Modeling the Instability of Mortgage-Backed Prepayments

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The U.S. mortgage market is the largest debt market in the world. As of the second quarter of 2002, residential mortgage debt outstanding was close to $6.5 trillion. Today, almost half of the debt is securitized and resold as mortgage-backed securities (MBS) by government and quasi-government institutions and private mortgage originators. Mortgage securitization has greatly enhanced liquidity in the mortgage market.

The mortgage market has also benefited from an expanding secondary market facilitated by the increased participation of loan brokers and private insurance mortgage companies. These entities provide a wide variety of services that allow mortgage originators and other investors to trade large portfolios of conforming or non-conforming loans in the secondary market (whole loan market).

In many respects, a mortgage security is similar to an ordinary bond. Like bonds, mortgage securities promise their holders a stream of payments over a number of periods. Mortgage pass-throughs, however, are different from a typical government bond because the promised cash payments depend on what happens with regard to prepayments. Mortgage borrowers in the United States have the right to prepay part of or the entire loan principal without penalty. This embedded option can drastically change the expected cash flows from a mortgage security. The effect of prepayment is amplified in more exotic mortgage-backed derivative products such as collateralized mortgage obligations (CMOs) and stripped MBS.

The prepayment experience of mortgage securities in the 1980s and 1990s has been quite bumpy. The mortgage market experienced several intense refinancing cycles prompted by sharp declines in interest rates. Most market participants were not surprised by the changes in interest rates. What took Street forecasters by surprise, however, was the precipitous rise (or fall) and the extremely volatile nature of pool prepayments.
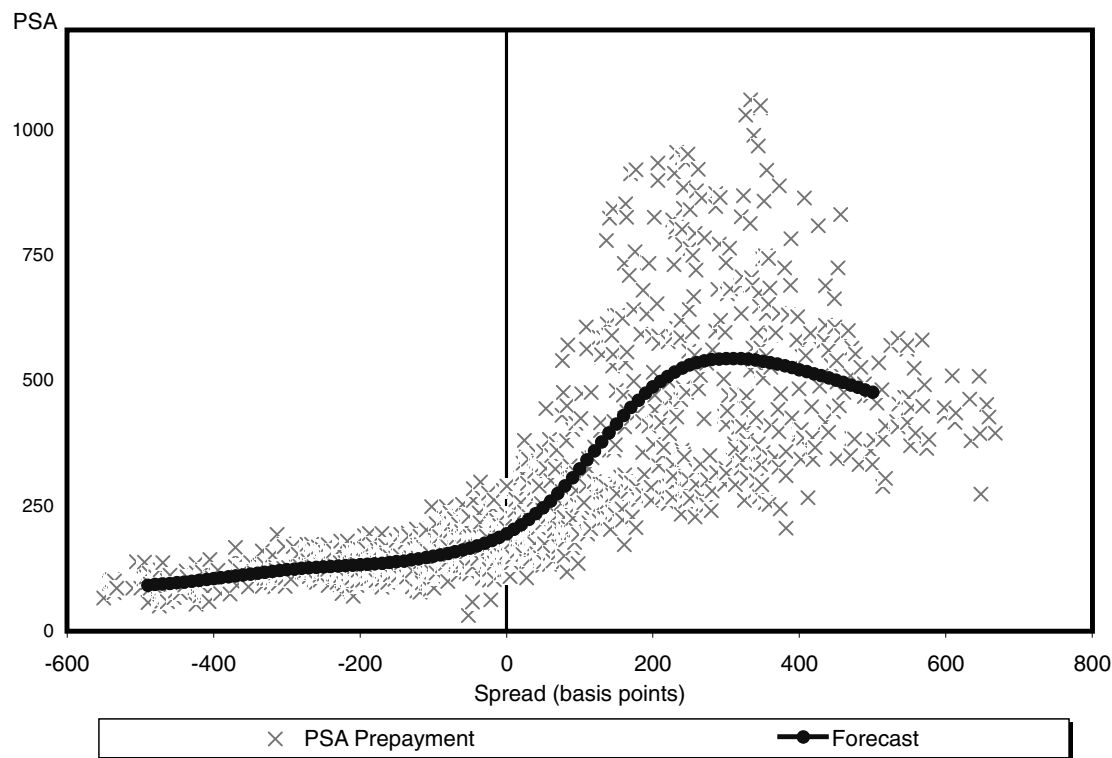
Some have attributed the changing intensity of prepayment cycles to the evolving character of the mortgage market. Mortgage banks and brokers increasingly dominated housing finance in the 1990s. These financial institutions are more agile than banks and thrifts because they face fewer restrictions and regulations. Aided by advances in information technology, mortgage bankers and brokers have expanded geographically by offering more attractive loan products.

The increased competition among lenders has reduced transaction costs for mortgage borrowers, which in turn has led to a rise in the propensity to refinance. Bennett, Peach, and Peristiani [2001] report that 12% of borrowers in the early 1990s prepaid their loan after five years. Under the same economic conditions, the prepayment rate in the 1980s would have been around 7%.

Lekkas [1993] provides further support for the changing nature of refinancings. He notes that the high-rate borrowers who

E X H I B I T   1
Prepayment Experience of FNMA 6s-12s 30-Year Pass-Throughs—1982-1994

reduced their monthly payments by refinancing into lower interest rate loans dominated the 1986-1987 experience. During 1992-1993, however, borrowers elected to shift into shorter-maturity mortgages.

Although borrowers appeared to be more responsive to interest rate movements in the 1990s, it is hard to rationalize how such a moderate shift in the incentive to refinance might have generated so much volatility in mortgage prepayments. Why are MBS prepayment speeds so erratic and unpredictable?

To investigate this question, we take a somewhat unorthodox approach. Most researchers analyze prepayment speeds at the aggregate (pool) level. We focus instead on the microstructure of the MBS. The prepayment experience of the pass-through security is simply the sum of all the individual prepayment decisions in the pool. We will argue that this process of aggregation makes the prepayment function inherently unstable.

I. BACKGROUND

To see the instability of MBS prepayment rates, consider a simple scatter diagram of prepayment speeds and

the relative interest rate differential between the weighted-average coupon (WAC) and the prevailing mortgage rate (*Exhibit 1*). The prepayment rates describe the experience of 30-year conventional Federal National Mortgage Association (FNMA) pass-throughs with coupon rates of between 7% and 12% over 1982–1994.

The solid curve in the scatterplot represents an in-sample forecast of PSA prepayment rates.[1] The S-shape configuration of the in-sample prediction illustrates the non-linear nature of prepayments. Homeowners are reluctant to refinance when spreads are negative because their mortgage option is out of the money. In this negative range, residual prepayments are small, resulting mostly from life events or other idiosyncratic factors.

The scatterplot shows that prepayment rates accelerate as interest rate spreads become more positive. The rising incentive to prepay at higher coupon spreads is best seen by the steepening slope of the forecast curve. Note, however, that at the same time prepayments become more dispersed at higher interest rate spreads. To put it another way, prepayments are *heteroscedastic* with respect to the coupon spread.

For example, FNMA prepayment rates range from 100 PSA to 300 PSA when the interest rate spread is zero.

They are more spread out when the spread is 200 basis points, ranging from 200 PSA to 900 PSA. This great disparity in prepayment rates is found in both individual FNMA coupon cohorts and single pools. Thus, the presence of heteroscedasticity cannot be attributed to the fact that the scatterplot portrays the prepayment experience of a wide class of FNMA securities.

Why is the relationship between prepayment rates and coupon spreads heteroscedastic? One possible explanation is that it may be the result of path-dependence. An MBS pool is made up of a finite number of mortgages. When a mortgage is prepaid, the servicer returns the principal to investors. Subsequent cash flows of the security are paid out of the remaining mortgages in the pool. Prepayment is therefore equivalent to sampling without replacement. This process introduces path-dependence because it changes the composition of the pool.

Consider, for example, an unseasoned mortgage pool that experiences two identical consecutive interest rate cycles. In both cycles, assume that the interest rate spread first rises by 200 basis points but eventually returns to its original level. In the first episode, rate-sensitive homeowners will rush to take advantage of favorable interest rates, and exit the pool, pushing prepayments higher. As mortgage rates decline for the second time around, however, prepayment rates will be lower because the pool now consists of constrained mortgagors who are less able to take advantage of the favorable interest rate environment.

I will offer a somewhat different interpretation of the heteroscedastic traits of prepayment. The unusual dispersion in prepayments is not necessarily caused by path-dependence. Rather, the phenomenon seems simply a statistical artifact of aggregation. In fact, pool-level prepayment rates continue to be heteroscedastic, even though an exactly identical individual replaces a prepaying mortgage holder in the pool.

To be sure, burnout is important. But its role is more critical in shaping the average propensity to prepay, which accounts for the non-linear S-shape of the prepayment function.

II. STATISTICAL MODEL FOR INDIVIDUAL PREPAYMENTS

A traditional mortgage loan is an amortized contract that requires the borrower to pay interest and repay the principal in equal installments. At the same time, the mortgagor is given the right to prepay part of or the entire principal before maturity without penalty. Like any contract with standardized payment streams, a mortgage loan obeys a well-developed mathematical framework (for more details, see Hayre and Mohebbi [1992]).

Assume that the i-th homeowner takes out a conventional fixed-rate mortgage loan at month $t = 0$. The mortgage rate is r_i, and the loan is amortized over T periods (typically, T equals 360 months). Let B_{ti} represent the remaining balance on the loan at month t (thus, B_{0i} represents the original balance of the loan). The remaining balance $\overline{B}_{ti}$ includes all partial (unscheduled) payments. When B_{ti} reaches zero, the loan is fully repaid at month t.

In the absence of any prepayment, the remaining balance of a mortgage is given by:

$$B_{ti} = B_{0i} \frac{(1 + r_i)^T - (1 + r_i)^t}{(1 + r_i)^T - 1} = B_{0i}\alpha_{ti} \qquad (1)$$

The term α_{ti} is known as the amortization factor. It follows that the proportion of the scheduled loan balance outstanding in any month is defined by:

$$q_{ti} = \frac{\overline{B}_{ti}}{B_{ti}} \qquad (2)$$

A mortgage loan that does not incur any partial payments will always have a q_{ti} ratio equal to 100%. The variable q_{ti} is useful for defining the standard measures of prepayment. The fraction of the outstanding loan balance that is prepaid each month is simply given by:

$$p_{ti} = \frac{\Delta q_{ti}}{q_{t-1,i}} \qquad (3)$$

In the monthly analysis, p_{ti} becomes the single monthly mortality rate (SMM). The SMM rate represents the proportion of the outstanding balance of mortgage i prepaid at month t. Typically, SMM, or its annualized version, the conditional prepayment rate (CPR), are used to measure pool-level prepayments, but these prepayment measures are also applicable to a single mortgage.

Mortgage prepayments can occur for three basic reasons: 1) refinancings, 2) property sale, and 3) default. Refinancings represent prepayment by non-mover occupants. The prepayment literature predicts a rational mortgagor will refinance when the intrinsic value of the loan, defined as the immediate benefit of refinancing measured in present value terms, is greater than the benefit from waiting to refinance later (the time value of the option plus

transaction costs). The decision to terminate a mortgage by moving or defaulting also depends on the moneyness of the mortgage option, although one would expect personal characteristics (income, education) and other idiosyncratic events (job loss, death, divorce) to also play an important role.

Recent studies have taken a more direct approach to modeling the cross-sectional heterogeneity in prepayment behavior. The rational prepayment model is characterized by an empirical specification that uses loan-level information on mortgage terminations (see Cunningham and Capone [1990], Caplin, Freeman, and Tracy [1997], and Peristiani et al. [1997]). These empirical studies find strong evidence that prepayments are driven by two particular factors: post-origination home equity, and homeowner creditworthiness.

The empirical methodology is also useful for defining a general stochastic model of individual prepayments. The decision to prepay can be simply expressed as:

$$p_{ti}^* = \beta_{0i} + x_t \beta_{1i} + \varepsilon_{ti} \qquad (4\text{-}1)$$

where

$$\begin{aligned}
p_{ti} &= 100 \quad \text{if} \quad p_{ti}^* \geq 100 \\
p_{ti} &= p_{ti}^* \quad \text{if} \quad 0 < p_{ti}^* < 100 \\
p_{ti} &= 0 \quad \text{if} \quad \text{otherwise}
\end{aligned} \qquad (4\text{-}2)$$

As before, the variable p_{ti} denotes a broad measure of actual prepayment (e.g., the SMM rate or the annualized conditional prepayment rate). For simplicity, we assume the value is bounded above by 100% (full prepayment) and below by zero (no prepayment). The variable p_{ti}^* represents the unobservable notional desire to prepay. Unlike actual prepayment, the notional desire is a continuous variable that can be negative or exceed 100%. If the notional desire to prepay is positive but less than 100%, the homeowner will partially prepay the loan.[2]

The willingness to prepay is determined by a systematic factor x_t, representing market conditions. For simplicity, here we let x_t be a scalar factor representing the spread between the coupon rate and the prevailing market rate. The parameters, β_{1i} and β_{0i}, capture homeowner or loan characteristics. Credit-constrained or collateral-constrained borrowers, on average, are expected to have small positive slope coefficients β_{1i} because they are less sensitive to economic conditions. In other words, a constrained borrower is less responsive to a rise in the interest rate spread x_t.[3]

The term ε_{ti} denotes the random error that accounts for all unexplained variation in the decision to prepay. We assume the error of the model is drawn from a normal distribution with zero mean and variance σ^2.

Equation (4) defines a two–limit censored regression model (see Maddala [1983]). The distribution of monthly prepayment in the two-limit model is determined by a mixture of discrete (unobserved) and continuous (observed) variables.

The probabilities of the three distinct outcomes of prepayment are given by:

$P(i\text{-th homeowner prepays fully in month } t)$

$$= 1 - \Phi(\lambda_{ti}^u)$$

$P(i\text{-th homeowner partially prepays in month } t)$

$$= \Phi(\lambda_{ti}^u) - \Phi(-\lambda_{ti})$$

$P(i\text{-th homeowner does not prepay in month } t)$

$$= 1 - \Phi(\lambda_{ti}) \qquad (5)$$

where $\lambda_{ti} = (\beta_{0i} + x_t\beta_{1i})/\sigma$ and $\lambda_{ti}^u = (100/\sigma) - \lambda_{ti}$. The function $\Phi(\lambda)$ is the standard normal cumulative distribution integrated between λ and ∞. Note that all three probability outcomes in Equation (5) sum to one.

In the censored regression model, the likelihood of prepayment is still determined by the homeowner's characteristics and interest rate conditions. The censored nature of individual prepayments complicates the error structure, however. We can show that:

$$E(\varepsilon_{ti}) = \sigma \frac{\varphi(\lambda_{ti}) - \varphi(\lambda_{ti}^u)}{\Phi(\lambda_{ti}^u) - \Phi(\lambda_{ti})}$$
$$= \sigma h(\beta_{0i}, \beta_{1i}, x_t) = \sigma h(\lambda_{ti}) \qquad (6\text{-}1)$$

$$\text{Var}(\varepsilon_{ti}) = \sigma^2 \left[1 - h(\lambda_{ti})^2 + 100 \frac{\varphi(-\lambda_{ti}^u)}{\Phi(\lambda_{ti}^u) - \Phi(\lambda_{ti})} \right]$$
$$= \sigma^2 \upsilon(\beta_{0i}, \beta_{1i}, x_t) = \sigma^2 \upsilon(\lambda_{ti}) \qquad (6\text{-}2)$$

The error in the mortgage prepayment model therefore has a non-zero mean, and its variance is heteroscedastic (that is, $\text{Var}(\varepsilon_{ti})$ is a function of x_t). It is not hard to understand why these unusual characteristics of individual prepayments are extremely important. An MBS comprises a finite number of borrowers. Since a borrower's decision function is heteroscedastic, this property would also transfer to the MBS prepayment function.

III. MBS PREPAYMENT FUNCTION

Consider a typical mortgage pass-through security consisting of a number of conventional mortgage loans. At origination ($t = 0$), the mortgage pool contains n_0 fixed-rate mortgages with maturity T. After the MBS is issued, there may be fewer mortgages in the pool (e.g., $n_{t+\ell} \,''\, n_t \,''\, n_0$). The overall size of the pool at origination equals B_0. Each loan in the pool contributes B_{0i}, so that $B_0 = \sum B_{0i}$. At origination, the WAC of the MBS is $\overline{r} = \sum \omega_{0i} r_i$, and the weighted-average maturity (WAM) is T months. The scaling factor ω_{0i} represents the relative weight of each mortgage loan at $t = 0$ (or, more generally, $\omega_{ti} = B_{ti}/B_t$).

At its inception, the stream of payments of the MBS is the cash flows of n_0 mortgage loans. Ultimately, the cash flows of the security are determined by the prepayment experience of the pool. To complete the model, we assume that an individual prepayment process is defined by Equation (4). The prepayment experience of the pool in any given month is the sum of all individual prepayments.

Algebraically, we can express this aggregate pool prepayment rate as

$$P_t = \beta_{0t} + \beta_{1t} x_t + \varepsilon_t \tag{7}$$

so that $P_t = \sum \omega_{ti} P_{ti}^*$, $\beta_{ik} = \sum \omega_{ti} \beta_{ik}$ ($k = 0, 1$), and $\varepsilon_t = \sum \omega_{ti} \varepsilon_{ti}$. We should note the parameters β_{0t} and β_{1t} are time-varying, meaning that the slope and intercept of the prepayment function change over time. Since mortgagors are not replaced when they exit the pool, the composition of the pool changes over time with prepayment.

In our simple framework, where prepayment rates are determined by a single factor (the coupon spread), the slope of the prepayment function will be fairly flat at negative values of the spread. In this range, we observe small residual prepayments resulting from idiosyncratic events. The slope of the prepayment function steepens as coupon differentials become positive and widen. Large positive spreads trigger rapid refinancing as borrowers with a higher propensity to prepay (those with high positive β_{1i}) are now in the money. Eventually, the slope of the prepayment function flattens at very high values of spread because the pool is burned out, meaning that the pool now includes mostly constrained borrowers (low-beta homeowners) who are unable to refinance at any rate.[4]

Since individual prepayments are heteroscedastic and have a non-zero mean, pool-level prepayments also have a similar structure. We can show that

$$\varepsilon_t = \sigma \sum_{i=1}^{n_t} \omega_{ti} h(\lambda_{ti}) \tag{8}$$

$$\mathrm{Var}(\varepsilon_t) = \sigma^2 \sum_{i=1}^{n_t} \omega_{ti}\, \upsilon(\lambda_{ti}) \tag{9}$$

The error structure of pool prepayments is again heteroscedastic in the sense that the variance depends on the level of coupon spread x_t.

Equations (8)-(9) can be simplified by linearizing the functions $h^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} h(\lambda_{ti})$ and $\upsilon^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} \upsilon(\lambda_{ti})$. Using a multivariate Taylor approximation rule, we can modify these functions to:

$$E(\varepsilon_t) \cong \sigma[\alpha_0 + \alpha_1 x_t + \alpha_2 x_t^2 + \ldots + \alpha_k x_t^k] = \sigma h(x_t) \tag{10}$$

$$\mathrm{Var}(\varepsilon_t) \cong \sigma^2[\gamma_0 + \gamma_t x_t + \gamma_2 x_t^2 + \ldots + \gamma_k x_t^k] = \sigma^2 \upsilon(x_t) \tag{11}$$

where k represents the polynomial order of the Taylor expansion. Thus, an additive form of heteroscedasticity, which depends on the scalar exogenous factor, can approximate the error structure of the prepayment function x_t.

Equation (11) reveals an interesting finding. The variance of the prepayment errors also depends on x_t. This relationship suggests that statistical inference is more uncertain at higher values of the coupon spread as the confidence interval for the prepayment forecast is wider. We can also use the Taylor approximation in the same manner to specify the theoretical structure of the prepayment function. The aggregate prepayment rate can be expressed as:

$$E(P_t) \cong [\,\beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \ldots + \beta_k x_t^k\,] \tag{12}$$

The aggregate prepayment function is therefore non-linear. But more important, this non-linear function can be easily approximated by a polynomial regression model, assuming that x_t is fully known.
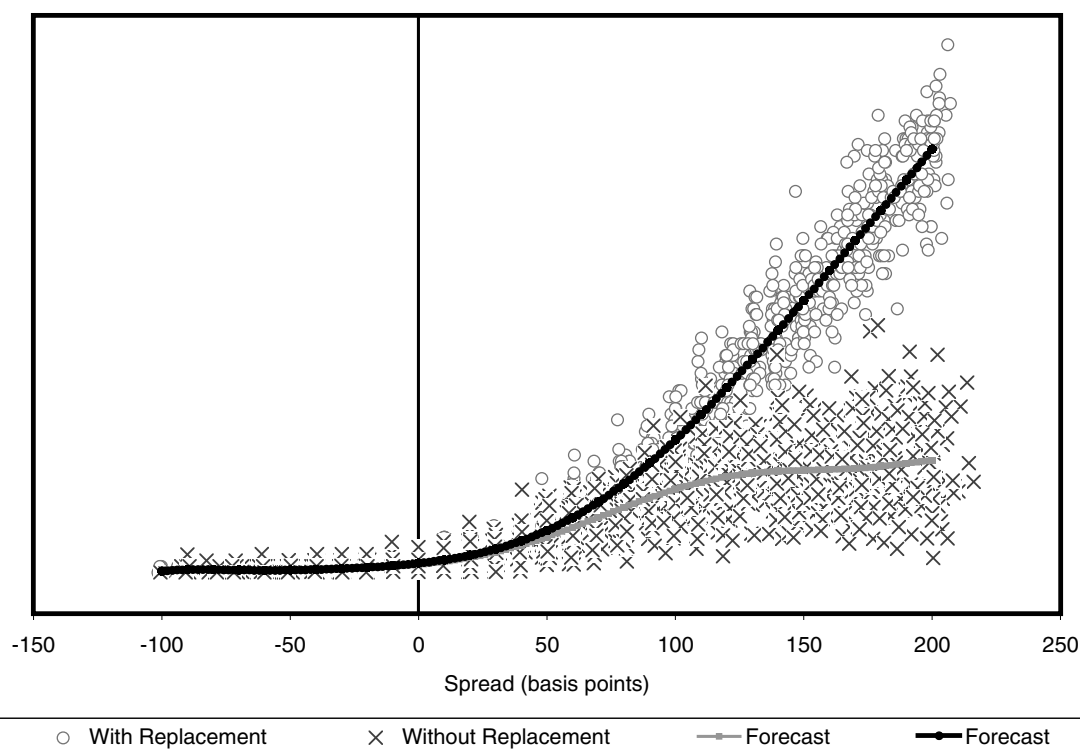
These findings can be easily generalized to the case where a borrower's decision to prepay is influenced by several variables represented by the row vector $x_{t\bullet} = (1, x_{t1}, x_{t2}, \ldots, x_{tp})$. In the multivariate case, we can show that prepayment errors would still be heteroscedastic, although the functional form of additive heteroscedasticity is more complicated.

An alternative way to illustrate the effects of aggregation in prepayments is through simulation. We first construct artificial pools of mortgages. The decision of

EXHIBIT 2
Simulated Pool Prepayments With and Without Replacement

Prepayments

Spread (basis points)

| ○ With Replacement | ✕ Without Replacement | —■— Forecast | —●— Forecast |

mortgage holders to prepay is determined by the interest rate spread defined by Equation (4). For simplicity, however, we assume that borrowers do not partially prepay their loans. In each period, mortgage holders are exposed to a different interest rate spread plus a random shock. The simulation example assumes that individuals in the pool have completely different prepayment functions. Each mortgagor has a different propensity to prepay (that is, mortgagers have unique β_{0t} and β_{1t}).

We perform two different simulation experiments. The first simulation experiment assumes that the mortgage holder exits the pool when the willingness to prepay p_{ti}^* is greater than zero; otherwise, the borrower does not prepay (remember, there are no curtailments). The second simulation experiment assumes again that a borrower would prepay when $p_{ti}^* > 0$; now the prepaying mortgage holder is replaced in the pool by an exactly identical borrower. In this way, we keep the size of the pool constant.[5]

The results of the two simulation examples are graphed in Exhibit 2. There are two distinct scatterplots in the exhibit. Observations marked by the symbol (✕) represent the prepayment experience of pools that prepay
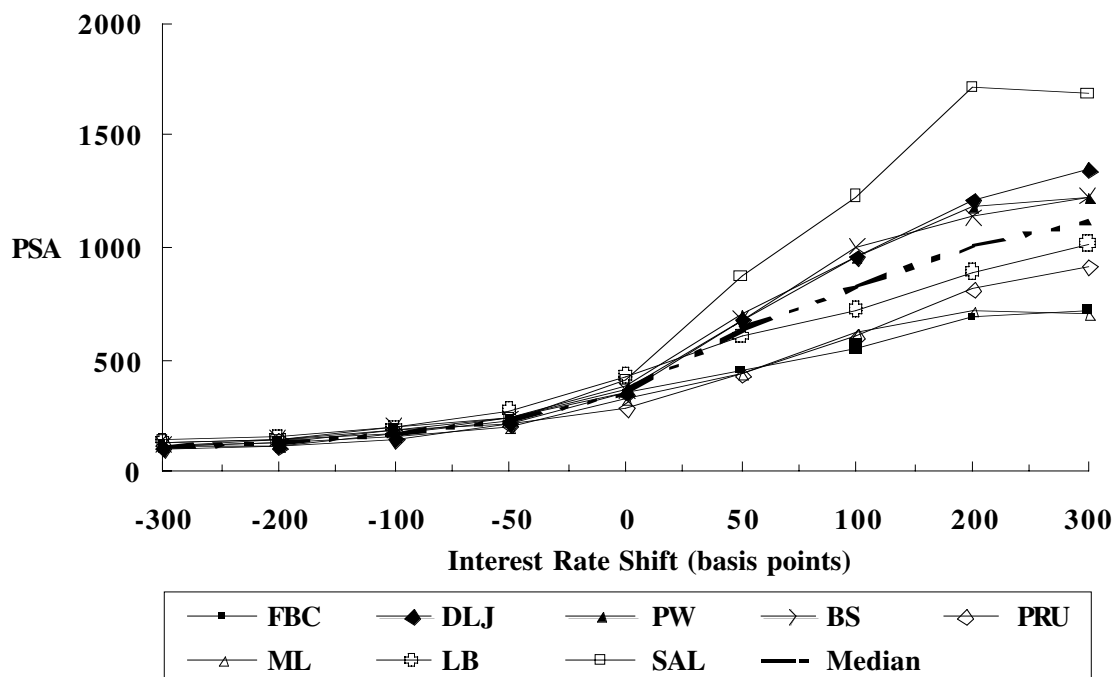
with replacement. The symbol (○) denotes pools that prepay without replacement. The solid curves in the figure represent in-sample predictions for aggregate prepayments estimated from a simple polynomial regression.

Prepayment rates are unstable for high values of the interest rate spread, indicating the heteroscedastic nature of the prepayment function. Note, however, that prepayments are heteroscedastic in both cases. This is an important finding because it helps to demonstrate that burnout (prepayment without replacement) alone cannot explain the heteroscedastic nature of prepayments. The pattern in prepayments remains heteroscedastic when we replace individuals who prepay in the pool.[6]

What clearly distinguishes these two simulation examples is the shape of the average prepayment function. Pool prepayment rates are, on average, much higher when prepaying borrowers are replaced in the pool. This outcome is not surprising, because in this case the composition of the pool is unchanged. At negative spreads, only a small fraction of these individuals wish to prepay. But as spreads become positive and widen, more mortgage holders are willing to prepay because the pool does

E X H I B I T   3
Dealer Prepayment Forecasts for FNMA 8s (as of January 1998)

Source: Bloomberg Financial.
FBC = First Boston Corporation, DLJ = Donaldson Lufkin Jenrette, PW = Paine Webber, BS = Bear Stearns, PRU = Prudential, ML = Merrill Lynch, LB = Lehman Brothers, SAL = Salomon.

not burn out. When borrowers are not replaced in the pool, however, aggregate prepayments tend to level off after a point, giving rise to the distinct S-shape.

The simulation findings suggest that pool burnout does not necessarily account for the phenomenon of heteroscedasticity in MBS prepayment rates. Burnout is solely responsible for the non-linear S-shape structure found in most pool prepayment functions.

IV. IMPLICATIONS

Our analysis provides a compelling theoretical argument that the MBS prepayment function is inherently heteroscedastic—prepayments are more likely to be scattered at higher positive coupon spreads. The unusual nature of prepayments raises a number of interesting questions. Should we be concerned with the heteroscedastic structure of the prepayment function? Can this distinctive error structure in prepayments distort pricing?

Broadly speaking, heteroscedastic errors diminish the power of statistical inference because the least squares regression estimator is inefficient. The impact of het-

eroscedasticity is quite evident in the wide discrepancy of published forecasts available from Bloomberg.

Exhibit 3 summarizes the prepayment forecasts made by eight firms for new FNMA 8s 30-year conventional pass-throughs. The graph clearly shows that forecast uncertainty (here measured by the range of the PSA forecasts) is significantly higher for large interest rate shifts.

The prepayment function is an indispensable part of any MBS pricing methodology. Prepayment assumptions allow investors to figure out cash flows and determine the price of the security. In theory, we expect that the value of an MBS would be influenced by interest rate dynamics and prepayment behavior.

We can formally define the price of a mortgage security j at time t as:

$$V_{tj} = V[\Omega_j, \Re_t, P(\beta_j, x_{t\bullet}, \varepsilon_t)] \tag{13}$$

where $\Re_t$ represents the interest rate process at time t, and Ω_j is a vector of security-specific attributes. We also assume that the exogenous vector $x_{t\bullet}$ and individual characteristics β_j drive prepayments. As shown previously, prepay-

ment errors are not identically distributed but are instead heteroscedastic [e.g., $\mathrm{Var}(\varepsilon_t) = \sigma^2 \upsilon(x_{t\bullet})$].

The great dispersion in prepayment errors introduces the potential for greater disparity in MBS prices. Thus, two MBS may end up with very different price realizations, although ex ante the securities are fundamentally similar. The extent of the price distortion depends essentially on the importance of the underlying prepayment assumptions. In some instances, the value of mortgage securities is extremely vulnerable to changes in prepayment projections.

To see the sensitivity of prices to prepayment assumptions, consider again the prepayment forecasts in Exhibit 3, and assume that the interest rate increases by 50 basis points. For this shift, prepayment forecasts range from a low of 434 PSA (forecast by First Boston) to a high of 868 PSA (forecast by Salomon). Using a Bloomberg pricing algorithm, we compute the option–adjusted spread (OAS) cost of a January 1998 TBA comparable pass–through under the different prepayment scenarios. When interest rates are unchanged (zero interest shift), the median OAS costs for the FNMA 8% pass–through are 75 basis points. Given the wide confidence bounds on prepayments, however, OAS values can range from 23 basis points to 112 basis points. Thus, a mere 50 basis point shift in interest rates produces a great disparity in prices.

V. CONCLUSION

This study has shown that mortgage prepayments become extremely unstable when the spread between the weighted-average coupon (WAC) and the prevailing mortgage rate is wide and positive. The customary view attributes this trait to path-dependence or burnout. According to this line of thought, prepayments are more dispersed (heteroscedastic) because often after a few bouts of refinancing the pool is made up of mostly constrained mortgagors. I provide an alternative interpretation of this phenomenon. Although burnout is an important determinant of prepayment, its role is more evident in the non-linear shape of the prepayment function. The wide dispersion in prepayments is not necessarily related to burnout, but is caused instead by statistical aggregation.

These findings highlight the unstable nature of the prepayment function. There is little dispersion in prepayments for negative or for narrow positive interest spreads. Prepayments, however, become increasingly more volatile when interest rate spreads cross a certain threshold. Consequently, forecasting MBS prepayments becomes more arduous in an economic environment marred by unan-

ticipated interest rate movements. Even a moderate shift in interest rates could alter cash flows in a way that would adversely affect the value of the mortgage security.

ENDNOTES

The author is grateful to Richard Stanton and to seminar attendees at the 2001 AREUEA Annual Conference for helpful comments and suggestions. He also thanks Gijoon Hong for research assistance. The views expressed here are those of the author and should not be interpreted as reflecting the views of the Federal Reserve Bank of New York or the Federal Reserve System.

[1]The Public Securities Administration (PSA) convention assumes pool prepayments rise 0.02% per month for the first 30 months of the life of the pool, and then remain constant at 6% (per year) from the 30th month until maturity.

We use the penalized least squares (PLS) method to estimate prepayment forecasts. This smoothing spline approximation technique is based solely on the coupon spread. Our objective here is to simply demonstrate the non-linear nature of prepayments. Later we show that a polynomial specification is also a good approximation of the prepayment function.

[2]The contribution of partial prepayments (or curtailments) to overall prepayment is generally quite small. For fixed-rate mortgages, partial prepayments contribute on average around 0.2% to conditional prepayment rates. As with partial prepayments, defaults make up a small portion of total prepayment. Usually, the homeowner default on fixed-rate mortgages is under 0.5% per year. Since most pass-throughs are insured against credit risk, we do not consider this option in the censored regression model, but one can explicitly include this outcome by using a more generalized version of the censored model.

[3]Caplin, Freeman, and Tracy [1997] and Peristiani et al. [1997] find evidence that strongly supports this premise. Using a large sample of homeowners, they estimate a qualitative model for the decision to refinance. The empirical findings suggest that credit quality and collateral value have a significant effect on the probability of refinancing.

[4]Another way to look at path-dependence in prepayments is to examine the stochastic properties of n_t, the number of mortgagors remaining in the pool at time t. Because loans are not replaced in the pool, the conditional expectation of n_t depends on n_{t-1}. In turn, this means that the conditional expectation of n_t depends on lagged values of x_t.

[5]Borrowers are heterogeneous in the sense that $\beta_{\ell i} = \overline{\beta}_{\ell i}(1 + \rho\zeta_i)$, $\ell = 0, 1$, where $\overline{\beta}_{\ell i}$ is the predetermined value for the intercept and slope, ρ is a small constant (usually, 0.05) and ζ_i is a random shock generated from a standard normal distribution.

[6]A simple F-test shows that the error sums of squares for the two experiments are not statistically significantly different.

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