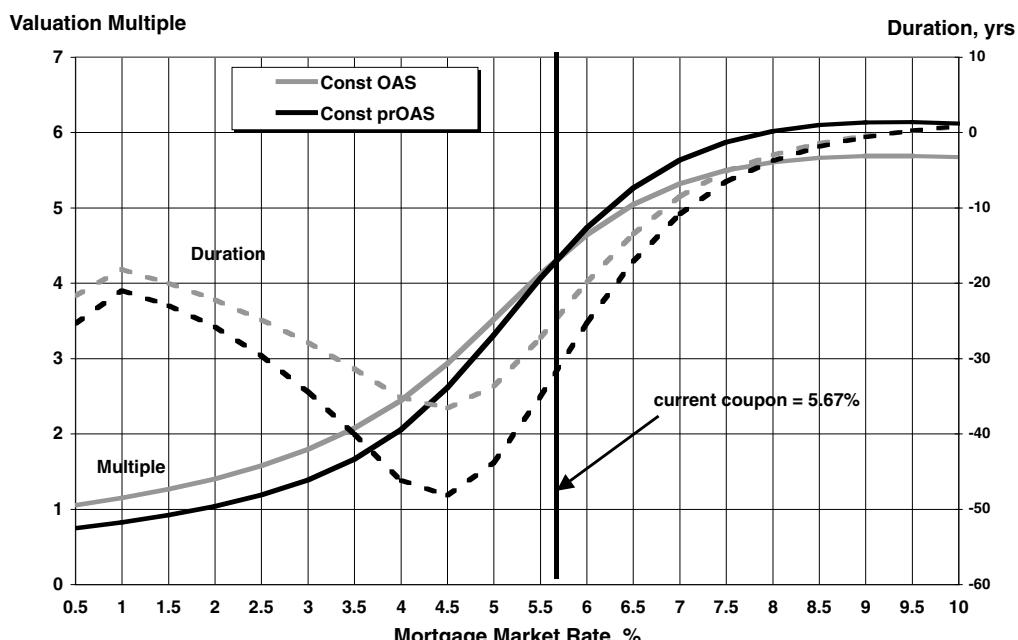
	FN4.5	FN5.0	FN5.5	FN6.0	FN6.5	FN7.0	FN7.5	FN8.0
OAD	4.10	2.90	1.96	1.71	1.67	1.59	1.83	2.12
prOAD	3.42	2.03	1.07	0.77	0.80	0.90	1.11	1.28
Difference	-0.68	-0.83	-0.89	-0.94	-0.87	-0.69	-0.72	-0.84

#### B. Market as of August 29, 2003 (MTGEFNCL = 5.670%)

	FN4.5	FN5.0	FN5.5	FN6.0	FN6.5	FN7.0	FN7.5	FN8.5
OAD	5.78	5.18	4.40	3.34	2.48	2.07	2.03	1.90
prOAD	6.11	5.44	4.54	3.32	2.35	1.98	1.81	1.68
Difference	0.33	0.26	0.14	-0.02	-0.13	-0.09	-0.22	-0.22

## EXHIBIT 7

### Valuation of 6.5% GWAC MSR as of August 29, 2003



<sup>2</sup>The mixed derivative term appears in the model even if risk factors are assumed independent.

<sup>3</sup>Arguably, TBAs should trade even richer than the agency debt curve because they 1) have superior liquidity, and 2) are collateralized, leading to a perception of higher credit quality. While a typical MBS is backed by properties, there is no formal legal mechanism that provides any enhanced protection of an agency MBS beyond the corporate guarantee of the agency.

<sup>4</sup>To account for the liquidity difference between IOs and TBAs, we apply 25 basis points prOAS.

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