A Simple, Transparent, and Accurate Mortgage Valuation Yield Curve

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This article implements a new simple, transparent, and accurate mortgage valuation curve estimation procedure useful for pricing mortgage loans and related derivatives with the following characteristics:

- The mortgage yield curve exactly matches the prices of new 15- and 30-year fixed-rate mortgages at par value, net of points.
- The primary mortgage market yield curve is obtained by deriving the maximum smoothness credit spread over the U.S. Treasury yield curve.
- The mortgage yield curve is based solely on data from U.S. government sources that are free to all users. The U.S. Treasury curve is based on the Federal Reserve's H15 statistical release, which is updated daily. The primary mortgage origination rates are based on the Federal Home Loan Mortgage Corporation's (FHLMC's) Primary Mortgage Market Survey, which is published weekly.

Although we could have used Eurodollars instead of Treasuries for the base yield curve, as was common practice for more than two decades, we elected not to do so. The Barclays LIBOR manipulation settlement with U.S. and British regulators on June 27, 2012, revealed that non-market clearing rates have been quoted since at least 2005. Using interest rate swaps (whose pricing is also largely controlled by dealers on the U.S. dollar LIBOR panel) as a basis for the long end of the mortgage yield curves is equally problematic. Indeed, a symptom of this difficulty is the observation that the 30-year interest rate swap rate has been more than 20 basis points below the 30-year U.S. Treasury rate for more than two years now.

We begin with the conceptual background underlying the mortgage yield curve procedure. The subsequent section provides an illustrative example of the computation, and the last section concludes.

CONCEPTUAL BACKGROUND

In the U.S. Treasury market, bonds are traded both on a coupon-bearing basis and on a "stripped" basis, in the form of zero-coupon bonds representing either interest or principal. As is well known, no arbitrage in a frictionless and competitive market ensures that the value of a default-free coupon bond equals the sum of its zero-coupon parts.

In a recent article, Jarrow [2004] addresses the conditions under which risky coupon bonds are equivalent to a portfolio of risky zero-coupon bonds. Under the assumptions that bond markets are arbitrage-free, competitive, and frictionless, and that there are no differential taxes on coupons versus capital gains income, Jarrow shows that the price of a risky coupon bond can be written as a linear combination of risky zero-coupon bonds if and only if the recovery rates of the zero-coupon bonds do not depend on the particular coupon bond's cash flow characteristics, except for the bond's seniority. Sufficient conditions for this to be true include the following:

- The recovery rate is constant, as explored by Jarrow and Turnbull [1995].
- The recovery rate is random and depends only on time and the seniority of the debt.
- The recovery, when using the approach of Lando [1998], is a fraction of the bond's price an instant before default.

These sufficient conditions are used often in the credit risk literature.

Applying this result, we see that even in light of the worldwide sovereign debt crisis in which market participants now acknowledge that there is no government issuer free of credit risk, a coupon-bearing U.S. Treasury bond's price can still be written as a portfolio of Treasury zero-coupon bond prices.

This article's key insight is the following theorem, which shows that this result can be extended to mortgage loans that include both default and prepayment risk. The proof is contained in the appendix.

Theorem (Mortgage Coupon Bonds As a Collection of Zero-Coupon Bonds)

Let the mortgage loan market be arbitrage-free, frictionless, and competitive, with no differential taxes on coupon versus capital gains income. Then, any coupon-bearing mortgage loan in a fixed collection of mortgage loans is equivalent to a portfolio of zero-coupon mortgage loans if and only if

- all the coupon-bearing mortgage loans in the collection reflect the same default and prepayment risks, and
- the recovery rate processes for default and prepayment on all the coupon-bearing mortgage loans in the collection are equal.

This is a reasonable set of conditions to impose on a collection of coupon-bearing mortgage loans when decomposing its payoff into a collection of similar risky mortgage zero-coupon bonds. Indeed, the "underlying" zeros need to reflect the common default and prepayment risks in all of the coupon-bearing mortgages included in the collection, and this can happen only if the necessary and sufficient conditions listed in the theorem are true.

This theorem is important because it enables us to use yield-curve smoothing techniques in the mortgage market, similar to those used in Treasury markets, for extracting zero-coupon bond prices consistent with the FHLMC Primary Mortgage Market Survey new issue yields for fixed-rate mortgages. These new issue rates are gathered carefully by FHLMC from mortgage lenders across the United States using a process summarized on the FHLMC website: http://www.freddiemac.com/pmms/. These zero-coupon bond prices can then be used to price arbitrary mortgage loans with the same default and prepayment risks.

The next section illustrates this procedure with an example.

EXTRACTING MORTGAGE ZERO-COUPON BOND PRICES

Step one in the procedure is to generate a smoothed U.S. Treasury yield curve. The inputs to the smoothing process are the constant-maturity Treasury rates reported in the Fed's H15 statistical release. Exhibit 1 shows the rates reported for August 23, 2012.

The U.S. Treasury zero-coupon yield curve and continuous forward rates consistent with this data, using the maximum smoothness forward rate procedure of Adams and van Deventer [1994], as corrected in van Deventer and Imai [1997], are shown in Exhibit 2.

Step two is to create the mortgage zero-coupon bond price curve. To do this, we use the FHLMC Primary Mortgage Market Survey data reported on August 23, 2012, as shown in Exhibit 3.

The "30-Yr FRM" is a 30-year amortizing fixed-rate loan, paid monthly, that can be prepaid at any time with no penalty. The average "new issue" coupon rate is 3.66%, and fees of 0.7% are paid at origination. For a 15-year maturity, the coupon is 2.89% on new mortgage loans with fees of 0.7% on average. The adjustable-rate mortgage (ARM) category is also important, but we do not use this data for two reasons:

Exhibit 1

U.S. Treasury Nominal Yield Inputs, Standard H15 Maturities: August 23, 2012

Observation Date U.S. Treasury Nominal Yield Inputs Standard H15 Maturities	8/23/2012 Percent
1 month	0.10
3 months	0.11
6 months	0.13
1 year	0.19
2 years	0.26
3 years	0.36
5 years	0.71
7 years	1.13
10 years	1.68
20 years	2.41
30 years	2.79

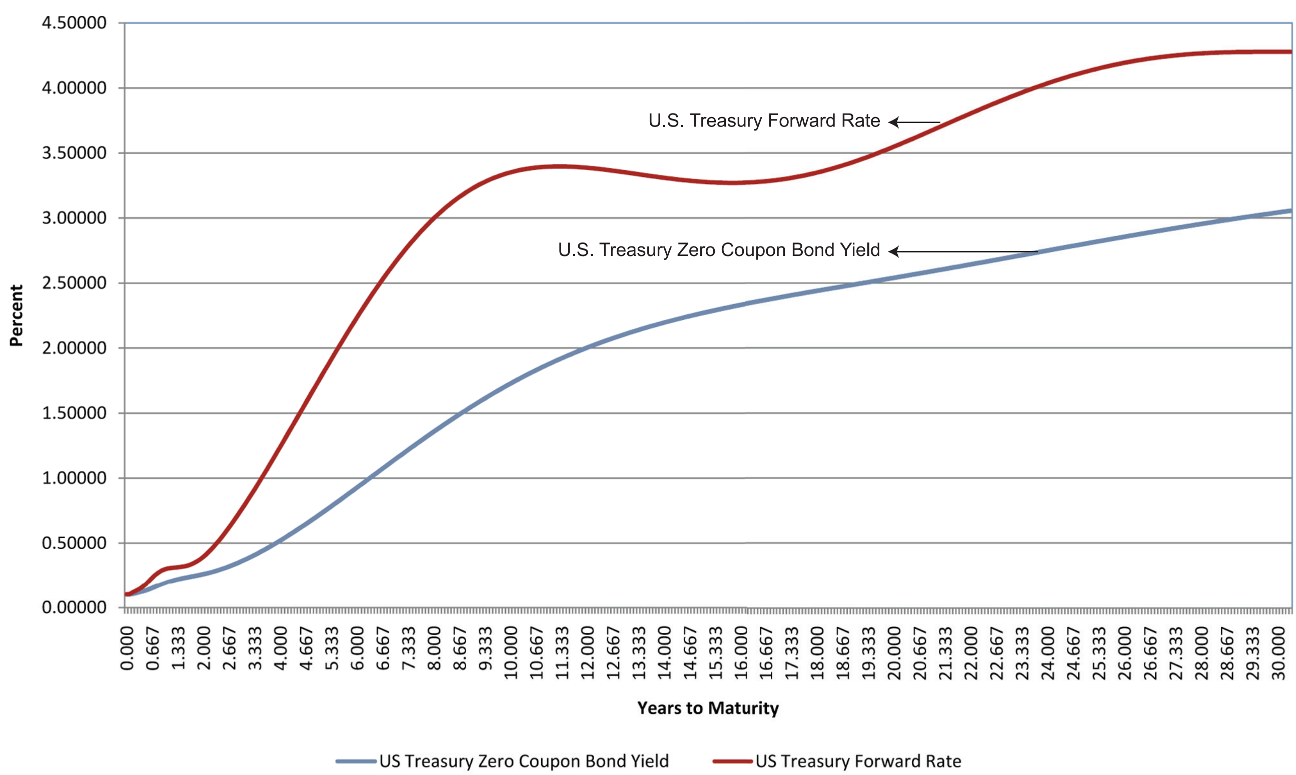
Source: Federal Reserve Release H15, http://www.federalreserve.gov/releases/h15/update/

1. ARMs include caps on the maximum change in the floating interest rate over three intervals: over the life of the loan, at the first interest rate adjustment period, and at all subsequent adjustment periods. The PMMS does not report these cap levels.
2. The nature of a floating-rate mortgage loan means that its default risk characteristics differ from fixed-rate mortgages. As such, this risk differential violates the necessary conditions for applying our theorem.

We next apply the maximum smoothness forward credit spread smoothing procedure discussed in van Deventer and Imai [1997] and van Deventer, Imai, and Mesler [2004] to these mortgage "bonds." This process generates the "best" credit spread for the mortgage valuation yield curve, where "best" is defined as the credit spread satisfying the following three conditions:

Exhibit 2

U.S. Treasury Forward Rates and Zero-Coupon Yields Derived from the Federal Reserve H15 Statistical Release Using Maximum Smoothness Forward Rate Smoothing



Source: Kamakura Corporation and the Board of Governors of the Federal Reserve.

EXHIBIT 3

Weekly Primary Mortgage Market Survey (PMMS), August 23, 2012

Weekly Primary Mortgage Market Survey® (PMMS®)

View Our PMMS News Releases

Compilation of Weekly Survey Data for 2012

2012 Weekly Mortgage Rates Data [XLS]

August 23, 2012

Regional Breakdown	30-Yr FRM	15-Yr FRM	5/1-Yr ARM	1-Yr ARM
Average Rates	3.66 %	2.89 %	2.80 %	2.66 %
Fees & Points	0.7	0.7	0.6	0.4
Margin	N/A	N/A	2.74	2.77

Source: FHLMC, http://www.freddiemac.com/pmms.

EXHIBIT 4

Summary of Zero-Coupon Bond Prices and Yields: U.S. Treasury and Fixed-Rate Mortgage Primary Market

	Number of Payments	Monthly Payment	Sum of Zero Coupon Bond Prices	Mortgage Value
15 Year Fixed Rate Mortgage	180	690.10	144.9071	100,000
30 Year Fixed Rate Mortgage	360	461.23	216.8113	100,000

1. The forward rate credit spread satisfies the maximum smoothness criterion.
2. Because there are only two observable prices, we use two knot points for the entire 30-year span of the credit spread, at the shortest maturity (0) and the longest maturity (approximately 30 years).
3. We impose the same constraints for smoothing the forward credit spread in the mortgage market used in the U.S. Treasury smoothing process; that is,
   a. The second derivative of the forward credit spread curve at time 0 is zero, $f'(0) = 0$.
   b. The second derivative of the forward credit spread curve at maturity = T is also zero, $f''(T) = 0$.
   c. The first derivative of the forward credit spread curve at maturity = T is also zero, $f'(T) = 0$.

The maximum smoothness continuous forward credit spread curve that fits these constraints is a fourth-degree polynomial:

$$\text{Credit spread forward rate} = c + d_1 t + d_2 t^2 + d_3 t^3 + d_4 t^4$$

where t is the time from the (current) origination date to the forward's maturity date (without loss of generality, expressed in "years" of 365 days in length) and $\{d_1, d_2, d_3, d_4\}$ are constants to be determined by the smoothing procedure.

Given the U.S. Treasury zero-coupon bond prices, yields and forwards, we solve for the coefficients such that the value of a new-issue 15-year and 30-year fixed-rate mortgage equals its par value, net of points. In doing this calculation, we make the following mechanical assumptions and calculations:

- The borrower of the mortgage borrows the points and pays them immediately from the loan proceeds. With points of 0.7% on August 23, 2012, that means that to net $100,000 in loan proceeds, the borrower borrows \$100,700 for both the 15- and 30-year maturities.

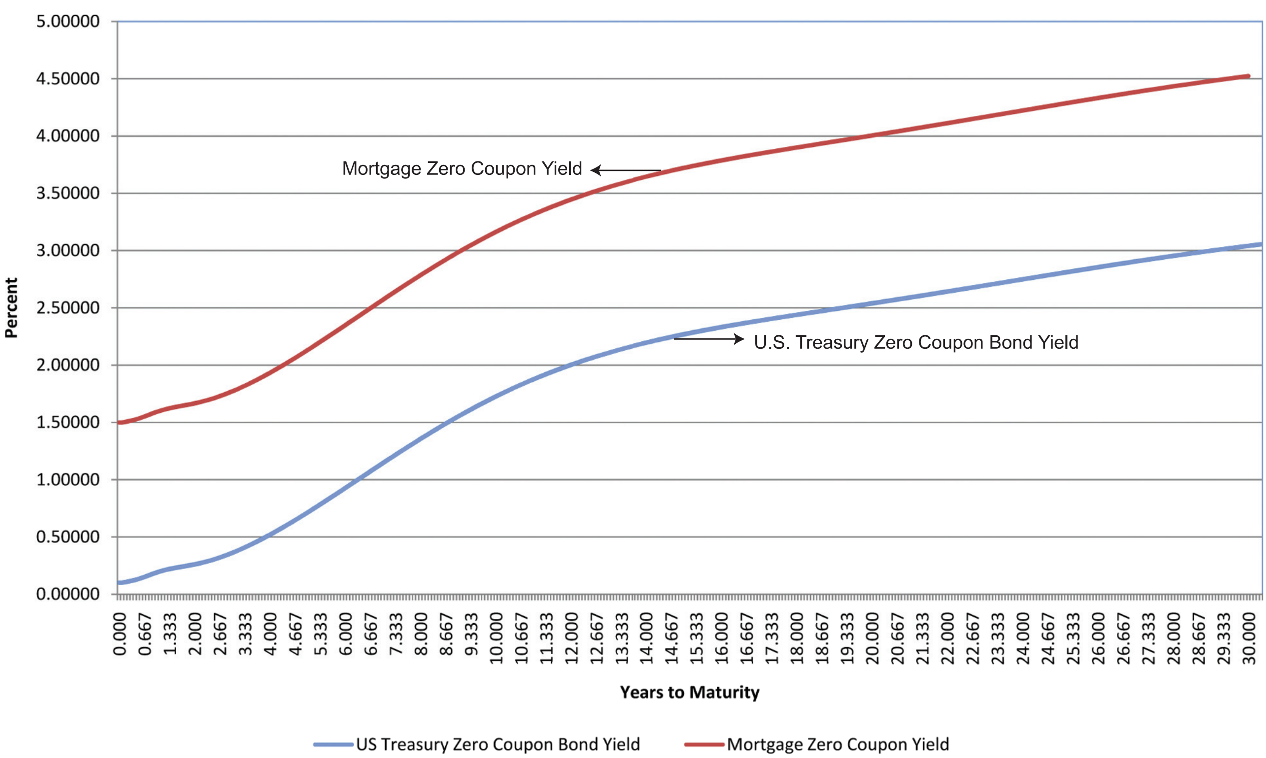
- Using standard mortgage payment calculations (no yield-curve smoothing), the monthly payments on a loan of \$100,700 will be \$690.10 for a 15-year mortgage and \$461.23 for a 30-year mortgage.
- We use the "solver" function in standard spreadsheet software to solve for the credit spreads at time 0 and at maturity, such that the value of both mortgages is equal to the net proceeds after the payment of points, \$100,000. This calculation involves nothing more than the solver function and inversion of a 5 × 5 matrix in a common spreadsheet. The calculated mortgage pricing, rounding to the nearest dollar, is given in Exhibit 4.

The zero-coupon mortgage credit spreads that produce these values are 1.3976% at time 0 and 1.4824% at maturity. Exhibit 5 provides the coefficients of the continuous forward credit spread function consistent with these zero-coupon credit spreads.

EXHIBIT 5
Forward Coefficients

Segment	c	d1	d2	d3	d4
1	0.0139755688	0.0000807547	0.0000000000	-0.0000000896	0.0000000015

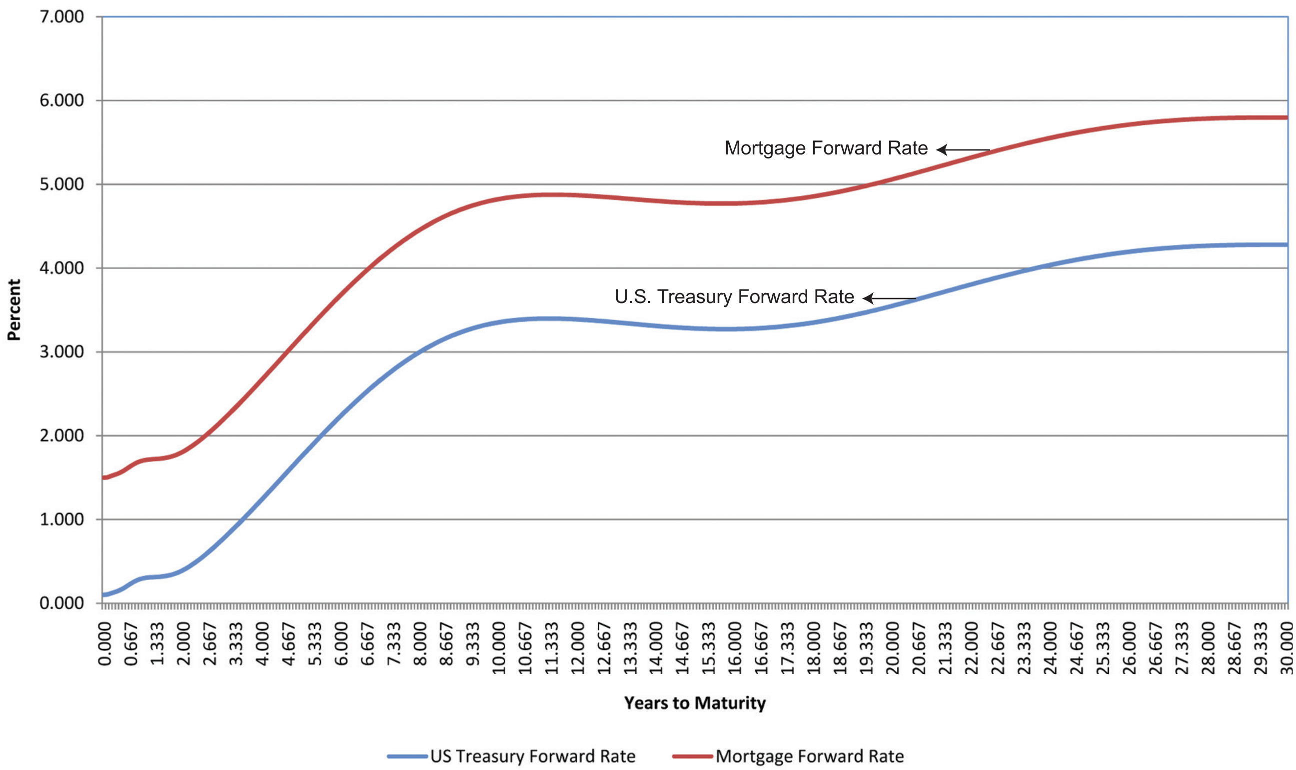
EXHIBIT 6
U.S. Treasury and Mortgage Zero-Coupon Yields Derived from the Federal Reserve H15 Statistical Release Using Maximum Smoothness Forward Rate Smoothing



Source: Kamakura Corporation and the Board of Governors of the Federal Reserve.

EXHIBIT 7

U.S. Treasury and Fixed-Rate Mortgage Forward Rates Derived from the Federal Reserve H15 Statistical Release Using Maximum Smoothness Forward Rate Smoothing



Source: Kamakura Corporation and the Board of Governors of the Federal Reserve.

The continuous forward credit spread function is nearly linear because the coefficients d_2, d_3, and d_4 are either zero or nearly zero.

The value of any coupon-bearing mortgage with the same default and prepayment risk can be obtained by using the theorem. The mortgage value is simply the monthly payment C multiplied by the appropriate mortgage zero-coupon bond prices over the bond's maturity; that is,

$$Value = C\sum_{i=1}^{m} P(t_i)$$

where m corresponds to the bond's maturity date and $P(t_i)$ correspond to the mortgage zero-coupon bond prices at the dates t_i for $i = 1, \ldots, m$.

Exhibit 6 shows the zero-coupon yields for U.S. Treasuries and the derived mortgage valuation yield curve. Because only two mortgage loans are used, the mortgage yield curve closely reflects the shape of the U.S. Treasury yield curve.

Exhibit 7 graphs the corresponding forward rate curves as well.

CONCLUSION

The standard method for computing the mortgage yield curve is a complex procedure involving the LIBOR/swaps curve. This article provides an alternative methodology for computing the mortgage yield curve that is simple and transparent. The method is based on an extension of the idea used to extract zero-coupon bond prices from coupon-bearing bond prices in both the Treasury and corporate debt markets. The extension includes both default and prepayment risk in the generation of a mortgage zero-coupon bond price curve.

Appendix

DERIVATION OF MORTGAGE LOAN ZERO-COUPON BOND RELATION

Consider a discrete time model with dates $t = 0, 1, 2, 3$. Let the market for mortgage loans be arbitrage-free, competitive, and frictionless. Let r_t be the default-free spot rate at time t.

Consider the mortgage loan with promised coupons C at time 1, C at time 2, and principal repayment of B at time 2. Let its time 0 value be denoted M. Prepayment at time 2 is equivalent to paying off the loan.

Let d and p represent the random default and prepayment times, respectively, for the mortgage loan under consideration.

Let δ_d be the recovery rate in the event of default, which can depend on the random default time.

Let β_p be the recovery rate if prepayment occurs, which can depend on the random prepayment time.

The mortgage loan's payoffs are as follows:

$$C\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\delta_d 1_{d\le 1}1_{d\le p}+\beta_p 1_{p<1}1_{p<d}\right]$$
$$+\left(B+C\right)\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\delta_d 1_{d\le 2}1_{d\le p}+\beta_p 1_{p<2}1_{p<d}\right]$$

It is always true that under no arbitrage, there exist risk-neutral probabilities such that

$$M = CP(0,1) + (C + B)P(0,2)$$

where

$E(\cdot)$ is expectation under the risk-neutral probability measure

$$P(0,1)=E\left(\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\delta_d 1_{d\le 1}1_{d\le p}+\beta_p 1_{p<1}1_{p<d}\right]e^{-r_1}\right)$$
$$P(0,2)=E\left(\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\delta_d 1_{d\le 2}1_{d\le p}+\beta_p 1_{p<2}1_{p<d}\right]\right)e^{-(r_1+r_2)}$$

In this expression, however, the zero-coupon bonds explicitly depend on the mortgage loan considered. To get a useful decomposition, one must guarantee that the zero-coupon bond prices are independent of the mortgage loans selected.

Consider a second mortgage loan with promised coupons c at time 1, c at time 2, c at time 3, and principal repayment of b at time 3. Let its time 0 value be denoted m. Let d and p represent the random default and prepayment times, respectively.

Let θ_d be the recovery rate in the event of default, which can depend on the random default time d.

Let μ_p be the recovery rate if prepayment occurs, which can depend on the random prepayment time p.

The mortgage loan's payoffs are:

$$c\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\theta_d 1_{d\le 1}1_{d\le p}+\mu_p 1_{p<1}1_{p<d}\right]$$
$$+c\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\theta_d 1_{d\le 2}1_{d\le p}+\mu_p 1_{p<2}1_{p<d}\right]$$
$$+(b+c)\left[\left(1_{3<d\le p}1_{3\le p<d}\right)+\theta_d 1_{d\le 3}1_{d\le p}+\mu_p 1_{p<3}1_{p<d}\right]$$

Again, under no arbitrage, there exist risk-neutral probabilities such that

$$m = cp(0,1) + cp(0,2) + (c + b)p(0,3)$$

where

$$p(0,1)=E\left(\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\theta_d 1_{d\le 1}1_{d\le p}+\mu_p 1_{p<1}1_{p<d}\right]e^{-r_1}\right)$$
$$p(0,2)=E\left(\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\theta_d 1_{d\le 2}1_{d\le p}+\mu_p 1_{p<2}1_{p<d}\right]e^{-(r_1+r_2)}\right)$$

The zero-coupon bonds are the same across the mortgage loans if and only if

$$P(0,t) = p(0,t) \quad \text{for} \quad t = 1,2$$

if and only if

$$E\left(\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\delta_d 1_{d\le 1}1_{d\le p}+\beta_p 1_{p<1}1_{p<d}\right]e^{-r_1}\right)$$
$$=E\left(\left[\left(1_{1<d\le p}1_{1\le p<d}\right)+\theta_d 1_{d\le 1}1_{d\le p}+\mu_p 1_{p<1}1_{p<d}\right]e^{-r_1}\right)$$

and

$$E\left(\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\delta_d 1_{d\le 2}1_{d\le p}+\beta_p 1_{p<2}1_{p<d}\right]e^{-r_2}\right)$$
$$=E\left(\left[\left(1_{2<d\le p}1_{2\le p<d}\right)+\theta_d 1_{d\le 2}1_{d\le p}+\mu_p 1_{p<2}1_{p<d}\right]e^{-r_2}\right)$$

This is true if and only if

- The mortgage loans have the same risk-neutral distribution over the default d and prepayment p times across both mortgage loans.

- The recovery rates in the event of prepayment and default on both mortgage loans are equal, that is, $\delta_d = \theta_d$, $\beta_p = \mu_p$.

This completes the proof. Note that in this case, the zero-coupon bond maturity at time 3 is unrestricted, because there are no other bonds with a payment at that date. Although the proof was done for only two mortgage loans, it is clear that it extends to a finite collection of mortgage loans with longer maturities.

REFERENCES

Adams, K., and D. van Deventer. "Fitting Yield Curves and Forward Rate Curves with Maximum Smoothness." *The Journal of Fixed Income*, 4 (1994), pp. 52-62.

Jarrow, R. "Risky Coupon Bonds as a Portfolio of Zero-Coupon Bonds." *Finance Research Letters*, 1 (2004), pp. 100-105.

Jarrow, R., and S. Turnbull. "Pricing Derivatives on Financial Securities Subject to Credit Risk." *The Journal of Finance*, 1 (1995), pp. 53-85.

Lando, D. "On Cox Processes and Credit Risky Securities." *Review of Derivatives Research*, 2 (1998), pp. 99-120.

van Deventer, D., and K. Imai, eds. *Financial Risk Analytics: A Term Structure Model Approach for Banking, Insurance, and Investment Management*. Chicago: Irwin Professional Publishing, 1997.

van Deventer, D., K. Imai, and M. Mesler, eds. *Advanced Financial Risk Management*. Singapore: John Wiley & Sons, 2004.