Prepayments on fixed-rate mortgage-backed securities

A model with innovations and attributes to extend the understanding of mortgage prepayments.

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Mortgage-backed securities (MBSs) are increasingly a part of the financial scene. To assess their relative value and the value of their derivatives, it is essential to predict mortgage prepayments accurately. It is important to understand how mortgage pass-through securities will prepay in today's interest rate environment as well as how prepayments will fluctuate as interest rates fluctuate.

This paper introduces our latest work on prepayment modeling. Recognizing that work in this area is never completed, we feel nevertheless that our model includes certain innovations and attributes that extend the understanding of mortgage prepayments. We begin by analyzing the economic theory underlying a homeowner's decision to prepay a mortgage. This option-theoretic analysis serves as the basis for our empirical model of prepayment rates.

Three aspects of our model are novel. First, we measure the mortgagor's refinancing incentive as the ratio of the mortgage coupon to the current refinancing rate, not as the difference between these two rates. This idea comes from our economic analysis of the mortgagor's prepayment decision. Second, we show that the seasoning process for mortgages depends importantly on this same ratio, the coupon relative to the refinancing rate. In particular, premium mortgages season more rapidly than current coupon mortgages, which, in turn, season more rapidly than discount mortgages. Finally, we examine the tendency of premium mortgages to slow or "burn out" over time. We introduce a measure of premium burnout that depends on the entire interest rate history since the mortgage was issued.

We try to provide sufficient details so that readers can understand how the model works. Examples in the figures explain the Goldman Sachs prepayment model, without bogging readers down with unnecessary mathematical detail. We also report a detailed summary of our model's predictions in relation to actual prepayment data.

PREPAYMENT THEORY

A homeowner with a new thirty-year fixed-rate mortgage has committed to make 360 fixed monthly payments. This stream of fixed payments is known as an annuity. As the mortgage ages, the market value of the annuity changes for two reasons: first, there are fewer payments left in the stream; second, the capitalization rate, as measured by the current mortgage rate, fluctuates.

The present value of the annuity, A, in month t, per dollar of monthly payment is given by the formula:

$$A = [1 - (1 + R)^{-360 + t}]/R,$$

where R is the mortgage refinancing rate in month t.[1]

We measure the homeowner's economic incentive to refinance by computing the annuity value per dollar of principal outstanding. In month t the outstanding principal, P, per dollar of monthly payment is given by

$$P = [1 - (1 + C)^{-360+t}]/C,$$

where C is the coupon rate. Hence the annuity value per dollar of principal in month t is

$$A/P = (C/R)\{[1 - (1 + R)^{-360 + t}]/[1 - (1 + C)^{-360 + t}]\}.$$

The value of A/P is well-approximated by C/R,

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the mortgage coupon rate divided by the mortgage refinancing rate, over a wide range of maturities. For small values of t (i.e., relatively new mortgages), the term in braces is trivial, so the dollar amount of refinancing incentive is nearly equal to C/R. Even for larger values of t, the term in braces is not very large. For example, for a fifteen-year-old mortgage (t = 180) with an 8% coupon (C = 0.08/12 = 0.00667) and a current mortgage rate of 10% (R = 0.1/12 = 0.00833), we find that the term in braces is 1.11. It is only as the mortgage nears maturity that the term in braces becomes large; for t = 359, C = 0.00667, and R = 0.00833, the term is 1.25.

The more commonly used measure of the refinancing incentive, the coupon rate less the refinancing rate C − R, is a poor approximation to A/P for most coupons and maturities. In fact, it is only as the mortgage nears maturity that A/P approximately equals 1 + C − R. For example, when t = 359, C = 0.00667, and R = 0.00833, we find that A/P = 0.99835, which is very nearly equal to 0.99833 = 1 + C − R. When t = 30, however, A/P = 0.842, and C/R = 0.800, but 1 + C − R still equals 0.998.

To understand why A/P measures the homeowner's refinancing incentive, we must turn to option pricing theory. Mortgagors have the option to prepay their mortgages. In order to decide whether to exercise this option, they must compare the cost of continuing the monthly payments to the cost of refinancing at the current rate. The cost of refinancing includes both explicit costs, such as title insurance and points, and implicit costs, such as qualifying difficulties if, for example, the mortgagor is currently unemployed. The costs of refinancing can actually be negative if the homeowner has a large incentive to move, as might occur with incentives offered by a new employer.

If mortgagors exercise the prepayment option, they retire their annuity with value A, but must repay the principal P plus the explicit and implicit costs of refinancing. In addition, they lose the right to exercise the option at a future date. This means that the homeowner's prepayment option is an American option on the annuity with strike price equal to the current principal plus the refinancing costs. If refinancing costs are proportional to the principal amount, then the option value per dollar of current principal depends on A/P. Hence the mortgage value per dollar of principal depends on A/P as well.

Standard option pricing theory for American options implies that it is rational for the mortgagor to prepay when A/P exceeds some critical value. This critical value of A/P in turn implies a critical value of refinancing costs, stated as a fraction of the principal. Because costs of refinancing are heterogeneous within

a mortgage pool, not all mortgagors will find it optimal to prepay simultaneously.[2] The number of mortgagors in a particular pool with refinancing costs below the critical value will determine the fraction of the pool that prepays. Hence the speed of prepayment in otherwise identical pools (in terms of maturity and coupon) will differ because of heterogeneous refinancing costs for mortgagors within the pools, even if every mortgagor is behaving rationally.

Furthermore, the data indicate that refinancing costs are time varying. To see this, consider a scenario where interest rates fall and then rise. As interest rates fall, A/P rises, causing some households to prepay because their refinancing costs are below the critical value of refinancing costs implied by the current A/P. When interest rates reach their (ex post) minimum, both A/P and the implied critical level of refinancing cost reach their maximum; denote this maximum refinancing cost, M. All households with prepayment costs below M will have prepaid, so only those households with prepayment costs above M will remain in a pool. Now assume interest rates rise so that A/P falls and the implied cost necessary to trigger a prepayment also falls.[3] If prepayment costs did not change over time, there would be no households with prepayment costs below M. As we continue to observe prepayments in this rising interest rate environment, however, we conclude that household prepayment costs must vary over time.

THE GOLDMAN SACHS PREPAYMENT MODEL

Our prepayment model is an empirical estimation of the mortgagor's financing decision. We try to uncover an explanation for prepayments by observing actual prepayments and relating them to the measurable factors suggested by our economic theory of prepayments.

Even a cursory empirical examination reveals that outstanding mortgages with very high coupons relative to current mortgage refinancing rates still do not entirely prepay. At the other end of the spectrum, some borrowers prepay when they could clearly receive higher returns by investing the same cash in safe money market instruments. These are both manifestations of the fact that the mortgagors have heterogeneous refinancing costs.

Factors That Explain Prepayments

The Goldman Sachs prepayment model captures four important economic effects:

1. The refinancing incentive;
2. Seasoning or age of the mortgage;
3. The month of the year (seasonality); and
4. Premium burnout.

We discuss the form and empirical measure-
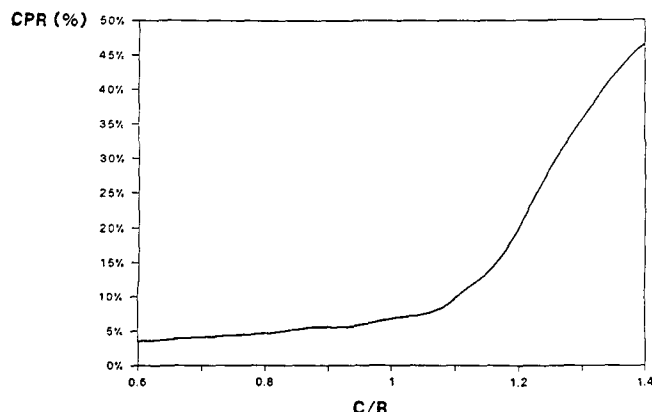
ment of each of these effects in turn.

THE REFINANCING INCENTIVE—We measure the refinancing incentive as the weighted average of recent values of C/R, the mortgage coupon rate divided by the mortgage refinancing rate.[4] Two comments are in order about this choice to measure the refinancing incentive. First, we use C/R instead of A/P because of the out-of-sample properties of these two measures. As a mortgage matures, A/P converges to one. This causes the refinancing incentive for premium mortgages to fall automatically and for discount mortgages to rise automatically. As we have no data on the end-of-life behavior of mortgage-backed security pools, we chose not to use A/P out of sample to avoid a possible source of bias. Within our sample, as we have noted, A/P is well approximated by C/R.

Second, we use a weighted average of recent values of C/R to capture the fact that homeowners exhibit varying delays in responding to refinancing incentives because of differences in processing times by mortgage lenders and, perhaps, differences in the time they need to react to a favorable interest rate environment. For convenience we will call this weighted average C/R. The lower recent interest rates have been, the higher the value of C/R. We know from our analysis of the homeowner's prepayment option that the higher C/R is, the higher prepayments will be.

We model the relationship between prepayments and C/R by using a curve-fitting technique. Figure 1 shows monthly prepayment rates (expressed in % CPR) for seasoned thirty-year GNMA single-family pools. The curve in Figure 1 reflects only the pure refinancing incentive for a seasoned pool without adjustment for path-dependent burnout. Notice that there is a highly non-linear relationship between the prepayment rate and C/R, which is typical of option pricing models.

For values of C/R below one, the homeowner's prepayment option is out-of-the-money, and the refinancing incentive is relatively small. If C/R is 0.8, for example, representing a seasoned 8% mortgage (with a pass-through rate of 7.5%) in a current mortgage refinancing rate environment of 10%, the prepayment rate is approximately 5% CPR. Conversely, when C/R exceeds one, the coupon is above the refinancing rate, and the incentive to refinance increases dramatically. A seasoned 13% mortgage in a 10% refinancing rate environment (C/R = 1.3), for example, has a base prepayment rate of about 36% CPR without adjusting for premium burnout.

We estimated the refinancing incentive independently for mortgage-backed securities besides GNMA (FNMA and FHLMC). The basic shape is similar across sectors, although the exact values differ to a certain extent.
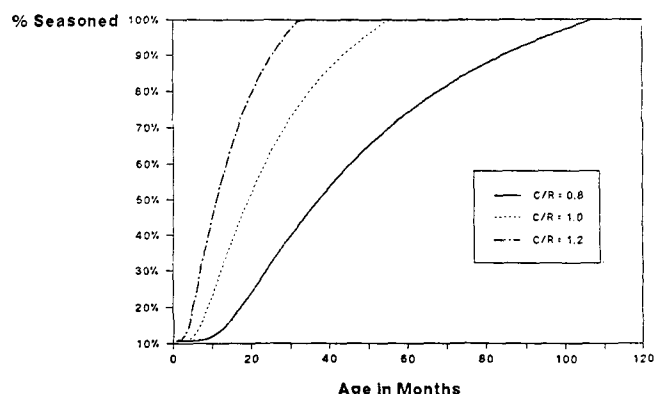
SEASONING—It is well known that mortgage prepayment rates rise from very low levels at issue to much higher levels as the mortgages age. This is the rationale for the PSA Standard Prepayment Model, whose base case models mortgage prepayment rates as increasing linearly from 0% CPR at issue to 6% CPR at thirty months and then remaining constant.

It is less well known that the mortgage process differs markedly depending on the coupon rate relative to current refinancing rates. Slight premium GNMA pools, for example, are typically fully seasoned in about thirty months, as suggested by the PSA standard. Current coupon pools, on the other hand, take nearer to five years to season fully, and discount pools can take considerably longer.

Our model captures the interaction between seasoning and coupon by making the seasoning effect a function of the mortgage's current C/R. In Figure 2 we show the relative seasoning effects for a discount pool with C/R = 0.8, a par pool with C/R = 1.0, and a premium pool with C/R = 1.2. Our model shows

FIGURE 1

GNMA REFINANCING INCENTIVE
WITHOUT PATH-DEPENDENCE

CPR (%)

C/R

FIGURE 2

GNMA SEASONING

% Seasoned

C/R = 0.8
C/R = 1.0
C/R = 1.2

Age In Months

that these pools season at remarkably different rates.

The discount pool (C/R = 0.8) takes about nine years to season fully, although it is 75% seasoned in about five years. This is not hard to understand in terms of the disincentive for homeowners to move when their mortgage rate is low compared to rates currently available. Moreover, the seasoning process for discount GNMA pools is slowed further by the fact that FHA/VA mortgages are assumable and that the incentive to assume is greater, the smaller C/R is. The par pool with C/R = 1.0 typically takes almost five years to fully season.

Even faster to season fully are premium pools; at a C/R value of 1.2, a pool typically seasons in just over thirty months, as prescribed by PSA. Unfortunately, the seasoning process is decidedly non-linear, and PSA is only an approximation throughout. Again we can see why premium pools season relatively quickly if we look at the homeowner's incentives: Homeowners will not be deterred from moving because they hold a premium mortgage, and it is implausible that anyone will want to assume a high premium mortgage.

After mortgages season fully, we assume that aging has no further effect on mortgage prepayment rates, as we have no data on the end-of-life prepayment behavior of mortgage-backed security pools. In the absence of a model of household prepayment costs, economic theory gives little guidance as to whether prepayments should increase or decrease as a mortgage nears maturity. In the absence of theory and data, we have made the neutral choice to assume no further aging effect.

MONTH OF THE YEAR—The seasonal pattern of mortgage prepayments is both important and somewhat surprising. It is commonly believed that prepayments peak in the summer months and trough in the winter because household moves follow a seasonal pattern. Figure 3 shows our model's estimate of the relative

month-of-the-year effect in the reporting month for GNMA thirty-year single-family MBSs. There is an obvious winter trough in February–March, but the peak occurs in the autumn in October–November. This is probably due to lags in passing through prepayments. The pattern for FNMAs and FHLMCs is similar to that of GNMAs.

PREMIUM BURNOUT—Because prepayment costs, both explicit and implicit, differ across households, not all mortgagors in a given pool prepay identically. When a given critical level of C/R is reached, only some of the households will have costs below that critical level, and only these households will choose to prepay. The other households will wait for interest rates to fall farther or for their prepayment costs to drop.

At different times households evidently face different prepayment costs. The first time a critical level of C/R is reached, all the households with prepayment costs below that level will prepay. If interest rates then rise and subsequently fall, this same critical level may be reached again. This will cause some more households to prepay, although not as many as the first time the critical level was reached.[5] Because they choose to prepay the second time a critical level of C/R is reached, but not the first time, we know that the households' costs have fallen over time. Of course, household costs can also rise over time. For example, the household may no longer qualify for a replacement mortgage because someone in the household may have become unemployed, or the value of the house may have decreased. While it is impossible to measure directly different household prepayment costs, it is certainly possible to estimate empirically the effect of these differing costs on prepayments.

The main empirical effect of household heterogeneity is "premium burnout," by which we mean the tendency of prepayments from premium pools to slow over time, all other things being equal. In the Goldman Sachs prepayment model we capture the effect of premium burnout through a complicated non-linear function. This function depends on the entire history of C/R since the mortgage was issued.

Roughly, we try to measure how much the option to prepay has been deep in-the-money since the pool was issued. The more the prepayment option has been deep in-the-money, the more burned out the pool is, and the smaller prepayments are, all other things being equal.

Figures 4A through 4C illustrate the effect of the adjustment for premium burnout. They show the burnout multiplier as a function of time for hypothetical, newly issued 11.5%, 12%, 12.5%, and 13% coupon GNMA pools (with underlying mortgage rates of 12%, 12.5%, 13%, and 13.5%, respectively) in
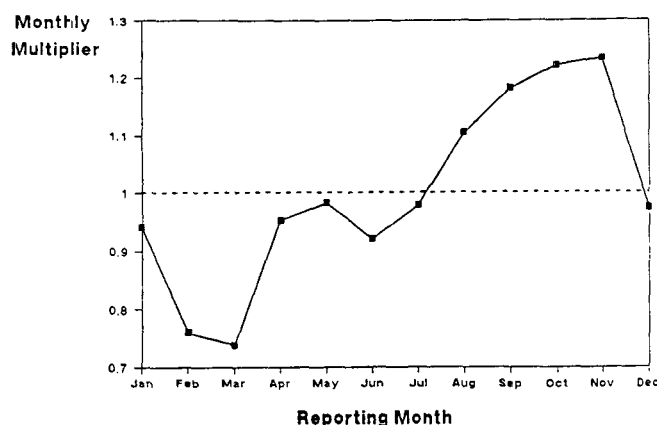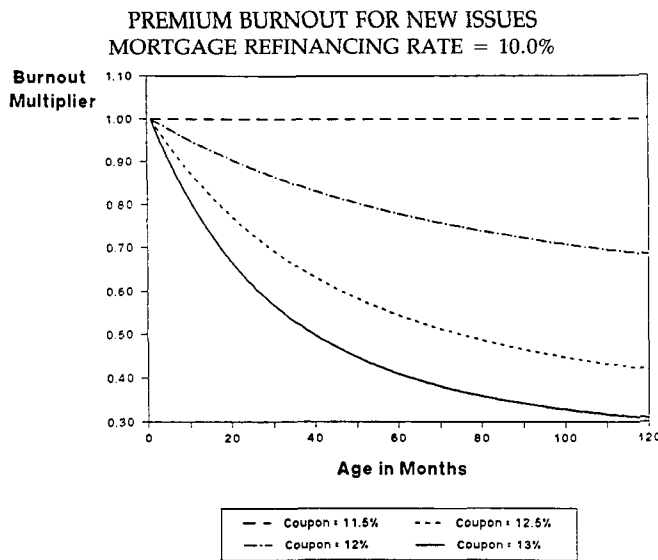
FIGURE 3

GNMA MONTHLY MULTIPLIERS

Monthly Multiplier

Reporting Month

FIGURE 4A

PREMIUM BURNOUT FOR NEW ISSUES
MORTGAGE REFINANCING RATE = 10.0%

FIGURE 4B

PREMIUM BURNOUT FOR NEW ISSUES
MORTGAGE REFINANCING RATE = 10.5%

constant 10%, 10.5%, and 11% mortgage refinancing rate environments, respectively.

To understand the effect of premium burnout, examine Figure 4A. We see that, in a constant 10% mortgage refinancing rate environment, newly issued GNMA pools with 13%, 12.5%, and 12% coupons will all experience premium burnout, but at decreasing rates, and that an 11.5% pool will not. The prepayment options for the higher coupon pools are more in-the-money than for the lower coupon pools.

From Figure 1, we know that the higher coupon pools have a higher refinancing effect. Over time, however, the households with lower prepayment costs in the 13% pool choose to prepay, leaving only the households with higher costs remaining in the pool. A similar effect occurs among households in the 12.5% and 12% coupon pools.

The reason the 11.5% pool experiences no premium burnout is that options for this group of homeowners are not as deeply in-the-money. After forty months in a constant 10% refinancing rate environment, the 13% pool has burned out to the point that prepayments are about half of what they would have been otherwise. Of course, this decline in prepayments is offset somewhat by the seasoning of the pool. For the same reasons, the 12.5% pool has a burnout multiplier of about 0.65, and the 12% pool a multiplier of about 0.85.

Turning now to Figures 4B and 4C, we see that the effect of premium burnout is very sensitive to the assumed refinancing rate. Figure 4B recasts Figure 4A in a 10.5% refinancing rate environment, omitting the 11.5% coupon line that experiences no burnout. We see that the 13% pool will experience burnout, but to a lesser extent than at a mortgage refinancing rate of 10%. The burnout multipliers therefore are higher in
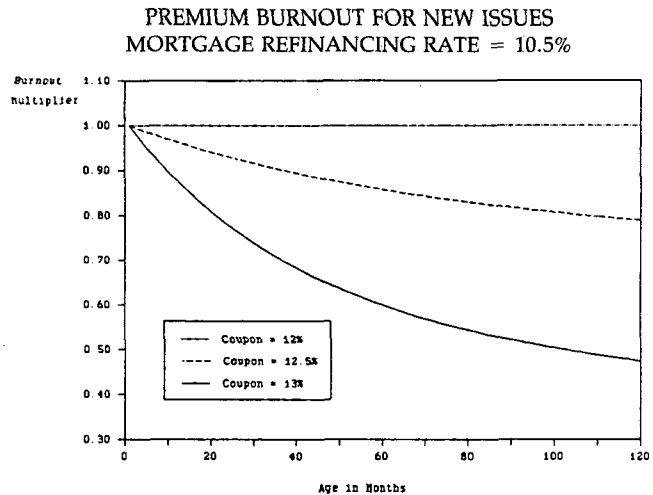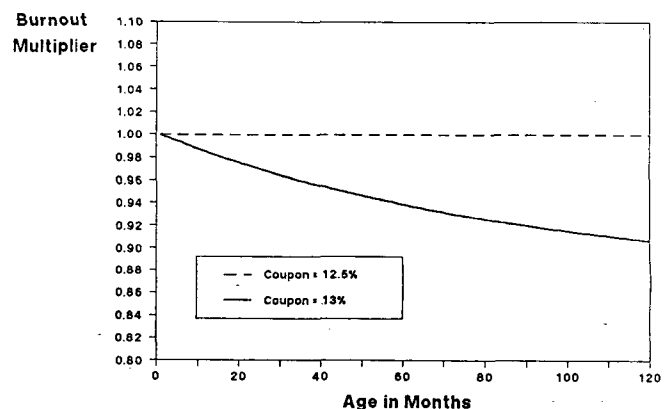
Figure 4B than in Figure 4A. For example, after forty months, a 13% pool has a burnout multiplier of about 0.7, and a 12.5% pool has a multiplier of about 0.9; at this higher refinancing rate coupons of 12% and below do not experience burnout.

Finally, Figure 4C shows that, in a constant 11% refinancing rate environment, only the 13% pool experiences burnout; pools of 12.5% and below are unaffected. The effect on the 13% pool is greatly diminished, however, as the prepayment option is not as deeply in-the-money. For example, after forty months the prepayment multiplier is about 0.95 for a 13% pool with an 11% refinancing rate.

We have experimented with other measures of heterogeneity besides the path-dependent measure of burnout. One obvious measure is the "pool factor," the fraction of the original aggregate balance of all mortgages in the pool that remains outstanding. Because the combined effects of scheduled amortization and prepayments reduce the factor, it is not an adequate measure of pool heterogeneity. That is, a well-

FIGURE 4C

PREMIUM BURNOUT FOR
NEW ISSUES
MORTGAGE REFINANCING RATE
= 11.0%

seasoned discount pool and a relatively new premium pool could have the same factor, which would be attributable mainly to the effect of scheduled amortization in the case of the discount pool and to the effect of prepayments in the case of the premium pool. Yet the two pools would have very different compositions, the discount pool retaining a representative sample of households at all levels of prepayment costs, but the premium pool retaining only those with relatively high costs. We would not expect these two pools to behave in the same way because of this difference. Our empirical experiments on GNMA prepayments bear out these expectations, as the explanatory power of the model is reduced if we use the pool factor as a measure of burnout.

Multiplicative Model

We combine the four effects — the refinancing incentive, seasoning, month of the year, and burnout — in a multiplicative formula to determine prepayment rates:

CPR = (Refinancing Incentive) × (Seasoning Multiplier) × (Month Multiplier) × (Burnout Multiplier).

This multiplicative formulation makes the effects interact proportionately. For example, with the monthly adjustment 10% higher in October than in August, the prepayment rates on all coupons in October will be 110% of their August levels, all other things being equal.

We can see how the effects combine by calculating prepayment rates for GNMA pools with various coupons in a constant refinancing rate environment. This is illustrated in Figure 5, which shows seasonally adjusted CPRs for hypothetical, newly issued GNMA pools with coupons between 8%

FIGURE 5

PREPAYMENTS OF SELECTED GNMA COUPONS
ALL ASSUMED TO BE NEWLY ISSUED
MORTGAGE REFINANCING RATE = 10.5%

Month (1 = August 1988)

—·— 8%	— — 10%	- - - 12%			
······· 9%	—·— 11%	—— 13%			

and 13% in a constant 10.5% mortgage refinancing rate environment (i.e., a GNMA coupon of 10% priced at its parity price). Notice that speeds of prepayment increase for all coupons due to seasoning, but that the 13% coupon experiences burnout that eventually slows its prepayment rate so that it falls below the prepayment rate on the 12% pool. In actuality, prepayment rates over time are much less smooth because of monthly multipliers and interest rate fluctuations, as we discuss in the next section.

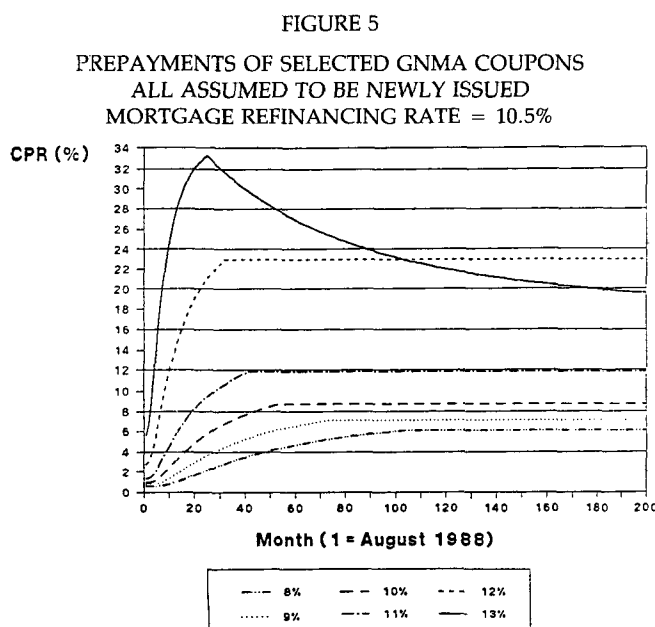### Other Possible Explanatory Variables

In constructing our model, we investigated other possible explanatory variables either used by others or reputed to be important. We discuss some of them here and explain why we decided not to include them.

VOLATILITY — It is well known that volatility is an important determinant of option prices: the higher the total return volatility of an asset, the more valuable an option on that asset, all other things equal. With homeowners retaining an option to prepay an annuity, we would expect an increase in the volatility of interest rates to cause an increase in the value of this option and a decrease in prepayments. Why, then, do we not include interest rate volatility as an explanatory variable?

The answer is that the model incorporates interest rate volatility implicitly through use of the current coupon mortgage rate. It turns out that the logarithm of the mortgage rate is well-approximated as a linear function of the logarithm of the yield on a ten-year zero coupon bond and the logarithm of the volatility of the ten-year zero's yield. Hence, an increase in interest rate volatility causes a proportional increase in mortgage rates, all other things being equal. As we use the mortgage rate to calculate C/R, an increase in mortgage rates causes C/R to decrease, which means prepayments will fall.

MACROECONOMIC VARIABLES — We explored the potential explanatory effect of several macroeconomic variables by regressing our model's residuals on them. Two of them had weak, but statistically significant, explanatory power in these regressions: industrial production and housing sales. We chose not to include them in our model for several reasons.

Industrial production measures aggregate economic activity. If it is higher than average, we would expect more prosperity than average and hence a greater propensity to trade up in housing. This should increase prepayments, at least for mortgages with a due-on-sale clause. In fact we found just the opposite effect: Industrial production has negative partial cor-

relation with prepayments. The reason for this anomaly can be found by examining our model's residuals,[6] which have trended down over the sample period. (Our coefficient estimates were corrected subsequently for serial correlation.) Industrial production, of course, trends upward, and it is this fact that explains the negative correlation. In fact, substituting a time trend for industrial production produces a regression with the same explanatory power.

Other researchers have found that lagged housing sales help explain prepayments. Hence, it is not surprising that lagged housing sales have some explanatory power for our model's residuals. It is also completely uninteresting and useless in forecasting. House sales *cause* prepayments, either because there is a due-on-sale clause (and the below-market rate mortgage cannot be assumed), or the mortgage is at a premium. Because of payment and reporting lags, house sales cause prepayments with a lag, however. Using house sales to explain prepayments is simply using prepayments to explain prepayments.

In general, we have avoided using macroeconomic variables in our prepayment model because they would not aid us in the long-term forecasts required for valuation. Our valuation model simulates interest rates over thirty years to generate prepayments over a pool's lifetime. If the prepayment model were to include a macroeconomic variable, we would have to simulate its behavior over thirty years also. By observing the term structure of interest rates, we can use the market's expectations to simulate possible interest rate scenarios, but we have no such guidance from the market in simulating other variables over a thirty-year horizon. Macroeconomic variables may have some limited use in short-term forecasting, but they cannot be useful in the long-range forecasts required for valuation purposes.

ESTIMATION AND EMPIRICAL RESULTS

The model is estimated using non-linear least squares. Data for each sector come from the Goldman Sachs Mortgage Database aggregated into cohorts each month. We form cohorts by taking the weighted average of all pools in a sector with equal average underlying mortgage rates (WACs) and remaining terms (WAMs). The weights are the outstanding pool balances. In each case, all cohort-months are included in the non-linear regression estimation.

The choice of weights in the non-linear regression has important effects on the estimated coefficients and in sample predictions. The weights we use are the outstanding pool balances adjusted for the age of the observation. Older observations receive less weight than newer observations: observations from one year ago receive 50% of the weight of an otherwise identical current observation; observations from two years ago receive 25% of the weight, and so on.

The purpose of this weighting scheme is twofold. First, we want our prepayment predictions to be more accurate for larger pools, so we use the outstanding pool balance as a weight. Second, we suspect there has been a secular change in prepayment rates and that recent observations have more validity for prediction than do older observations, which we discuss after reporting our results.

Consider the results for the GNMA single-family thirty-year MBSs for the period May 1979 through May 1988, inclusive. There are 103,694 observations in the Goldman Sachs data base for the period. The R-squared is 94.6%, which is the proportion of cross-cohort and cross-month variability in prepayment rates explained by the four effects in the regression. That is, in the GNMA thirty-year sector, roughly 95% of the prepayment differences over time and across coupons can be attributed to refinancing incentives, seasoning, seasonality, and premium burnout.

The table compares our model's in-sample predictions to the actual prepayment rates. For each coupon and each range of mortgage refinancing rates we show five statistics: our prediction of the weighted average prepayment rate (weighted by the outstanding principal balance in the cohort), the actual weighted average prepayment rate, the standard deviation of the actual prepayments in the cell, the number of observations, and the weight of each observation. In general, our predictions are close to the actual observed prepayment rates in each cell. Our prediction is within two standard deviations of the actual prepayment rate in all but two cells and within one standard deviation in all but four of the cells.

We now return to the issue of a possible secular change in prepayment rates. If we recast the table without using the time-dependent portion of the weighting scheme, we find that the comparison of actual versus predicted results changes in a systematic way. Generally, the model then substantially underpredicts prepayment rates for pools with coupons of 7.5% and 8%. This indicates that prepayments on discount GNMA pools have slowed recently compared to the last decade. This secular slowing of prepayment rates on discount GNMA pools may be attributable to two changes in the mortgage market.

First, adjustable rate mortgages (ARMs) have become more readily available over the last decade. Home buyers with more chance of moving soon may be more inclined to finance with an ARM because of the lower initial interest rate. This self-selection will reduce observed prepayments on fixed-rate mortgages. Second, the market for home equity credit lines has become well organized. Where once the second

TABLE

Prepayments for GNMAs in Years 1979–1988

		Refinancing Rate Range				
Coupon	Statistic	8.5%-9.5%	9.5%-10.5%	10.5%-11.5%	11.5%-12.5%	12.5%-13.5%
7.5%	Predicted CPR (%)	4.6	4.1	3.2	4.1	2.4
	Actual Average CPR (%)	6.2	4.9	3.2	4.3	2.8
	Standard Deviation of CPR (%)	4.5	3.7	2.8	1.6	1.3
	Number of Observations	593	1448	663	716	1877
	Weight	19	24	21	4	2
8%	Predicted CPR (%)	3.8	3.3	2.4	4.8	2.9
	Actual Average CPR (%)	4.7	3.7	2.4	4.4	3.0
	Standard Deviation of CPR (%)	4.5	3.4	2.5	1.3	1.2
	Number of Observations	809	2031	918	1003	2601
	Weight	47	66	60	8	4
8.5%	Predicted CPR (%)	2.0	2.0	1.5	5.4	3.4
	Actual Average CPR (%)	1.9	2.4	1.6	4.7	3.3
	Standard Deviation of CPR (%)	3.4	2.4	1.8	2.2	2.0
	Number of Observations	597	1467	622	650	1666
	Weight	48	69	66	2	1
9%	Predicted CPR (%)	2.6	3.0	2.3	5.5	3.3
	Actual Average CPR (%)	2.3	3.2	2.2	3.9	2.6
	Standard Deviation of CPR (%)	2.9	2.3	1.6	1.2	1.0
	Number of Observations	629	1621	639	650	1462
	Weight	207	210	208	10	6
9.5%	Predicted CPR (%)	5.4	3.5	2.8	6.2	3.5
	Actual Average CPR (%)	4.5	3.9	2.9	3.6	2.5
	Standard Deviation of CPR (%)	4.1	3.1	2.0	1.0	0.8
	Number of Observations	475	1236	459	417	850
	Weight	109	168	153	13	8
10%	Predicted CPR (%)	9.5	4.1	3.4	6.1	3.5
	Actual Average CPR (%)	7.5	4.6	3.6	3.4	2.3
	Standard Deviation of CPR (%)	4.5	3.5	2.2	2.2	1.5
	Number of Observations	580	1445	577	556	992
	Weight	52	83	71	3	2
10.5%	Predicted CPR (%)	17.7	6.0	3.8	3.6	1.8
	Actual Average CPR (%)	18.8	7.1	4.3	2.3	1.6
	Standard Deviation of CPR (%)	11.1	5.3	3.6	3.6	2.9
	Number of Observations	452	1103	458	386	551
	Weight	24	36	36	1	1
11%	Predicted CPR (%)	29.6	13.3	7.0	5.2	4.2
	Actual Average CPR (%)	27.2	13.7	7.6	3.5	3.2
	Standard Deviation of CPR (%)	9.0	5.4	3.6	2.6	1.6
	Number of Observations	589	1445	530	617	1056
	Weight	75	82	83	16	8
11.5%	Predicted CPR (%)	38.0	22.7	10.8	5.9	2.7
	Actual Average CPR (%)	38.3	25.3	12.5	5.0	2.8
	Standard Deviation of CPR (%)	8.0	9.6	4.7	2.6	1.6
	Number of Observations	546	1359	491	573	1003
	Weight	56	60	58	26	15
12%	Predicted CPR (%)	41.2	29.2	13.4	4.7	2.1
	Actual Average CPR (%)	41.1	30.3	15.3	4.2	2.4
	Standard Deviation of CPR (%)	6.4	10.1	5.4	3.1	2.0
	Number of Observations	469	1166	407	463	862
	Weight	50	53	53	27	10
12.5%	Predicted CPR (%)	41.8	34.9	18.3	8.2	4.1
	Actual Average CPR (%)	41.4	34.3	18.9	7.8	4.2
	Standard Deviation of CPR (%)	6.8	11.6	5.6	3.8	3.1
	Number of Observations	504	1239	430	510	996
	Weight	26	28	28	15	7
13%	Predicted CPR (%)	41.1	38.2	22.5	12.2	4.7
	Actual Average CPR (%)	41.9	37.0	22.0	12.9	5.3
	Standard Deviation of CPR (%)	7.8	12.7	7.2	5.8	3.7
	Number of Observations	434	1061	374	434	840
	Weight	15	17	17	10	6

mortgage market was poorly organized and reputed to be expensive, now a homeowner who wants to increase a mortgage balance, but does not want to move, can borrow in the home equity credit market. This will reduce observed prepayments on discount mortgages, as homeowners no longer need to sacrifice the gain on their discount mortgages in order to increase their mortgage borrowing.

Our model's very high degree of explanatory power was obtained in a sample that includes all coupons and all maturities of thirty-year GNMA single-family pools. Another approach is to estimate a separate model for each coupon and for each maturity year, a procedure apparently followed by some other investigators. We have examined the differences and found that explanatory power can be increased to as much as 99% if a separate model is estimated for each coupon and maturity year. These separate models, however, have different coefficients in each case, and we suspect they would fit poorly out-of-sample. Furthermore, estimating separate models ignores much useful information because pools with equal C/R, age, and burnout should behave similarly. (Our high R-squared shows that, in fact, they do behave similarly.) Hence there is important information about the prepayment characteristics of a 12% pass-through in a 10% refinancing rate environment that can be inferred from the prepayment behavior of a 10% pass-through in an 8.25% refinancing rate environment.

This cross-coupon information is particularly important when using a prepayment model for valuation purposes. Valuation models consider prepayments in many different interest rate scenarios, usually through Monte Carlo simulations. Frequently the prepayment model is forced to forecast out-of-sample because the simulated interest rate scenario lies outside the bounds of observed interest rates. For example, we have no observations on 7.5% GNMA pools in a 6.5% refinancing rate environment (C/R = 8/6.5 = 1.23), although these are frequently observed in simulation models. We have, however, many observations of 11.5% coupons with a 9.75% refinancing rate or a 12% coupon with a 10.2% refinancing rate (both also have C/R = 1.23). Guided by our analysis of the homeowner's prepayment option, and lacking any data to the contrary, we anticipate that the 7.5% coupon pool in a 6.5% mortgage refinancing rate environment will behave much as an 11.5% coupon pool does in a 9.75% mortgage refinancing rate environment.
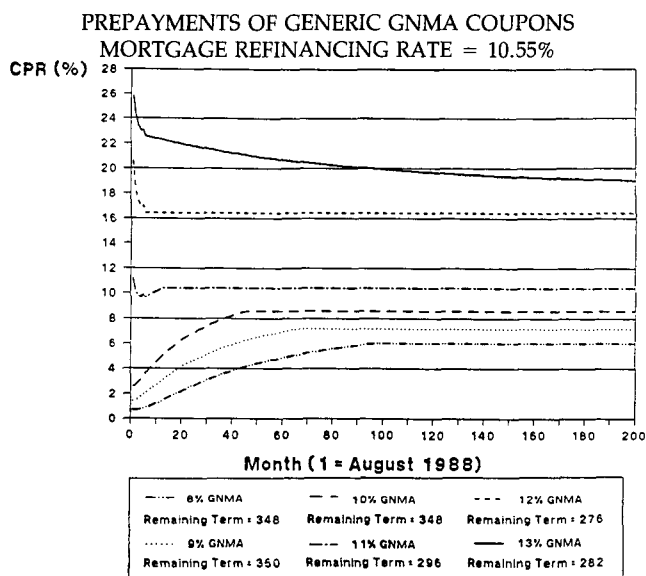
USING THE RESULTS TO PREDICT PREPAYMENTS

To predict prepayments over the remaining life of a mortgage-backed security, the model requires two sets of inputs: 1) the characteristics of the MBS itself, i.e., its sector, WAC, and WAM; and 2) the actual historical path of the mortgage refinancing rate since origination and its assumed path from now to maturity. The actual path since origination is a matter of public record, but, of course, the assumed path could take any shape.

One commonly used assumption in generating prepayment forecasts is that the refinancing rate will remain constant at its current level. This is called a static scenario. The static scenario forecasts for generic GNMA coupons are shown in Figure 6 for a constant

FIGURE 6

PREPAYMENTS OF GENERIC GNMA COUPONS
MORTGAGE REFINANCING RATE = 10.55%

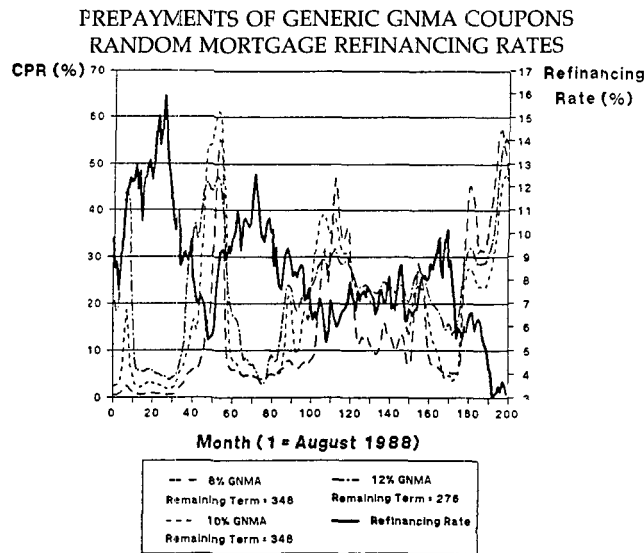| ――・― 8% GNMA
Remaining Term ¤ 348 | ― ― 10% GNMA
Remaining Term ¤ 348 | ― ― ― 12% GNMA
Remaining Term ¤ 276 |
| ・・・・・・ 9% GNMA
Remaining Term ¤ 350 | ――・― 11% GNMA
Remaining Term ¤ 296 | ―――― 13% GNMA
Remaining Term ¤ 282 |

10.55% mortgage refinancing rate. (These forecasts are seasonally adjusted by removing the effect of the monthly multiplier.) The assumed remaining terms are 353, 350, 330, 312, and 296 months for the 8%, 9%, 10%, 11%, 12%, and 13% generic coupons, respectively.

Static scenarios are highly unrealistic, because interest rates do not follow simple, deterministic paths over time, let alone constant paths. In a model such as ours, with path-dependent prepayment rates, a constant interest rate scenario will not generate the proper behavior of premium burnout. Hence, if prepayments are forecast under the unrealistic assumption that the refinancing rate is not random, then we cannot expect to obtain very realistic forecasts.

Figure 7 illustrates prepayment behavior along a random path. It shows prepayment rates for the same generic GNMA pools with 8%, 10%, and 12% coupons as shown in Figure 6, but with a particular random path for the refinancing rate. The random path was generated by letting the refinancing rate follow a random walk with 8% annual variance. Other random paths will generate very different prepayment rates over time.

The contrast between the paths in Figures 6

FIGURE 7

PREPAYMENTS OF GENERIC GNMA COUPONS
RANDOM MORTGAGE REFINANCING RATES

Month (1 = August 1988)

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-- -  8% GNMA              --·-  12% GNMA
Remaining Term = 348      Remaining Term = 276
- - -  10% GNMA           ——  Refinancing Rate
Remaining Term = 348
```

and 7 is striking. The prepayment rates in the static scenario are monotonically ordered by coupon. No such simple order prevails in the random scenario because of premium burnout. In fact, the 12% coupon experiences substantial burnout when interest rates first fall below 10% and its CPR climbs to over 40%. The next time interest rates fall, they fall to about 6% (in month 50), and again the CPR for a 12% coupon soars to over 40%. But these CPRs are eclipsed by the prepayment rates on the 8% and 10% coupons, which have experienced no burnout since issue. In the third rally, interest rates again fall to about 7% in month 110, but this time the 12% GNMAs prepay at only a 30% CPR. Clearly, the static interest rate scenario does a poor job of capturing the type of fluctuations in prepayments that we anticipate will actually occur.

CONCLUSION

We have presented a model of prepayments for mortgage-backed securities that has some unique features and a high level of explanatory power. Among the novel features, three are key in understanding mortgage prepayments.

First, we have shown that option pricing theory suggests that the homeowner's incentive to refinance is best measured in our sample by the ratio of the mortgage coupon rate to the mortgage refinancing rate. By measuring this refinancing incentive properly, we find that the empirically measured effect of the refinancing incentive corresponds well with the type of effect predicted by option pricing theory.

The other two key effects modify the pure refinancing incentive. We have shown that the rate of seasoning of a mortgage pool depends importantly on whether it is a premium, par, or discount pool. Specifically, premium pools season quickly, typically in thirty months or less. GNMA current coupon pools take longer to season, about five years. Finally, discount pools can take substantially more time to season

fully, even as long as ten years.

The third key effect is premium burnout, and it is the most difficult to measure empirically. We have offered an explanation in terms of household refinancing costs for why prepayment rates for premium coupon mortgage pools tend to burn out or slow over time (relative to prepayment rates on lower coupon pools). This effect is measured by the cumulative refinancing incentive experienced by the mortgage pool when homeowners' prepayment options are deep in-the-money. Proper measurement and use of premium burnout are vital for explaining mortgage prepayments accurately.

The Goldman Sachs mortgage prepayment model has a good fit to the data available over the last ten years. A note of caution is in order, nevertheless. There are no reliable data on some aspects of mortgage-backed security prepayments, such as end-of-life prepayment rates or burnout on slight premium mortgages over longer periods. We must extrapolate out-of-sample to forecast these effects. While we feel the extrapolations are sensible, we look for the passing of time to produce further data to refine the model.

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[1] Actually this formula understates the market value of the homeowner's annuity because the mortgage rate is determined for new loans with the right to prepay. Ideally, we should use the rate that would prevail for new loans without the right to prepay, i.e., mortgages with lifetime lockouts. In equilibrium, the lockout mortgage rate would be lower than the actual mortgage rate, as the homeowner gives up the prepayment option in return for a lower interest cost. Capitalizing the mortgage cash flows at the lower lockout mortgage rate would give a higher annuity value.

[2] For an option-theoretic model of household prepayment decisions based on prepayment costs, see K. B. Dunn and C. S. Spatt, "The Effect of Refinancing Costs and Market Imperfections on the Optimal Call Strategy and Pricing of Debt Contracts," Carnegie-Mellon University Working Paper, March 1986; and G.C. Timmis, "Essays in Applied Economics," Unpublished Doctoral Dissertation, Carnegie-Mellon University, 1988.

[3] There is an offsetting effect caused by the aging of the mortgage. As the mortgage ages, the prepayment option's maturity becomes shorter. This shortening can make it optimal to exercise an in-the-money prepayment option, even if refinancing costs do not change. Experiments with rational option pricing models indicate that this effect is small for thirty-year mortgages with more than fifteen years to maturity.

[4] The term "refinancing rate" refers to the mortgage refinancing rate and not the yield on the current coupon GNMA. The mortgage refinancing rate would be fifty basis points above the yield on the current coupon GNMA trading at its parity price.

[5] As we noted, there is an alternative explanation that has to do with the aging of the mortgage. See footnote 3.

[6] The model's residuals are the differences between the actual and predicted prepayment rates in our data set.