The Term Structure of Mortgage Rates: Citigroup's MOATS Model

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Mortgage valuations require the calculation of projected prepayment speeds along given interest rate paths. These paths are typically of benchmark rates such as swaps or Treasuries. However, prepayments are determined by primary market mortgage rates, as these are what the borrower sees. Given a certain path for swap or Treasury rates, how do we obtain a corresponding path for mortgage rates?

Constant Spread to the 10-Year Mortgage

Since it has been traditionally assumed that the 10-year part of the interest rate curve drives mortgage rates, a common assumption has been to hold the spread between mortgage and 10-year swap or Treasury rates constant. For example, if the current 30-year mortgage rate is 6% and the 10-year swap rate is 4.70%, we assume that the spread between the two stays constant at 6–4.70%, or 130 basis points (bp). Then, along each future swap rate path, we assume that the mortgage rate is always the 10-year swap rate plus 130 bp.[1]

Although this is a simple and intuitive approach, it has a basic problem. Nowadays, primary market mortgage rates depend on secondary market MBS current-coupon yields. Hence, mortgage rates depend not just on the 10-year part of the curve, but also on other factors that affect MBS current-coupon yields. Such factors include

- the shape of the yield curve
- indeed, all of the points on the yield curve
- volatilities.

Regression Models

Regression models have been used to incorporate factors other than the 10-year rate into the estimation of mortgage rates. Such models assume that the mortgage rate is a function of two or more points on the yield curve and, in some cases, one or more volatilities. The regression coefficients are estimated using historical data.

This approach overcomes the major limitation of the constant spread method, as it incorporates factors other than the 10-year rate. However, it is a completely empirical approach with the major assumption that the regression parameters are constant regardless of the level of rates and volatilities or the shape of the curve. In addition, the dependency of the mortgage rate on benchmark rates is likely non-linear, making the regression unstable.

Citigroup MOATS Model

To overcome the limitations of these traditional approaches, Citigroup developed a new methodology called the Mortgage Option-Adjusted Term Structure (MOATS) model.[2] The MOATS model assumes a constant OAS for the current-coupon mortgage-backed security (MBS) at all times and across all interest rate paths. From this assumption, it calculates the current-coupon yield at all nodes.

The primary mortgage rate is then calculated from the current-coupon yield.

Although the MOATS model has been available on Citigroup's analytic system, the Yield Book®, for several years, we regularly receive questions on the model, and it remains a black box for many accounts. A recently released report attempts to open up the black box by showing MOATS-projected changes in mortgage rates for changes in the level and slope of the swap curve and for changes in volatility.[3] In addition, we have recently conducted historical data analysis to check the empirical validity of the assumptions underlying MOATS. Hence, this seems an opportune time for a comprehensive overview of the MOATS method.

The rest of this article is organized as follows:

1. In the section titled The MOATS Method, we provide a more detailed, but generally non-technical description of the MOATS approach. This also allows us to highlight the key assumptions behind this approach.
2. In the section titled Empirical Validation of MOATS, we compare the assumptions underlying MOATS with the actual behavior of MBS prices and mortgage rates. The historical data tend to validate the MOATS method.
3. In the section titled Impact of MOATS on MBS Valuation, we analyze the impact on MBS valuation and hedging numbers of using MOATS versus traditional methods of deriving mortgage rates. We compare OASs, durations, convexities, and partial durations and argue that MOATS provides numbers that are more intuitive and accurate.
4. Finally, in the Conclusion, we discuss some nuances of the MOATS method.

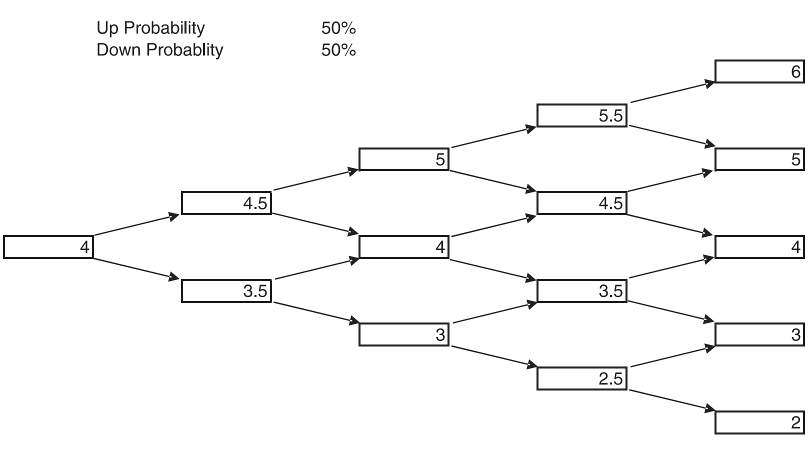
THE MOATS METHOD

To understand MOATS, we must first revisit how MBSs are priced. MBSs have embedded options that are sensitive to the uncertainty of future interest rates, usually expressed in terms of volatility of interest rates. To value this optionality we first need to develop a stochastic model of the term structure of interest rates. This model generates rate scenarios at future dates and endows each scenario with a probability. To avoid arbitrage, the model chooses scenarios and probabilities in such a way as to match benchmark securities such as swaps, swaptions, and caps. The Citigroup 2f-skew model is such a term structure model. In Exhibit 1, we show a simplified example of a binomial term structure model. For simplicity, the model is not calibrated to any particular set of benchmarks.

EXHIBIT 1

Example of a Binomial Term Structure Model

Interest Rate Tree: One-year risk free interest rate (%)



Source: Citigroup.

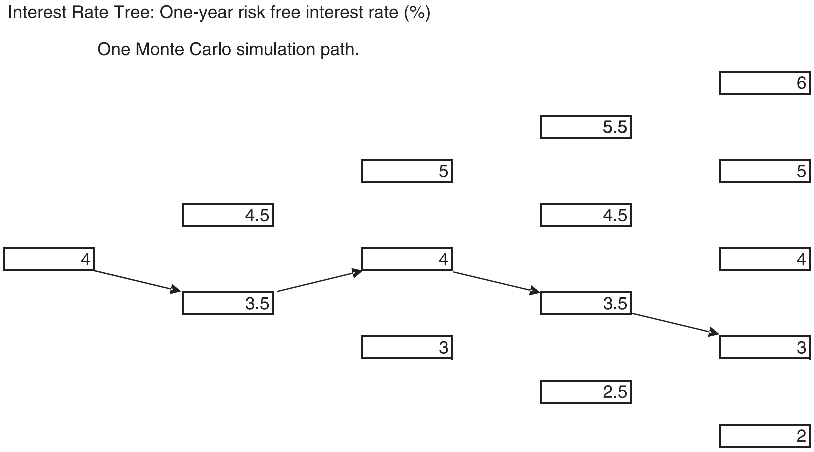
Given a term structure model, the value of an option is simply the sum of the payoff for each scenario multiplied by the probability that the scenario will occur and by the appropriate discount factor, which would include the OAS. A popular method to do this averaging is a Monte Carlo simulation of the interest rates (Exhibit 2).

In particular, MBSs are priced using Monte Carlo simulations. In fact, the path-dependent nature of mortgage prepayments[4] makes this the only practical method. For each path, the term structure model provides the month-to-month discounting and longer tenor rates. Using a prepayment model, we compute the cash flow of the MBS on each path and discount it back, including the OAS in the discounting. Then, we average the value of the MBS over all of the paths.

The only wrinkle in this pricing method is that the MBS prepayments depend on the mortgage rate, which is dependent on

EXHIBIT 2

Monte Carlo Simulation of the Binomial Term Structure Model



Source: Citigroup.

benchmark rates in a nontrivial way. As discussed in the previous section, current-coupon MBS yields in the secondary market tend to drive primary market mortgage rates. Thus, our problem boils down to determining the current coupon in each scenario generated by the term structure model. To do so, MOATS makes a fundamental assumption:

Assumption

At any future time and in any interest rate scenario, the OAS of the current-coupon MBS is equal to the OAS of the current-coupon MBS at the current time and market conditions.

This assumption requires us to find in each scenario that coupon which, when using today's OAS, prices to par.

How MOATS Derives MBS Current-Coupon Yields

For each interest rate path, we need to calculate the MBS current-coupon yield at each point in time from now until the maturity of the MBS. In theory, we could, at each point on each path, generate a whole new set of interest rate paths (with the yield curve at that node as the starting point), and use Monte Carlo simulation to estimate the yield of a brand new MBS priced at parity (by definition, the current-coupon yield). However, this would be computationally prohibitive; if, for example, there were 200 paths, we would need to do 200 × 360, or 72,000 simulations.

Because implementing MOATS using Monte Carlo simulation would be close to computationally impossible, we make an approximation. Instead of using the full Citigroup Prepayment Model, which is path dependent, we use a simplified path-independent prepayment model. A path-independent prepayment model allows us to use a grid method, such as the binomial tree, instead of a Monte Carlo simulation to price MBSs and, in particular, the current-coupon mortgage. This transforms our problem of finding the current coupon on each grid point in our tree a much more manageable task. The simplified prepayment model is "calibrated" to the full path-dependent model by matching the initial current coupon that the simplified model generates to the actual initial current coupon, as observed in the market.

Steps in the MOATS Method

To price 30-year MBSs, we need 30-year MBS current-coupon yields 30 years into the future. Hence, to implement MOATS, we propagate the interest rate tree *60 years* into the future. At year 30 in our tree, we need to compute the value of a 30-year mortgage, but we assume that at year 31 only 29-year mortgages are available and that at year 59 only 1-year mortgages are available. This approximation is needed to have a terminal time from which we can move backward on our tree.[5] Essentially, our methodology computes the current coupon of a 1-year mortgage at year 59, the current coupon of a 2-year mortgage at year 58, and so on, until, at year 30, we compute the value of a 30-year mortgage. To price 30-year mortgages, this is exactly the time that we need a 30-year current coupon.

Assuming we have a term structure model on a tree that extends out to 60 years, we are now ready to travel back in (tree) time and calculate current coupons as we move along by backward induction. To simplify the description of the algorithm, we assume here that the mortgages have yearly cash flows. The steps in the MOATS method are as follows:

1. Select an arbitrary value for the pricing OAS, called OAS*.
2. At year 59, as mentioned, we seek the current-coupon 1-year mortgage. Because there is only one

period of discounting and no possibility of prepayment, the current coupon at year 59 is simply the 1-year interest rate plus OAS* (because the current coupon must price to par).

3. At year 58, at every node, we know that the price of the MBS depends on only two following nodes at year 59. All of the current-coupon rates have been calculated for year 59, and together with the prepayment model, we can easily value any 2-year mortgage at year 58. At each node we choose the coupon that prices to par after adding OAS* to all discount factors.
4. At year 57, we have already determined the current-coupon values at all nodes in years 58 and 57 and along with the prepayment model, we can price a 3-year mortgage. Again, we find at each node the coupon that prices to par.
5. We repeat this procedure until year 30. At this point, we are pricing a 30-year mortgage for which 29 years of current coupon have been determined. We now wish to find current-coupon rates for the first year of this mortgage.
6. We repeat this procedure until we come to year 0, when we use only the current-coupon rates up to 30 years. At year 0, there is only one node corresponding to the initial yield curve. Again we follow the procedure and find the coupon that prices to par using our chosen OAS*.[6]
7. Compare the calculated current coupon from step 6 against the current coupon implied by TBA prices. If the two do not match, adjust the OAS*, return to step 1, and repeat the procedure until the two do match.

A Simple Example

In Exhibits 3 and 4 we present a stylized version of this algorithm. Again we assume that the mortgage pays interest and prepays annually. We choose an arbitrary distribution of rates (not calibrated to the swap and swaption markets), and we further assume that the current-coupon OAS* is 0 (if it were not zero we would only need to add to the discounting throughout the example). The example presents a four-step binomial tree and we wish to find a 3-year current-coupon rate. We start our procedure at year 3, where we assume the existence of only a 1-year mortgage. Because there is no prepayment (and the current-coupon OAS is 0), the current coupon at year 3 must be the 1-year rate.

Now we focus on the uppermost node in year 2, where the 1-year rate is 5%. We guess at a current coupon and see if it prices at par. If not, we iterate until we find a coupon that prices at par. Our first guess is 4.75% at the top node in year 2. Using the path–independent prepay model, we compute the prepayment at the two upper nodes in year 3, because we know the current-coupon rates for all nodes in year 3. Let's say that the prepay model predicts a 50% paydown when the current-coupon rate is 4.5% and a 0% paydown when the rate is 5.5%. The price of the 4.75% coupon at the top node in year 2 depends only on the two top nodes in year 3 and each has a 50% chance of occurring. It follows that

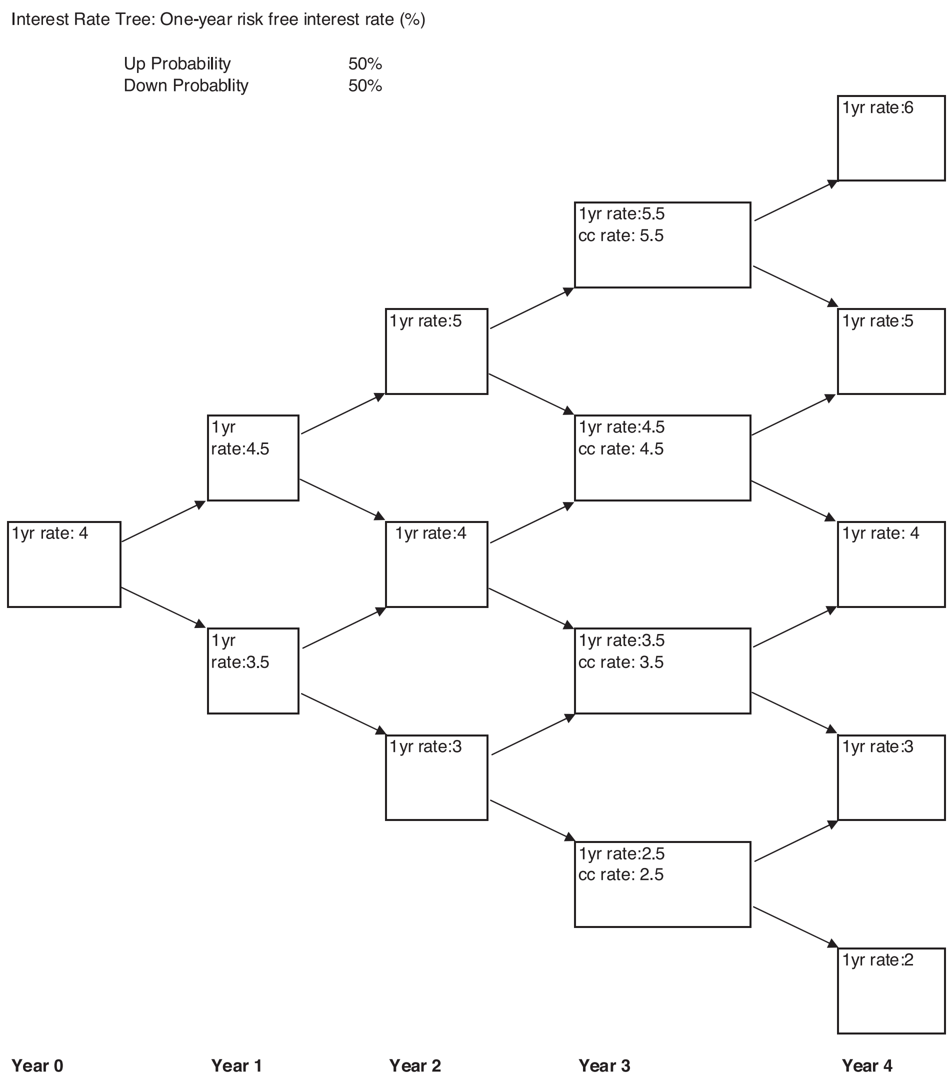
$$\begin{aligned} \text{Px\_of\_4.75} &= 1/2(4.75(1.05) + 104.75/(1.05*1.055)) \\ &\quad + 1/2(54.75(1.05) + 52.375/(1.05*1.045)) \\ &= 99.48. \end{aligned}$$

Because 4.75% did not price to par, we iterate until we find a coupon that does. We repeat this procedure for each node in year 2. All that we require is the current-coupon rate at the nodes that follow in year 3. With this method, we can fill in all the current-coupon rates in year 2.

Now we step back to year 1 and start with the top node. Here again we choose a current-coupon rate; let's say 4.3%. This node will depend on the top two nodes in year 2 and those in turn will depend on the top three nodes in year 3. Using the prepay model, we can compute the prepays at each of the nodes in years 2 and 3 (because we already computed the current-coupon rate at all of those nodes). To value a MBS with a coupon of 4.3% at year 1 we again start at year 3 and do a 1-year discounting to arrive at year 2. In year 2, some prepayment occurs and it reduces the total that we discount from year 3. We discount back to year 1 and obtain a price for the 4.3% coupon MBS. Again, we iterate until we obtain a par price and apply the same methodology to all nodes in year 1.

Finally, we arrive at year 0. When we now compute the value of a 3-year mortgage, the current-coupon rate at year 3 will be immaterial, because the MBS has matured by that time. However, the procedure is the same as that described previously and results in a current coupon at year 0. If we had TBA prices in our stylized world, we would compare the current coupon found at year 0 against the market and adjust the OAS* until the market and the model matched.

EXHIBIT 3
Example of MOATS Back Propagation



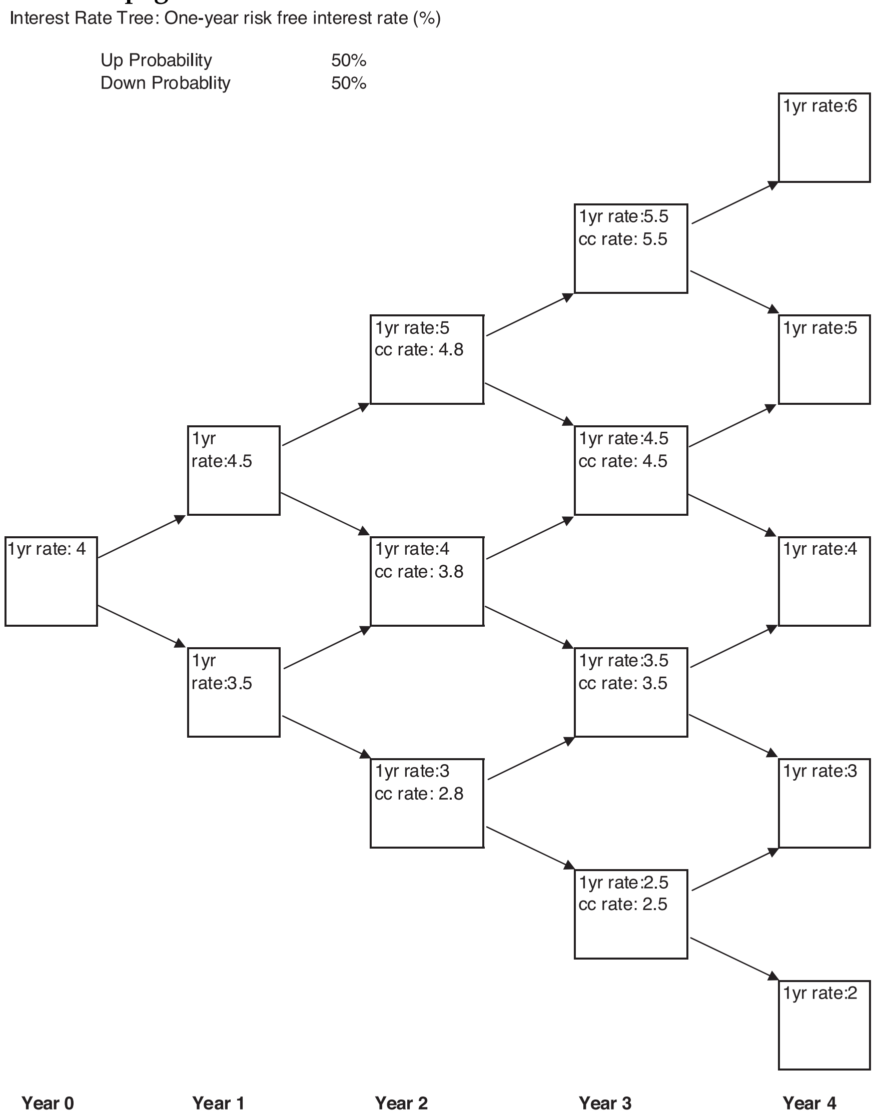
Source: Citigroup.

MOATS in Interest Rate Scenarios

The procedure described in this section produces current-coupon rates everywhere on the tree, with a calculated initial current coupon that matches the market-observed initial current coupon. We can now use the full Citigroup Prepayment Model and Monte Carlo simulation, along with the arrays of MBS current coupons generated by MOATS, to obtain the OAS for any MBS. In particular, let "OAS-full" be the resulting OAS for the current-coupon MBS.

The OAS-full is important when defining interest rate scenarios for use in risk calculations. The difference between OAS* and OAS-full is a function of how well the path-independent prepay model approximates the full Citigroup Prepayment Model. Under various interest rate scenarios, the relationship between the full Citigroup Prepayment Model and the approximation might change, and the difference between OAS* and OAS-full might change as a result. Thus, we assume that in scenarios in which market conditions have changed, that OAS-full is unchanged.

EXHIBIT 4
MOATS Back Propagation to Year 1



Source: Citigroup.

So for the scenarios we follow the procedures outlined above, with the only change that at time 0 we do not seek to match an observable current coupon. Instead, we force the OAS-full to remain unchanged.

To recap, in the base case, the challenge is finding an OAS* and OAS-full to match an observed current coupon. In an interest rate scenario, no current coupon can be observed at time 0. So, the challenge becomes finding a current coupon at time 0 to match the OAS-full of the base case.

EMPIRICAL VALIDATION OF MOATS

The MOATS model is theoretically appealing and consistent with the general OAS framework used in MBS valuation and hedging. However, the ultimate test comes from its performance in practice and how it compares with actual market behavior of mortgage rates and MBSs. In this section we show that the assumptions underlying MOATS are consistent with historical data and that its predictive power is superior to that of traditional methods of deriving mortgage rates.

MOATS Report

The first step in validating the MOATS model is to make its output more transparent. A report, shown in Exhibit 5 and available daily on manifold MB727, on Citigroup's fixed income website, F1 Direct, attempts to do this. This report provides projected mortgage rates computed for 25 bp increments of the 10-year and the 2s/10s slope under different rate and volatility scenarios. The numbers in bold represent the base case. For example, in the top left matrix, the mortgage rate of 5.56% corresponds to the 10-year swap rate of 4.62% and the 2–10-year slope of 63 bp as of the close on May 4, 2005.

Parallel Yield Curve Shift

The columns of the top left matrix in Exhibit 5 show the MOATS-projected changes in the mortgage rate for parallel shifts in the yield curve. For example, an increase of 25 bp in the 10-year swap rate from 4.62% to 4.87% gives a MOATS-projected increase of 26 bp in the mortgage rate, from 5.56% to 5.82%.

A drop of 50 bp in the 10-year swap rate generates a projected drop of 55 bp in the mortgage rate, from 5.56% to 5.01%. This is counter to the traditional belief that mortgage spreads widen when rates rally. However, one of the assumptions underlying this view, that volatilities increase when rates drop, is not a factor here, because the 2f-skew model that we use already incorporates a widening in volatilities as rates drop.

Coupon effects are important and, in fact, explain most of the 5 bp tightening in mortgage spreads. Let's assume that in the base case, 5s are the current coupon, with a price of 99.531 and an OAS of −17 bp.[6] If swap rates rally 50 bp, 4.5s become the new current coupon. If we price 4.5s at the same OAS of −17 bp, with swap rates 50 bp lower, the price is 99.784. At a −17 bp OAS, the coupon that gives us the parity price of around 99.53 is 4.45%, rather than 4.50%, explaining the extra 5 bp drop in the mortgage rate versus swaps. The faster amortization of the 4.5s (they have less IO) makes the 4.5s worth more in a 50 bp rally than the 5s in the base case.

Slope of the Curve

The rows of the top left matrix in Exhibit 5 show the MOATS-projected change in the mortgage rate if the 2–10-year slope changes while the 10-year remains unchanged. For example, if the slope flattens from 63 to 38 bp, with the 10-year swap curve remaining unchanged at 4.62%, the

EXHIBIT 5

MOATS Base Mortgage Rate by 10-Year Swap Rate and 2–10-Year Slope (%)[a]

Two- to Ten-Year

4 May 05 Close

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.22	5.11	5.01	4.91	4.83
4.37	5.49	5.38	5.28	5.19	5.10
4.62	5.76	5.65	**5.56**	5.47	5.38
4.87	6.01	5.91	5.82	5.73	5.65
5.12	6.25	6.16	6.07	5.99	5.90

Up 1% Vol for 4 May 05 Close

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.28	5.17	5.07	4.98	4.89
4.37	5.56	5.45	5.35	5.25	5.16
4.62	5.82	5.72	**5.62**	5.53	5.44
4.87	6.07	5.98	5.88	5.79	5.71
5.12	6.31	6.22	6.13	6.05	5.97

Down 1% Vol for 4 May 05 Close

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.15	5.04	4.95	4.86	4.77
4.37	5.42	5.32	5.22	5.13	5.04
4.62	5.69	5.59	**5.49**	5.40	5.32
4.87	5.95	5.85	5.76	5.67	5.59
5.12	6.19	6.10	6.01	5.93	5.85

4 May 05 Close Using 27 Apr 05 Volatilities

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.22	5.11	5.01	4.92	4.83
4.37	5.49	5.38	5.28	5.19	5.10
4.62	5.76	5.65	**5.56**	5.47	5.38
4.87	6.01	5.92	5.82	5.73	5.65
5.12	6.25	6.16	6.07	5.99	5.91

4 May 05 Close Using 27 Apr 05 CCOAS

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.23	5.12	5.02	4.92	4.84
4.37	5.50	5.39	5.29	5.20	5.11
4.62	5.77	5.66	**5.57**	5.48	5.39
4.87	6.02	5.92	5.83	5.74	5.66
5.12	6.26	6.17	6.08	6.00	5.91

27 Apr 05 Close

10-Yr	0.13	0.38	0.63	0.88	1.13
4.12	5.21	5.11	5.00	4.91	4.83
4.37	5.49	5.38	5.28	5.18	5.10
4.62	5.76	5.65	**5.55**	5.46	5.38
4.87	6.01	5.91	5.82	5.73	5.64
5.12	6.24	6.15	6.06	5.98	5.90

[a]The report shows the mortgage rate (a constant spread of 50 bp is added to the secondary rate) as projected by the Citigroup's MOATS model for varying levels and slopes of the yield curve and volatility scenarios. The change in current-coupon OAS over the past week was −1.0 bp. The changes in the 10-year and in the (the two- to ten-year) slope were −6.0 bp and −2.8 bp, respectively, over the past week.
Source: Citigroup.

mortgage rate is projected to increase from 5.56% to 5.65%. This is not surprising, as higher swap rates for shorter tenors decrease the value of an MBS because the current coupon has to increase to preserve a constant OAS.

Changes in Volatilities

The top middle and top right matrices in Exhibit 5 show the MOATS-projected impact on mortgage rates of a parallel change in volatilities. For example, for May 4, 2005, closes, a parallel 1% increase in volatilities is projected to lead to an increase in the mortgage rate from 5.56% to 5.62%. A rise in volatility increases the option cost, and because the OAS is constant, the nominal spread (which is approximately the OAS plus the option cost) rises producing a higher mortgage rate.

The other tables in Exhibit 5 display additional diagnostics. For example, the bottom middle matrix takes last night's curve and volatilities but uses the current-coupon OAS from the previous week. The change in the current-coupon OAS over the week should explain the difference in the mortgage rate between the top left and the bottom middle matrices.

Is the Constant OAS Assumption Valid?

The MOATS model assumes that the current-coupon OAS remains constant through time and for different rate environments. Strictly speaking, this is not a correct assumption because the OAS tends to vary from day to day. However, to paraphrase Winston Churchill's comment on democracy, it may be the least worst assumption. Exhibit 6 shows the current-coupon OAS versus the 10-year swap rate over the past 4 years. While the OAS has varied widely, there is little systematic dependence on the level of rates, with the correlation a weak −0.14.

Exhibit 7 provides further insight into OAS behavior. The top panel shows a scatterplot of current-coupon OASs versus 10-year swap rates from the beginning of 2003 to May 2005. As in Exhibit 6, there is no clear relationship between OASs and swap rates. The lower panel shows the current-coupon OASs and 10-year swap rates plotted over time for the same period. The OAS was fairly stable from the beginning of 2003 to the fall of 2004, even though interest rates varied considerably over this period. The OAS then declined over the last few months of 2004, though rates were stable, but has been stable since early 2005. Thus, the OAS often seems to be stable for long periods of time, until a *regime change* occurs, after which the OAS is again stable for a while. Because such regime changes are difficult to predict, we feel that assuming a constant OAS is better than the alternatives.

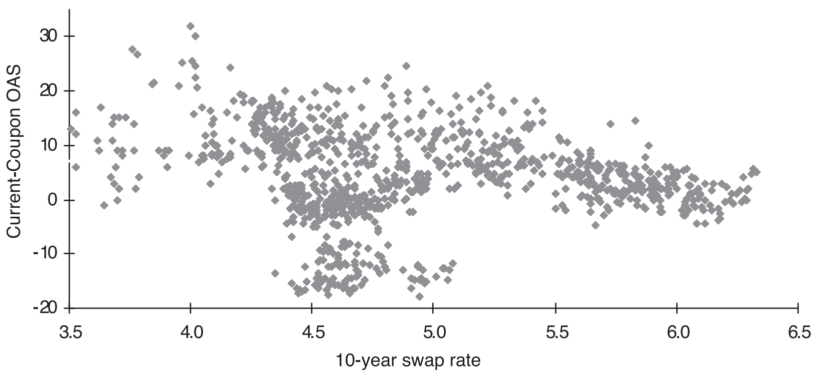
Is the MOATS Dependence on Volatility Consistent with History?

Theory suggests that in addition to the level and slope of the yield curve, the mortgage rate should also depend on volatility. For example, an increase in volatility tends to raise the option cost of current-coupon mortgages, resulting in higher mortgage rates. This effect can be observed empirically, as displayed in Exhibit 8 where we regress the current-coupon spread to the 10-year against a swaption volatility.

Exhibit 8 indicates that a 1% increase in the 5×10 swaption volatility has corresponded to a roughly 7.2 bp increase in the mortgage rate over the past 3 years. The MOATS report currently projects a 6–7 bp increase in the mortgage rate for a one vega increase in volatilities (see Exhibit 5). For example, the mortgage rate in the base case (10-year, 4.62%; 2s/10s, 63 bp) for the May 4, 2005 close (upper left matrix) is 5.56%, while for the same closing date and a 1% bump up in volatility (matrix immediately to the right), the mortgage rate increases to

EXHIBIT 6

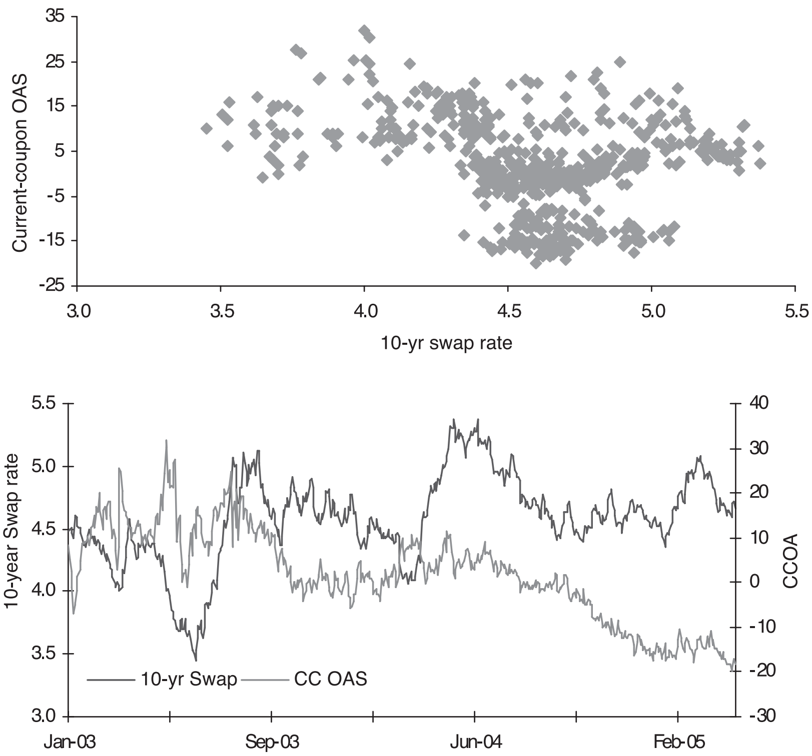
Conventional Current-Coupon OAS versus the 10-Year Swap Rate, May 2001–2005



Source: Citigroup.

EXHIBIT 7

Conventional Current-Coupon OASs versus 10-Year Swap Rates (Top Panel) and Current-Coupon OASs and 10-Year Swap Rates Across Time (Bottom Panel), January 2003–May 2005



Source: Citigroup.

5.62%. *Hence, the MOATS-predicted dependence of mortgage rates on vols is close to observed behavior.*

The Predictive Power of MOATS

Ultimately, any mortgage rate model must be measured by how well it predicts mortgage rate changes as yield curve and volatilities change. For the "constant spread to the 10-year" model, this performance is easy to measure. It is simply the change in the spread to the 10-year from one day to the next. For example, if the spread to the 10-year tightens by 5 bp, then the constant spread to 10-year model overestimated the mortgage rate change by 5 bp, and we can say that the error (or residual) is 5 bp.

For MOATS, the predictive power is determined by the validity of the constant current-coupon OAS assumption. Changes in this OAS would result in under- and/or overpredictions of the mortgage rate, which are the residuals. Exhibit 9, which compares the standard deviation of the errors of MOATS and the constant spread to 10-year models over the past year, indicates that the constant OAS assumption generally performs better than the traditional constant spread approach. As one can see, the MOATS method has a lower standard deviation of errors and, hence, has been a superior predictor over this period.[7]

IMPACT OF MOATS ON MBS VALUATION

In this section, we discuss the effect of MOATS on mortgage valuation measures. We will start with examining the impact on OASs, durations, and convexities, and then discuss partial and volatility durations, both for pass-throughs and mortgage derivatives. As we will see, the major impact of MOATS is in the accurate calculation of partial and volatility durations, where traditional methods can give misleading results.

OASs, Durations, and Convexities

Exhibit 10 shows OASs, durations, and convexities for several conventional TBA pass-throughs, using the traditional spread to 10-year swap and MOATS methods for deriving mortgage rates.

The impact of MOATS on pass-through OASs is negative, but generally small, with OASs declining about 1–2 bp for discounts and 2–4 bp for premiums. The mortgage rate generated by the traditional spread method depends solely on the 10-year swap, while the one from the MOATS model depends on the entire swap curve and, hence, incorporates the higher volatilities associated with the shorter end of the curve.

The durations and convexities are hardly affected. In fact, MOATS durations are generally a little smaller, but the difference is generally well under 0.1 year, and so, often does not show up when the durations are rounded to the nearest tenth of a year. The small difference is caused mainly by the greater projected decline in the mortgage rate if the swap curve is shocked down 25 bp (from Exhibit 5, MOATS projects the mortgage rate to decline 28 bp, versus 25 bp for the constant spread method). This leads to a great prepayment-induced price compression effect, so that the projected price increase is lower with MOATS.[8]

Exhibit 11 shows OASs, durations, and convexities for several IOs. As might be expected, the MOATS effect is much larger, with OASs dropping by 30–40 bp. This larger drop simply reflects the greater sensitivity of the IO to mortgage rate volatility.

The more negative durations and convexities of the IOs using MOATS again reflect the larger mortgage rate changes projected by MOATS if rates are shocked up and down 25 bp. As an illustration, the projected changes of the Trust 354 IO under both methods are shown in Exhibit 12. The price using MOATS declines more in the down 25 bp scenario because of the larger mortgage rate drop under MOATS, and it increases more in the up 25 bp case because of the (slightly) larger mortgage rate increase.

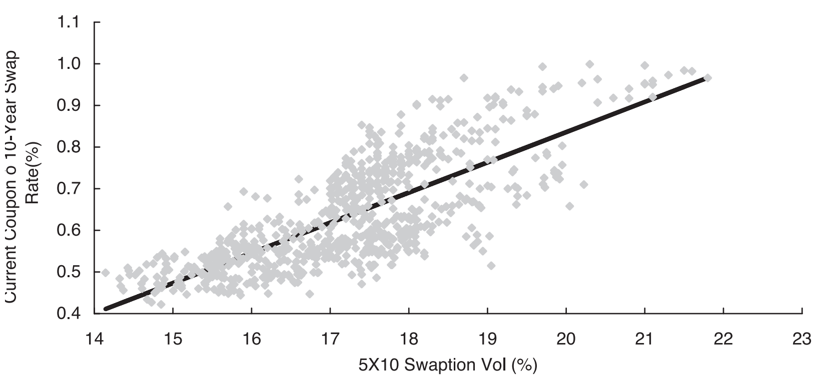
Partial Durations

Exhibit 13 displays effective and partial duration for several mortgage pass-through coupons. Because the main difference between the two methods is in the projected mortgage rate, the differences in partials are most noticeable for premiums. Taking 6.5s as an example, note that MOATS gives a *smaller* partial duration for the 5-year part of the curve, but generates a *larger* partial duration for the 10-year segment. Why is this? The discount rates in the ± 25 bp scenarios used to calculate the partials are the same under both methods. The differences arise from differences in the mortgage rates:

1. For the 10-year partial duration, the constant spread method projects larger changes in mortgage rates (in fact, the same as the change in the 10-year swap rate) than MOATS. Let's first consider the down 25 bp scenario. The projected price change consists of two parts: (1) the positive change due to lower discount rates, and (2) the adverse impact of higher speeds. The second effect will be lower for MOATS, so that the projected price change down 25 bp will be higher for MOATS. There is a similar but lower impact in the up 25 bp scenario, generating a larger 10-year partial using MOATS.
2. For the 5-year partial, the constant spread method projects no mortgage rate change in the scenarios,[9] while MOATS forecasts some change. Hence, similar to the 10-year partial duration example, MOATS gives a larger 5-year partial.

Exhibit 14 displays effective and partial durations for several Trust IOs. While the differences in partial durations between MOATS and the constant spread to 10-year swap rate are meager for pass-throughs, the differences are more glaring for strips. The

EXHIBIT 8
A 1% Increase in 5 × 10 Swaption Volatility Corresponds to a 7.2 bp Increase in the Mortgage Rate, May 2002–2005



Source: Citigroup.

EXHIBIT 9
Standard Deviation of Daily Changes in Current-Coupon OAS (MOATS) versus Spread to 10-Year Swap Rate, May 2004–2005

Period	MOATS	Constant Spread to Ten-Year Swap
May 04-May 05	1.33	2.22

Source: Citigroup.

EXHIBIT 10
OASs, Durations, and Convexities for TBAs, May 4, 2005

		OAS (bp)			Effective Duration (Years)			Effective Convexity		
Coupon (%)	Price	Spd/Swp	MOATS	Diff	Spd/Swp	MOATS	Diff	Spd/Swp	MOATS	Diff
4.5	96.688	-11	-12	-1	5.2	5.2	0.0	-1.1	-1.0	0.1
5.0	99.156	-16	-18	-2	4.3	4.3	0.0	-1.9	-1.9	0.0
5.5	101.156	-19	-21	-2	3.1	3.1	0.0	-2.7	-2.7	0.0
6.0	102.781	-20	-23	-3	1.9	1.8	-0.1	-2.6	-2.6	0.0
6.5	104.094	-7	-10	-3	1.6	1.6	0.0	-2.1	-2.1	0.0

Source: Citigroup.

EXHIBIT 11
OASs, Durations, and Convexities for IOs, May 4, 2005

		OAS (bp)			Effective Duration (Years)			Effective Convexity		
Trust	Price	Spd/Swp	MOATS	Diff	Spd/Swp	MOATS	Diff	Spd/Swp	MOATS	Diff
PC 227 (5s)	23.79	-124	-164	-40	-31.8	-32.7	-0.9	-24.5	-25.5	-1
TR 354 (5.5s)	21.46	-201	-235	-34	-53.8	-55.9	-2.1	-24	-26.3	-2.3
TR 344 (6s)	19.39	41	10	-31	-41.9	-43.5	-1.6	-13.4	-13.6	-0.2

Source: Citigroup.

EXHIBIT 12
Projected Price Changes for Trust 354 IO for a 25 bp Parallel Yield Curve Change, May 4, 2005

	-25bp	0bp	25bp
Spread/Swap	18.224	21.46	24.364
MOATS	18.045	21.46	24.466

Source: Citigroup.

partials to the 10-year are less negative for MOATS, as the model diffuses the projected change in the mortgage rate across the curve, rather than concentrating it at the 10-year point, as the constant spread method does. In contrast, the partial durations to other parts of the curve are lower for MOATS, as it projects some change in the mortgage rate while the constant spread method projects no change.

Hedging Implications for IOs

The choice of the mortgage rate process becomes particularly important for hedging the curve exposure of IOs. For instance, consider a bear flattener in which the short end sells off significantly while the long end drops marginally. In this case, longer-dated cash flows are discounted at a lower rate, increasing the value of the IO.

EXHIBIT 13

Partial Durations of Conventional 30-Year Pass-Throughs, May 9, 2005

	Price	Model	Eff. Dur.	Two-Year	Five-Year	Ten-Year	30-Year
30-Year FN 4.5	96.13	Constant Spread to Ten-Year Swap	5.3	0.6	1.3	2.6	0.9
		MOATS	5.3	0.6	1.3	2.6	0.9
30-Year FN 5	98.66	Constant Spread to Ten-Year Swap	4.4	0.7	1.2	1.8	0.7
		MOATS	4.4	0.7	1.1	2.0	0.6
30-Year FN 5.5	100.63	Constant Spread to Ten-Year Swap	3.4	0.8	1.0	1.2	0.4
		MOATS	3.4	0.8	0.9	1.4	0.3
30-Year FN 6	102.41	Constant Spread to Ten-Year Swap	2.1	0.8	0.7	0.5	0.1
		MOATS	2.1	0.7	0.6	0.7	0.1
30-Year FN 6.5	103.84	Constant Spread to Ten-Year Swap	1.8	0.8	0.9	0.3	0.1
		MOATS	1.8	0.7	0.6	0.5	0.0

Source: Citigroup.

EXHIBIT 14

Partial Durations for Trust IOs, May 9, 2005

	Price	Model	Eff. Dur.	Two-Year	Five-Year	Ten-Year	30-Year
PC 227 (5s)	24.59	Constant Spread to Ten-Year Swap	-29.4	5.4	5.1	-31.1	-8.5
		MOATS	-30.1	3.6	-3.9	-18.6	-11.3
TR 354 (5.5s)	22.67	Constant Spread to Ten-Year Swap	-50.3	6.5	4.3	-50.9	10.1
		MOATS	-51.9	1.3	-9.5	-29.8	-14.5
TR 344(6s)	20.38	Constant Spread to Ten-Year Swap	-39.8	5.2	3.7	-40.9	-7.6
		MOATS	-41.0	1.2	-7.4	-24.2	-10.8

Source: Citigroup.

However, a flatter curve results in lower forward mortgage rates, leading to faster prepayments along the forward curve and decreasing the value of the IO. Given the leveraged exposure of IOs to prepayments, the impact of lower forward mortgage tends to dominate the discounting effect and should lead to a decrease in the value of the IO. In this example, MOATS should do a better job of hedging the curve exposure of the IO than 10-year swap rate method, which would concentrate most of the duration at the 10-year point making the impact of faster prepayments overwhelmingly strong.

Volatility Durations

Exhibit 15 shows the volatility durations with and without MOATS. Volatility durations are defined as the exposure of the instruments to a parallel shift of the whole volatility surface by −1%.

Exhibit 15 highlights the subtle way in which volatility changes affect MOATS. For a constant spread to swaps mortgage rate model, the effect of volatility increases is easy to understand. Both P/T and IO are short an option (they have negative gamma (almost) everywhere), so an increase in volatility hurts both the P/T and the IO, and this is reflected in the volatility duration numbers.

However, in switching from the constant spread to swaps to MOATS, an offsetting factor is introduced. Because MOATS assumes that the current-coupon OAS is constant in all scenarios, any increase in volatility depresses the price of the current coupon, increasing the current-coupon rate. This has the effect that as volatility rises so does the mortgage rate, helping both the P/T and the IO. Hence, volatility duration decreases for both the P/T and

EXHIBIT 15
Volatility Durations With and Without MOATS for P/T and IO

	MOATS	Constant Spread to Swaps
FN 4.5	0.247	0.253
FN 5.0	0.236	0.272
FN 5.5	0.202	0.271
FN 6.0	0.127	0.214
FN 6.5	0.101	0.188
PC 227	-0.461	1.798
TR 354	-2.149	1.469
TR 344	-1.680	1.013

Source: Citigroup.

the IO. Because the IO is very sensitive to the mortgage rate, volatility duration is negative for the IO when using MOATS. Thus, as volatility increases, the mortgage rate increases, in turn making the IO more valuable.

CONCLUSION

To our knowledge, MOATS represents the first effort to model the mortgage rate in a manner that is consistent with the pricing of mortgages. Mortgage pricing takes into account interest rate uncertainty and enforces the absence of arbitrage by fitting the model to take into account swaps, caps, and swaptions. MOATS does the same for the mortgage rate.

MOATS can be opaque and difficult to understand, but we hope that the description in this report is sufficiently detailed to allow users to understand the nuts and bolts of the model. In addition, a daily report published on MB727 shows MOATS mortgage rate predictions for many scenarios and should be used by investors to monitor how it is working.

MOATS does make some assumptions, notably that the current-coupon OAS stays constant in all scenarios and across all time. While the current-coupon OAS does vary over time, empirical studies have shown it to be uncorrelated with interest rates. Hence, the assumption that the current-coupon OAS never changes is valid, on average.

The MOATS feature most likely to be contentious is how volatility changes affect mortgage rates. MOATS assumes that as volatility increases, the mortgage rate rises as well. This feature can also be observed empirically, further validating the model.

The effect that MOATS has on valuation and risk of MBS varies, depending on market conditions. However, a significant and consistent change is the shortening of volatility durations in P/T and IOs.

In conclusion, MOATS not only has theoretical merits, but also outperforms the constant spread to swaps model in predicting mortgage rate changes.

ACKNOWLEDGMENTS

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ENDNOTES

[1]A variation on this approach is to assume that the spread reverts to some assumed long-term average over time. For example, if we believe that the spread between mortgage and 10-year swap rates has an average or "fair" value of 120 bp, and the current value of 130 bp incorporates a temporary 10 bp widening, we assume that the spread will tighten by 10 bp over some specified number of months.

[2]See "Mortgage Option-Adjusted Term Structure Model," *Bond Market Roundup: Strategy, Citigroup*, January 21, 2001.

[3]The report is on manifold MB727. A description of the report is provided later in this article and is included in "Analysis of the MOATS Method," *Bond Market Roundup: Strategy*, March 24, 2005.

[4]See our November 20004 publication Guide to Mortgage-Backed Securities for a general introduction to MBSs. A more detailed discussion of prepayments may be found in our April 2004 publication, Anatomy of Prepayments. Both publications are available on Citigroup's Fixed Income web site, FI Direct.

[5]If we had to continually price 30 mortgages we would have to go forward in time indefinitely.

[6]These numbers approximate the market at the time of writing. We thank Robert Young for providing this example.

[7]We have not developed regression models for the OAS as a function of several points on the yield curve and volatilities, so could not test MOATS against such models. However, conversations with other analysts indicate that the coefficients of such a regression model become misspecified over time.

[8]The corresponding effect in the +25 bp case is minimal.

[9]This is not quite true because the Citigroup Prepayment Model uses shorter mortgage rates in addition to 30-year ones, and the shorter rates are assumed to depend on the 5-year part of the curve. However, the effect is minor.

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