# Incorporating the Dynamic Link Between Mortgage and Treasury Markets in Pricing and Hedging MBS

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The mortgage-backed securities (MBS) market has increased substantially over the last few years to become one of the largest sectors of the fixed-income securities market. Prior to 2002, the MBS market was comparable in size to the U.S. Treasury market but is now substantially larger. As of September 2005, the outstanding volume of MBS pools/trusts was $5.519 trillion compared to $4.066 trillion for the Treasury market. This larger MBS market has spurred higher levels of interaction between the MBS and Treasury markets, so much so that now it is believed that the "mortgage tail is wagging the Treasury dog."

Concerns about mortgage hedging activity's impact on financial market's stability have been expressed by U.S. regulators (see, Frame & White [2004]). The primary derivative instruments for duration hedging MBS portfolios are interest-rate swaps and swaptions. The Bank for International Settlements (2003) argues that the swap market is exposed to potential loss of liquidity because of the high degree of concentration of hedging activity by MBS holders with a few swap dealers. The loss of liquidity occurs because of a reduction of market-making activities by swap dealers due to risk limits resulting from mortgage hedging activities. Swap rates can then fall below their long-run level as a result of a decline in the effective duration of MBS.

Duarte [2004] proffers three theories about the relationship between mortgage refinancing and interest-rate volatility: 1) the fixed-income market is perfect and complete without MBS and therefore any increase in prepayments due to refinancing should have no impact on interest-rate volatility; 2) dynamic hedging activity[1] of MBS investors in the swap and Treasury markets should increase the volatility of interest rates; and 3) the swaptions market is imperfect and an increase in demand for swaptions to hedge prepayment risk due to refinancing will increase the volatility implied by swaptions. The first theory suggests no impact of mortgage hedging activities on interest-rate volatility while the last two theories suggest an increase in interest-rate volatility from the use of swaps and swaptions for hedging. In fact, volatility in interest rates leads to higher hedging requirements by holders of huge MBS portfolios and higher hedging in turn, leads to higher volatility in interest rates. This can be intuitively explained by the following causal relationship. During a low interest-rate environment, prepayment risk increases resulting in shorter durations for most of the large MBS portfolios. MBS portfolio holders hedge these shorter durations by buying duration from the market. They do so by either going long Treasuries or entering into an interest swap (receiving fixed/paying floating). In either situation, Treasury market yields are pushed down further (increasing volatility) because even if they enter into a swap their counterparty would most likely hedge its exposure by going long Treasuries. As a consequence, there is a dynamic link between the MBS and Treasury markets.

Empirical support for an increase in interest-rate volatility resulting from mortgage hedging activities is provided by Perli and Sack [2003] who present evidence that hedging MBS prepayment risk amplifies movements in the 10-year swap rate (as implied by swaptions). While this can be of considerable magnitude, this amplification effect is generally expected to persist only for several months.[2] This finding is also supported by Goodman and Ho [2004] but they have different estimates from Perli and Sack of the extent of the amplification attributable to mortgage hedging activities.

There are also theories put forth that suggest that the hedging activities of major players in the mortgage market reduce the volatility of mortgage spreads (see, Gonzalez-Rivera [2001] and Naranjo & Toevs [2002]).

A recent study by Chang, McManus, and Ramagopal [2005] examines a wide range of interest rate derivatives to assess the impact of mortgage activities on interest-rate volatility. After removing the influences of the collapse of Long-Term Capital Management and the market reaction to September 11, 2001, they find that mortgage hedging activity appears to stabilize interest-rate volatility over some periods while exacerbating it in others. They conclude that there is no simple relationship between mortgage hedging and interest-rate volatility.

The fact that mortgage hedging activities can increase interest-rate volatility requires that risk managers of entities with mortgage portfolios and portfolio managers take this into consideration in developing models for hedging. Since MBS and Treasury note prices are determined simultaneously in the market, it is likely that they have an equilibrium or a long-term relationship defined as a function of time. Statistically, this process of feedback between the two markets can be described by cointegration modeling (see, Engle & Granger [1987]). This implies that if either price moves away from equilibrium, the prices would tend to revert to equilibrium. As noted earlier, empirical evidence suggests that the volatility in Treasury note and MBS prices are quite persistent, implying that they follow a Generalized Autoregressive Conditionally Heteroskedastic (GARCH) process; that is, their volatility at time $t$ is dependent on volatility at time $t-1$ and error term at time $t-1$.

Unfortunately, current MBS hedging strategies do not incorporate the above-mentioned market characteristics. A typical MBS hedging strategy involves a hedge instrument and is based on empirical duration (linear regression). The strategy assumes that the MBS and hedge instrument volatilities are constant and the MBS price is the dependent variable. However, in real markets neither assumption is true. Also, Koutmos and Pericli [1999] have shown that traditional regression-based hedge ratios are inferior to hedge ratios based on the time-varying joint distribution of GNMA MBSs and 10-year Treasury note futures contracts. As a consequence, the trading and hedging strategies of MBS and Treasury notes need to be redesigned.

In this study, we analyze pricing and hedging strategies that incorporate the dynamic link between MBS and Treasury markets and the persistence in MBS and Treasury note price volatilities. We introduce the changes in total outstanding dollar amount of in-the-money MBS market (moneyness) as a variable to capture the increased hedging requirements in a low interest-rate environment. A cointegration GARCH model that generates dynamic hedge ratios using the variance-covariance structure of all the securities included is then employed and the hedging effectiveness of the model compared to traditional regression-based hedging is then evaluated.

## SECURITIES ANALYZED

In an effort to capture the MBS market's responsiveness to changes in 10-year Treasury rates and vice-versa, we choose four 30-year MBS coupons and a 10-year Treasury note futures contract. Of the four MBS coupons selected, two are lower than the estimation period's average current coupon and two are higher. For instance, if the estimation period's average current coupon is 6.23, then the two lower coupons will be FNMA 5.5 and 6.0 to-be-announced (TBAs), and the two higher ones will be FNMA 6.5 and 7.0 TBAs. The 10-year Treasury note futures contract selected is the most actively traded contract at any given time. We define "moneyness" of the mortgage market with respect to the 30-year current coupon and the mortgage market is proxied by the two government-sponsored enterprises, Fannie Mae and Freddie Mac, since they comprise about 70% of the outstanding total.

## COINTEGRATION *GARCH* MODEL

The model assumes all five security prices are simultaneously determined by the same set of explanatory variables. The log prices of the five securities are assumed to be cointegrated, implying that any deviation from the long-term relationship is expected to be corrected. In the model, this deviation is referred to as the error-correction term. The log prices of all five securities are also expected to be affected by changes in moneyness of the mortgage market. In addition, the volatilities of these five securities are assumed to follow a GARCH(1,1) process.

The long-term relationship of the five securities is estimated with ordinary least squares regression using the following equation

$$\begin{aligned} y_t^{t-note} &= \lambda_0 + \lambda_1 y_t^{tba5.5} + \lambda_2 y_t^{tba6.0} \\ &\quad + \lambda_3 y_t^{tba6.5} + \lambda_4 y_t^{tba7.0} + Corr_t \end{aligned} \tag{1}$$

where $y_t^{security}$ is the log price of the respective security at time $t$ and $Corr_t$ is the error-correction term or the deviation of the five securities from the long-term relationship of the MBS and Treasury markets at time $t$.

The changes in the log prices of the five securities are explained by the following set of equations

$$\begin{aligned} \Delta y_t^{tba5.5} &= \alpha_t^{tba5.5} + \beta_t^{tba5.5}\left(Corr_{t-1}\right) + \gamma_t^{tba5.5}(M\,\text{var}_{t-1}) \\ &\quad + \varepsilon_t^{tba5.5} \\ \Delta y_t^{tba6.0} &= \alpha_t^{tba6.0} + \beta_t^{tba6.0}\left(Corr_{t-1}\right) + \gamma_t^{tba6.0}(M\,\text{var}_{t-1}) \\ &\quad + \varepsilon_t^{tba6.0} \\ \Delta y_t^{tba6.5} &= \alpha_t^{tba6.5} + \beta_t^{tba6.5}\left(Corr_{t-1}\right) + \gamma_t^{tba6.5}(M\,\text{var}_{t-1}) \\ &\quad + \varepsilon_t^{tba6.5} \\ \Delta y_t^{tba7.0} &= \alpha_t^{tba7.0} + \beta_t^{tba7.0}\left(Corr_{t-1}\right) + \gamma_t^{tba7.0}(M\,\text{var}_{t-1}) \\ &\quad + \varepsilon_t^{tba7.0} \\ \Delta y_t^{t-note} &= \alpha_t^{t-note} + \beta_t^{t-note}\left(Corr_{t-1}\right) + \gamma_t^{t-note}(M\,\text{var}_{t-1}) \\ &\quad + \varepsilon_t^{t-note} \end{aligned} \tag{2}$$

where $\Delta y_t^{security}$ is the change in log price of the respective security at time $t$, $Corr_{t-1}$ is the error-correction term at time $t-1$, $M\,\text{var}_{t-1}$ is the change in moneyness at time $t-1$, and $\varepsilon_t^{security}$ is the error term generated for the security at time $t$.

The error terms of equation (2) are assumed to follow a multivariate normal distribution and are given by

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^{tba5.5} & \varepsilon_t^{tba6.0} & \varepsilon_t^{tba6.5} & \varepsilon_t^{tba7.0} & \varepsilon_t^{t-note} \end{bmatrix} \tag{3}$$

Finally, the variance-covariance structure for the five securities is given by

$$H_t = A'A + B'\varepsilon_{t-1}\varepsilon'_{t-1}B + C'H_{t-1}C \tag{4}$$

where $H_t$ is the variance-covariance matrix for the variance equations of the GARCH processes for the five securities at time $t$. $A$ denotes the diagonal matrix of intercepts and $B$ and $C$ denote the diagonal matrices of slope coefficients of the variance equations.

Once the variance-covariance structure is determined, a multivariate maximum likelihood estimation[3] procedure is used to jointly estimate the parameters of the matrices $A$, $B$, and $C$ and the coefficients of the set of equations (2).

In addition, the variance-covariance matrix generated in equation (4) can be used to calculate the hedge ratios for various permutations and combinations of the securities in the system. For instance, the hedge ratio, or the proportion to be invested in FNMA 6.0 to hedge FNMA 5.5, is calculated by

$$w_{tba6.0} = -\frac{\sigma_{tba5.5,tba6.0}}{\sigma_{tba5.5}^2} \tag{5}$$

where $\sigma_{tba5.5,tba6.0}$ and $\sigma_{tba5.5}^2$ are part of the variance-covariance matrix generated by the model and are the covariance of FNMA 5.5 with FNMA 6.0 and the variance of FNMA 5.5, respectively.

## EMPIRICAL METHODOLOGY FOR HEDGING *MBS*

The daily observations on weekly returns of the 10-year Treasury note futures contract and FNMA TBAs are used for estimating the parameters of the model. Preliminary stationarity tests[4] required for cointegration are done using the bid prices of FNMA coupon TBAs and closing prices of the 10-year Treasury note futures contract, both sourced from Bloomberg. The monthly outstanding positions data for all coupons, sourced from a proprietary database of Countrywide Securities, is converted to daily observations by assuming that it changes uniformly over the whole month.

The sample period used for testing the model is January 2002 through December 2004 and is divided into two parts based on the average current coupon. In the first part, January 2002 through December 2003, the average current coupon was between 6.0 and 6.5 and in the second part, January 2004 through December 2004, it was between 5.5 and 6.0. As a result, the first part system is FNMA 5.5, 6.0, 6.5, and 7.0 and 10-year Treasury note futures contract and the second part system is FNMA 5.0, 5.5, 6.0, and 6.5 and 10-year Treasury note futures contract. Though the current coupon was low in June 2003, it could not be incorporated since the model needs at least 180 observations on the lowest coupon.

After estimating the variance-covariance structure of the five securities, we can devise many hedging strategies with different permutations and combinations of securities. For the purpose of this study, we selected three strategies that involve hedging an MBS using a 10-year Treasury note futures contract and another MBS (i.e., a cross hedge). These strategies enable us to test the model's hedging effectiveness and its ability to capture the co-movement of two MBS. The strategies are tested under daily and weekly holding-period assumptions.

The model's hedging methodology is shown in the following illustration. The first hedged portfolio is created at the beginning of the first week of January 2002 with the estimation period, defined as 180 observations before the sample period, from March 2001 through December 2001. We hedge one unit long position in FNMA 5.5 with short positions of proportions $w_{t-note}$ in the 10-year Treasury note futures contract and $w_{tba6.0}$ in FNMA 6.0 calculated using equation (5). After the first estimation, the estimation period is rolled forward one day or week depending on the holding-period assumption. Using the new estimated weights, $w_{t-note}$ and $w_{tba6.0}$, the hedge portfolio is readjusted. The procedure is repeated until we reach the end of the sample period.

The model's hedging effectiveness is measured by the standard deviation of the hedged portfolio's return over the sample period where the return is given by

$$R_{hp} = R_{tba5.5} - w_{t-note}(R_{t-note}) - w_{tba6.0}(R_{tba6.0}) \qquad (6)$$

where $R_{security}$ denotes the log return on the security.

The hedging effectiveness is tested for both assumptions for the holding period. In addition, the standard deviation from the cointegration GARCH model is compared to the standard deviation from regression-based hedging.

## EMPIRICAL METHODOLOGY FOR PRICING *MBS*

The model estimates the changes in log prices of the five securities using the set of equations given by (2) for each iteration of the estimation period, as explained above. Using the changes in the log prices we derive the price at time $t+1$ given the price at time $t$. The procedure is repeated for each daily roll forward of the estimation period. The pricing effectiveness for the five securities is tested using a Root Mean Square Error (RMSE) statistic by comparing the model price with the realized price.

## RESULTS

The model's hedging effectiveness can be seen in Exhibits 1 and 2 where the standard deviation of the hedged portfolio's return from the cointegration GARCH model is compared with that of the regression-based model. The one-day holding period results are reported in Exhibit 1; Exhibit 2 shows the results for the one-week holding period.

Based on the results reported in Exhibits 1 and 2, it appears that the cointegration GARCH model is substantially better at hedging a MBS with another MBS (cross hedge) and a 10-year Treasury note futures contract than the regression-based model. For both holding-period assumptions, the improvements are substantial in all cases, except when hedging an in-the-money coupon. For the one-day holding-period assumption (Exhibit 1), the hedged portfolio's standard deviation improves by approximately 18% to 43%, except when hedging the higher coupons. Similarly, assuming a one-week holding period (Exhibit 2), the hedged portfolio's standard deviation improves by approximately 20% to 43%, except when hedging the higher coupons.

These results seem in line with the fact that as we move through the higher end of the FNMA coupon stack,

## EXHIBIT 1

**Standard Deviation Of Hedged Portfolio's Return: Daily Rebalancing**

**a: Period Tested: January 2002–December 2003**

| Security Hedged | Securities used for Hedging | Regression-Based Model | Cointegration GARCH Model |
|---|---|---|---|
| FNMA 5.5 | FNMA 6.0 and T-note futures | 0.979 | 0.802 |
| FNMA 6.0 | FNMA 6.5 and T-note futures | 0.801 | 0.619 |
| FNMA 6.5 | FNMA 7.0 and T-note futures | 0.361 | 0.364 |

**b: Period Tested: January 2004–December 2004**

| Security Hedged | Securities used for Hedging | Regression-Based Model | Cointegration GARCH Model |
|---|---|---|---|
| FNMA 5.0 | FNMA 5.5 and T-note futures | 0.648 | 0.409 |
| FNMA 5.5 | FNMA 6.0 and T-note futures | 0.682 | 0.389 |
| FNMA 6.0 | FNMA 6.5 and T-note futures | 0.313 | 0.306 |

we move up along the so-called S-shape curve for prepayments that shows the relationship between the incentive to refinance and prepayment rates. This increases the interest-rate sensitivity of the prepayments, which in turn introduces negative convexity in MBS prices. Although we have included a higher coupon cross hedge in the hedged portfolio to account for negative convexity, it does not account for the negative convexity of the lower coupon being hedged appropriately.

The model's pricing effectiveness is measured using RMSE in percentages for the prices of the five securities in dollars. The RMSE for the four MBS and the 10-year Treasury note futures contract for the two sample periods are presented in panel a and panel b of Exhibit 3, respectively.

The minimal RMSEs shown in Exhibit 3, ranging from approximately 0%–0.4%, indicate that the cointegration GARCH model is also an effective tool to price MBS and 10-year Treasury note futures contracts.

## CONCLUSION

Based on the results reported, it seems that the cointegration GARCH model presented in this article successfully captures the dynamic link between the MBS and Treasury markets. The success of the model can be attributed to three factors. The first factor is the inclusion of moneyness of the whole mortgage market as an explanatory variable which appears to capture the hedging requirements of the MBS portfolio holders in a low interest rate environment. The second factor is the generation of a dynamic variance-covariance matrix which results in dynamic hedge ratios. The last factor is the use of a GARCH process to capture the persistence of volatilities in MBS and Treasury note futures prices. The model's effectiveness can be measured by the marked reduction in the standard deviations of the hedged portfolios' returns when compared with a traditional regression-based technique. In addition, the results of the study suggest that in order to hedge higher coupons that exhibit negative convexity, it is not sufficient to use Treasury note futures contracts or a higher coupon MBS. Though one would not expect a model like this to perform well in a high interest rate environment, it still needs to be tested whether the hypothesis carries any weight when market conditions allow such testing.

## ENDNOTES

[1]Dynamic hedging activity is one in which the holder of mortgage-related securities rebalances its portfolio as interest rates change. In contrast, a static hedging strategy used by the

## EXHIBIT 2

**Standard Deviation Of Hedged Portfolio's Return: Weekly Rebalancing**

**a: Period Tested: January 2002–December 2003**

| Security Hedged | Securities used for Hedging | Regression-Based Model | Cointegration GARCH model |
|---|---|---|---|
| FNMA 5.5 | FNMA 6.0 and T-note futures | 0.979 | 0.773 |
| FNMA 6.0 | FNMA 6.5 and T-note futures | 0.800 | 0.636 |
| FNMA 6.5 | FNMA 7.0 and T-note futures | 0.363 | 0.357 |

**b: Period Tested: January 2004–December 2004**

| Security Hedged | Securities Used for Hedging | Regression Based Model | Cointegration GARCH Model |
|---|---|---|---|
| FNMA 5.0 | FNMA 5.5 and T-note future | 0.650 | 0.449 |
| FNMA 5.5 | FNMA 6.0 and T-note future | 0.681 | 0.389 |
| FNMA 6.0 | FNMA 6.5 and T-note future | 0.314 | 0.294 |

two major holders of mortgage-related securities, Fannie Mae and Freddie Mac, is to issue callable debt or synthetic callable debt.

[2]Perli and Sack argue that by assuming that hedging flows are determined by the duration, convexity, or actual refinancing activity taking place in the market, the amplification factor can be empirically estimated using a GARCH model for the volatility of interest rates.

[3]The negative log likelihood function for a multivariate normal is given by

$$L = \frac{1}{2}\log\left(\det\left[H_t\right]\right) + \frac{1}{2}\varepsilon_t H_t^{-1}\varepsilon_t' + \frac{N}{2}\log(2\pi)$$

where $N$ is the number of observations.

[4]The price series used should be integrated to the order of one.

## EXHIBIT 3

**Root Mean Squared Error in Percentages for the Prices of the Five Securities (in dollars)**

**a: Period Tested: January 2002–December 2003**

| | T-Note Future | FNMA 5.5 | FNMA 6.0 | FNMA 6.5 | FNMA 7.0 |
|---|---|---|---|---|---|
| **RMSE** | 0.358 | 0.155 | 0.073 | 0.041 | 0.099 |

**b: Period Tested: January 2004–December 2004**

| | T-Note Future | FNMA 5.0 | FNMA 5.5 | FNMA 6.0 | FNMA 6.5 |
|---|---|---|---|---|---|
| **RMSE** | 0.303 | 0.225 | 0.105 | 0.060 | 0.026 |

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