Modeling the Dynamics of MBS Spreads

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The U.S. mortgage-backed securities (MBS) market is much larger in dollar terms than the total amount of Treasury debt outstanding. Huge amounts of MBS are held by individual and institutional investors domestically and internationally. Even though principal and interest payments on MBS are default-free due to implicit government guarantees, the timing of cash flows is extremely uncertain because of prepayment options that allow mortgagors to prepay part or all of an outstanding mortgage loan at any time. The prepayment option makes the duration of MBS quite volatile.

One of the most important variables determining MBS yields and prepayment tendencies is the yield on government securities in general and the yield on the ten-year Treasury note in particular. Understanding the spread between MBS yields and Treasury note yields is an important first step in the direction of managing mortgage portfolios in terms of both risk and asset allocation.

Goodman and Ho [1997, 1998] in an investigation of the factors determining MBS-Treasury spreads find that the ten-year Treasury rate, the shape of the yield curve, and the five-year cap volatility are important determinants of fixed-rate MBS yields. And the LIBOR effect, measured by ten-year swap spreads, has become very important in recent years. Arora, Heike, and Mattu [2000] suggest that a five-factor model can explain almost 60% of mortgage excess returns (i.e., spreads).

While we have important information regarding the determination of MBS spreads, no research has been conducted on the time series properties of MBS spreads. For example, are the distributions of spreads constant or time-varying? Are they predictable on the basis of past information? Is there a tendency for them to revert to some long-term mean, and if so what are the dynamics of the mean reversion? Is the volatility of spreads constant or time-dependent?

I attempt to shed some light on these questions using an extended version of the Longstaff and Schwartz [1992] model. The extension allows testing of asymmetric mean reversion in MBS spreads (i.e., the possibility that the process of mean reversion behaves asymmetrically in response to increases and decreases in past spread, as well as time variation and asymmetry in the volatility process.[1]

The empirical evidence supports use of a two-factor framework for study of the dynamics of MBS spreads. Mean reversion is asymmetric during spread increases and decreases. Specifically, spread change behavior is non-stationary following spread increases, but strongly mean-reverting following spread decreases. The mean-reverting component is statistically and economically stronger, thus offsetting non-stationarity. Volatility is time-varying, depending on past innovations, past volatility estimates, and the past level of spreads. Its behavior is asymmetric, rising more in response to positive innovations.

EXHIBIT 1
Unit Root Tests

	Phillips-Perron (PP)	Augmented Dickey-Fuller (ADF)
30-Year Fixed MBS Rate	–1.8409	–2.1175
10-Year Treasury Rate	–2.2360	–2.2868
7-Year Treasury Rate	–2.3369	–2.4148
5-Year Treasury Rate	–2.4544	–2.5459
(MBS – 10-Year Treasury) Spread	–5.8076*	–5.9157*
(MBS – 7-Year Treasury) Spread	–6.2492*	–6.3528*
(MBS – 5-Year Treasury) Spread	–6.2770*	–6.2906*

**Significant at the 5% level.*
PP = Phillips-Perron statistic; ADF = augmented Dickey-Fuller statistic.
Sample period 4/2/71–3/22/02 (1,617 weekly observations).

I. DATA AND PRELIMINARY STATISTICS

The primary data set consists of weekly annualized yields to maturity on 10-year, 7-year, and 5-year Treasury notes as well as weekly annualized yields on 30-year fixed-rate conventional mortgages. The data set is obtained from the Federal Reserve H.15 database. Variables are characterized as follows:

DGS10YR = 10-year Treasury constant maturity;
DGS7YR = 7-year Treasury constant maturity;
DGS5YR = 5-year Treasury constant maturity; and
WMORTG = 30-year conventional mortgage rate (contract rate on commitments for fixed-rate first mortgages).

The time period extends from March 2, 1971, through March 22, 2002, giving a total of 1,617 weekly observations for each variable. Using the variables above, three MBS spreads are constructed with respect to the ten-year, seven-year, and five-year Treasury yield.

Studies have found that interest rates follow processes resembling random walks. For example, Nelson and Plosser [1982] find that interest rates have a unit root. Similarly, in Koutmos [2001] I find that MBS rates have unit roots. The implication is that these variables are non-stationary in their levels. No results have been reported thus far with regard to the properties of spreads, even though knowing whether spreads have unit roots is extremely important for the purpose of modeling them properly.

The test measures used to test for unit roots are Dickey-Fuller [1981] (ADF) and Phillips-Perron [1988] (PP) statistics. The ADF statistic is calculated as follows:

$$\Delta y_{i,t} = a_0 + a_1 t + a_2 y_{i,t-1} + \Sigma^{p}_{s=1} C_s \Delta y_{i,t-s} + u_{i,t}$$

where Δ is the first-difference operator, t a time trend, $y_{i,t}$ the variable to be tested for a unit root, and a_0, a_1, a_2, and c_s are fixed parameters. The null hypothesis is H_0: $a_2 = 0$ versus H_1: $a_2 < 0$.

A somewhat different approach is adopted by Phillips and Perron. The PP test is based on the regression:

$$y_{i,t} = b_0 + b_1(t - T/2) + b_2 y_{i,t-1} + v_{i,t}$$

Their null hypothesis is H_0: $b_2 = 1$ versus H_1: $b_2 < 1$, where T is the sample size.

In either model, acceptance of the null hypothesis would imply that the variable $y_{i,t}$ is non-stationary or, equivalently, there is a unit root in the univariate representation.

The results for unit root testing are reported in Exhibit 1. Both the Phillips-Perron and the augmented Dickey-Fuller statistics confirm that there is a unit root for all three Treasury yields as well as the 30-year mortgage yield. Of more direct interest for my purposes, however, are the properties of the spreads. For all three spreads, both the PP and the ADF tests reject the hypothesis that there is a unit root. The implication is that spreads are stationary and mean-reverting so that standard statistical hypothesis testing is applicable.

EXHIBIT 2
Descriptive Statistics

	$\Delta_{10,t}$ (MBS Yield – 10-Year Treasury)	$\Delta y_{10,t}$ First-Difference (MBS Yield – 10-Year Treasury)	$y_{7,t}$ (MBS Yield – 7-Year Treasury)	$\Delta y_{7,t}$ First-Difference (MBS Yield – 7-Year Treasury)	$y_{5,t}$ (MBS Yield – 5-Year Treasury)	$\Delta y_{5,t}$ First-Difference (MBS Yield – 5-Year Treasury)
μ	1.6714*	–0.0000	1.7344*	–0.0000	1.8918*	0.0000
σ	0.5854	0.1478	0.5749	0.1554	0.6064	0.1639
Skewness	1.6347*	1.7759*	1.4664*	1.7291*	0.8556*	1.6324*
Kurtosis	4.9462*	21.3935*	4.8051*	20.5196*	2.5391*	17.5997*
ρ	0.9681*	0.1989*	0.9635*	0.1915*	0.9636*	0.1751*
LB(5)	6353.4*	73.7061*	6133.6*	67.8963*	6175.3*	58.5739*
$LB^2(5)$	5772.1*	448.0718*	5662.9*	482.7238*	5837.6*	484.7261*

**Significant at the 5% level at least.*

μ = sample mean; σ = sample standard deviation; ρ = first-order autocorrelation; LB(5) = Ljung-Box statistic for y_t and Δy_t; $LB^2(5)$ = Ljung-Box statistic for $(y_t)^2$ and $(\Delta y_t)^2$.

Sample period 4/2/71–3/22/02 (1,617 weekly observations).

Exhibit 2 reports several descriptive statistics for the spreads, as well as their first differences. The average spread ranges from 1.67% for the ten-year yield to 1.89% for the five-year yield. In all cases, these spreads are both statistically and economically significant.

The estimated standard deviations suggest that there is considerable risk in the spread structure. Measures of skewness and kurtosis suggest that both spreads and their first differences fail to be normally distributed processes. The first-order autocorrelation is very close to unity for the spreads, but as reported earlier they do not have unit roots.

The Ljung-Box statistic indicates that autocorrelations are significant up to five lags. This holds for spreads, first differences, and their corresponding squares, providing some early indication of time variation in the conditional first and second moments of MBS spreads.

II. TWO-FACTOR MODEL AND ECONOMETRIC SPECIFICATIONS

The Longstaff and Schwartz [1992] continuous-time two-factor model can be written as follows:

$$dy_t = (a - by_t)dt + c\sigma_t dz_{1,t} \quad (1)$$
$$d\sigma^2_t = (k - e\sigma^2_t)dt + f\sigma_t dz_{2,t} \quad (2)$$

where a, b, c, k, e, and f are fixed parameters, and $z_{1,t}$ and $z_{2,t}$ are Wiener processes. The stochastic differential Equations (1) and (2) describe the dynamics of the MBS spreads (in our case) y_t and its variance σ^2_t.

The discrete approximation used by Longstaff and Schwartz is given by:

$$y_t - y_{t-1} = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2\sigma^2 t + \varepsilon_t \quad (3)$$
$$\sigma^2_t = \beta_0 + \beta_1\varepsilon^2_{t-1} + \beta_2 y_{t-1} + \beta_3\sigma^2_{t-1} \quad (4)$$

where y_t is the spread; σ^2_t its conditional variance; α_0, α_1, α_2, β_0, β_1, β_2, and β_3 are fixed parameters; and ε_t is the error term or innovation.

The two-factor model allows both spread changes and volatility to influence the evolution of MBS spreads. Mean reversion is captured by α_1. If $\alpha_1 < 0$ ($\alpha_1 \geq 0$), the interest rate process is mean-reverting (non-stationary). The conditional mean of spread changes depends on the level of past spreads y_{t-1} and the level of current volatility σ^2_t. The conditional variance follows a standard GARCH(1,1) process with the exception that the past value of the spread is allowed to influence the variance. The sensitivity of variance to the level of the last period's spread is measured by β_2.

The econometric specifications given in (3)-(4) have the limitation that they cannot be used to test for asymmetric

EXHIBIT 3
Least Squares Estimates of Mean Reversion

	MBS Yield – 10-Year Treasury Yield (= $y_{i,t}$)	MBS Yield – 7-Year Treasury Yield (=$y_{i,t}$)	MBS Yield – 5-Year Treasury Yield (=$y_{i,t}$)
Constant (c)	0.0533	0.0634	0.0690
	(4.826)*	(5.2030)*	(5.2160)*
Mean Reversion (b)	–0.0319	–0.0365	–0.0365
	(–5.1140)*	(–5.4820)*	(–5.474)*
Adjusted R^2	0.0153	0.0176	0.0176
F (1,1614)	26.1577*	30.0497*	29.9635*

**Significant at the 5% level at least.*
t-statistics are in parentheses.
Sample period 4/2/71–3/22/02 (1,617 weekly observations).

mean reversion and asymmetric volatility. Consequently, I modify the model along the lines of Bali [2000] as follows:

$$y_t - y_{t-1} = \alpha_0 + I_t\alpha_{1,p}y_{t-1} + (1-I_t)\alpha_{1,n}y_{t-1} + \alpha_2\sigma^2_t + \varepsilon_t \quad (3\text{–A})$$

$$\sigma^2_t = \beta_0 + \beta_1\varepsilon^2_{t-1} + \beta_2 u^2_{t-1} + \beta_3 y_{t-1} + \beta_4\sigma^2_{t-1} \quad (4\text{–A})$$

where I_t is the Heaviside indicator function, defined as:

$$I_t = 1 \text{ if } y_t - y_{t-1} \geq 0$$
$$I_t = 0 \text{ if } y_t - y_{t-1} < 0 \quad (5)$$

and

$$u_{t-1} = \min(0, \varepsilon_{t-1}) \quad (6)$$

If $\alpha_{1,p} \neq \alpha_{2,p}$, mean reversion is asymmetric. Furthermore, overall mean reversion is assured if $\alpha_{1,p} + \alpha_{2,p} < 0$. β_1 measures the sensitivity of σ^2_t to past squared errors, while β_2 captures potential asymmetries due to the sign of past errors. The contribution of a positive (negative) error to the variance will be equal to $\beta_1 + \beta_2$ (β_1). Assuming $\beta_2 > 0$, positive past errors (i.e., spread increases) will increase the conditional variance proportionately more than negative past errors (i.e., spread decreases). The opposite holds for $\beta_2 < 0$. The sensitivity of variance to the level of the spread is measured by β_3, and the persistence is measured by β_4.

The vector of parameters describing the conditional mean and the conditional variance of the two-factor model can be obtained via maximum-likelihood estimation (MLE). Assuming that innovation ε_t follows a conditional normal distribution, the sample likelihood to be maximized can be expressed as:

$$L(\Theta) = \Sigma_{=1} \log f(\mu_t, \sigma^2_t) \quad (7)$$

where Θ is the parameter vector, $f(\cdot)$ is the conditional normal density, and μ_t and σ^2_t are the conditional mean and the conditional variance, respectively.

Most studies dealing with financial time series find that the normality assumption is violated but the violations are much less frequent when conditional heteroscedasticity is accounted for (see, for example, Bollerslev, Chou, and Kroner [1992]). I use numerical maximum-likelihood estimation based on the algorithm suggested by Berndt et al. [1974].

III. MAIN EMPIRICAL FINDINGS

Exhibit 3 reports the results of a test for mean reversion based on a simple regression model.[2] The mean reversion parameter b has the right sign and is statistically significant, implying that MBS spreads are mean-reverting. The F-statistic is also significant, suggesting that the model produces significant results overall, even though the adjusted R^2 is very low. As a first approximation, the simple regression model is useful—but it cannot be used to provide answers to the questions I raise earlier.

EXHIBIT 4
Maximum-Likelihood Estimates of Two-Factor Model

	MBS Yield – 10-Year Treasury Yield (= $y_{i,t}$)	MBS Yield – 7-Year Treasury Yield (=$y_{i,t}$)	MBS Yield – 5 -Year Treasury Yield (=$y_{i,t}$)
α_0	0.0232	0.0306	0.0277
	(58.379)*	(37.304)*	(33.409)*
$\alpha_{1,p}$	0.0235	0.0165	0.0164
	(28.419)*	(11.111)*	(20.516)*
$\alpha_{1,n}$	–0.05478	–0.0570	–0.0532
	(–31.745)*	(–44.699)*	(–50.252)*
α_2	0.3343	0.2201	0.6166
	(1.248)	(1.150)	(9.980)*
β_0	0.0000	0.0004	0.0003
	(0.000)	(0.111)	(1.167)
β_1	0.0643	0.0662	0.0717
	(7.169)*	(7.563)*	(9.650)*
β_2	0.0883	0.1230	0.2257
	(5.684)*	(6.310)*	(9.772)*
β_3 (10^4)	0.3577	0.6378	0.8075
	(2.977)*	(2.458)*	(3.973)*
β_4	0.8858	0.8741	0.8321
	(168.71)*	(155.65)*	(89.405)*
t-statistic H_0: $\alpha_{1,p} = \alpha_{2,p}$	38.0953*	34.9767*	49.4051*
t-statistic H_0: $\alpha_{1,p} + \alpha_{2,p} \geq 0$	–15.2390*	–19.2871*	–26.0283*

**Significant at the 5% level at least.*
t-statistics in parentheses.
Sample period 4/2/71–3/22/02 (1,617 weekly observations).

Exhibit 4 reports the results from estimation of the two-factor model with asymmetric mean reversion. The maximum-likelihood estimates of the model present an entirely different picture from results of the simple regression model that postulates symmetric mean reversion. There is strong evidence that the spreads follow asymmetric mean-reverting processes. This result is consistent across all three MBS spreads under examination.

Using a t-test, we can easily reject the hypothesis that $\alpha_{1,p} = \alpha_{2,p}$. Interestingly, $\alpha_{1,p}$ is positive and significant, while $\alpha_{1,n}$ is negative and significant in all instances. This implies that changes in spreads exhibit non-stationary behavior following spread increases, but are strongly mean-reverting following spread decreases.

It is reassuring that the mean-reverting component more than offsets the non-stationary component, given that for all three spreads the estimated parameters satisfy the condition for overall mean reversion, i.e., $\alpha_{1,p} + \alpha_{2,p} < 0$. These findings are in agreement with Bali [2000], who finds that U.S. interest rates exhibit asymmetric mean reversion.

There is little evidence that the conditional variance affects the spread changes. With the exception of the MBS spread with respect to the five-year Treasury yield, α_2 is insignificant. The conditional variances are functions of past squared residuals (β_1), their past history (β_4), and the level of past spreads (β_3). The response of the conditional variance to past innovations is also asymmetric, given that β_2 is positive and statistically significant.

The type of asymmetry documented here is the same as that reported for stock returns; that is, negative innovations (i.e., spread declines) increase volatility more than positive innovations (spread increases). Using as a

EXHIBIT 5
Model Diagnostics

Model Estimated Standardized Residual	MBS Yield – 10-Year Treasury Yield (= $y_{i,t}$)	MBS Yield – 7-Year Treasury Yield (=$y_{i,t}$)	MBS Yield – 5-Year Treasury Yield (=$y_{i,t}$)
$E(\varepsilon_t/\sigma_t)$	0.0167	0.0299	0.0150
$E(\varepsilon_t/\sigma_t)^2$	1.0322	1.0241	0.9936
LB(5)	5.7335	5.2031	4.1323
$LB^2(5)$	9.6863	3.1904	1.1720
Sign Bias Test	0.6057	0.7162	0.1070
Negative Size Bias	–1.6465	–0.6316	0.0083
Positive Size Bias	0.2652	–0.1056	–0.7092

Sign Bias Test: $(\varepsilon_t/\sigma_t)^2 = a + b\,S + e_t$
Negative Size Bias Test: $(\varepsilon_t/\sigma_t)^2 = a + b\,S\varepsilon_{t-1} + e_t$
Positive Size Bias Test: $(\varepsilon_t/\sigma_t)^2 = a + b\,(1 - S)\varepsilon_{t-1} + e_t$

$E(\varepsilon_t/\sigma_t)$ and $E(\varepsilon_t/\sigma_t)^2$ are the mean and variance of the standardized residuals obtained from the two-factor model.
LB(5) = Ljung-Box statistic for (ε_t/σ_t); $LB^2(5)$ = Ljung-Box statistic for $(\varepsilon_t/\sigma_t)^2$.
Sample period 4/2/71–3/22/02 (1,617 weekly observations).

metric for asymmetry the ratio $(\beta_1 + \beta_2)/\beta_1$, it can be said that for the ten-year spread the asymmetry factor is 2.3 (i.e., spread decreases increase volatility 2.3 times more than spread increases) and for the seven-year spread the asymmetry factor is 2.8. For the five-year spread the asymmetry factor is 4.1.

Exhibit 5 reports some diagnostics using model estimated standardized errors. The first and second moments of the standardized errors are zero and unity, respectively, as implied by the model. There is no evidence of autocorrelation up to five lags in either the standardized errors or the squared standardized errors.

Correct specification of the conditional variance is extremely important to price derivatives products correctly and to manage spread risk. Diagnostics such as the LB statistics for the squared normalized residuals are useful in terms of detecting any remaining non-linear structure, but they are not designed to test how well a model captures asymmetry in the conditional variance, or the impact of positive and negative innovations on volatility.

For this purpose I use some recently suggested diagnostics proposed by Engle and Ng [1993]. These tests are based on the news impact curve implied by the particular model used. If the volatility process is correctly specified, the squared standardized residuals should not be predictable on the basis of observed variables. The tests include 1) the sign bias test, 2) the negative size bias test, and 3) the positive size bias test.

The sign bias test examines the impact of positive and negative innovations on volatility not predicted by the model. In this case, the squared normalized residuals are regressed against a constant and a dummy S that takes the value of unity if the residual ε_{t-1} is negative and zero otherwise. The test is based on the t-statistic for S.

The negative size bias test examines how well the model captures the impact of large and small negative innovations. It is based on regression of the squared standardized residuals against a constant and $S\varepsilon_{t-1}$. The calculated t-statistic for $S\varepsilon_{t-1}$ is used in this test.

The positive size bias test examines possible biases associated with large and small positive innovations. Here, the squared standardized residuals are regressed against a constant and $(1 - S)\varepsilon_{t-1}$. Again, the t-statistic for $(1 - S)\varepsilon_{t-1}$ is used to test for possible biases.

Results of these tests are also reported in Exhibit 5. The normalized residuals show no evidence of misspecification. The t-statistics for the individual tests are statistically insignificant for all three interest rates. Thus, the estimated volatilities from the two-factor model fully incorporate all past information.

IV. CONCLUSION

This research has explored the dynamics of MBS spreads using a two-factor model. The empirical evidence presented supports the hypothesis of asymmetric mean reversion. More specifically, spread changes exhibit non-stationary behavior following spread increases, but they are strongly mean-reverting following spread decreases. Mean reversion is statistically and economically stronger, thus offsetting non-stationarity.

Volatility depends on past innovations, past volatility, and the level of the past spread. With respect to past innovation, volatility is asymmetric, rising more in response to negative innovations. This is the same type of asymmetry that is reported for stock return volatility.

Various model diagnostics performed on the standardized residuals show that the two-factor model describes interest rate dynamics quite well.

ENDNOTES

[1]The Longstaff and Schwartz [1992] model is a two-factor general equilibrium model used to describe the yield curve. The two factors are the short-term interest rate and the instantaneous variance of changes in short-term rates. Longstaff and Schwartz find support for the two-factor model using one-month U.S. Treasury bill yields.

[2]The regression model is given by $y_{i,t} - y_{i,t-1} = c + b\, y_{i,t-1} + u_{i,t}$. Mean reversion implies that b is negative and statistically significant. It should also be noted that the least squares estimates for this model are consistent even if the variance is time-dependent.

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