Markets Quantitative Analysis
Citi MBS Focus

MARCH 2010

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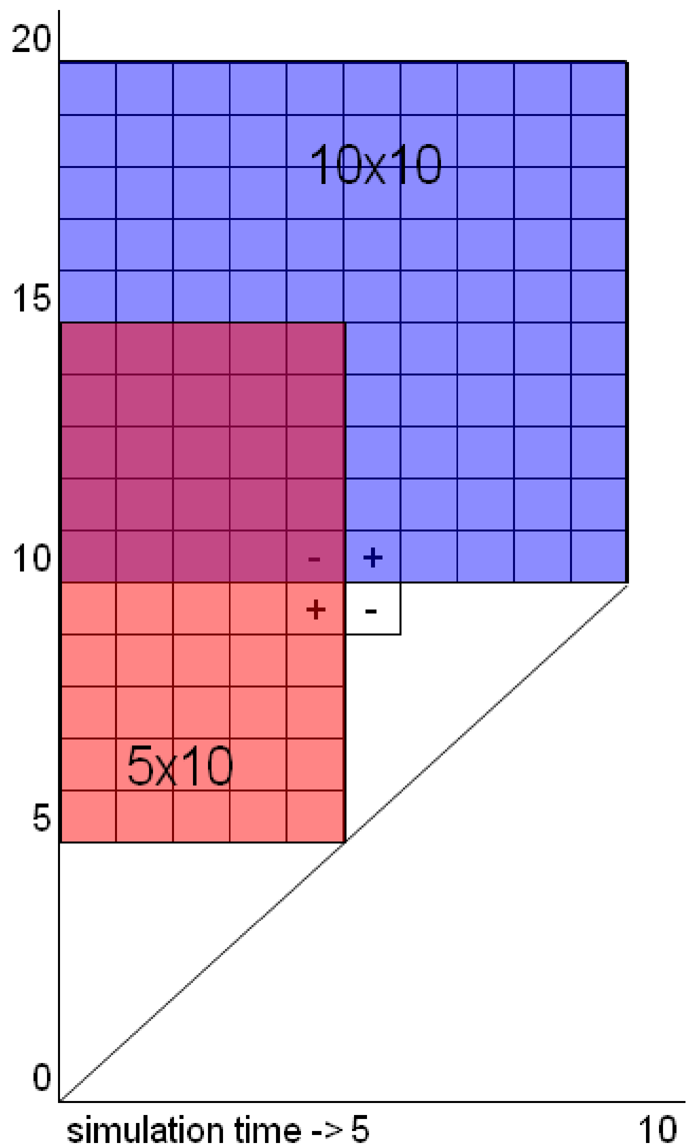
Introducing the Citi LMM Term Structure Model for Mortgages



Introducing the Citi LMM Term Structure Model for Mortgages

This commentary has been prepared by Markets Quantitative Analysis ("MQA"), which is part of Citigroup Global Markets' sales and trading operations.

Summary

- New term structure model improves control over the volatility and correlations structure of forward LIBORs. More realistic correlations of swap rates.
- More accurate calibration to the entire volatility surface, with consequences for serial correlation of swap rates and mortgage option volatility.
- MOATS re-implemented to work with LMM, retaining all desirable properties.
- Wider OAS and longer durations than in the current model; comparable partial durations and vegas.

Contents

Acknowledgements. The authors thank Ranjit Bjattacharjee and Mikhail Teytel for many vigorous discussions.

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The LMM Model

The Citi Libor Market Model (LMM) is a term structure model that deals directly with market-observable objects, rather than the infinitesimal short rate, as in the current production (2fskew) model. It belongs to a class of term structure models previously known as BGM, but "Market Model" is the more current terminology. The new model has two random factors, as does the 2fskew, but the meaning of these factors is different. In LMM the factors essentially constitute the principal components of the forward LIBOR movements. The 2fskew stochastic factors drive the short rate and the slope of the curve.

- The main appeal of LMM is its control over the volatility and correlation structure of forward rates.

LMM defines an evolution of forward LIBOR rates spanning the entire forward curve, making it easy to obtain other quantities of interest, such as the swap rates. Concretely, the term structure of interest rates at time t can be expressed in terms of the forward rates $L_i(t) = L(t, T_i, T_{i+1})$ from T_i to T_{i+1}, for a fixed set of maturities T_i. The terminal value $L_i(T_i)$ is just a LIBOR of tenor $T_{i+1} - T_i$. Each $L_i(t)$ is a stochastic process governed by a law of type

(*) $$\frac{dL_i}{f(L_i)} = \mu_i(t)dt + \sigma_i(t)dW_i,$$

with W_i not necessary independent Brownian motions, one for each LIBOR maturity.

A crucial advantage of LMM lies in its ability to specify the instantaneous correlations $\rho_{ij}(t)$ among the forward rates, as well as the general shape of the volatility structure, which defines the local volatilities σ_i during the model's calibration. These, in turn, determine the drift terms μ_i through a no-arbitrage condition. The only remaining item is the function *f*, which controls the skew; for example, $f(L) = L$ defines the log-normal case, and $f(L) = 1$ yields normally-distributed rates.

- The correlation and volatility structure of forward LIBORs determines the corresponding quantities for the forward swap rates. A good choice of this structure for LIBORS results in more realistic swap correlations.

This is because the forward swap rates are naturally linear combinations of forward LIBOR rates,

$$S^{N\times M}(t) = \sum_{i=\alpha}^{\beta} w_i(t)L_i(t),$$

and a well-established LMM approximation[1] allows one to freeze the weights and obtain swap volatilities and correlations in terms of the ρ_{ij} 's and σ_i 's .

The following examples highlight LMM's control over correlations.

- The LMM model maintains correlations between forward 2Y and 10Y swap rates, and between 1M LIBOR and the 10Y swap rate close to their long-term historical levels (94% and 86%, respectively).[2]
- The model is consistent with historical observations of forward swap rates, which retain a substantial degree of correlation even at increasingly distant maturities: in LMM the correlation between the forward LIBORs of different maturities falls to a positive floor, as the maturity gap grows.
- The model's volatility parameterization is flexible enough to enable a close fit to the entire volatility surface. This feature, together with better control over the forward LIBOR correlation and volatility structure, is the main source of differences with the current production model.

[1] See Rebonato's formula on p.248 of *Interest Rate Models: Theory and Practice*, D. Brigo and F. Mercurio, Springer 2001.
[2] In 2fskew the first correlation is close, but the second is substantially lower for the first 5 years of simulation.

There is a choice of several realistic skews; here we focus on the LMM Skew, representing the long-term historical average of the ATM skew. The LMM model produces LIBOR and swap rates that are non-negative by design with every choice of the skew.

As with most Market Model implementations, the Citi LMM is entirely Monte-Carlo-based, unlike the 2fskew, which employs Monte Carlo sampling from a lattice. This has some advantages by preventing unwanted periodicities, though it also creates a challenge for MOATS, Citi's approach to arbitrage-free mortgage rate modeling, which is lattice-based in 2fskew. The new version reproduces all of the desirable features of MOATS in a pure Monte Carlo setting.

Comparison with the Production Model

The following table (Figure 1) summarizes the main advantages of LMM over the current production model.

Figure 1. LMM vs. 2fskew

LMM	2fskew
Built on market-observable forward LIBORs, with a straight-forward connection to swap rates.	Built on the less intuitive short rate.
Calibrates to full swaption volatility surface, achieving a close overall fit.	Calibrates to x10 swaptions and caps. In the current rates environment, typically, elevates short volatility, leading to lower serial correlations among the 10Y swap rates
Ability to specify realistic volatility and correlation structure of simulated rates.	Less control over correlations and the shape of forward volatility. Broadly speaking, 2fskew inflates volatilities and undervalues correlations.
More stable calibration over time.	
Greater flexibility in the skew implementation.	
Non-negative rates with every choice of the skew.	Some negative rates are possible, though this is not a significant effect.

Source: Citi

These features are discussed in detail in the rest of the article. We conclude this section by mentioning which differences are most relevant to the model's end user.

Main sources of valuation differences between LMM and 2fskew:

- Lower serial correlation among the 10Y swap rates in 2fskew.
- Somewhat different skews.

Calibration Fit and Stability

LMM is calibrated to the entire swaption volatility surface and achieves a close fit (under 1% volatility RMSE on a typical day), see Figure 2. By contrast, the 2fskew model in production is calibrated to the 10Y tails only, plus a set of caps. Short volatilities in 2fskew can widen significantly from their market values, partly due to incomplete swaption volatility surface information, and partly due to a misalignment between the cap and swaption markets.

Figure 2. LMM Skew Calibration Fit on 16 Feb 2010

Vol Diff (%)	tenor						
Expiry	1	2	3	4	5	7	10
1	-1.3	-0.2	0.0	0.2	0.0	0.1	-0.2
2	-0.2	-0.1	0.2	0.3	0.3	0.1	-0.2
3	-1.2	-0.4	0.0	0.2	0.1	-0.1	-0.2
4	-0.3	0.0	0.1	0.1	0.0	-0.3	-0.4
5	0.0	0.1	0.1	0.0	-0.1	-0.3	-0.4
7	0.0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2
10	0.0	-0.1	0.0	0.0	0.0	-0.1	-0.2

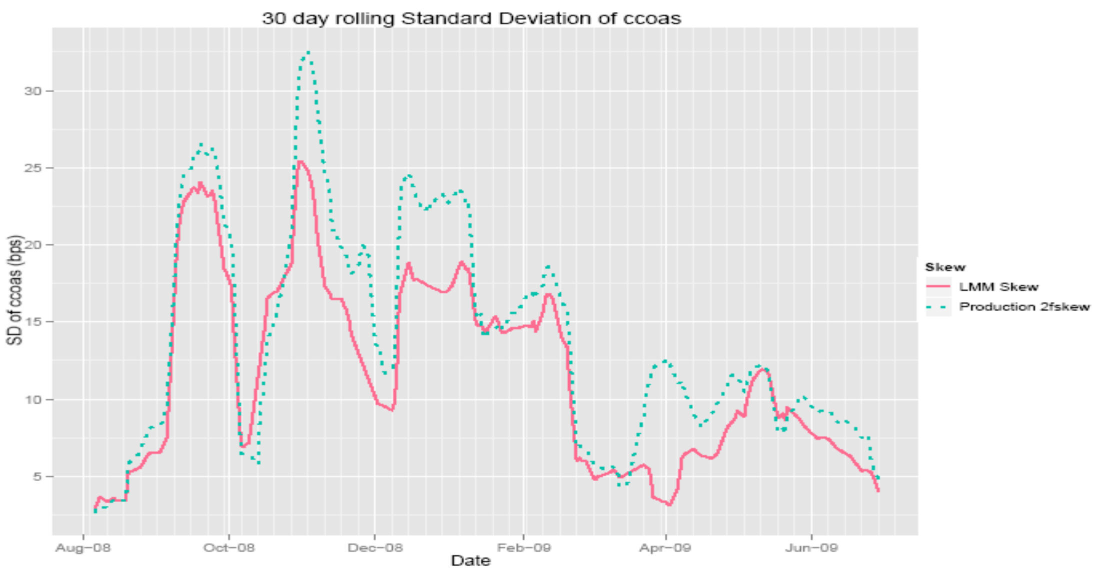
Source: Citi

Note the relatively balanced nature of the LMM Skew fit, with the over- and under-calibration distributed fairly evenly.

Inflated short volatility in 2fskew calibration must have consequences, since an earlier Citi study[3] shows that MBS are exposed to the entire volatility surface. The next section explains the mechanism by which the inflated short volatility impacts the mortgage option embedded in a pass through.

It is also important to know how stable the calibration is over time. Figure 3 compares the two models by calculating the 30-day rolling standard deviation of the Current Coupon OAS, an essential ingredient in the MOATS calculations.

Figure 3. Stability of Current Coupon OAS in the Two Models



Source: Citi

CCOAS is observed over a year that included the most dramatic period in the market history, and LMM Skew shows materially lower volatility than 2fskew across the entire period.

Serial Correlation of CMS10Y and the Mortgage Option

In this section we will explain how short volatility influences the mortgage option embedded in a pass through.

- Accurate calibration to short volatility is important even when the mortgage rate is modeled as a constant spread over 10Y swap rate, because it impacts the serial correlation of the 10Y swap rate – an underappreciated factor affecting all MBS. In turn, serial correlation affects the mortgage option.
- It is even more important with MOATS, which makes the mortgage rate depend on shorter-tenor rates
- Prepayment models include features that magnify the effects of serial correlation, and of the short volatility itself.

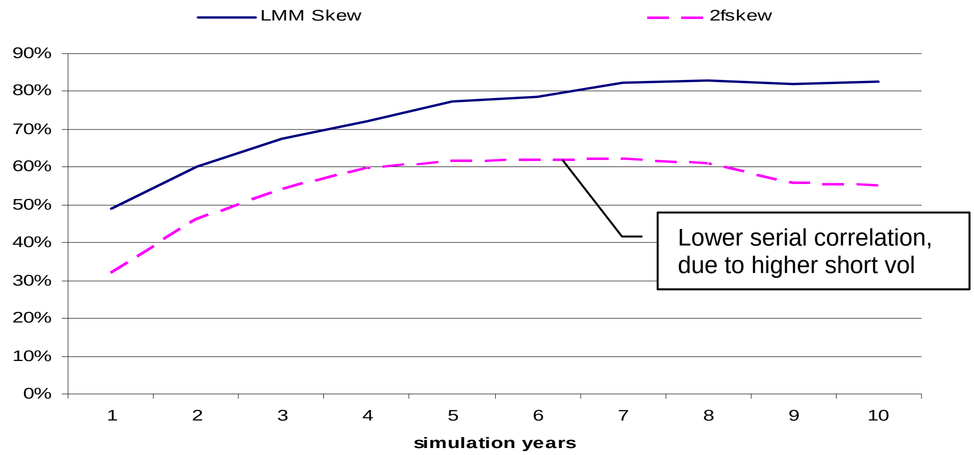
To focus on the key issues, assume that the mortgage rate is at a fixed spread to the 10Y swap rate. Then one may think that an error in calibrating the swaptions of other tails should not seriously affect MBS valuation. A pass through has a set of embedded prepayment options, exercisable at multiple dates. Each option taken individually depends only on the mortgage rate, i.e. on the 10Y swap rate, at the exercise date. And the volatilities of the x10Y rates achieve a close fit to the market in both 2fskew and LMM. But calibrating to the x10Y swaptions does not fix the **serial correlation**, or the *autocorrelation*, of the simulated 10Y swap rates at different horizons. In Appendix 1 we show that

- shorter-expiry, shorter-tenor swaption volatility lowers the serial correlation of the 10Y swap rate.

[3] *Vega Partials of Mortgage Securities*, B. Radak and R. Bhattacharjee, Citigroup, February 2005.

Elevated short volatility in 2fskew leads to commensurate reduction of the 10Y swap rate's serial correlation. For example, setting the lag to 5 years, the correlation difference between the two models can grow to 20-25% (see Figure 4).

Figure 4. Serial Correlation of the 10Y Swap Rate at 5-year Lag in the Two Models on 16 Feb 2010



Source: Citi

The reason this is relevant to MBS pricing is straight-forward: the bond's value depends not only on volatility, but also on the joint distribution of the mortgage rates at all possible refinancing dates. And

- serial correlation among the 10Y swap rates at different horizons "helps" the bond by reducing the overall value of the embedded option.

A simple example illustrates this point. Suppose there are only two possible refinancing dates, t_1 and t_2. Assume also that the entire balance is either prepaid in full, if the prepayment option is in the money, or not exercised. If the 10Y swap rates S_1 and S_2 at these two dates are perfectly correlated, then those Monte Carlo paths which are in the money at t_1 are still in the money at t_2. But the balance remaining after the exercise at t_1 is 0. And the other paths have non-zero balance at t_2, but are not in the money. Thus, perfect correlation between S_1 and S_2 destroys the value of the option at t_2. On the other hand, if S_1 and S_2 are independent, then a Monte Carlo path not in the money at t_1 still has a chance of being in the money at t_2 with a non-zero balance, and so the total embedded option value is greater than just the option at t_1.

This behavior is aggregated over many loans. The result of lower correlation between the rates S_1 and S_2 is an increase in the notional with which the simulation paths enter in-the-money states at time t_2. In turn, the higher volume of prepayments shortens the average path-wise WAL of the bond.

Volatility

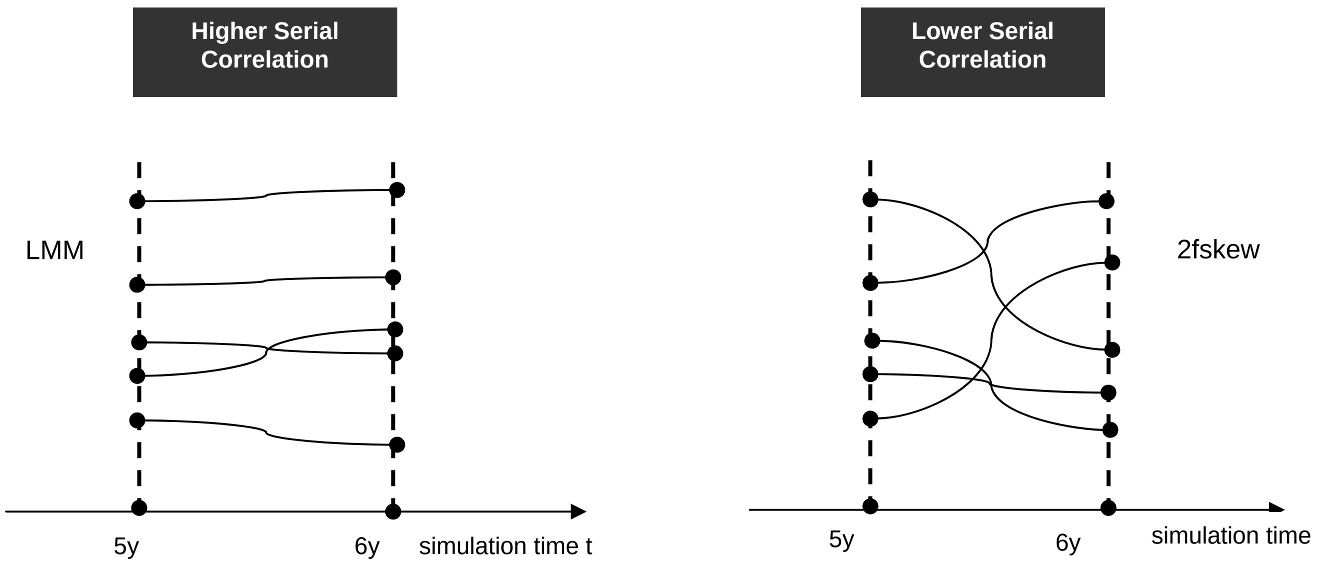
Continuing as above, with just two possible refinancing dates, the overall option value can be viewed as a sum of two European options exercising on these dates, with weights reflecting the notional to which the option applies. Evidently, the weight at t_2 must be lower for the model with higher serial correlation. If we simplify the total optionality into a single European put, the same weights can be used to estimate its effective volatility, since the option value is approximately linear in volatility, at least near at-the-money. Thus,

- higher serial correlation, as in LMM vs. 2fskew, leads to lower effective volatility of the embedded mortgage option.

More on Serial Correlation

Moving on from just two possible refinancing dates, the general case is similar, if one focuses on two horizons, say 5y and 6y. Figure 5 illustrates Monte Carlo paths for 10Y swap rates simulated between these horizons, exhibiting higher or lower serial correlation.

Figure 5. Visualizing Serial Correlation



Source: Citi

Higher serial correlation means that the rates do not change much from 5y to 6y. Therefore, any path in the money at 6y was likely in the money already at 5y; hence, little new prepayment can be expected. Conversely, lower serial correlation makes it likelier that a high-rate out-of-the-money path at 5y drops in the money by 6y, creating new refinancing opportunities.

The Role of the Prepayment Model

Before leaving the issue of the overcalibrated short volatility in 2fskew, we should mention that certain features of the prepayment model and of MOATS magnify its effects. Most important is the burnout, which postpones some of the prepayment, and hence gives the serial correlation among the CMS10Y rates more time to have an impact. Also, the Citi Prepayment Model uses the 1Y rate alongside CMS10Y, making the inflated 1Y volatility directly contribute to the overpricing of the mortgage option. Finally, Citi's MOATS method of projecting mortgage rates involves all tenors, further boosting the effects of elevated short volatility.

The Volatility Skew

Swaption markets have long recognized the existence of the volatility skew: the dependence of the implied volatility on the strike (OTM skew) and on the current level of the forward swap rate (ATM skew).

The importance of ATM and OTM volatility skews to mortgage valuation and hedging is well established.[4] In particular, the duration of a security is directly affected by the model's ATM skew:

(**)
$$Dur_{skew} \approx Dur_{\log normal} + (\text{ATM Skew Slope}) \times (Vol\ Dur)\ .$$

Both ATM and OTM skews arise from the same source: the skewness (in the standard statistical sense) of the rates distribution.

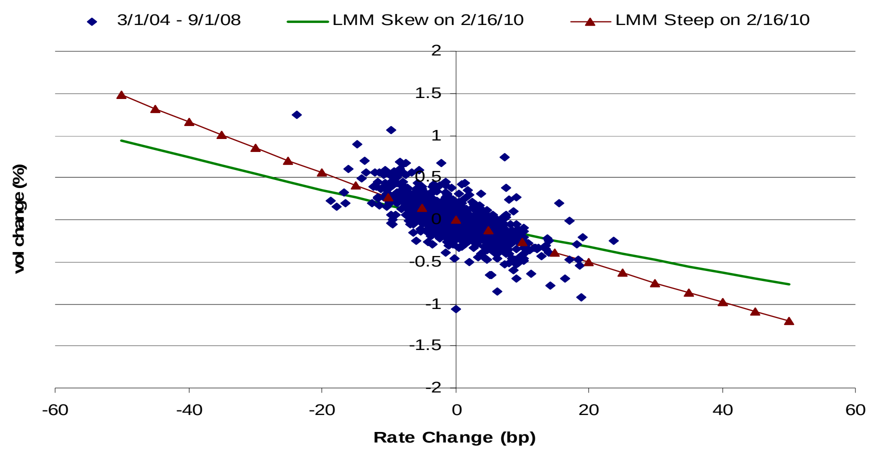
LMM is implemented with a choice of three skews:

- The default choice, **LMM Skew**, and **LMM Steep** approximate the historical ATM skew. See Figure 5 for their current relationship to the 3.5 years preceding the crisis.
- One more choice, **LMM Flat,** can be used to gauge the skew impact, or to express a view.

Figure 6 presents some historical data that went into the choice of LMM Skew.

[4] See *Volatility Skew and the Valuation of Mortgages*, R. Bhattacharjee, B. Radak, R.A. Russell, Citigroup, December 2005.

Figure 6. The 5x10 ATM Skew: Pre-Crisis History and LMM

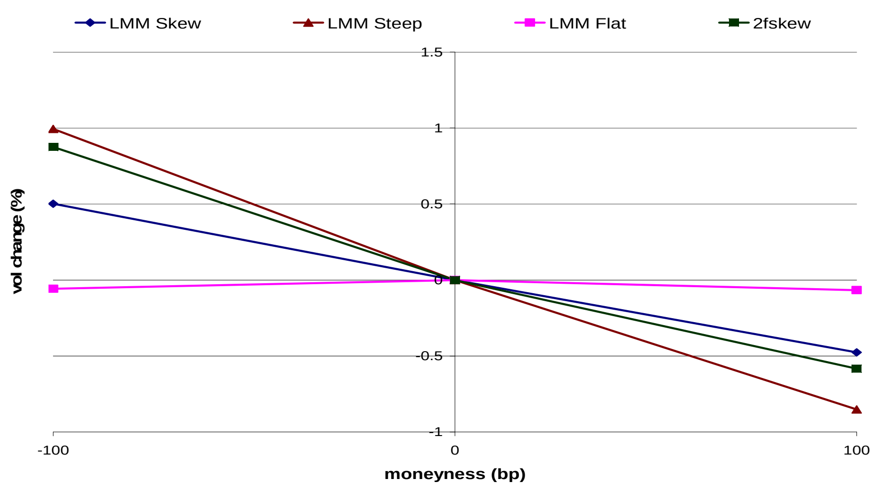


Source: Citi

One may hope to regularly calibrate the term structure model to the market (OTM) skew, but this is neither feasible nor desirable. The OTM skew is highly variable and poorly observable. However, a historically-based choice of the ATM skew, as above, determines the internal model parameters which, in turn, define the OTM skew, as well.

Figure 7 displays the three OTM skews provided with the LMM model, together with the production model.

Figure 7. The 5x10 Swaption OTM Volatility Skew on 16 Feb 2010



Source: Citi

LMM Skew is close to 2fskew, particularly in the money (on the right-hand side of Figure 7, where moneyness is positive). In the out-of-the-money region, 2fskew falls in between LMM Skew and LMM Steep.

The skews in the two models also differ in the details of their implementation. In 2fskew, the skew is achieved by using a shifted log-normal distribution, resulting in some negative rates. While this is not a serious problem, it leads to some awkward moments, such as a floor struck at zero having a small positive value. By contrast, each skew provided with the Citi LMM model has its defining function $f(L)$ in the equations (*) above designed to ensure that the rates are non-negative.

Impact on Valuation: Pass throughs

This section discusses the differences between the production 2fskew term structure model and LMM from a user's perspective, starting with the comparison of the two models on the FNMA 30Y TBA coupon stack. First, the numbers.

Figure 8. FNMA TBA OAS Measures in the Two Models, with MOATS, on 16 Feb 2010

Coupon	Price	Model	OAS	Duration	Convexity	Mean WAL
4	98-04	2fskew	-13	5.9	-1.7	8.2
		LMM Skew	-11	6.2	-1.9	8.6
4.5	101-02	2fskew	-17	4.7	-2.5	7.7
		LMM Skew	-12	5.1	-2.8	8.1
5	103-26	2fskew	-35	2.9	-3.5	6.1
		LMM Skew	-25	3.4	-3.9	6.6
5.5	105-18	2fskew	-35	2.0	-3.0	5.4
		LMM Skew	-20	2.4	-3.8	5.9
6	106-12	2fskew	-12	1.8	-2.0	4.8
		LMM Skew	3	2.1	-2.6	5.2
6.5	107-04+	2fskew	20	1.8	-1.4	4.6
		LMM Skew	38	2.0	-2.1	5.0
7	107-26	2fskew	41	1.4	-1.3	4.1
		LMM Skew	59	1.4	-2.0	4.5

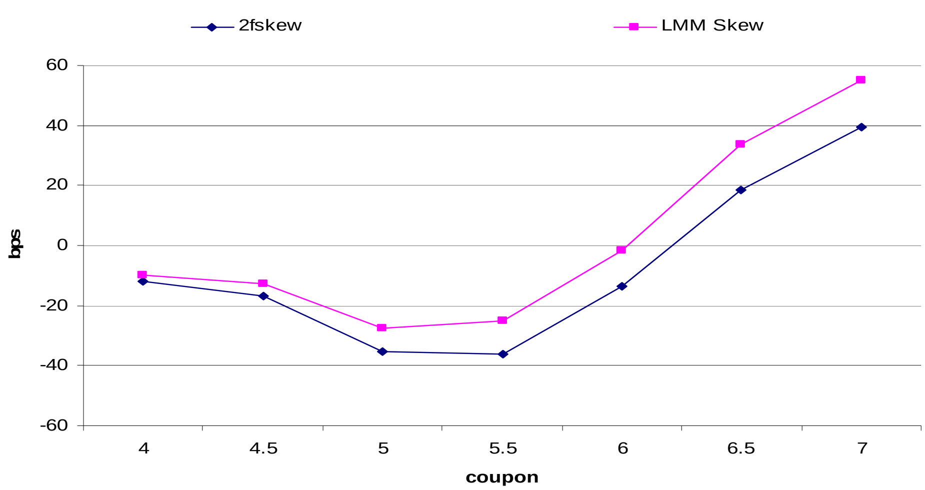
Source: Citi

Very little of the differences is due to MOATS, because the magnitude of the MOATS impact is similar in production and in LMM. Accordingly, as we review each model output, we continue to model the mortgage rate as a fixed spread to the 10Y swap rate, leaving the MOATS effects for a later section.

OAS

The pass through OAS under LMM Skew is close to 2fskew for discount coupons, but the difference widens to 15-17 bps for premiums; see Figure 9.

Figure 9. Pass Through OAS in the New and Old Models with Constant Mortgage Spread, on 16 Feb 2010



Source: Citi

Tighter OAS in 2fskew vs. LMM Skew is an immediate consequence of the higher value of the mortgage option in 2fskew. As explained earlier, the root cause is the lower serial correlation of 10Y swap rates, deriving from the overcalibrated short volatility in 2fskew.

The OAS difference is greater for premium coupons in part because the spread duration declines with coupon, reflecting a shortening WAL.

It is instructive to note that running both models with a simplified prepayment model, with no burnout, and whose incentive function has only the 10Y swap rate as its input, the differences shrink dramatically. This highlights two additional details alluded to above.

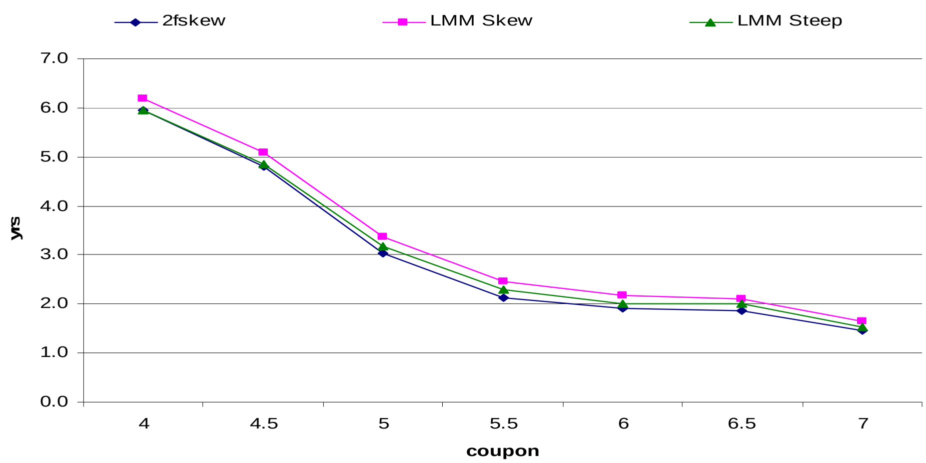
1. Lower serial correlation of the 10Y swap rate in 2fskew is the most significant explanatory factor, but it is not the only one. Even with the mortgage rate modeled as a constant spread to 10Y rate, the Citi prepayment model brings the 1Y rate into the mix it uses for the mortgage rate. Higher volatility for the 1Y rate in 2fskew directly contributes to a higher option cost, hence wider OAS gap with LMM.

2. The burnout feature of the prepayment model amplifies the significance of the serial correlation of CMS10Y by shifting more prepayment from earlier to later dates, hence increasing the value of the refinancing option at later dates.

Duration

The LMM Skew duration is longer by 0.3-0.4 year than in 2fskew; see Figure 10.

Figure 10. Pass Through Duration in the New and Old Models with Constant Mortgage Spread, 16 Feb 2010



Source: Citi

Once again, the difference arises from the lower serial correlation of CMS10Y in 2fskew, but the argument is more involved.

A partial explanation goes as follows. Lower serial correlation spurs prepayment at later dates, hence shortens the average of path-wise WAL numbers in 2fskew, compared to LMM Skew. However, one cannot make a blank assumption that any model that shortens the average WAL automatically shortens durations as well. For example, if two term-structure models differ only in their skews, then the one with the flatter skew will produce shorter average WAL, yet longer pass through durations.[5]

A more complete qualitative picture of duration differences between the two models comes from an analysis of the mortgage option embedded in a pass through; see Appendix 2.

Duration and the Skew

Another factor contributing to the duration difference is visible in Figure 7: 2fskew falls half-way between LMM Skew and LMM Steep for near-at-the-money and the out-of-the-money strikes. As a result, the 2fskew durations are closer to LMM Steep (included in Figure 10). They get even closer if one recalibrates LMM for the elevated ATM swaption volatilities implied by 2fskew.

[5] This holds both in production and LMM. The skew effect on pass through durations follows from formula (**) in the Volatility Skew section. And WAL shortens because a flatter skew shifts more probability to the rates paths which are slightly in the money, which is precisely the region where prepayment reacts to the incentive most strongly. And out of the money, or deeply in the money, prepayment is relatively insensitive to volatility differences.

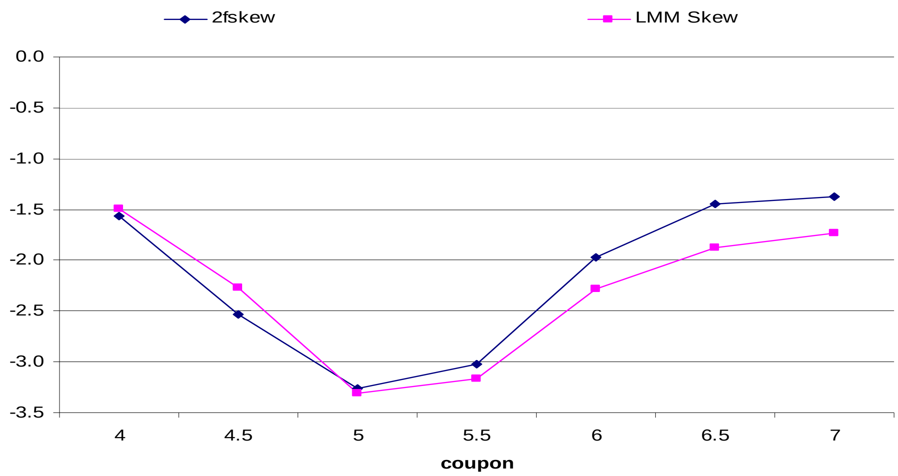
Impact of the Prepayment Model

In line with the OAS observations, the duration effects of short volatility overcalibration and serial correlation of CMS10Y are amplified by the prepayment model, through its use of the 1Y rate and the burnout feature.

Convexity

Figure 11 compares pass through convexity in LMM and the production model.

Figure 11. Pass Through Convexity in the New and Old Models with Const Mortgage Spread, on 16 Feb 2010



Source: Citi

Pass through's (negative) convexity derives from the embedded put's gamma, which exhibits a similar pattern, when the option's attributes are adjusted to reflect the differences between the models. See Figure 19 in Appendix 2 for more details.

Impact on Valuation: IOs and POs

Here we extend the above comparisons to several representative trusts.

Figure 12. IO/PO in Old and New Models, with MOATS, on 16 Feb 2010

	MOATS			IO					PO			
CPN	Deal	Age	Model	Price	OAS	Dur	Conv	Mean WAL	Price	OAS	Dur	Conv
4.5	TR.396	10	2fskew	25-00	147	-26.8	-34.7	7.9	76-14	-64	15.7	8.0
			LMM Skew		213	-22.3	-36.5	8.3		-80	14.7	8.3
5	TR.377	50	2fskew	20-26	201	-36.3	-25.1	5.8	83-22+	-81	13.3	2.4
			LMM Skew		295	-32.4	-32.6	6.1		-99	12.9	4.1
5.5	TR.379	36	2fskew	18-14	547	-35.8	-16.4	5.4	87-22	-167	11.3	-0.4
			LMM Skew		659	-32.8	-24.3	5.7		-183	11.2	1.1
6	TR.391	23	2fskew	15-02	1593	-29.5	-10.9	5.5	91-15	-279	9.8	-1.6
			LMM Skew		1717	-27.5	-20.0	6.0		-287	10.0	-0.2
6.5	TR.380	41	2fskew	16-00+	1448	-23.3	0.1	5.0	91-22	-237	8.6	-2.1
			LMM Skew		1556	-23.0	-7.1	5.3		-244	8.9	-1.1

Source: Citi

The most significant difference is in the IO OAS: the LMM numbers are substantially higher than in the production model. As with the pass throughs, this is because the embedded option in IO is priced higher in 2fskew, due to the lower serial correlations of the 10Y swap rates. The PO OAS exhibits the reverse pattern, with LMM shorter than 2fskew, because the PO is long its embedded option.

Most of the time, the LMM Skew durations are slightly longer (less negative) for IOs, and shorter for POs, than in 2skew, but the differences are relatively small. It is interesting to note that even when this relationship begins to invert (the highest-coupon IO or the lower-coupon POs), the average path-wise WAL is still longer in LMM than in 2fskew, as expected from the serial correlation discussion.

MOATS

MOATS is Citi's methodology projecting mortgage rates consistent with the pricing of mortgages.[6] Constant spread over swaps and other regression-type approaches create a serious discrepancy between the mortgage rate at some future time, and the current coupon interpolated from a coupon stack priced at the same time. With MOATS, the projected mortgage rate and the interpolated current coupon are in agreement throughout the simulation.

While the overall concept of MOATS in LMM remains the same as in the current model, the execution has to be different. In 2fskew the MOATS algorithm is implemented on a lattice. LMM is implemented via Monte Carlo simulation, and the MOATS method had to be adapted to this setting.

The properties of LMM MOATS are similar to those in 2fskew. In fact, MOATS can be thought of as a natural way to establish proper dependence of the projected mortgage rate on the rates of different tenors:

- MOATS redistributes some of the current coupon's rates dependence from 10Y to other tenors. It also induces positive volatility dependence.

This reworking of the current coupon dependence on is realized not only for projections in a Monte Carlo simulation, but also in the instantaneous or horizon rates and volatility scenarios, based on the idea of keeping the current coupon OAS constant. This has important consequences for vegas and for partial durations of all MBS, examined in the next section.

Valuation Impact

In both 2fskew and LMM Skew the MOATS pass through durations are 0.2-0.3 year shorter than the constant spread version, with the duration gap between LMM Skew and 2fskew almost unaffected by MOATS. The OAS gap between LMM Skew and 2fskew widens by a couple of bps for the cusp coupons.

The MOATS impact on IO and PO durations is also relatively small. The effect on the IO OAS is more pronounced, in part because of the high spread duration.

The real differences arise when we come to partial durations and vega, in the next section.

Partial Durations and Vega

In LMM the MOATS impact on partial durations and vega is similar to the production model, but it is sufficiently complex to warrant a review.

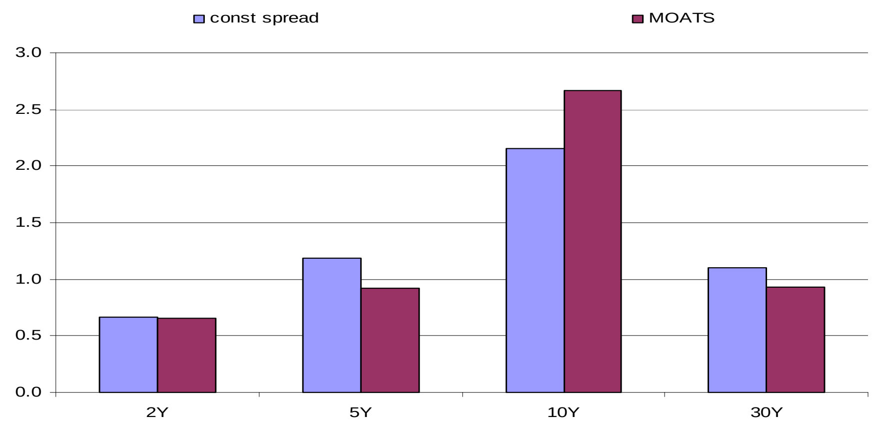
MOATS Effects for Common MBS Types

- For pass throughs[7] MOATS brings greater concentration of partials in the 10Y bucket and lower volatility durations.

For example, on 2/16/10 the FN4.5 volatility duration was 0.34 with constant mortgage spread, and 0.26 with MOATS. Its partial durations are displayed in Figure 13 below.

[6] See *Projecting Mortgage Rates for MBS Valuation: Citigroup's MOATS Model*, R. Bhattacharjee, L.S. Hayre, P. Jha, Citigroup, June 2005.
[7] We may need to exclude deep discounts, which imposes no restrictions in the current rates environment.

Figure 13. FN4.5 Partials in LMM Skew, with and without MOATS, on 16 Feb 2010

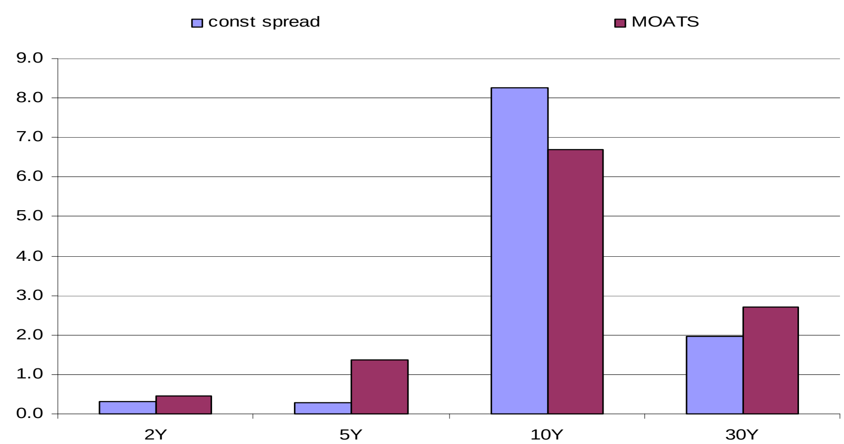


Source: Citi

- For POs, MOATS redistributes partial durations away from 10Y, and increases the volatility duration.

Thus, the TR 379 PO's LMM Skew volatility duration on 2/16/10 went from 0.10 with constant mortgage spread, to 0.42 with MOATS. The partials are shown in Figure 14.

Figure 14. TR 379 PO Partials in LMM Skew, with and without MOATS, on 16 Feb 2010

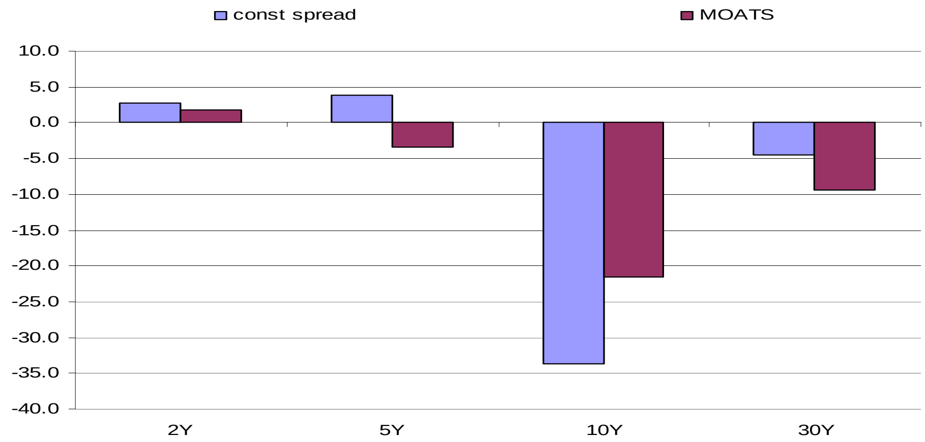


Source: Citi

IO durations are negative, but the MOATS effect is similar to that on pass throughs, in the absolute sense:

- For IOs, MOATS makes the 10Y partial duration less negative, while other partials are (more) negative; see Figure 15.

Figure 15. TR 379 IO Partials in LMM Skew, with and without MOATS, on 16 Feb 2010



Source: Citi

Importantly – **with MOATS**, – both models assign negative volatility duration to IOs, i.e. **volatility helps IOs, even as it hurts POs and pass throughs**. In particular, the LMM Skew volatility duration for TR 379 IO on 2/16/10 was 1.1 with constant mortgage spread, and -0.9 with MOATS.

Explanation

These effects may appear disparate, but they flow from the same cause. The price P of any mortgage-backed security is a function of the current coupon spread ccs, volatility vol, and the rates of various tenors, e.g. y_2, y_5, y_{10} and y_{30}, among other market factors:

$$P = f(y_2, y_5, y_{10}, y_{30}, vol, ccs, \dots)\,.$$

In the constant spread case, by definition, $ccs = ccs_0$, the initial current coupon spread. With MOATS it changes in any rates or volatility scenario. Thus the MOATS ccs can be thought of as some more complicated function of ccs_0, vol, and of all the rates:

$$ccs = g(y_2, y_5, y_{10}, y_{30}, vol, \dots) + ccs_0\,.$$

Total price sensitivity to any variable z is given by

$$\frac{dP}{dz} = \frac{\partial f}{\partial z} + \frac{\partial f}{\partial ccs} \cdot \frac{\partial ccs}{\partial z}\,,$$

the second summand becoming non-zero only under MOATS. This gives a way to relate the duration with respect to any market factor z under MOATS and under the constant spread model:

$$Dur_z^{MOATS} = Dur_z^{Const\,Spread} + Dur_{ccs} \cdot \frac{\partial ccs}{\partial z}\,.$$

Now, $\partial ccs / \partial y_{10} < 0$ and $\partial ccs / \partial y_i > 0$ for i = 2, 5 and 30, and $\partial ccs / \partial vol > 0$, because of the principal property of MOATS highlighted in the previous section. And the current coupon spread duration Dur_{ccs} is negative for pass throughs[8] and IOs, and positive for POs. Going through $z = y_2$, y_5, y_{10}, y_{30} and vol, and keeping track of the Dur_{ccs} sign for each MBS type, produces the following table (Figure 16), which is nothing but a summary of the previously highlighted MOATS effects.

[8] It is here that we may have to exclude deep discounts, because of the turnover effects.

Figure 16. MOATS Impact on Partial Durations and Volatility Durations for Pass Throughs and Trusts

		MOATS effect on duration with respect to		
MBS Type	CC Spread Dur	10Y	2Y, 5Y, 30Y	vol
Pass through	-	up	down	down
IO	-	up	down	down
PO	+	down	up	up

Source: Citi

LMM Partials and Vegas vs. the Production Model

Compared with 2fskew, the LMM Skew partials are slightly shifted toward longer maturities, which is consistent with the slightly longer average WAL.

Volatility duration for pass throughs and POs is almost indistinguishable in the two models, while the IO volatility duration is lower (more negative) by 0.1-0.2.

Appendix 1. Short Volatility and the Serial Correlation of the 10Y Swap Rate

We will sketch how for most shorter-tenor/shorter-expiry swaptions,

- increasing short volatility decreases the serial correlation of CMS10Y.

Concretely, we focus on the 5x10 and 6x10 forward swap rates, and show how their correlation is linked to the volatility of the 5x1 forward swap rate. First, the 5x10 forward swap rate must be very close to the 5x11 rate; so

$$corr(5\times 10,\, 6\times 10) \approx corr(5\times 11, 6\times 10) = \frac{\mathrm{cov}(5\times 11, 6\times 10)}{\sigma(5\times 11)\sigma(6\times 10)}.$$

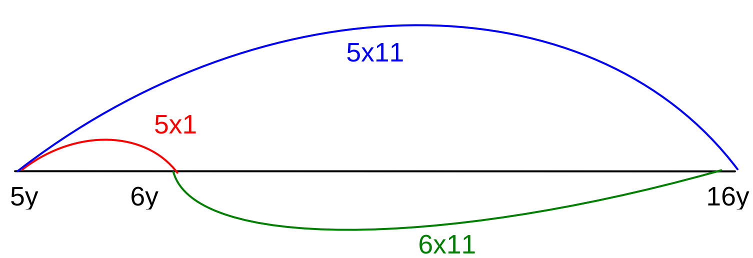
The volatilities in the denominator are assumed fixed by calibration. Therefore, the correlation in question moves together with the covariance in the numerator.

The 6x11 forward swap rate can be decomposed into the forward 5x11 and 5x1 rates (see Figure 17),

$$6\times 10 \approx 6\times 11 = w_1 5\times 11 - w_2 5\times 1,$$

with suitable weights w_1 and w_2.

Figure 17. Decomposing Forward Swap Rates



Source: Citi

The decomposition remains valid, now as an approximation with the weights frozen, even if we introduce observation time evolving from 0 up to 5y. Applying this decomposition to the variance of the 6x10 rate yields

$$\mathrm{var}(6\times10) = (\text{fwd var from 5y to 6y}) + (\textbf{mid-curve var} \text{ up to 5y}) =$$
$$= (\text{fwd var}) + w_1^2\,\mathrm{var}(5\times11) - 2w_1w_2\,\mathrm{cov}(5\times11, 5\times1) + w_2^2\,\mathrm{var}(5\times1)$$

And the same decomposition implies

$$2w_1\,\mathrm{cov}(5\times11, 6\times10) = 2w_1^2\,\mathrm{var}(5\times11) - 2w_1w_2\,\mathrm{cov}(5\times11, 5\times1)\,.$$

Together, the two calculations show that

$$2w_1\,\mathrm{cov}(5\times11, 6\times10) = \{(\text{fwd var}) + w_1^2\,\mathrm{var}(5\times11) + \mathrm{var}(6\times10)\} - w_2^2\,\mathrm{var}(5\times1)\,.$$

The terms within the braces are fixed by calibration.[9] Then, if the 5x1 volatility goes up, the correlation between 5x11 and 5x10 rates must go down.

Variance of Change in the 10Y Swap Rate

There is another way to express the consequences of higher short volatility for the 10Y swap rate. Declining serial correlation increases the variance of the change in CMS10Y between two horizons,

$$Var(S_2 - S_1) = Var(S_2) + Var(S_1) - 2Cov(S_1, S_2)\,,$$

since the volatility of CMS10Y is fixed at every horizon when each model is calibrated to the x10 swaption volatilities.

Intuitively, because of the way in which swaption volatility scenarios are defined, the increased short-rate volatility spills into increased variance of the CMS10Y change.

Appendix 2. The Mortgage Option in the Black-Scholes World

- The embedded option can be approximated by a single vanilla put on the mortgage rate.

Pass Through Duration and the Embedded Option's Delta

Breaking up the full pass through value $V = V_0 - Put$ into its optionless part V_0 and the embedded short put, and letting y denote the level of rates, one can write[10]

(***)
$$Dur(\text{Bond}) = -\frac{100}{V}\frac{dV}{dy} = \frac{100}{V}\left(-\frac{dV_0}{dy} + \frac{dPut}{dy}\right) = \frac{100}{V}\left(-\frac{dV_0}{dy} + \Delta_{Put}\right).$$

So comparing durations in LMM vs. 2fskew reduces to comparing deltas for two options, which comes down to their basic characteristics.

- As discussed above, the put's volatility is higher in 2fskew.
- Higher serial correlation slows prepayment, on average. Therefore, the simplified put's exercise time should be longer in LMM than in 2fskew.
- Exercise delay also implies a higher forward rate, since the yield curve is upward-sloping.

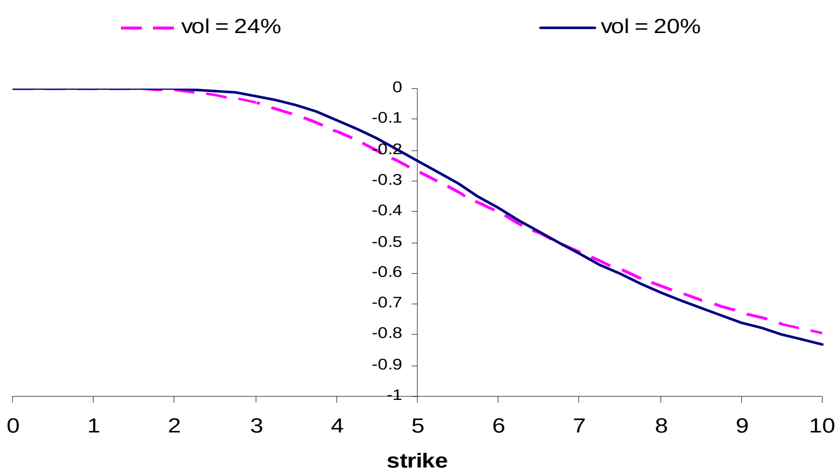
This motivates the following comparison: one put with the volatility of 20%, expiry in 4.4 years, and the forward rate of 6.2%; and another with volatility 24%, expiry 4 years, and the forward rate 6%. The example will illustrate how, with suitable choice of expiries and forward rate values, the higher volatility of the embedded option in 2fskew vs. LMM Skew

[9] When the 5x1 volatility goes up, the forward volatility of the 6x10 rate between 5y and 6y may go down, to compensate for keeping the longer-expiry volatilities constant. But this would only strengthen the argument.
[10] The constant mortgage spread assumption is used here to identify the put's derivative with respect to the rates level with its delta, i.e. the sensitivity to the (forward) mortgage rate.

- shortens the pass through duration;
- results in less negative convexity for much of the coupon stack.

The higher-volatility put in Figure 18 has lower (more negative) delta for all but the highest strikes. This directly compares with the duration chart in Figure 10, which shows shorter duration in 2fskew vs. LMM Skew for most coupons.

Figure 18. Delta of Two Puts over a Range of Strikes; the Low-Vol Option has Longer Exercise Time and Higher Fwd Rate

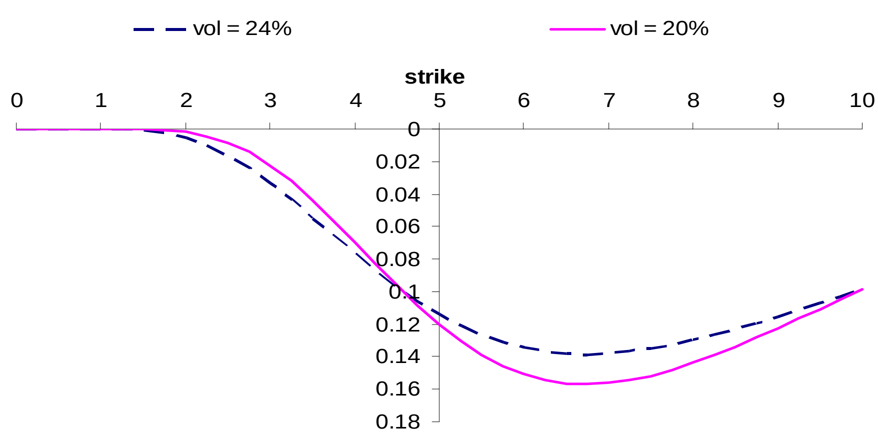


Source: Citi

Convexity and the Embedded Option's Delta

A pass through's negative **convexity** arises from (the negative of) the embedded option's **gamma**.[11] Figure 19 compares the gamma of the same two puts as above, with the gamma axis inverted to make the relationship to the convexity chart in Figure 10 easier to observe.

Figure 19. Gamma of Two Puts (inverted axis) over a Range of Strikes; the Low-Vol Option has Longer Exercise Time and Higher Fwd Rate;



Source: Citi

[11] Analogously to (***) for duration, and with the same notation, *Conv*(Bond) = $\frac{100}{V}\left(\frac{d^2V_0}{dy^2} - \Gamma_{Put}\right)$, at least under the constant mortgage spread assumption.

The two graphs cannot match exactly, since each coupon's embedded option requires its own translation into a single put.[12] Even taking the bond's coupon to be the put's strike, as we do, is clearly a simplification. But in the above picture, the gammas of the higher- and the lower-volatility puts, restricted to the strikes between 4% and 7%, display the same relationship as the convexity in 2fskew vs. LMM Skew in Figure 11.

[12] Even closer fit can be achieved at the cost of added complexity. Explicitly recognizing the S-curve-driven nature of the prepayment option suggests modeling it as bear put spread or, more precisely, as a put struck at the bond's coupon, minus a multiple of the put struck at a somewhat lower strike.

Disclaimer

This commentary has been prepared by Markets Quantitative Analysis ("MQA"), which is part of Citigroup Global Markets' sales and trading operations.