

On convergence of MOATS Mortgage Rate Model

by

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Abstract

A number of mortgage prepayment models require a specification of the mortgage rate process. In 2006, Citigroup published its MOATS model where the prepayment model was based on the endogenously defined mortgage rate process. MOATS was based on the “End of Finance” (EOF) assumption, which stated that the term for a new mortgage is 30 years or the rest of time until EOF (assumed to be in sixty years ”from now”), whatever is less. This article shows that MOATS gives a sufficient approximation to the “real” endogenous mortgage rate (i.e., without the EOF assumption). Next, we show a number of techniques to significantly reduce the high computational price of MOATS (which is very high even for one stochastic factor). Some of these techniques require practically no coding effort if the original MOATS is already implemented. The result is based on our simple and transparent generalized formulation of the MOATS model which is not tied to a specific numerical method. In particular, in our model set-up Monte Carlo methods can be used as well.

Mortgage prepayment models try to predict the borrowers' prepayment behavior based on information available to the market. There is still no agreement on the "correct" prepayment model. The major trend on Wall Street is to model the prepayment behavior empirically. For example, the prepayment rates are regressed against some explanatory variables, and the 10-year Treasury yield (or similar factor) is often considered as one of the most influential predictors.

If we ask "why 10-year Treasury"? then the most common answer would be that this rate closely tracks the 30-year fixed-rate mortgage rate. So, in fact, the empirical models use some benchmark for the mortgage rate. Why not to use a real (implied by the model) mortgage rate instead of the benchmark? One of the common comments here is that why bother solving a complicated problem (finding the endogenous mortgage rate) if we can define the prepayment model based *directly* on the interest rate (e.g., 10-yr Treasury yield) and calibrate the prepayment model accordingly (exactly what many prepayment modelers do), thus, completely avoiding the problem of the mortgage rate computation. The answer is similar to the answer to "why bother to model the prepayment if we could regress the mortgage prices based on history directly". By creating an adequate model we get a reliable

predictive ability in out-of-sample situations. If a market situation is changed, then an empirical model requires re-calibration and old data might not be very helpful. Models are created to deal with these problems. Assume we have correctly separated borrower's and market components of prepayment, i.e., we have represented (modeled) the prepayment as a function (the borrower's component, describes a human behavior) of the refinancing incentive (the market component, driven by the efficient market). The borrower's component is a "low-frequency", i.e., we do not expect people's behavior to change on a short time-scale.¹ The market component, on contrary, is "high-frequency" and might experience a dramatic change in a short period of time. A change in the market would lead to the change in the market component but not in the borrower's component, thus making the model robust: the change can be handled using standard fixed-income techniques.

This idea was implemented in the mortgage-rate based (or MRB for short, see [3]) approach to prepayment modeling. It assumes that the borrowers' refinancing decision is based mainly on the compari-

¹There might be significant "revolutionary" changes as the introduction of Internet to the public, but still the change is smooth and not as dramatic as changes on market.

son of the *contract* and *current* (available for refinancing) mortgage rates (the Internet is abundant with “calculators”, which based on this comparison, tell one how much one “saves” if he/she refinances). In this case, the refinancing incentive (the market component) is assumed to be a comparison (e.g., difference) of mortgage rates (which are driven by the market). If this approach to model the refinancing incentive is taken, then the investor needs to model the *mortgage rate* to be able to model the prepayment process in order to price or hedge mortgage-backed securities. This mortgage rate model needs to be in agreement with the mortgage rates implied by the resulting prepayment behavior and the underlying interest rate model (we call such mortgage rate models *endogenous*).

The first general endogenous mortgage rate model under sub-optimal prepayment assumption was developed by Goncharov in [3] and [4], where the endogenous mortgage rate is formulated as a fixed-point of a functional operator. Pliska [6] investigated the problem in a discrete time setting. Goncharov, Okten, and Shah [5] applied a randomized quasi-Monte Carlo method for the mortgage rate computation.

Independently, Citigroup developed its mortgage rate model (called MOATS) which was published by Bhattacharjee and Hayre [1]. The

model formulation is based on a tree technique: the authors illustrate the model with a help of an example which considers what is happening at each node in a binomial tree.² In this paper we present a concise and generalized formulation of MOATS model. This lets us have a certain insight which helps to deal with a number of MOATS problems. In particular, in our set-up the computation of the expectations is not restricted to any specific method and Monte Carlo can be applied as well.

In this paper we show that MOATS mortgage rates converges to the endogenous mortgage rate function (see [4], [3], and [5] for details) as EOF day goes to infinity. Therefore MOATS can be viewed as a certain approximation technique applied to the fixed-point mortgage rate problem formulated in Goncharov [3], [4].³ MOATS [1] chose the EOF day to be 60 years in future. As we show, this choice gives

²From the example it seems that MOATS gives the rate for an interest rate only mortgage; regrettably, it is not clear if the real MOATS implementation uses this assumption or not. As we show, the standard mortgage rate behavior is quite different from the interest-only one.

³The fixed-point mortgage rate equation is formulated for a time-homogenous processes there. However, this does not reduce generality since the time itself might be included as a factor to make a non-time-homogenous process time-homogenous.

a satisfactory result for most applications, but might be insufficient for valuation of such complicated securities as Z-bonds, for which the projected cash flow is distributed in the “far future”. Next, we show a number of ways to make MOATS more computationally efficient. Some of them require virtually no coding effort (thus, those who have implemented MOATS might reduce its computational time without almost any change of their implementation), some require a code overhaul of various degrees.

Endogenous mortgage rate

We assume that our mortgage model is driven by a Markovian process X_t and the prepayment time is modeled as a first jump of a generalized Poisson process (so called Cox process) with an intensity process γ_t (for our purpose it is safe to think about γ_t as a prepayment rate model) which is adapted to a natural filtration of the state process X_t (i.e., knowledge of X_t is sufficient to determine the value of γ_t). In the definition of our prepayment rate model we emphasize the dependence on the contract and current mortgage rates as the most influential prepayment predictors (borrowers compare their mortgage rate with the rate available for refinancing): $\gamma_t = \gamma_t(m^{t_0}, m^t)$, where m^t is the

mortgage rate at time t , t^0 is the time of the mortgage origination, and in the notation $\gamma_t(\cdot, \cdot)$ we hide the other possible dependencies such as the time since the origination of the mortgage (e.g., to account for seasoning), house prices, etc.

Goncharov showed in [3, 4] that the mortgage rate process $m^t = m(X_t, t)$ must satisfy the following non-linear functional equation:

$$m(x, t) = \frac{\mathbf{E} \left[\int_t^{t+T} r_u P(u - t, m(x, t)) e^{-\int_t^{t+u} (\gamma_\theta(m(x, t), m(\theta, X_\theta)) + r_\theta) d\theta} du \mid X_t = x \right]}{\mathbf{E} \left[\int_t^{t+T} P(u - t, m(x, t)) e^{-\int_t^{t+u} (\gamma_\theta(m(x, t), m(\theta, X_\theta)) + r_\theta) d\theta} du \mid X_t = x \right]}, \quad (1)$$

where $r_t = r(X_t, t)$ is the instantaneous risk-free interest rate and the function $P(t, m)$ is the outstanding principal of an m -rate mortgage after time t since the origination.

Let us define the operator $\mathcal{A}[f(\cdot, \cdot)](x, t; T)$ which returns an implied mortgage rate for a T -year mortgage originated at time t conditioned that the “true” mortgage rate model for time $\theta > t$ is known and given by $f(\theta, X_\theta)$. That is, we define the “implied mortgage rate operator” $\mathcal{A}[f(\cdot, \cdot)](x, t, T) := \tilde{m}(x)$, where $\tilde{m}(x)$ is a solution to the

following equation for fixed t and T :

$$\tilde{m}(x) := \frac{\mathbf{E} \left[\int_t^{t+T} r_u P(u-t, \tilde{m}(x)) e^{-\int_t^{t+u} (\gamma_\theta(\tilde{m}(x), f(\theta, X_\theta)) + r_\theta) d\theta} du \mid X_t = x \right]}{\mathbf{E} \left[\int_t^{t+T} P(u-t, \tilde{m}(x)) e^{-\int_t^{t+u} (\gamma_\theta(\tilde{m}(x), f(\theta, X_\theta)) + r_\theta) d\theta} du \mid X_t = x \right]}, \quad (2)$$

Given this operator, the endogenous mortgage rate for a T -year mortgage must satisfy $\mathcal{A}[m(\cdot, \cdot)](x, t, T) = m(x, t)$.

Generalized MOATS

MOATS introduces a horizon time T_H . If a mortgage originated at a time t which is too close to this time (namely $T_H - T < t < T_H$) then MOATS assumes that the only mortgage term which is available is the time left to T_H . Otherwise, a standard $T = 30$ year mortgage is available for refinancing. In other words, the MOATS assumption is that the market does not exist beyond T_H (that is the reason for EOF name for this day).

Next, let us assume we measure time in Δ units in our model. For example the popular choice for Δ can be one month, in this case for a standard 30-year mortgage we would have $T = 360$. This Δ time unit can be viewed as a time-step in the time-discretization of the

expectations in the model. We are looking for an approximation of the mortgage rate in the form⁴

$$\hat{m}(x, t) = \sum_{s=1}^{T_H} m_s(x) \mathbf{I}_{\{s-1 < t \leq s\}}.$$

Given the notation above, MOATS can be generalized as a two-step algorithm:

1. for t from $T_H - 1$ to $T_H - T$ do

$$\hat{m}(x, t) := \mathcal{A}[\hat{m}(\cdot, \cdot)](x, t; T_H - t)$$

2. for t from $T_H - T - 1$ to 0 do

$$\hat{m}(x, t) := \mathcal{A}[\hat{m}(\cdot, \cdot)](x, t; T)$$

⁴We do not reformulate the problem in a discrete-time setting to avoid unnecessary for our purpose technicalities and additional notations. Given the Δ -time discretization, the computation of the expectations in the model will not (“usually”) depend on the way $m(x, t)$ is interpolated between t and $t + 1$. This particular continuous form for $\hat{m}(x, t)$ avoids necessity to refer to the set of $\{m_s(x)\}_{s=t}^{T_H}$ every time we use the mortgage rate in formulae later. Advanced high-order techniques to treat the problem in time are beyond the scope of this paper.

Notice that the operator $\mathcal{A}[\hat{m}(\cdot, \cdot)](x, t; \cdot)$ in the *formulas* in this procedure depends on $\hat{m}(\cdot, s)$ for $s > t$ only and does not depend on $\hat{m}(x, t)$. That is, the mortgage rates are found consequently for time t from T_H to 0, where a solution for time t depends on all “previous” solutions for later times $t + 1, \dots, T_H$.

As one can see, the difference between these steps is the mortgage term. In the first step the term is growing as we are stepping back (an auxiliary problem solving for hypothetical mortgages) and in the second step the term stays constant T (as it should be for real mortgages).

Let us make several observations about the computation of the operator \mathcal{A} . Firstly, \mathcal{A} is defined as a fixed point of (2) for each x . Secondly, the computation of expectations in (2) for two different x and y -values (even for the same time t) should be done separately because different behavior of f -function (as implied by X_t given two initial values x and y) implies different behavior of γ -function (prepayment). Additionally, we have different (implied by x and y) \tilde{m} -values (the contract mortgage rates) which makes the expectations different as well (through the outstanding principal and prepayment function). Thirdly, if the underlying state variables are not time-homogenous,

the results will be different for different times. But even in the case of time-homogeneity, MOATS will produce a time-inhomogeneous term structure of mortgage rates.

Taking into account these observations, MOATS computes the implied mortgage rates operator \mathcal{A} using a binomial tree for *each* value of x and for *each* value of t . At each node an iteration procedure is employed to solve (2). The high price of MOATS computation is clear.

Note that in our description of MOATS, we did not specify a numerical technique for evaluation of the expectations in the operator \mathcal{A} . This is can be tree techniques, PDE, or Monte Carlo methods (see [5] for latter).

One simple model specification

We consider a one-factor CIR interest rate model⁵. That is, the interest rate is given by a solution of the following time-homogenous stochastic differential equation

$$dr_t = \alpha(\mu - r_t)ds + \sigma\sqrt{r_t}dW_t \quad (3)$$

⁵Other interest rate models produce the same qualitative picture. In particular we test Vasicek and Black-Karasinski models

and where W_t is a Brownian motion. We consider the following parameters for the CIR interest rate model: $\alpha = 0.3$, $\mu = 0.07$, and $\sigma = 0.115$.

The question of approximating the expectations is beyond the scope of this paper. For this paper we used a Crank-Nicholson finite-difference scheme for associated PDEs⁶ (instead of a binomial tree used by Citigroup). For application of Monte Carlo methods in this context, see [5].

We assume the following special case of the prepayment intensity function:

$$\gamma_t = \begin{cases} \gamma_1, & \text{if } m^0 > m^t + \delta \\ \gamma_0, & \text{if } m^0 \leq m^t + \delta, \end{cases} \quad (4)$$

where γ_0 , γ_1 , and δ are constants and represent the intensity of prepayment due to exogenous reasons, the intensity of prepayment in the case when it is financially justifiable, and the transaction costs of refinancing in terms of mortgage percentage. This function is look-alike of an empirically known S-shaped function.

We assume $\gamma_0 = 0$, $\gamma_1 = 0.65$, and $\delta = 1\%$. This choice implies that borrowers check their refinancing opportunities in stochastic time intervals with an average “waiting” time of $1/\gamma_1 \approx 1.5$ years. They

⁶We refer a reader to [2] for the explicit formulation of these PDEs

require about 10% of decrease on their monthly mortgage payments (if we compute these “savings” implied by 1% mortgage rate decrease) to refinance and they do not prepay for exogenous reasons.

The reason for considering a time-homogenous case in this paper is that we can study the convergence of MOATS procedure without the “contamination” of the effect of time-varying parameters. In this case a solution to the equation for the endogenous mortgage rate does not depend on time and can be computed with the method⁷ proposed in [3], [4], and [5] which utilizes the monotonicity of the mortgage rate to boost the computational efficiency. The time-homogeneity transforms the MOATS and real mortgage rate term structures but does not change the qualitative relation between them, i.e., all the conclusions stay the same.

MOATS convergence.

The Exhibition 1 shows the MOATS term structure for a standard 30-year fixed rate mortgage. The Exhibition 2 on shows the endoge-

⁷The method can be applied to time-inhomogeneous case as well. For this, the time should be treated as a parameter.

nous mortgage rate term structure, i.e., the true solution in our time-homogenous set-up (no changes with respect to time).

To view the dynamics of the convergence of mortgage rate functions, let us look at the r - and t -traces of the MOATS graph. On Exhibition 3 we show graphs of the mortgage rate (as given by MOATS) as functions of interest rate for various times. The Exhibition 4 shows the mortgage rate for a 30-year fixed-rate *interest-only* mortgage⁸. As we can see, MOATS converges to a certain function. In fact, this function is virtually indistinguishable from the solution of the endogenous mortgage rate equation obtained in [3, 4] (which can be regarded as a true endogenous mortgage). Another useful observation is that the behavior of interest-only mortgage rate is *very* different (with more accented non-linearity which is located in a different interest rate region) and, therefore, should not be used instead of the standard amortized mortgage rate.

Exhibition 5 shows MOATS mortgage rates as functions of time for various interest rate values. We see a steep change in the mortgage values initially (to be expected since the influence of the mortgage

⁸MOATS does not state it explicitly, but the example in the paper implies that MOATS computes an interest-only mortgage term structure

term is significant in relative terms). It is interesting to note the sharp change in the behavior after 30 years, that is the time when MOATS switches from the first (auxiliary) to the second (fixed 30-year terms) step. The region where the curves depart from each other corresponds to the formation of the “jump.” We can see that on the interval $(0, 15)$ -years the curves are “flat”, i.e. changes are negligible and the convergence is “achieved” after stepping 45 years in MOATS procedure. Therefore, MOATS get an accurate approximation of the mortgage rate term structure for the first 15 years. However, after 15 years the mortgage rates depart from their true values and the closer to 30 years, the greater is this misspecification. The prices of mortgage pass-through securities will not be significantly affected by these long-term discrepancies⁹, but some other sophisticated securities (like Z-bonds) might be mispriced. Therefore, the safe choice for EOF date should be $T_H = 75$ years (instead of MOATS’s choice of 60 years) in this case if complex CMO’s tranches are to be priced.

In practice, the convergence depends on the particular model specification and should be analyzed by the end user. For example, the

⁹It is indicated by the “achieved” convergence for the initial 15 years because the mortgage rate equation is, fact, pricing the mortgage “at par”.

inclusion of a “burn-out” has a smoothing effect (as shown in [2]) and a desired tolerance might be achieved for smaller T_H .

Speeding-up “MOATS”

The misspecification of the mortgage rates for (15, 30)-year time interval is caused by the mortgage rate values for, say, 33 years are based on mortgage rates which are far from desired true rates (see the mortgage rates over (3, 33)-time period). This distortion wears off only after going 45 years backward from T_H (as we noticed before) in MOATS procedure. The large misspecification of mortgage rates produced in the MOATS’s auxiliary first step shows that the computations for these first time-steps are not very useful. How can we improve the efficiency?

What if we skip the first step altogether and use a 10-yr Treasury rule of thumb instead? It is hard to be worse than the auxiliary MOATS step and Treasury yield computation is practically free (compared to mortgage rate computation).¹⁰ Let us modify the prepayment incentive the following way: we use 10-yr Treasury yield

¹⁰The computation of 10-yr Treasury yields is very cheap compared to the computation of the endogenous mortgage rates (values of the yield for one time-step is computed at

benchmark instead of the mortgage rates if the prepayment model is called for time interval $(T_H - T, T_H)$ (initialization interval), i.e., the way it is commonly done in the financial industry today. However, as soon as we need the prepayment incentive at any other time, we switch to the mortgage rate. Given this prepayment incentive modification, the need for the auxiliary MOATS step disappears and we can proceed with the mortgage rate computation directly (without EOF assumption).

The Exhibition 6 illustrates the performance improvement of this modification. For this graph we chose $T_H = 90$ but graphed on $(0, 60)$ time interval to make it comparable to the Exhibition in a sense of computational price.¹¹ A satisfactory convergence is achieved after only 15-20 years of backward computations and we can see formation of the “jump” very early. Therefore, with this modification we can pick T_H to be 80 and perform computations over 50 year interval to get “true” mortgage rate term structure throughout the desired

once, i.e., there is no need to repeat computation for all level of the underlying factor).

Moreover, a closed form solutions are available for many popular models.

¹¹Since the computation load is on $(0,60)$ -time interval, and over $(60,90)$ -time interval we need to know only 10-yr Treasury yield, which is very simple to compute.

(0,30)-time interval (as opposed to MOATS required 75-years long time interval to get the same result).

An endogenous way to boost the efficiency is to update the mortgage rates as soon as new values are computed in the first MOATS step (why use the mortgage rate of an artificial 11-year mortgage if we have computed the mortgage rate for a more realistic 15-year mortgage?) That is given mortgage rates over the first $(n - 1)$ time-steps, the mortgage rate for the n^{th} time-step is used to update all the previous mortgage rate values. The update can take into account (in an ad-hoc fashion) the time change of the interest rate parameters to make the mortgage rates closer to the true ones (tracking, for example, the 10-yr Treasury yield and its relation to the mortgage rate). In our *time-homogenous* example the update is simple: we can simply “overwrite” all the computed “future” mortgage rates with the “current” n^{th} one in the first MOATS’ step. That is in this case the auxiliary MOATS step is

1. for t from $T_H - 1$ to $T_H - T$ do

$$\hat{m}(x, t) := \mathcal{A}[\hat{m}(\cdot, \cdot)](x, t; T_H - t)$$

$$\hat{m}(x, s) := \hat{m}(x, t) \text{ for all } s > t.^{12}$$

The result is shown on Exhibition 7. We can see that this technique gives the convergence after 30 years (as opposed to 45 years in pure MOATS).

Note, that for the first MOATS's step the time-homogeneity is not an unreasonable assumption in practice since the situation when the market expects/predicts a noticeable change in the market environment from, say, 32 to 35 years (i.e., certain time-varying parameters in an interest rate model) looks suspicious. That is, if $\theta(t; t_0)$ is a vector of parameters for an interest rate model, where t_0 is a date of calibration and t is the time parameter in a model ($t > t_0$), one should not expect $\theta(t, t_0)$ to vary for large values of t on the same (or large) order then for small values of t for the same t_0 .¹³ Information “to-day” has more input for near future and very scarce input for distant future. Additionally, the effect of possible changes (of parameters of the state process) for long times is “worn out” for our solution even

¹²In code there is no need to actually overwrite it. Reference to the $(t - 1)$ -time-step result when computing t -time-step does the trick.

¹³That is, it is not to say that, for example, $\theta(35, t_0)$ will not have noticeable variation in t_0 . The market might have sharp changes in its perception about the future behavior even for the distant future given shocks to the economy.

by gross misspecification as illustrated, for example, by Exhibition 5. Thus, even in the presence of time-varying parameters in the state process, it is better to base the solution of the second MOATS step on the “wrong” (due to time-varying parameters) stationary solution of the step one, then on the grossly misspecified solution of an auxiliary problem of the first MOATS step (see the left graph on Exhibition 5 on the time interval $(30, 60)$ -years). Note, that the time stationary implementation of a “mortgage rate update” on the first step requires no effort in terms of coding if MOATS is already implemented.

If a modeler agrees that the time-homogenous assumption is adequate, then the best streamline of the MOATS auxiliary step is its change to a simple endogenous mortgage rate computation as in [?, 5], where there is no need of computation of the operator \mathcal{A} on every time-step at all. The mortgage rate computation can formally be written as (assuming T_H is the time from which the time-homogeneity is an adequate assumption)

1. Find $\hat{m}_{T_H}(x)$ such that

$$\hat{m}_{T_H}(x) := \mathcal{A}[\hat{m}_{T_H}(\cdot)](x, T_H; T)$$

$$\hat{m}(x, t) = \sum_{s=1}^{T_H-1} m_s(x) \mathbf{I}_{\{s-1 < t \leq s\}} + m_{T_H}(x) \mathbf{I}_{\{T_H-1 < t < \infty\}};$$

2. for t from $T_H - T - 1$ to 0 do

$$\hat{m}(x, t) := \mathcal{A}[\hat{m}(\cdot, \cdot)](x, t; T)$$

In this case we use this “auxiliary” (time-homogenous) solution for the second MOATS step, where the change due to inhomogeneity is accounted for. The second step can be shorter than the MOATS’ choice of 30 years (e.g., in the case of time-homogeneous parameters it is zero!) and, in general, its length depends on the degree of time dependence in the parameters.

An even more efficient approach to compute the endogenous mortgage rate process with time-varying parameters of the underlying factor process is to consider the problem as in [3, 4] and treat time as an additional parameter. The state variable tend to be asymptotically time-homogeneous (thus, mortgage rate behavior does not change much for large time) and, therefore, there is no reason to keep a uniform time-step for mortgage rates. This time parameter might be treated monotonically from T_H to 0 (as in MOATS), but time-discretization might be chosen to be adaptive (wider time-steps

for longer times) because A smooth dependence of a solution on the parameters (see [5] for analysis of the mortgage rate behavior with respect to varying parameters in the model) tells us that we do not need to solve a full-scale mortgage rate problem on every time-step. It is sufficient to solve the mortgage rate problem on a wider (then used in the estimations of expectations) mesh and intermediate mortgage rate values (which are needed in the expectations) can be interpolated. If one desires, to check if the time-step is too large (or interpolation is inadequate), then after performing a time-step, one might solve the problem on a half-time-step problem to check how it agrees with the interpolation. To make the computations efficient and to deal with the dimensionality plague, for every fixed time the fixed-point equation should be solved with the new technique described in [2].

Conclusions. Q&A

- Does the limit $T_H \rightarrow \infty$ produces a convergence to the real mortgage rates, i.e., without EOF assumption.

Answer: yes.

- How the EOF assumption affects the mortgage rates? Does the Citigroup’s choice of $T_H = 60$ years gives a satisfactory approximation to the real mortgage rates?

Answer: yes, but one should be careful using this choice for mortgage securities for which the long dated cash flow is prevailing (e.g., Z-bonds) because MOATS does not approximate well the term structure of mortgage rates for times longer than 15 years.¹⁴

- What makes the MOATS so computationally expensive? Can we compute the mortgage rates more efficiently?

Answer:

1. The first auxiliary step. It takes roughly the same computational time as the second MOATS step but produce a result which is inferior to the “10-yr. Treasury yield” rule of thumb. As we showed in this paper, this step is rudimentary and can be handled in various ways. For example, in

¹⁴The answer, of course, depends on the specifications used in the model. We leave it to Citigroup to test if the change of T_H from 60 to 75 (or more) years gives significant changes for such securities under their specifications.

one time-step (versus $T_H - T$ time-steps in MOATS) if one treats the problem as time-homogenous (see [4, 5]).

2. The repetition of the mortgage rate computations for each time-step in the time-mesh used in the mortgage valuation. Generally, an interpolation over adaptive time mesh must be used.
3. The repetition of the mortgage rate computations for each state factor value. This basically forbid usage of high-dimensional state process (because the number of expectations growth exponentially with the number of factors) which are must for mortgage securities. The cure of this curse is to use new techniques proposed by Goncharov in [2].

- Is it possible to use Monte Carlo for the mortgage rate computation?

Answer: yes, the procedures in this paper can be repeated with Monte Carlo methods for the operator \mathcal{A} computation. For the application of the correlated sampling Monte Carlo to the mortgage rate evaluation see [5].

References

- [1] Bhattacharjee, R., and L.S. Hayre, *The Term Structure of Mortgage Rates: Citigroup's MOATS Model*, Journal of Fixed Income, March 2006.
- [2] Goncharov, Y., *Computating the Endogenous Mortgage Rate without Iterations*, to appear, Quantitative Finance, 2008.
- [3] Goncharov, Y., *Mathematical Theory of Mortgage Modeling*, Ph.D. dissertation, University of Illinois at Chicago, 2003.
- [4] Goncharov, Y., *An Intensity-Based Approach to the Valuation of Mortgage Contracts and Computation of the Endogenous Mortgage Rate*, International Journal of Theoretical and Applied Finance, Vol. 9, pp. 889-914, 2006.
- [5] Goncharov, Y., Okten, G., and M. Shah, *Computation of the Endogenous Mortgage Rates with Randomized Quasi-Monte Carlo Simulations*, Mathematical and Computer Modelling, in press, available on elsevier.com.
- [6] Pliska, S., *Mortgage Valuation and Optimal Refinancing*, Proceedings of Stochastic Finance 2004, Lisbon, Portugal (Springer Verlag).

**Exhibit 1. The entire MOATS mortgage rate term
structure of mortgage rates**

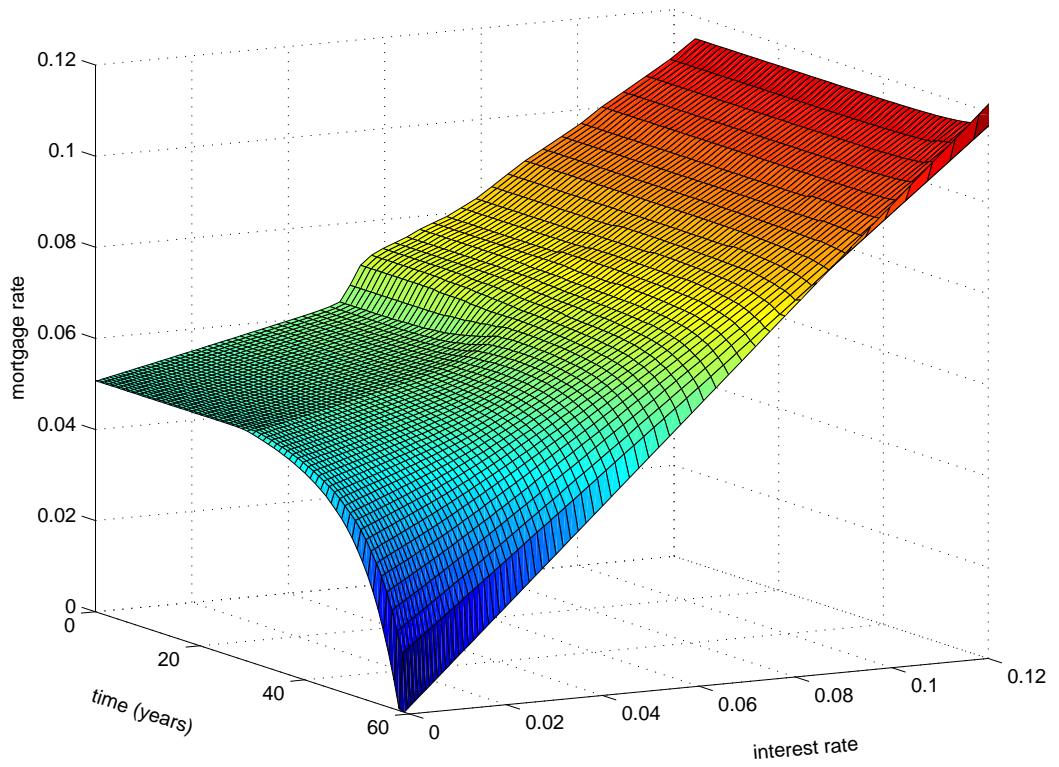
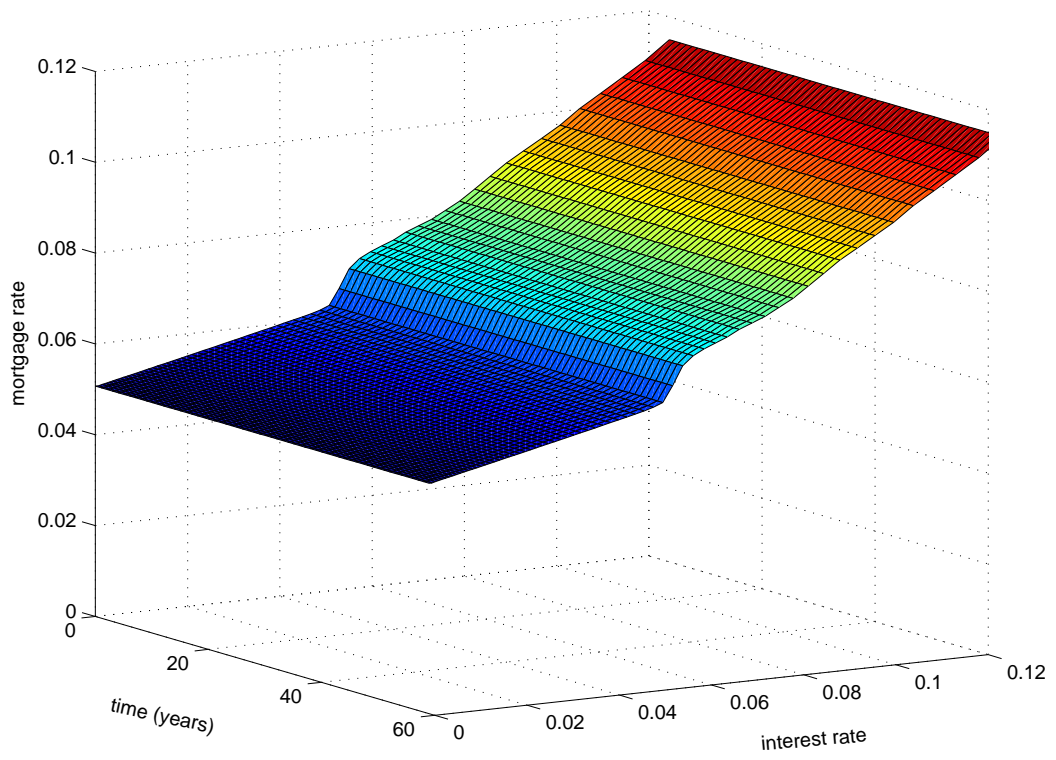
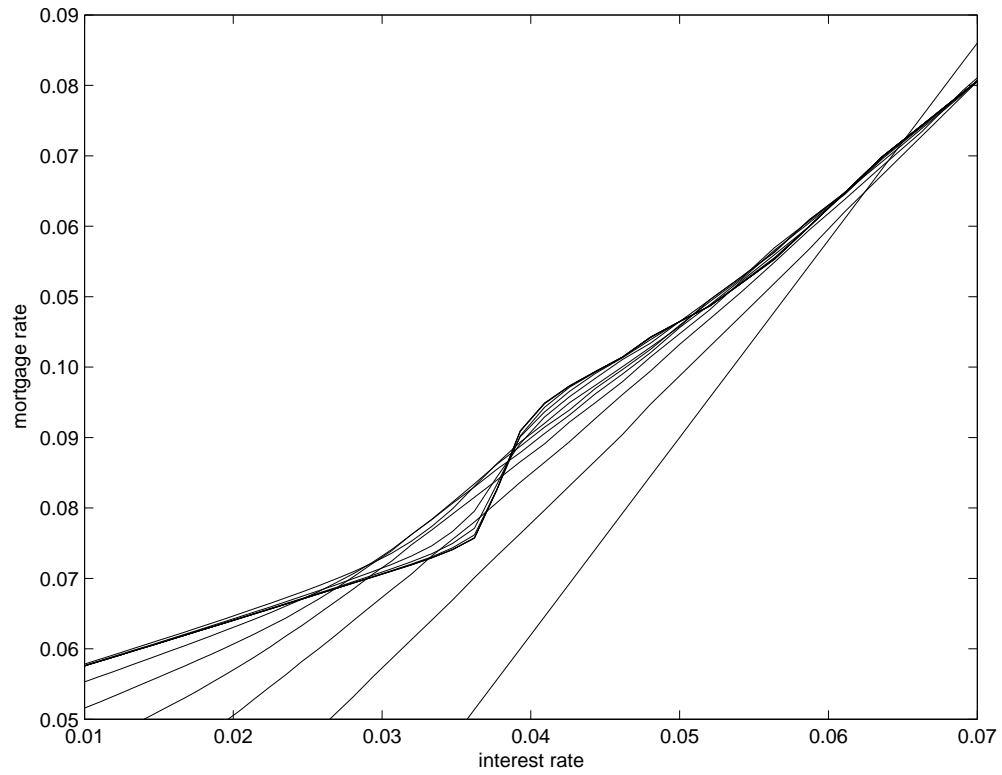


Exhibit 2. The entire endogenous mortgage rate term structure



**Exhibit 3. The mortgage rate as a function of the
interest rate for various steps of MOATS procedure**



**Exhibit 4. The mortgage rate as a function of the
interest rate for various steps of MOATS procedure for
interest-only mortgage**

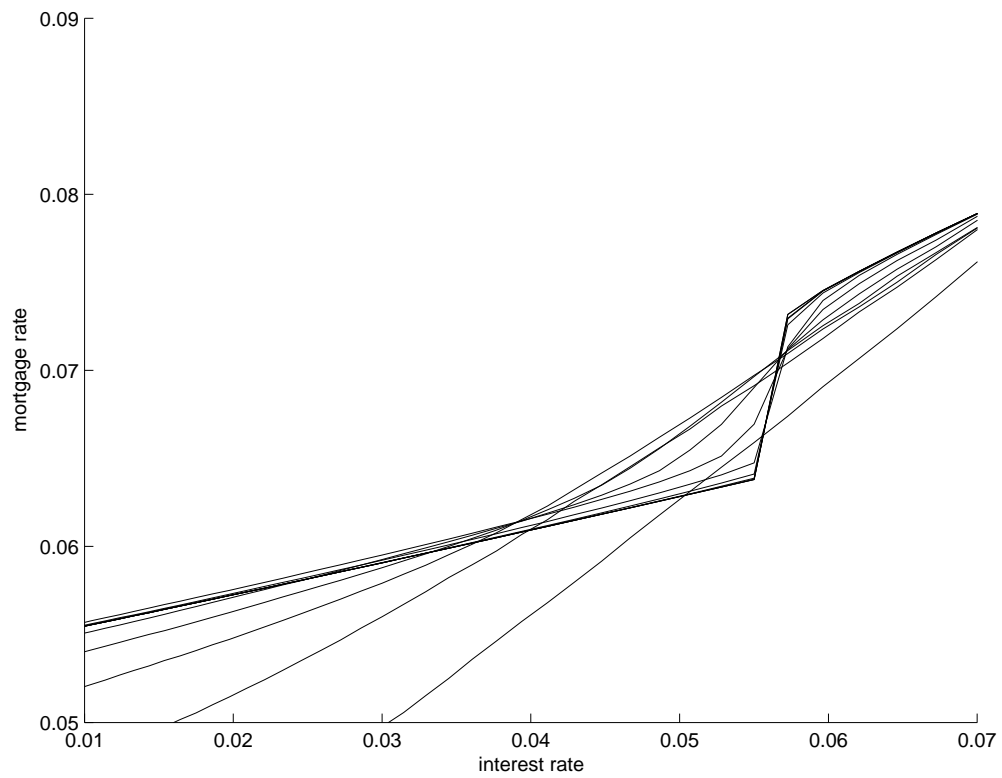


Exhibit 5. t -traces of the MOATS mortgage rate term

structure

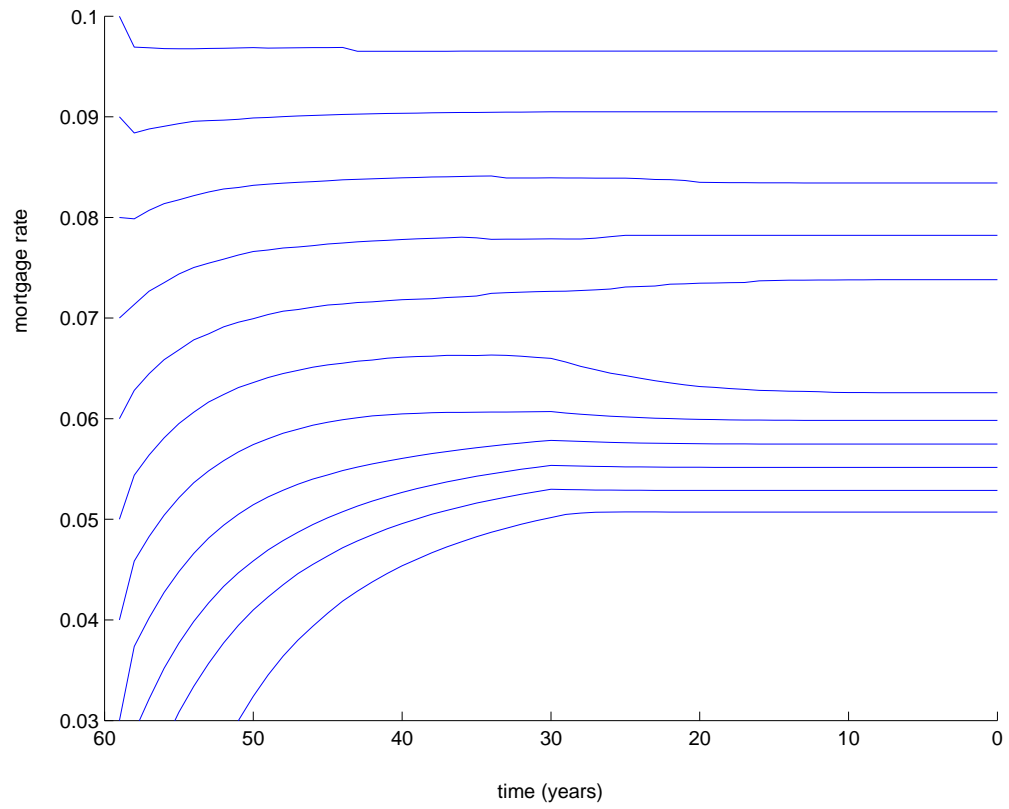
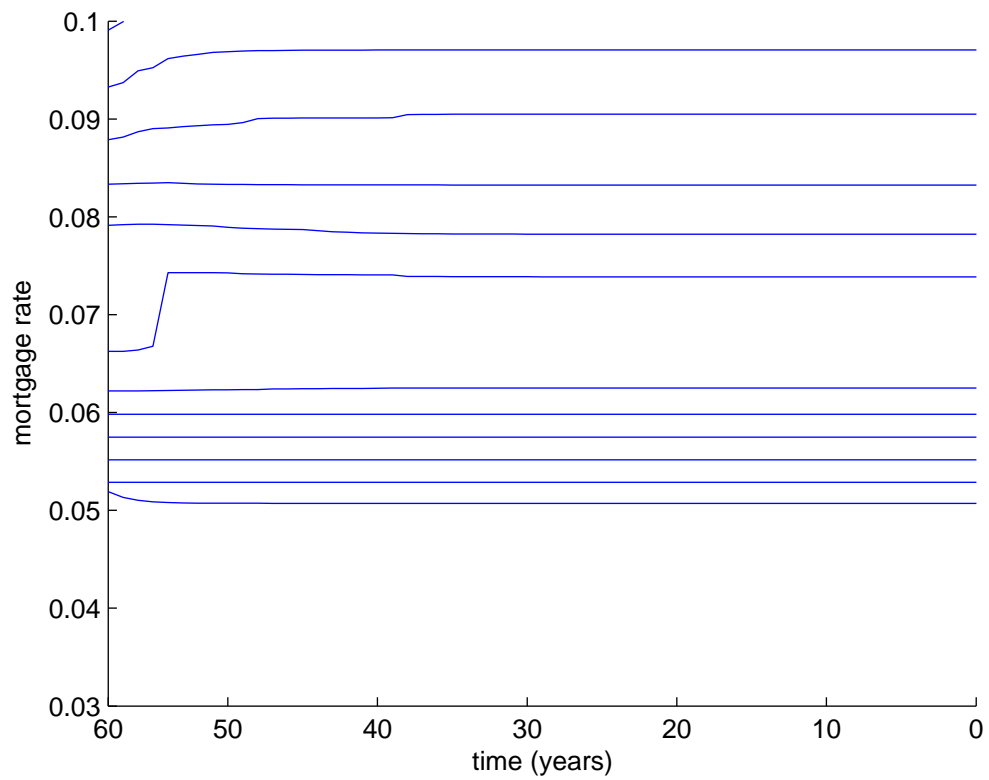


Exhibit 6. The t -traces of the mortgage rate term structure given the 10-year Treasury rule of thumb for the initialization



**Exhibit 7. t -traces of the mortgage rate term structure
of modified procedure**

