MORTGAGE HEDGE RATIOS: WHICH ONE WORKS BEST?

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Mortgage performance quality is, to a very large extent, in the eye of the beholder. The trickiest problem for many portfolio managers is to determine the duration, or interest rate sensitivity, of their mortgage holdings and how this sensitivity changes over time. For example, an investor who owns \$1 million current face of FNMA 7s instead of an equal amount of the ten-year Treasury may find that the pass-throughs outperform when the market sells off and underperform when the market rallies. This makes sense, because FNMA 7s have a much shorter duration than a ten-year Treasury.

The real question a mortgage investor must focus on is: Which Treasury security — or combination of securities — on a market-neutral basis, is my mortgage-backed security a substitute for? Clearly, as interest rates change, hedge ratios in the mortgage market have to be adjusted. But, by how much?

A bond's duration measures simply the relative change in price that would be expected for an instantaneous, small change in its required yield. Quantifying duration in a mortgage security is not the straightforward exercise it is in non-callable Treasury or corporate securities. In Treasury or corporate securities, a bond's duration is simply the first derivative of price with respect to yield. The embedded prepayment option in a mortgage-backed security, however, means that as interest rates change, the prepayment rate is also changing, and hence so too are the mortgage cash flows. And the duration or interest rate sensitivity of a mortgage security must reflect both the effect of an interest rate change on the cash flows and the changes in the cash flows themselves.

Mortgage market participants commonly use three techniques to compute the interest rate sensitivity of mortgage instruments: option-adjusted (OA), empirical, and option-implied duration. We explore the calculation of each of these measures, as well as the advantages and disadvantages of each.

The question we are always asked is: Which duration measure works best? We attempt to answer this question by looking at which measure has historically minimized the hedging error.

This issue is especially important to portfolio managers who tightly control the duration of their portfolios, either by matching it to an absolute number or by matching it to the duration of an index. Many of these managers want their trades executed on a duration-neutral basis, whether they are trades between Treasury securities or trades within the mortgage market. And while managers will still want to look at *all* the duration measures, this research offers guidelines for which measure to emphasize when results diverge.

I. A QUICK REVIEW OF MBS DURATIONS

There are a variety of ways to express the interest rate sensitivity of a mortgage security. The simplest is *cash flow duration*, which captures simply the change in the price of a security as its yield changes, assuming the prepayment rate remains the same. This is obviously completely unrealistic; as interest rates change, prepayment rates will also change. Failing to account for the response of prepayment rates to interest rate changes generally puts an upward bias on cash flow duration, causing the mortgage security to appear to be longer than the market actually treats it. Thus, this indicator can be extremely misleading and should be used only with extreme caution.

Option-Adjusted Duration

Many investors rely on *option-adjusted durations* to measure the interest rate sensitivity of their MBS holdings. This number is calculated by assuming some small shift in the yield curve, holding constant the option-adjusted spread (OAS) of the mortgage security. OAS is typically calculated using a Monte Carlo simulation, a methodology that captures the bond's performance over a range of scenarios. A large number of interest rate paths are chosen for the simulation from an interest rate distribution centered on forward rates. For each path, the cash flows of the mortgage security are determined using a prepayment model that gives a prepayment rate at each point in time along the path.

An OAS is found by using an iterative search procedure. That is, a spread is assumed (guessed); each path is raised by this amount; and the present value of the cash flows along each path is calculated. The present value across all paths is averaged and compared to the market price of the security; if they are equal, the assumed spread is the OAS. If the values are not equal, other spreads are tried until one is found that equates the market price of the bond to the average present value of the modeled cash flows.

Once the OAS is found, the OA duration is captured by raising the interest rate curve a small amount and reflecting this shift in the paths. The cash flows are then generated, and the average price, given the OAS, is found. The process is repeated for a small downward shift. The average absolute price change, stated as a percentage of the current price, is the OA duration.

Unfortunately, the OA durations of mortgage securities can vary markedly from model to model. The differences can stem from generation of the interest rate curve, the volatility used, and, most important, the prepayment model. Some dealer models generate the interest rate curve using only active Treasury issues. This, of course, means that a good deal of interpolation must be employed, as there are active issues only in three-month, six-month, and one-year bills, as well as two-, three-, five-, ten-, and thirty-year bonds. In addition, there can be distortions in this curve if some of the issues are very special in the repo market.

Other dealer models may use either the entire Treasury universe or Treasury strip data to estimate the curve. Either approach introduces further opportunities for difference. For example, if the entire Treasury universe is used, should all observations be weighted equally, or should infrequently traded issues be weighted less heavily than frequently traded issues? If Treasury strip data are used, technical distortions can occur from time to time in particular issues.

Given the increasing number of investors who look at their funding costs in LIBOR terms, it is also becoming more common to fit the model to the LIBOR curve. This is the methodology used to calculate the duration numbers used in this article. We use Eurodollar futures out to one year and swap rates thereafter.

The advantage of this is that the swaps market provides a rich data set. Swaps are actively quoted for each year out to fifteen years, as well as for twenty- and thirty-year terms. Thus, versus the active Treasury issues, this universe is much richer, supplying data points for four-, six-, seven-, eight-, nine-, eleven-, twelve-, thirteen-, fourteen-, fifteen-, and twenty-year maturities. In practice, the difference in the curve used tends to raise or lower the absolute levels of the OAS numbers, but leaves relative values roughly similar.

Another potential source of differences among option pricing models centers on the volatilities used to calibrate the interest rate process. Some dealers continue to use a single fixed volatility, but most use a term structure of volatilities, calibrated to cap rates or swaption rates or both. The model used in this article uses a term structure of cap prices to calibrate the model. In practice, the exact methodology used to calibrate the term structure of volatilities tends not to make much of a difference in mortgage pass-through durations.

The biggest variations in OA durations across dealers arise from differences in the prepayment model used. Prepayment models differ considerably across firms, so the same security could have very different option-adjusted durations as measured by different dealers, each based on a particular prepayment model. Investors generally do not have enough information to determine the biases in each dealer's model, and may prefer one prepayment model over another for different types of pass-throughs. The option-adjusted calculations used in this article employ a prepayment model estimated from consensus prepayment projections incorporating the models of eleven dealers.

Our casual observation has been that pass-throughs do not actually behave as the model predicts, particularly if the market's expectations about prepayments are very different from those incorporated in the prepayment model. We have often observed that OA durations overstate the interest rate sensitivity of higher-coupon mortgages, because the market tends to build in a steeper prepayment function than most models project in any given interest rate environment. Results we describe later in the article tend to substantiate this casual impression.

Empirical Duration

A third way to measure the interest rate sensitivity of MBS prices is to calculate empirical hedge ratios and durations from historical price information. We do this on a daily basis, calculating two-week, four-week, and eight-week empirical hedge ratios for each pass-through using a simple linear regression. The regression takes the form shown in Equation (1):

$$\Delta_{\text{Mortgage Price}} = \alpha + \beta \times \Delta_{\text{10yr Price}} \tag{1}$$

The beta (β) calculated from the regression is the hedge ratio. If the beta equals 1, this implies that the minimum variance hedge ratio is 1:1. That is, the best hedge for an investor who is long a security with this hedge ratio is to go short the same face amount of the ten-year Treasury notes. The four-week empirical hedge ratio for FNMA 7s is 0.66, indicating that the best hedge is to short \$66 face of ten-year notes for each \$100 current face of FNMA 7s.

We can easily calculate the empirical duration from the empirical hedge ratio. Empirical duration measures the percent change in price of the security for a small change in the yield of the ten-year note. We know the duration of the mortgage is given by Equation (2):

$$\text{Duration}_{\text{Mortgage}} = \frac{\left(\dfrac{\Delta_{\text{Mortgage Price}}}{\text{Full Price}_{\text{Mortgage}}}\right)}{\Delta_{\text{10yr Yield}}} \tag{2}$$

Using the relationship above, we can substitute $\beta \times \Delta_{\text{10yr Price}}$ [from Equation (1)] for the change in the mortgage price. Thus, the duration is given by Equation (3):

$$\text{Duration}_{\text{Mortgage}} = \frac{\beta\left(\dfrac{\Delta_{\text{10yr Price}}}{\Delta_{\text{10 yr Yield}}}\right)}{\text{Full Price}_{\text{Mortgage}}} \tag{3}$$

Note that these empirical durations are estimated from the actual movement of market prices. There is no embedded prepayment estimate of any kind. The empirical duration measure works well as long as the market remains in roughly the same price range over the measurement period. If the market stays for a while in a trading range and then moves into, say, a lower price range, the actual duration will be longer than the empirical duration suggests. Similarly, if the market moves into a higher price range, empirical duration may be overstated relative to actual price behavior.

II. CASH FLOW, OA, AND EMPIRICAL DURATIONS COMPARED

Exhibit 1 compares the cash flow duration, the

EXHIBIT 1 ■ Various Measures of Mortgage Duration and Hedge Ratios

	5/30/96	Cash Flow		Option-Adjusted		Empirical		Option-Implied	
	Price	Dur	HR	Dur	HR	Dur	HR	Dur	HR
GNSF 6.5	92:26	6.57	0.85	6.59	0.85	5.92	0.77	6.46	0.84
GNSF 7	95:20	6.40	0.85	6.18	0.82	5.27	0.70	5.79	0.77
GNSF 7.5	98:07	6.05	0.83	5.75	0.79	4.63	0.63	5.10	0.70
GNSF 8	100:19	5.57	0.78	5.26	0.74	3.94	0.55	4.28	0.60
GNSF 8.5	102:20	4.77	0.68	4.59	0.66	3.04	0.44	3.36	0.48
GNSF 9	104:18	3.90	0.57	3.75	0.55	2.15	0.31	2.44	0.36
FNCL 6.5	93:05+	6.10	0.79	5.96	0.77	5.74	0.75	6.02	0.78
FNCL 7	95:26	5.87	0.78	5.57	0.74	4.95	0.66	5.43	0.72
FNCL 7.5	98:10	5.61	0.77	5.16	0.71	4.35	0.60	4.73	0.65
FNCL 8	100:15+	4.76	0.67	4.66	0.65	3.68	0.52	3.99	0.56
FNCL 8.5	102:12	4.11	0.59	3.96	0.57	2.73	0.39	3.10	0.44
FNCL 9	104:04	3.38	0.49	3.17	0.46	1.89	0.27	2.28	0.33
FNCI 6.0	94:03	4.55	1.04	4.64	1.06	4.77	1.09	4.72	1.08
FNCI 6.5	96:10	3.98	0.94	4.42	1.04	4.18	0.98	4.25	1.00
FNCI 7	98:12+	3.39	0.81	4.17	1.00	3.57	0.86	3.69	0.89
FNCI 7.5	100:07	2.69	0.66	3.88	0.95	2.83	0.69	3.01	0.74
FNCI 8	101:26	2.01	0.50	3.47	0.86	2.13	0.53	2.36	0.59

Note: Hedge ratio to 10yr for 30yr mortgages, to 5yr for 15yr mortgages.

option-adjusted duration, and the four-week empirical duration for an assortment of thirty- and fifteen-year pass-throughs. These numbers are calculated as of the close on May 29, 1996. As can be seen from this table, on thirty-year product, virtually across the board the cash flow duration is the longest of the measures. While this generally holds true, market participants know better than to rely on this static measure.

Note also that, as of this writing, the OA duration is longer than the empirical duration for most pass-throughs, particularly the higher-coupon thirty-year pass-throughs. In other words, the market is treating the higher-coupon securities as if they were much shorter than would be predicted by the Street consensus prepayment model.

III. OPTION-IMPLIED DURATION

We believe that market prices embody a great deal of information, and that investors should incorporate this information into their valuation of mortgage securities. In particular, the mortgage options market, a very active over-the-counter market, gives us highly useful information about the durations of mortgage securities. In this market, dealers buy and sell at-the-money and in- and out-of-the-money puts and calls on mortgage pass-throughs to both investors and mortgage originators. The many pass-through investors who are not actively trading options tend not to look at options prices; as a result, they miss out on valuable information.

One bit of information is particularly useful: the ratio of the fee on an at-the-money forward option for a given pass-through to the fee for an option on an at-the-money forward ten-year Treasury note. It is important because it actually reflects the options markets' expectation of the relative price volatility of these two securities. We refer to this as the *fee ratio* or the *option-implied hedge ratio*.

The intuition behind this assertion is straightforward. Consider a mortgage option that is struck at the money forward. That is, assume the strike price for a three-month option (which expires on the TBA settlement date in August 1996) is the price of the security for August settlement.* Now, for a security struck at the money forward, the price of the put and the price of the call will be roughly the same. This option has no intrinsic value when the contract is arranged. In other words, the forward price is exactly equal to the strike price. Thus, the prices of the put and the call reflect simply the time value of the security. The cost of this option, for a fixed period to expiration, will be determined by one variable: the price volatility of the security. If the security has no price volatility, the option will have no value.

Similarly, we can observe the price of an option on the ten-year note struck at the money forward. The forward price on the ten-year note is simply the break-even price on a cash and carry transaction. That is, an investor who buys a ten-year note by financing the security in the repo market, earns the carry, and sells the security forward has no economic risk and has not invested any money. Thus, the forward price must be such that the investor earns nothing on this transaction, or the investor will have an arbitrage profit.

Although Treasury options are most commonly struck at the current spot price, options dealers often quote prices on the ten-year Treasury note with an at-the-money forward strike. Again, the options price reflects only the time value, which is determined by volatility.

We can make this more concrete. Exhibit 2 lists forward prices for the ten-year Treasury and various mortgage pass-throughs for August settlement. It also lists premiums for at-the-money forward options on these securities. (Puts and calls are not shown separately because, for an at-the-money forward strike, the prices of the put and call will be identical.)

EXHIBIT 2 ■ Evaluating Mortgage Options

	Aug-96 Forward Px	ATM Option Premium	Fee Ratio
GNSF 6.50	92:18	1:06+	0.84
GNSF 7.00	95:08	1:035	0.77
GNSF 7.50	97:25	1:002	0.70
GNSF 8.00	100:02	0:276	0.60
GNSF 8.50	102:04	0:221	0.48
GNSF 9.00	104:06	0:221	0.36
10-yr*	100:08	1:140	
FNCL 6.50	92:29+	1:022+	0.78
FNCL 7.00	95:15	0:316	0.72
FNCL 7.50	97:26	0:28+	0.65
FNCL 8.00	99:31+	0:24+	0.56
FNCL 8.50	101:28	0:193	0.44
FNCL 9.00	103:28	0:14+	0.33
10-yr*	100:09+	1:117	
FNCI 6.00	93:29	0:297	1.08
FNCI 6.50	96:01	0:27+	1.00
FNCI 7.00	98:00+	0:243	0.89
FNCI 7.50	99:25	0:202	0.74
FNCI 8.00	101:12	0:161	0.59
5-yr*	98:15+	0:27	

*Forward/expiration dates corresponding to respective TBA settlement dates.

Note: Hedge ratios to 10yr for 30yr mortgages, to 5yr for 15yr mortgages.

As can be seen from the table, the August 1996 settlement price of thirty-year FNMA 7s is 95:15. An option struck at 95:15 for August settlement would sell at a premium of 0:316 (that is, 31 and 6/8 ticks). For the ten-year note, the current price (as of the close on May 29, 1996) is 100:23, and the August forward price is 100:09+. The price of the three-month option on the ten-year note is 1:117. Thus, the fee ratio on the FNMA 7s (that is, the ratio of the price on the FNMA 7 option to the price on the option on the ten-year note) is 0:316/1:117 or 0.72.

Exhibit 2 shows fee ratios for the various pass-throughs. Note that the settlement dates on the FNMAs and GNMAs are different. Thus, the forward prices and premiums for an at-the-money option on the ten-year Treasury are slightly different for the FNMAs from those for the GNMAs.

We have now established that this fee ratio measures the implied volatility of FNMA 7s relative to the implied volatility of the ten-year note. That is, the three-month fee ratio captures the beliefs of market participants regarding the relative volatility of these instruments over the next three months. Since the price volatility of all these instruments is driven by one factor — interest rates — the relative volatility between pass-throughs and Treasuries is essentially the options market's estimate of the correct hedge ratio.

Investors should be aware that just as the other measures of duration have their drawbacks, the fee ratio will have its drawbacks, as well. The fee ratio approach assumes that the options market is correctly priced. To the extent that there are technical imbalances driving options prices, the fee ratio may be distorted — if demand for mortgage options is strong, prices may be artificially high; if demand is weak, prices may be artificially low.

It should be noted that, like empirical duration measures, option-implied durations are based solely on prices generated by the market. The results are not dependent on the myriad of individual assumptions and decisions embodied in models.

These fee ratios — hedge ratios, in effect — are translated into duration numbers and included in Exhibit 1 with the other measures of duration. This permits us to compare the option-adjusted and empirical durations to those durations derived from the options market. For most securities, the option-implied duration is currently higher than the empirical duration, but lower than the option-adjusted duration. The configuration does not always hold, however.

IV. PERFORMANCE CONTEST: MODEL VERSUS MARKET

Which duration measure, then, is a mortgage portfolio manager to use? Ultimately, this is an empirical question — how have the hedge ratios performed over time? We attempt to answer this question in the remainder of this article. We ignore cash flow duration, since using it would merely be setting up a straw man. We compare the results of the other three duration measures, option-adjusted duration, empirical duration, and option-implied duration, for thirty-year product. Our conclusion is that option-adjusted duration measures tend to be less accurate than either empirical or option-implied measures. Option-implied measures tend to perform slightly better than empirical measures.

Exhibit 3 tracks the option-adjusted and empirical hedge ratios and the fee ratio on thirty-year FNMA 7.5s from September 1994 through May 1996. We choose the 7.5s because, over that period, 7.5s were — on average — the current coupon. That is, the average price on this security was 98.40 points over the period, making it on average the mortgage selling closest to but below par. This masks a wide price variation. The high price on the security was 102:21+ points on January 19, 1995. The low on FNMA 7.5s was 92:13+ points on November 4, 1994.

Exhibit 3 also tracks the level of the ten-year note over the time period. Note that the three hedge ratios were fairly close when rates were high. When the market rallied, however, the empirical and option-implied durations (empirical and fee ratios) declined more quickly than the option-adjusted durations. In fact, the lower the level of the ten-year notes, the greater the difference.

As rates have risen since January 1996, the empirical and option-implied hedge ratios have expanded far more quickly than the option-adjusted ratios. While the empirical duration is still considerably shorter than the option-adjusted hedge ratio, the difference has narrowed considerably.

Exhibit 4 compares scatter plots of the three hedge ratios calculated for FNMA 7.5s for each day of the period as a function of the ten-year yield on that date. The pattern is what we would have expected from Exhibit 3. The option-adjusted hedge ratio is far more stable than the other two hedge ratios over the period. That is, the option-adjusted hedge ratios for 7.5s are higher than those derived from market information when rates are low and lower when rates are high. In other words, durations generated by a model are far more stable than durations implied by market prices.

EXHIBIT 3 ■ FNCL 7.5% — Hedge Ratio Time Series

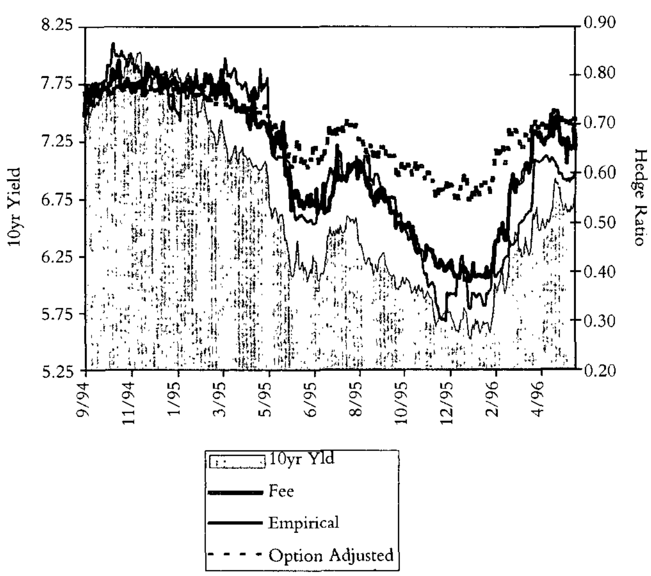
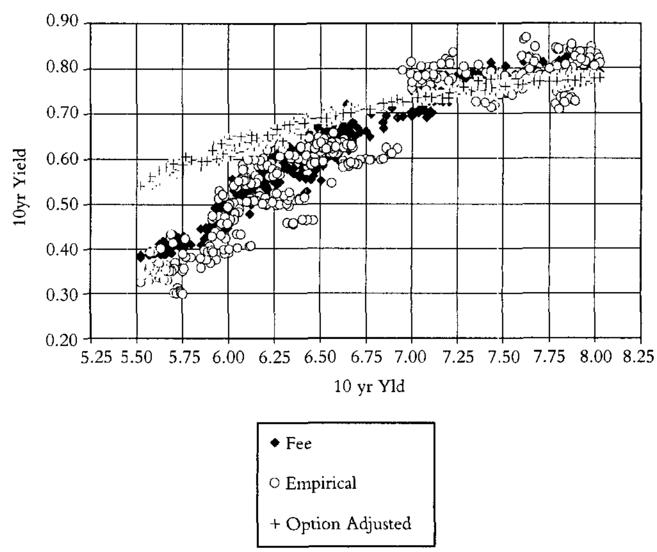


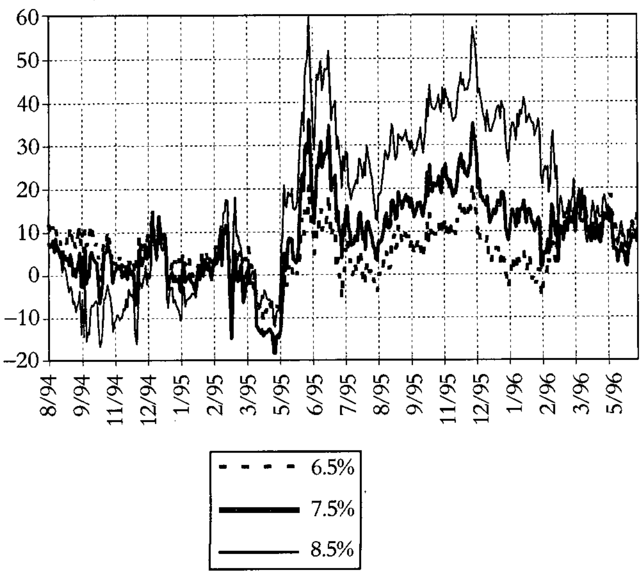
EXHIBIT 4 ■ FNCL 7.5% — Hedge Ratios Versus Ten-Year Yield



Several explanations are possible. We focus on three:

- *Market sentiment is heavily conditioned by the current situation, and sentiment rules in establishing market prices.* The market does not price off forward rates, as in a model. Furthermore, if the market is rallying and sentiment is positive, mortgage securities are priced as if the market will continue to rally. Thus, when rates are low and prepayments are high, the market behaves as if the rally will continue, and prepayments will accelerate further. When rates are high and prepayments are low, the market behaves as if rates will drift higher, and even normal housing turnover will be curtailed.
- *The market anticipates a much steeper prepayment curve than is embedded in OAS models.* That is, when mortgages are non-refinanceable, speeds are very slow — slower than model consensus; when mortgages are refinanceable, speeds are very fast — faster than model consensus.
- *The market demands the highest OAS on securities that arouse the greatest prepayment fear.* Exhibit 5 illustrates this point quite clearly. At low rates — in the second half of 1995 and early 1996, for example — the highest OAS were on the 8.5s, the lowest on the 6.5s. When rates were high — as in 1994 — and much of the mortgage market was at a considerable discount, the market's fear was that speeds would slow. Thus, the securities judged to be the most risky, and hence deserving of the highest OAS, were the lowest coupons. Likewise, at high rates, the lowest OASs belonged to the higher coupons.

EXHIBIT 5 ■ OAS History for Thirty-Year FNMA 6.5, 7.5, and 8.5%



Some readers may be surprised to note how low the absolute levels of these OAS are. Because the OAS are stated relative to the LIBOR curve, they are lower than most investors are used to seeing. Crudely speaking, it is possible to obtain the OAS to the Treasury curve by adding the swap spread.

V. WHY THE COUPON DISTRIBUTION OF OUTSTANDINGS MATTERS

Current-mortgage outstandings in the late spring of 1996 were heavily concentrated in thirty-year 6.5s through 8.0s and in fifteen-year 6s through 7.5s, as Exhibit 6 shows. Note that the thirty-year pass-through market was far larger than the fifteen-year market. When interest rates were low, much of this market was

EXHIBIT 6 ■ Outstandings in the Mortgage Market

		Current Balance	
	Cpn	GNMA	Conventional
30yr	6.0	3,642	16,721
	6.5	27,603	111,669
	7.0	67,342	178,303
	7.5	64,177	125,064
	8.0	56,964	90,278
	8.5	27,902	46,157
	9.0	35,248	28,573
	9.5	18,436	15,180
15yr	5.5	206	5,812
	6.0	3,019	46,330
	6.5	6,850	72,885
	7.0	7,310	55,694
	7.5	3,729	31,528
	8.0	2,325	16,766
7yr	5.0		202
	5.5		3,309
	6.0		11,573
	6.5		13,030
	7.0		8,911
5yr	5.0		1,466
	5.5		4,696
	6.0		7,825
	6.5		5,303

refinanceable, or close to it. As a rule of thumb, the rate offered to a mortgage holder is typically 120-140 basis points above the ten-year note rate. Thus, a 5.6% ten-year note would have put primary mortgage rates at 6.8%-7.0%. Mortgages in which the homeowner does not have to pay points (so-called no-points mortgages) would then be available at around 7.25%.

We have observed that market participants begin to refinance when there is a 60-basis point difference between the rate they are paying on their mortgage and the no-point rate that is obtainable. Thus, a mortgagee who is paying 8.0% on a mortgage (put into a 7.5% pool) would consider refinancing into a no-points 7.25% mortgage. When the ten-year Treasury was yielding 5.60%, thirty-year 8s were fully refinanceable, and 7.5s were somewhat refinanceable. At that point, the market feared faster speeds, and very little in the market was "safe" from prepayments (only the 6.5s and below were considered to be safe). As a consequence, the OAS were high on mortgages that were refinanceable and lower on mortgages that were not.

VI. WHICH MEASURE PERFORMED BEST?

Having established that the durations or hedge ratios produced by these three measures vary considerably in different rate environments, we may now judge which one works best. For the purposes of this analysis, we consider just six securities: thirty-year FNMA 6.5s, 7.5s, and 8.5s, and thirty-year GNMA 6.5, 7.5s, and 8.5s. We first consider the cumulative hedging error for a one-day holding period. This analysis is far more applicable to trading desks — which rebalance every day — than to investors, who generally do not.

For each day in the period, we hedge the mortgages with the ten-year note, using the hedge ratio dictated by each of the three duration measures. The next day, we record the error (gain or loss on the position) in hedging and rebalance the position. This exercise is run for the period September 12, 1994, through May 29, 1996.

Focusing on the FNMA 7.5s, Exhibit 7 depicts the cumulative performance against the hedge for each of the three hedge ratios. For example, if we make \$0.20 on the hedge one day and lose it the next, the cumulative differences total 0. Note that, of the three measures, the cumulative error is the highest (that is, the most different from zero) on the option-adjusted duration, while it is very similar on the fee and empirical hedge ratios.

Exhibit 8 shows the cumulative *absolute* hedge error for each of the three hedge ratios. In this case, if we make \$0.20 on the hedge one day, and lose it the next, the cumulative absolute difference is \$0.40.

Note that the alternative methodologies all produce similar results when the hedge ratios were close in

EXHIBIT 7 ■ FNCL 7.5% — Cumulative Performance Against Hedge (one-day holding period)

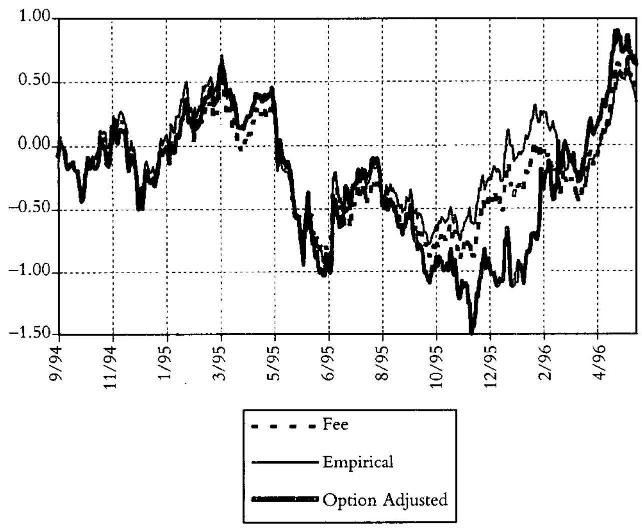
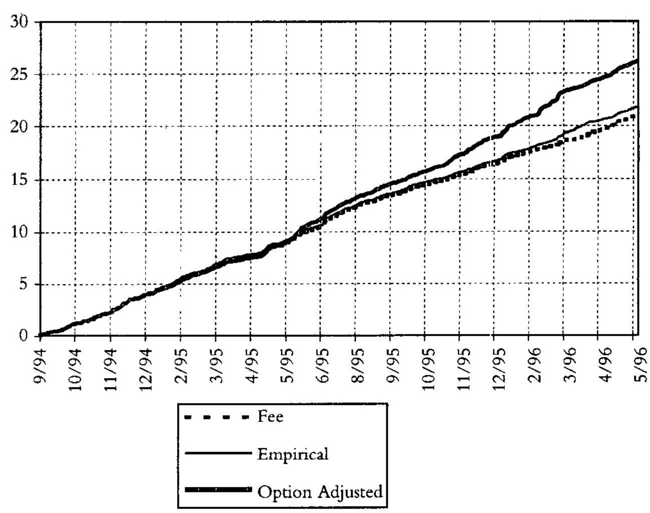


EXHIBIT 8 ■ FNCL 7.5% — Cumulative Absolute Hedge Error (one-day holding period)



late 1994 and early 1995, but very different results when the hedge ratios began to diverge. As is clear from this figure, the lowest hedge error occurs using the fee ratio; the empirical hedge ratio is very close. The highest absolute hedging error occurs when hedging is based on the option-adjusted duration. Per $100 face on FNMA 7.5s, rebalanced daily, the cumulative absolute hedge error would be $26.20 with the option-adjusted hedge ratio, $21.90 with the empirical hedge ratio, and $21.10 with the fee ratio.

Exhibit 9 expands the example and compares the results for a one-day holding period for the FNMA 6.5s, 7.5s, and 8.5s. The cumulative absolute error (or the sum of the absolute errors) is the smallest on the fee

EXHIBIT 9 ■ Measures of Hedging Error

	Fee				Empirical				Option-Adjusted			
Coupon	6.5	7.5	8.5	Sum	6.5	7.5	8.5	Sum	6.5	7.5	8.5	Sum
					30yr FNMA Pass-Throughs (starting 9/12/94)							
1-day holding period												
sum abs err	24.0	21.1	21.7	66.7	24.3	21.9	21.6	67.8	25.6	26.2	28.5	80.4
sum err sqr	2.2	1.8	2.0	6.0	2.3	2.0	1.9	6.2	2.5	2.9	3.5	9.0
var err	0.0050	0.0043	0.0047	0.0140	0.0054	0.0046	0.0045	0.0145	0.0059	0.0068	0.0082	0.0209
20-day holding period												
sum abs err	116.1	109.1	105.6	330.8	111.4	117.6	103.3	332.2	104.2	131.3	146.6	382.1
sum err sqr	54.0	46.0	46.2	146.2	50.6	56.5	48.3	155.3	38.4	64.0	86.0	188.3
var err	0.1313	0.1117	0.1130	0.3560	0.1223	0.1380	0.1180	0.3783	0.0938	0.1544	0.2097	0.4578
60-day holding period												
sum abs err	169.1	175.1	141.7	485.9	176.8	205.8	225.2	607.9	195.3	198.4	274.8	668.5
sum err sqr	134.0	151.4	87.6	373.0	154.0	210.1	204.3	568.4	205.9	195.0	313.0	713.9
varr err	0.2704	0.4033	0.5434	1.2171	0.3062	0.5239	0.5071	1.3372	0.2227	0.5520	0.8424	1.6171
					30yr GNSF Pass-Throughs (starting 9/12/94)							
1-day holding period												
sum abs err					27.5	23.6	23.4	74.5	30.3	30.0	33.1	93.3
sum err sqr					3.0	2.4	2.4	7.7	3.6	3.9	5.0	12.5
var err					0.0069	0.0055	0.0056	0.0180	0.0083	0.0092	0.0116	0.0290
20-day holding period												
sum abs err					110.5	117.1	111.2	338.8	100.1	141.9	175.2	417.3
sum err sqr					57.8	62.4	58.5	178.6	35.0	81.2	139.1	255.4
var err					0.1412	0.1524	0.1430	0.4366	0.0827	0.1966	0.3400	0.6193
60-day holding period												
sum abs err					185.3	219.1	226.0	630.5	140.1	265.7	344.2	750.0
sum err sqr					162.7	213.6	254.4	630.7	73.1	267.4	515.5	855.9
var err					0.3905	0.5464	0.6739	1.6109	0.1963	0.7226	1.3816	2.3005
					30yr GNSF Pass-Throughs (starting 6/30/95)							
1-day holding period												
sum abs err	13.8	11.6	11.5	36.8	14.0	12.0	11.9	37.9	16.5	17.7	19.9	54.1
sum err sqr	1.4	1.1	1.1	3.5	1.5	1.2	1.2	3.9	2.0	2.5	3.3	7.8
var err	0.0061	0.0046	0.0046	0.0153	0.0065	0.0050	0.0053	0.0169	0.0089	0.0110	0.0141	0.0340
20-day holding period												
sum abs err	58.1	49.9	49.0	157.1	59.9	52.5	48.6	161.1	56.9	76.1	99.6	232.6
sum err sqr	30.2	17.6	18.0	65.8	33.9	20.2	18.2	72.3	21.6	38.1	71.4	131.1
var err	0.1443	0.0824	0.0808	0.3075	0.1621	0.0967	0.0831	0.3419	0.1025	0.1716	0.3233	0.5974
60-day holding period												
sum abs err	89.6	69.7	84.9	244.2	103.2	88.7	85.5	277.4	67.5	135.6	194.7	397.8
sum err sqr	71.5	44.1	55.1	170.7	109.3	61.9	57.3	228.4	37.0	129.3	280.3	446.5
var err	0.3095	0.2471	0.3179	0.8745	0.4908	0.3035	0.3354	1.1297	0.2080	0.7509	1.6012	2.5601

ratio for the 6.5s and 7.5s; on the 8.5s, the empirical hedge ratio is very marginally better. Overall, the sum of the absolute errors on the three coupons is 66.7 using fee ratios, 67.8 using empirical hedge ratios, and 80.4 using option-adjusted hedge ratios.

We also look at the sum of the squared errors. This imparts a proportionately greater penalty for larger errors than for smaller errors. As can be seen, the squared errors are also smaller when hedging with fee ratios, largest using the option-adjusted numbers.

Finally, we look at the variance of the errors, which is the squared difference between the actual gain or loss on the hedged position and the average gain or loss on the position each day, divided by the number of observations. We use this measure in addition to the sum of the squared errors, as we do not want to penalize a hedge that has consistently made money day after day.

Specifically, our concern was that the cumulative performance against the hedge was highest for the option-adjusted hedge ratios. It is not clear whether this is because option-adjusted hedge ratios provided a great hedge that made money day in and day out, or whether it coincidentally made money because the hedges were too long as the market backed up. We want to use a measure that would not penalize the hedge for excess profits. As can be seen in Exhibit 9, the variance of the option-adjusted hedge is higher than the variances on the empirical hedge ratios or the fee ratios.

Similar results on these three conventional coupons obtain over longer periods, as Exhibit 9 also shows. For example, we perform exactly the same exercise for the same three coupons (the FNMA 6.5s, 7.5s, and 8.5s), assuming this time that the hedge is rebalanced every twenty business days (approximately once a month). We choose this time frame to more closely approximate the practice of a portfolio manager.

Each day we put on a new position and take it off twenty business days later; we hedge using each of the alternative hedge ratios. This means that, at any given time, there are twenty positions on. Note that the last observation is April 30, 1996.

Exhibit 10 shows the cumulative absolute hedging error on each of the three alternative hedge ratios, assuming the twenty-day holding period. Note that the fee ratio is again the hedge that produces the lowest cumulative hedge error.

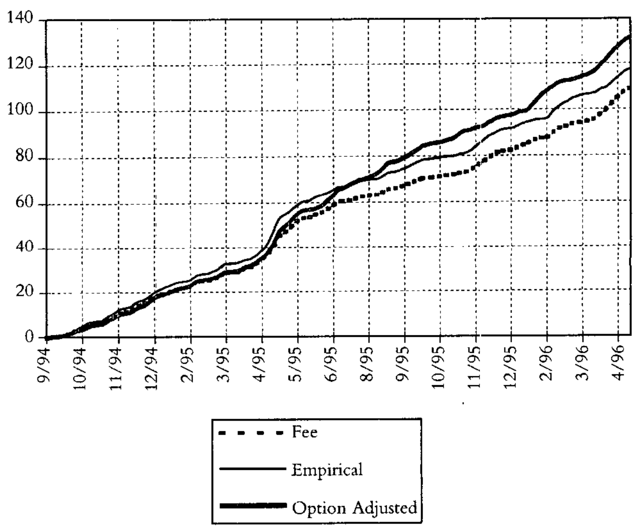
Exhibit 9 shows the results of this analysis in detail. The cumulative absolute error is smallest on the FNMA 6.5s using the option-adjusted hedge ratios, smallest on the FNMA 7.5s using the fee ratio, and smallest on the FNMA 8.5s using the empirical duration. Overall, the sum of the three error terms is smallest using the fee ratios. Squaring each of the error terms — this is in order to penalize large errors more than small errors — the result is the same: The fee ratios have the best performance overall. When we calculate the variance rather than simply squaring the error terms, the fee ratios still have the best overall performance.

In the twenty-day results, the fee ratios and the empirical hedge ratios clearly show less hedging error than the option-adjusted hedge ratios, although the results are closer than they are with the one-day rebalancing. Consequently, we decided to stretch out the hedging period even more and examine the robustness of the results. We used a sixty-day rebalancing. That is, each day we put on a new hedge, taking it off sixty business days later.

As can be seen in Exhibit 9, with a sixty-business day rebalancing period, the fee ratio is still the best hedge. Note that for the sixty-day rebalancing period, the fee ratio unequivocally provides the best hedge for each of the three coupons tested. The empirical hedge ratio is second-best, and the option-adjusted hedge ratio again provides the worst hedge.

Exhibit 9 also shows the results of the same analysis for GNMA 6.5s, 7.5s, and 8.5s. We compared all

EXHIBIT 10 ■ FNCL 7.5% — Cumulative Absolute Hedge Error (twenty-day holding period)



three measures going back only through June 30, 1995, as we did not have earlier data on fee ratios. We were able to compare the empirical hedge ratios versus the option-adjusted fee ratios for a period comparable to the comparison on the FNMAs: from September 12, 1994, through May 29, 1996.

As can be seen when we compare all three measures, regardless of the rebalancing period, the fee ratios dominate both of the other alternatives. When we look back over the longer period from September 12, 1994, to the end of the data, the empirical hedge ratios dominate the option-adjusted hedge ratios.

VII. SUMMARY

These results must be regarded as preliminary, as we have confined the analysis to three thirty-year conventional and GNMA coupons. We have not examined fifteen-year GNMA or conventional securities. Nonetheless, the results appear quite compelling.

The best duration to use for mortgages appears not to be that given from OAS models, but rather the durations generated from market prices. That is, both the fee ratios calculated from over-the-counter options prices and the empirical hedge ratios seem to produce considerably smaller hedging errors than do the option-adjusted hedge ratios. The fee ratio appears to be slightly more effective than the empirical hedge ratio.

ENDNOTES

This article has been adapted from Laurie S. Goodman and Jeffrey Ho, "Hedging Issues Related to Mortgages," in *Controlling and Managing Interest Rate Risk*, Anthony Cornyn, Robert Klein, and Jess Lederman, Editors. 1997, adapted with permission of Prentice-Hall.

*Mortgage options generally expire on the TBA settlement date and have a one-week notification period.

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