The Yield Curve and Mortgage Current Coupons

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A dynamic prepayment model is an integral component of an option-adjusted valuation and risk management framework. Such models typically require a forecast of future mortgage rates and possibly index rates such as prime and Cost of Funds Index. Term structure models are used to forecast rates of various maturities on the LIBOR/swap yield curve or on the Treasury yield curve. It is therefore necessary to model mortgage rates as functions of these yield curve rates.

We discuss an approach to forecasting mortgage current coupons as a function of either Treasury or swap rates. We compare several statistical approaches to the problem and discuss software implementation issues. In addition, we compare using a single Treasury rate, two Treasury rates, or two swap rates to forecast mortgage current coupon rates.

I. DATA

The current coupon is the semiannual equivalent of the parity price interpolated coupon, according to the two bond prices bracketing the parity price. The mortgage current coupon rates we use are based on month-end closing prices. The mortgages are Fannie Mae 7-, 15-, and 30-year; Ginnie Mae I 15, and 30-year; and Freddie Mac 5-, 7-, 15-, and 30-year. These rates can be viewed on Bloomberg for the FNMA 30 rate.

The Treasury rates used are month-end closing par bond-equivalent yields for the two- and ten-year on-the-run Treasuries or the two- and ten-year zero-coupon Treasuries. The swap rates are either the two- and ten-year LIBOR or the two- and ten-year zero-coupon LIBOR, which are derived from the par swap rates using a boot-strapping method with linear interpolation. The period of analysis covers November 1991 through August 2000.

II. PRELIMINARY ANALYSIS

Exhibit 1 is a scatterplot of FNMA 30-year current coupons versus both ten-year Treasuries and ten-year swap rates. This plot suggests that ten-year swap rates may be a better predictor of current coupon than ten-year Treasury rates, because the swap versus current coupon points show much less variation about a line through their center.

Because we also need to predict rates of shorter maturities, such as five- and seven-year balloon rates, it is useful to examine the relationship between ten-year rates and a balloon rate. Exhibit 2 displays scatterplots of a five-year balloon rate versus both the ten-year Treasury and swap rates. In this case, neither Treasury nor swap rates give a tight fit.

Exhibit 3 shows Freddie Mac Gold (FHG) five-year balloons plotted against two-year swaps and Treasuries. This appears to be a much better fit than that in Exhibit 2.

EXHIBIT 1

FNMA 30 and LIBOR Fit versus FNMA 30 and Treasury Fit

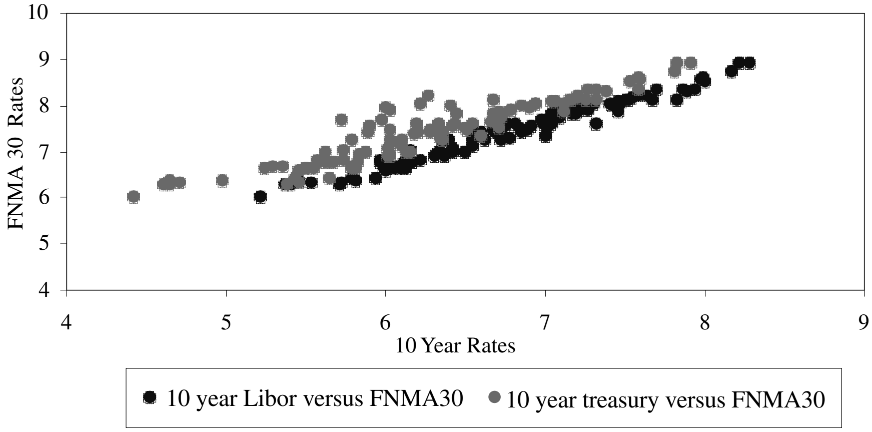


EXHIBIT 2

FHG 5 and LIBOR Fit versus FHG 5 and Treasury Fit

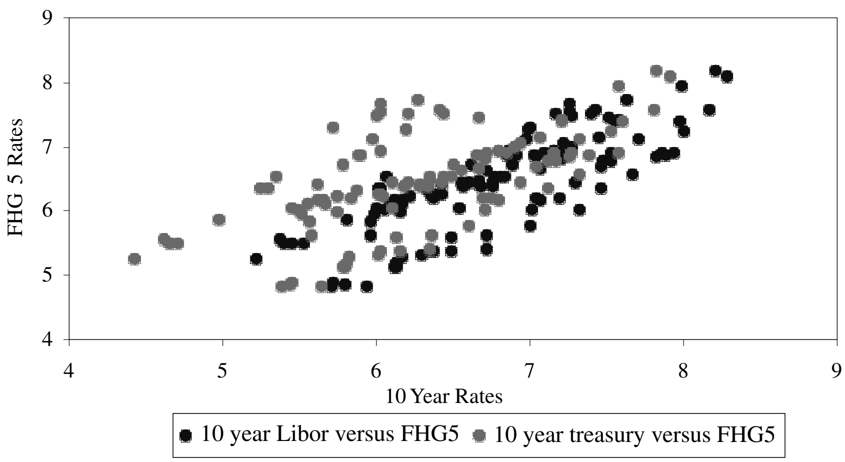
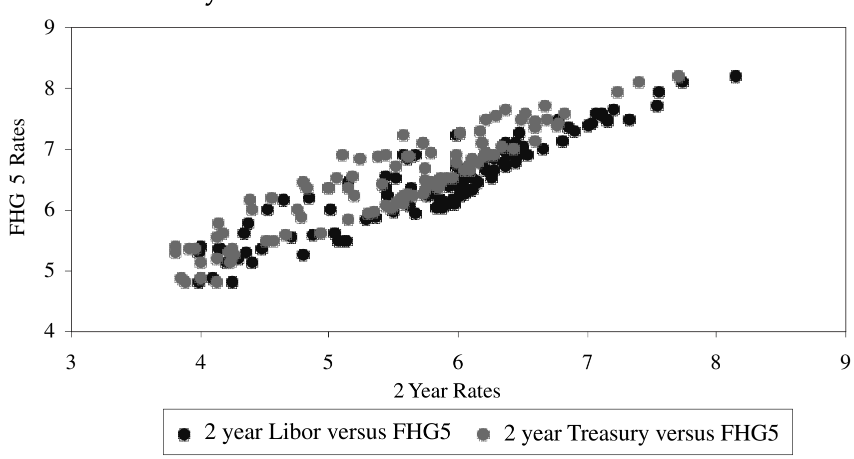


EXHIBIT 3

LIBOR and FHG 5 Fit versus Treasury and FHG 5 Fit



Before deciding on the yield curve maturities to use, it is helpful to look at correlations between rates of different maturities and some mortgage current coupon rates. Ideally, the rates chosen to model mortgage current coupon would come from liquid points on the yield curve. In addition, they should have high correlations with mortgage current coupon rates, but low correlations with each other. A high correlation with mortgage current coupon indicates that there is a strong linear relationship between the two rates; a low correlation with each other implies that the additional rate adds to the strength of the relationship.

The final set chosen should maximize fit with as simple a set of rates as possible. The correlation matrix of two- and ten-year LIBOR and one-, two-, five-, and ten-year Treasury rates versus the thirty-year FNMA and five-year Freddie Mac balloon mortgage rates is shown in Exhibit 4.

The correlation matrix tells us several things. First, using the two-year rate is better than using the one-year rate because the two-year rate has both higher correlations with mortgage current coupon rates and low correlation with the ten-year rate. Second, while the five-year Treasury has a stronger relationship with the thirty-year mortgage rate than does the ten-year Treasury, it has a weaker relationship with the five-year balloon rate than does the two-year Treasury. Because the five-year Treasury has a high correlation with both of these Treasury rates, it is inadvisable to include the five-year in a regression model with either the two-year or ten-year Treasury.

The combination of the scatterplots, the extent of the errors that result from using only ten-year rates, and the correlation analysis prompts us to use the two-year and ten-year rates in our regression.

III. REGRESSION APPROACHES

In a standard regression, error terms are assumed to be uncorrelated with each other and are derived from the same random distribution with zero mean and equal variance. Modeling time series data often violates the first assumption, because error terms are autocorrelated, which means that the error in a given period adds explanatory power to the error in the next period. It is well known that fitting a standard regression in such a circumstance can lead to substantial bias in the parameter estimates, resulting in regression lines that do not fit the data as well as a model that corrects for autocorrelation (see Box and Jenkins [1976]). In addition, ignoring autocorrelation can yield measures of fit that overstate the actual strength of the relationship.

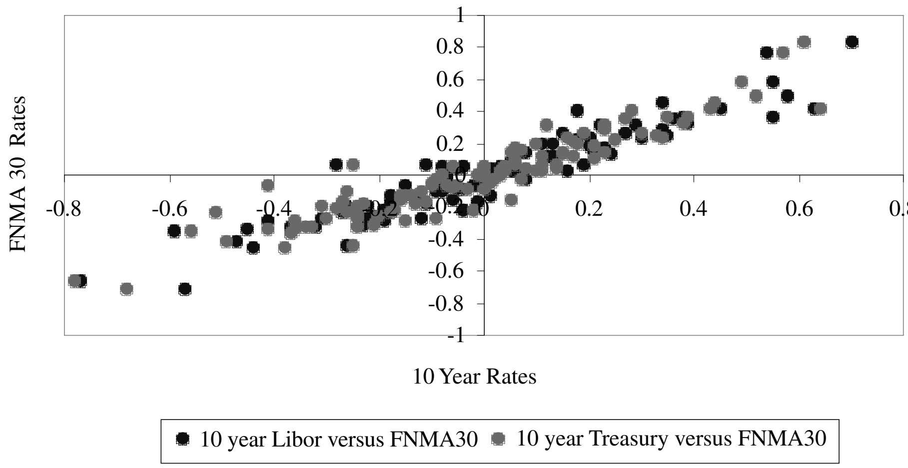
One way to analyze time series data with autocorrelation is to perform a regression on differences between observations, rather than on observation values themselves. This technique, commonly called the method of first differences, adjusts for autocorrelation to a significant degree. Exhibit 5 shows a scatterplot of the differences between the points in FNMA 30s and those of ten-year LIBOR and Treasuries. The correlation coefficients from the first-differences method are 0.942 for ten-year LIBOR and FNMA 30 and 0.932 for the ten-year Treasury and FNMA 30.

Another approach to regression with autocorrelated data involves estimation of ρ using the Durbin-Watson statistic [1951]. Then, in the regression equations, the coefficients are estimated using a variation of first differences, where the estimated ρ is multiplied by the first data point and then subtracted from the second data point, and so on throughout the whole data set in the same manner

EXHIBIT 4
Correlation Matrix

	LIBOR 2 yr.	LIBOR 10 yr.	1 yr. Treas	2 yr. Treas	5 yr. Treas	10 yr. Treas	FHLMC5	FNMA30
LIBOR 2 yr.	1	0.604	0.937	0.979	0.739	0.422	0.932	0.662
LIBOR 10 yr.		1	0.386	0.647	0.932	0.932	0.798	0.983
1 yr. Treas			1	0.927	0.580	0.242	0.791	0.464
2 yr. Treas				1	0.818	0.526	0.911	0.708
5 yr. Treas					1	0.914	0.835	0.945
10 yr. Treas						1	0.609	0.915
FHLMC							1	0.837
FNMA								1

E X H I B I T 5
FNMA 30 and LIBOR Fit versus FNMA 30 and Treasury Fit



as the first-differences method. This is known as the Hildreth-Liu method (see Pindyck and Rubinfeld [1991]).[1]

IV. ANALYSIS

We are interested in comparing par coupon Treasury rates and swap rates to determine which are more accurate predictors of mortgage current coupons. Because many interest rate term structure models output zero-coupon Treasury or swap rates, we also consider zero-coupon rates as predictors of mortgage current coupon rates.

When we perform a parallel yield curve shift, we would like all the mortgage current coupons to shift by the same amount as Treasuries or swap rates. We can achieve this by placing a restriction on the model that the sum of the coefficients in the regression must be equal to 1.0.

In all, we have 72 models to compare; there are nine different agency and maturity combinations, four covariate groupings, and two model estimation techniques (first differences and Hildreth-Liu) per covariate grouping. We use the SAS system for Windows V8.

The first measure of error used to compare the four sets of independent variables and the two estimation techniques is the square root of the sum over all nine coupon types of all of the squared errors divided by the number of current coupons.[2] This measure of error uses the constant term from the regression, which is equivalent to using an average spread.

Increases in this measure of error can be caused either by the non-spread coefficients (and bad explanatory variables) or by changes in this historical spread. To distinguish between the two, we use an additional measure of error based on an examination of only the changes in the predictors and predicted variable, and not on the spread. This second measure of error is simply the standard deviation of the differences between the predicted and actual values of changes in the underlying variables.

V. RESULTS

Exhibit 6 graphs the first measure of error as a function of time for the four covariate sets using the first-differences estimation approach. Exhibit 7 displays the results of the modified Hildreth-Liu method.

Two features of these graphs are very striking. First, the overall difference in error between the two estimation methods is significant. The range for Exhibit 6 is 20 to 60 basis points, while the range for Exhibit 7 is only about 2 to 40 bp. This indicates that estimation by the Hildreth-Liu method is much more accurate than the first-differences method.

The second interesting result is that the swap rate errors are consistently lower than Treasury errors after March 1994. Between March 1994 and 1995, the errors for both swap models drop to about 5 bp, and hover around that mark for the rest of the time period covered. Errors using the Treasury model, however, remain more than twice as high until July 1998, when there is a very dramatic increase that continues until August 2000.

The appendix shows the coefficient estimates and R^2 statistics for the four estimations performed using the

EXHIBIT 6

Estimated and Actual Current Coupon for First-Differences Method Using Spread Term

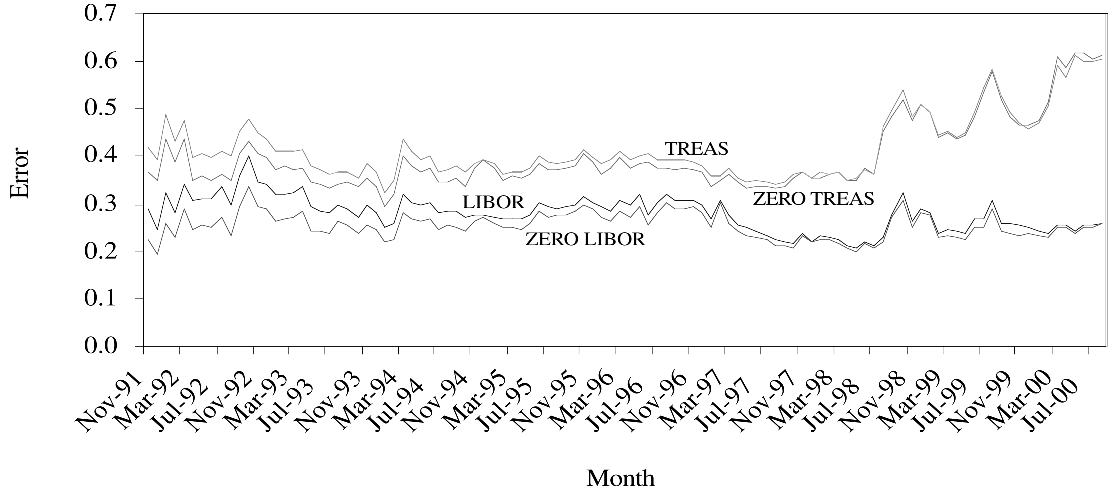
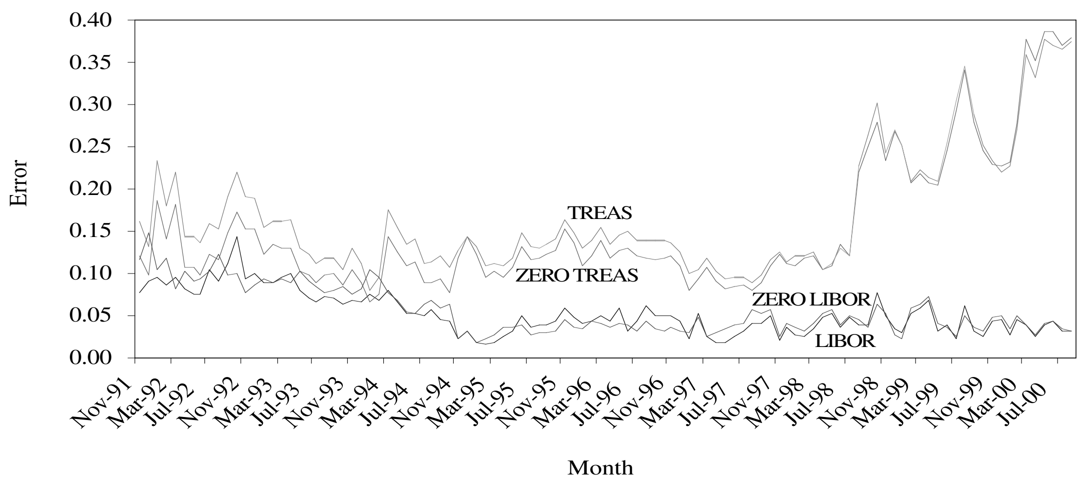


EXHIBIT 7

Estimated and Actual Current Coupon for Hildreth-Liu Method Using Spread Term



modified Hildreth-Liu method (par/zero, swap/Treasury). For a given yield curve type, there appears to be no significant difference between using zero or par rates, as one would expect. In general, as the maturity of the mortgage rate modeled increases, the coefficients shift from a heavy weight on the two-year rate and a low weight on the ten-year rate, to a low weight on the two-year rate and a higher weight on the ten-year.

Moreover, there does not appear to be a great amount of variation in the measures of fit within a particular method. These results add to the evidence in Exhibit 6; not only are the average errors significantly lower using swap rates, but the fit is also better for each and every current coupon type.

It remains unclear, however, whether this difference in the fit is due to shifts in the spread level (the constant term in the regression) between Treasuries and the other rates, or simply that none of the coefficients is as good when we use Treasuries. In order to answer this question, we examine the standard deviation of errors using monthly movements in swap rates and Treasury rates to predict monthly movements in mortgage rates using the same coefficients, without the constant terms.

EXHIBIT 8
Errors for FNMA 30 Using Treasury versus Swap Rates

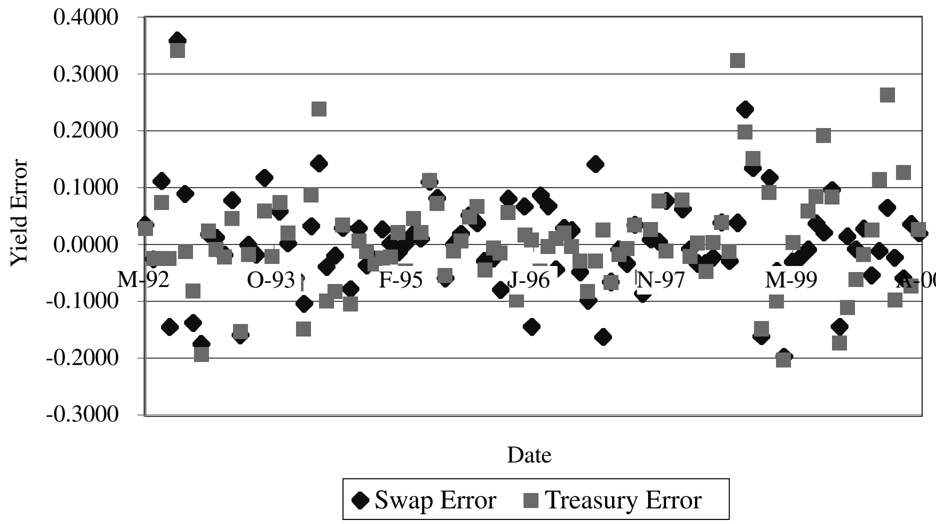


EXHIBIT 9
Comparison of Errors

Period	Swap	Treasury	Conclusion	P-Value
1992-1994	0.114339351	0.116064038	No difference	0.464
1995-1997	0.07000653	0.047830825	Treasury lower	0.014
1998-2000	0.083611199	0.12240061	Swap lower	0.019

Exhibit 8 displays these errors over time for the FNMA 30-year current coupon. Comparing Exhibit 8 to Exhibits 6 and 7, we see no obvious difference in the magnitudes of Treasury-based error and swap-based error. There are times when both are roughly the same and other times when one is somewhat higher than the other. T-tests on the means of the errors confirm that the average error is not different from zero in a statistically significant way; F-tests confirm that the standard deviations of the errors are not different.[3]

If we divide the error analysis into three distinct periods, however, we begin to see differences. Exhibit 9 shows that over 1992-1994 both Treasuries and swaps had higher errors, and that the level of error is not statistically distinguishable. For 1995-1997, both had much lower errors, and Treasuries had lower errors than swap rates. Finally, in 1998-2000, swap errors continue to hold their 1995-1997 levels, while Treasury errors are much higher.

Exhibit 10 displays the total outstanding U.S. government debt as a function of time.[4] In the summer of 1998, U.S. government debt outstanding peaked at around $3.8 trillion, and then began to decline as the U.S. Treasury retired debt using rising budget surpluses. Both the three- and seven-year Treasury bonds were eliminated, and market expectations of future surpluses and debt paydown began to grow. The Russian debt crisis and the collapse of Long-Term Capital Management occurred later that year.

It is within this context that we should look at the evidence. The errors graphed in Exhibit 8 appear to break at this time and increase from their historical levels. A closer look at the errors shows that around the summer of 1998, the equations begin to consistently underpredict mortgage current coupon; before that period, errors of both signs occur. This is consistent with a premium paid for Treasuries, which would have reduced their yields without affecting yields on swaps or mortgages.[5]

EXHIBIT 10
Publicly Held Outstanding U.S. Debt

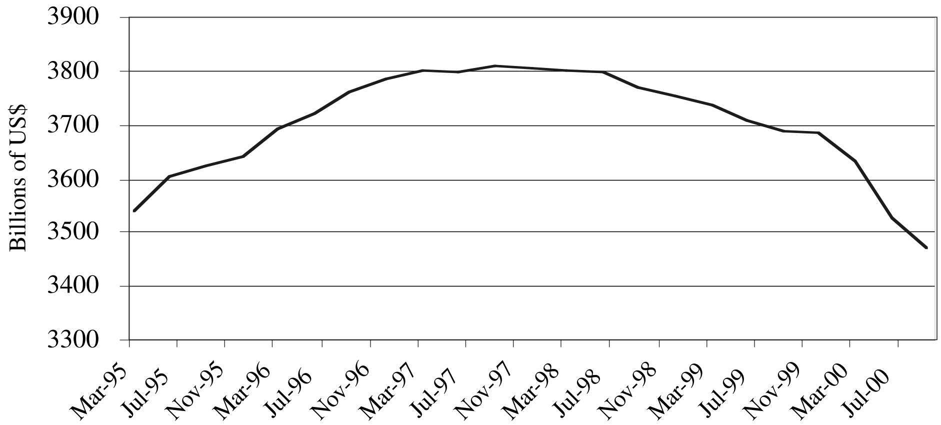


EXHIBIT 11
GNMA 30 Forecasting Errors September 2000-January 2001

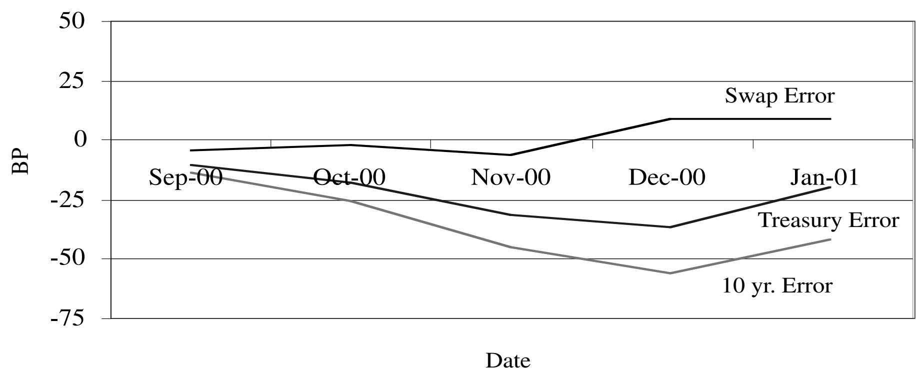
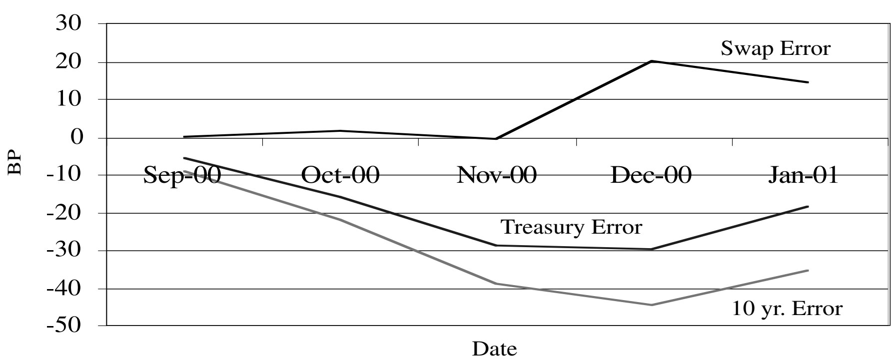


EXHIBIT 12
FNMA 30 Forecasting Errors September 2000-January 2001



VI. OUT-OF-SAMPLE TESTS

We perform out-of-sample tests of fit for FNMA 30 and GNMA 30 coupons using par Treasury and swap rates in order to compare historical fit with forecasting fit on market data generated after model estimation. To compare against a "control," we use the original ten-year-plus spread method discussed in the preliminary analysis.

In all three methods, the spread is calculated on the basis of August 2000 rates. These are the assumptions that someone running an analysis as of August 2000 would make for mortgage rate forecasts for September 2000 through January 2001, and with access to perfect Treasury forecasts for this period. Exhibits 11 and 12 display these errors.

In both graphs, it is clear that the ten-year-plus spread method performs the most poorly, with errors reaching 40-50 basis points in just the first few months. Using two Treasury rates is somewhat better; errors do not exceed 35 bp. Both methods, however, have systematically negative errors over the period, while the errors using swaps are both negative and positive. Moreover, the magnitude of the errors using swaps is considerably lower. The majority are under 10 basis points.

VII. CONCLUSION

Comparison of two related regression methodologies and four sets of yield curve variables indicates that

Appendix

Regression Coefficients and R^2 Using Modified Hildreth-Liu

		Par Swap			Zero Swap		
	Agency	**2-Year**	**10-Year**	**R^2**	**2-Year**	**10-Year**	**R^2**
5	**FHG**	0.56651	0.43349	95.04%	0.61794	0.38206	94.85%
7	**FNMA**	0.34450	0.65550	97.70%	0.41912	0.58088	97.27%
	FHG	0.36634	0.63366	93.61%	0.43865	0.56135	92.99%
15	**FNMA**	0.17570	0.82430	93.80%	0.27112	0.72888	93.13%
	FHG	0.19701	0.80299	91.31%	0.28708	0.71292	90.81%
	GNMA	0.18884	0.81116	89.18%	0.28787	0.71213	89.07%
30	**FNMA**	0.10213	0.89787	95.40%	0.20684	0.79316	95.18%
	FHG	0.10206	0.89794	95.17%	0.19977	0.80023	94.73%
	GNMA	0.09880	0.90120	92.59%	0.20272	0.79728	92.43%

		Par Treas			Zero Treas		
	Agency	**2-Year**	**10-Year**	**R^2**	**2-Year**	**10-Year**	**R^2**
5	**FHG**	0.518357	0.481643	82.45%	0.55953	0.44047	82.45%
7	**FNMA**	0.504595	0.495405	88.58%	0.54015	0.45985	88.55%
	FHG	0.510901	0.489099	86.20%	0.55953	0.44047	86.02%
15	**FNMA**	0.357817	0.642183	87.63%	0.42110	0.57890	87.35%
	FHG	0.345772	0.654228	89.72%	0.40540	0.59460	89.70%
	GNMA	0.335591	0.664409	79.99%	0.39987	0.60013	79.85%
30	**FNMA**	0.236148	0.763852	80.24%	0.30429	0.69571	80.14%
	FHG	0.18466	0.81534	87.04%	0.26141	0.73859	87.32%
	GNMA	0.18445	0.81555	86.39%	0.25495	0.74505	86.62%

zero-coupon or par LIBOR swap rates are better predictors of mortgage current coupon than the corresponding Treasury rates. There are significant changes in the predictive power of Treasuries beginning around July 1998. Both the in-sample fits and out-of-sample forecasting support this conclusion.

Given ongoing uncertainty about the impact of supply-related technical factors on the Treasury market, we believe that option-adjusted valuation based on swap rates is preferable at this time. We also find that explicitly taking into account the autocorrelation structure leads to better parameter estimates and significantly lower overall error than using a pure first-differences approach.

ENDNOTES

[1]The original Hildreth-Liu method uses least squares to determine ρ, so using the D-W statistic should really be called a modified Hildreth-Liu method.

[2]This is one measure of average distance over coupons, which does not provide information on whether some coupons are better predicted and others less well predicted.

[3]Both at the 90% and the 95% significance levels.

[4]The source is Bloomberg. This index does not include the Social Security IOUs.

[5]The retirement of debt would create a scarcity premium while a flight to quality would prompt an increase in the Treasury credit premium. Both would increase spreads.

REFERENCES

Box, G.E.P., and G.M. Jenkins. *Time Series Analysis: Forecasting and Control.* San Francisco, CA: Holden-Day, 1976.

Durbin, J., and G.S. Watson. "Testing for Serial Correlation in LS Regression.II." *Biometrika,* 38 (1951), pp. 159-178.

Theil, H., and A.L. Nagar. "Testing the Independence of Regression Disturbances." *Journal of the American Statistical Association,* 56 (1961), pp. 793-806.

Pindyck, R.S., and D.L. Rubinfeld. *Econometric Models and Economic Forecasts*, 3rd ed. New York: McGraw-Hill, 1991.