

# Interest Rate Model Selection

*A conscientious choice for mortgage investors.*

Alexander Levin

**I**ntensive developments in interest rate modeling have delivered a bold but confusing model selection choice to financial engineers, risk managers, and investment analysts. Do these modeling issues sound familiar?

- Should a mortgage bank assess interest rate risk using the lognormal Black-Karasinski [1991] model or the normal Hull-White [1990] model?
- Can a portfolio be hedged using different pricing models for assets and derivatives?
- Is there any historical evidence that one model is better than another?
- What does the market think about the interest rate distribution? (It must have some idea, or how would interest rate options be traded?)

We show that selecting the best term structure model is becoming more of a conscientious task than a matter of taste. Recent historical rates, the implied volatility skew for swaptions, and general volatility levels confirm rate normalization and reject the idea of lognormality. We propose valuing mortgages using the Hull-White [1990] model, which can be quickly and accurately calibrated to both the yield curve and the swaption volatility matrix.

## LOGNORMALITY: THE OLD DAYS

Those who read research on how interest rates performed in the 1980s and the early 1990s are accustomed to the conjecture of lognormality. It is that interest rates

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are lognormally distributed; i.e., their logarithm is normally distributed. The rates therefore cannot become negative, and their randomness should be naturally and steadily measured by relative volatility. This conjecture underlies the validity and applicability of the Black-Scholes pricing model to the interest rate option market. A good introductory treatment of the Black-Scholes model and the notion of Black volatility can be found in Hull [2000].

Following Wilmoth [1998], we will measure volatility by plotting the averaged daily increments versus the rate level. We can collect all daily rate increments and store them in “buckets,” each bucket corresponding to some rate level. For example, a 7% bucket includes all daily increments when the rate was between 6.5% and 7.5%. After the data are collected, we average increments using the root mean square formula applied within each bucket, and then annualize them. Although the U.S. Treasury rates are currently not the best benchmark for mortgages, they have the longest history (*Exhibit 1*).

In Exhibit 1, let us first disregard the bars and look only at the line depicting historical volatility measured by

annualized deviations (right axis). The absolute historical volatility seems to be very much independent of rates in the left half of the chart (west of 10%). When the rates are in double-digits, the same absolute volatility measure grows with the rate level.

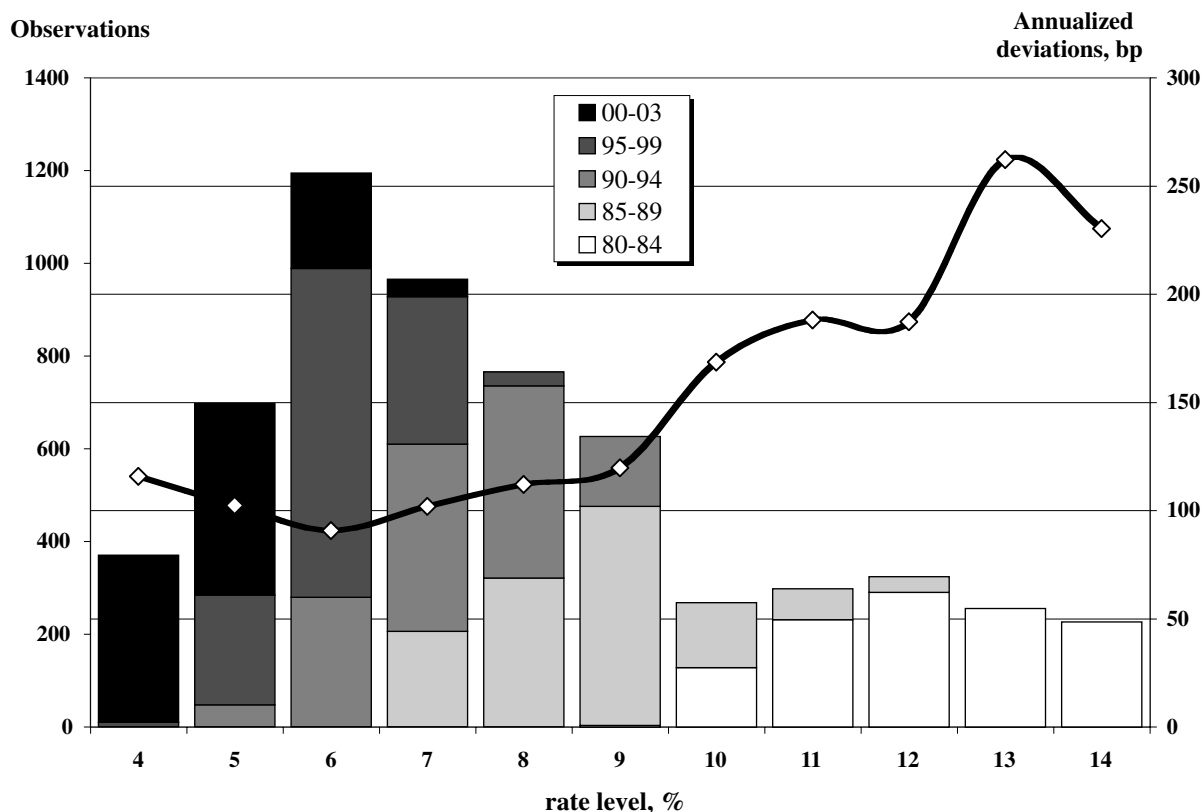
Now, reading the historical labeled bars, we conclude that the absolute volatility has become rate-independent since the late 1980s. This conclusion is generally confirmed by a similar analysis performed for the ten-year swap rate history dating back to 1989 (*Exhibit 2*).

A weak or absent relation between absolute volatility and rate level is a sign of normality rather than lognormality. It also prompts quoting rate uncertainty (and therefore option prices) in terms of absolute volatility (such as 110 basis points) rather than relative volatility (say, 20%). Recently, many brokers have begun communicating in exactly that way.

## WHAT DOES THE SWAPTION MARKET THINK?

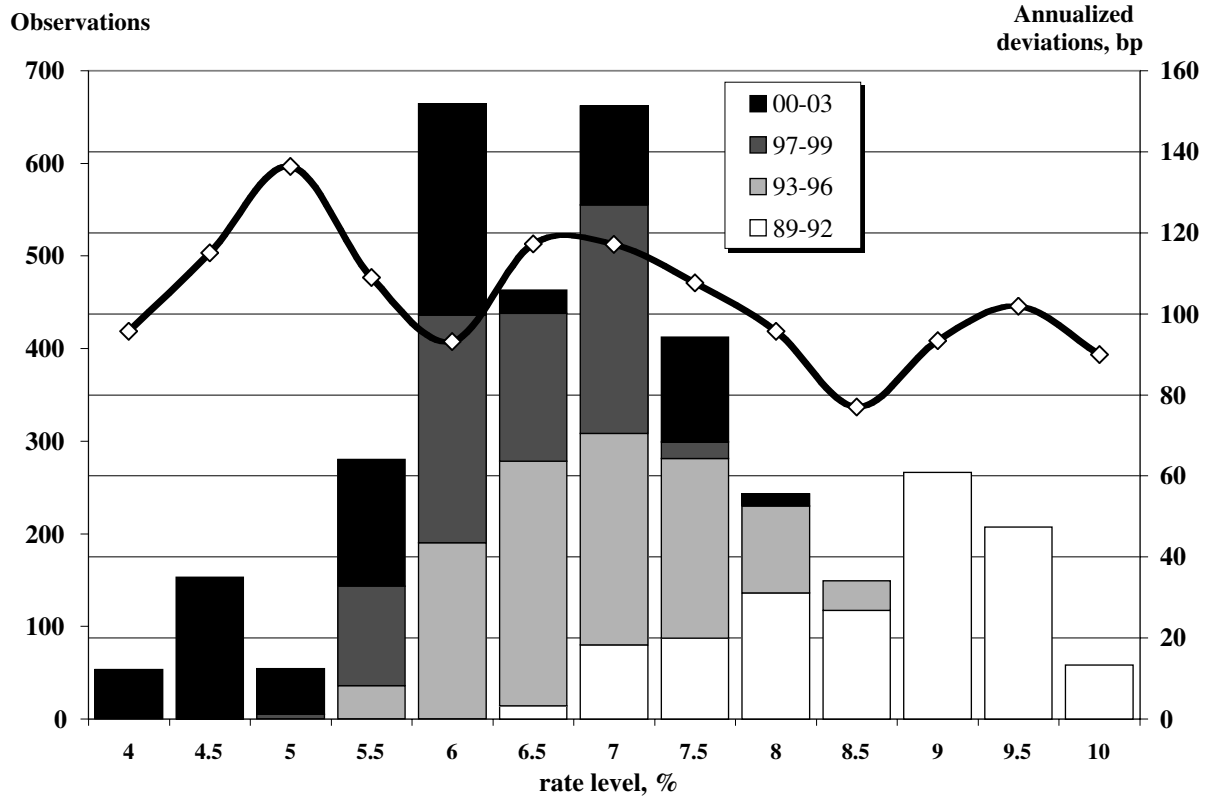
Can we recover the rate distribution from the way interest rate options trade? A simple way is to measure the

**EXHIBIT 1**  
Daily Volatility versus Level for 10-Year Treasury Rate



## EXHIBIT 2

### Daily Volatility versus Level for 10-Year Swap Rate



implied volatility skew, i.e., the dependence of the implied Black volatility on the strike level. If market participants believe in lognormality, there would be little reason for the implied volatility to change with the option's strike. A volatility skew testifies against lognormality by the very fact of its existence.

To discuss a simple skew measurement method, let us first introduce a setup that generalizes many known and popular single-factor models, a constant elasticity of variance (CEV) model:

$$dr = (\text{Drift})dt + \sigma r^\gamma dz \quad (1)$$

where  $r$  is some modeled rate,  $\sigma$  is the volatility coefficient, and  $\gamma$  is the CEV constant. As usual,  $t$  is time,  $z(t)$  is the Brownian motion that disturbs the market, and the exact specification of the drift term is not very important for our purposes. The CEV concept has no specific economic meaning but can be viewed as a convenient way to generalize and compare all known popular models.<sup>1</sup>

For  $\gamma = 1$ , the absolute volatility is proportional to the rate, and we have a lognormal model (with a properly

selected drift term), such as Black, Derman, and Toy (BDT) [1990] or Black and Karasinski (BK) [1991]. For  $\gamma = 0$ , the absolute volatility is rate-independent and can lead to a normal model, such as Hull and White (HW) [1990]. If  $\gamma = 0.5$ , we may have a popular family of square root models, such as the squared Gaussian model (SG) (see James and Webber [2000]), or the model of Cox, Ingersoll, and Ross (CIR) [1985]. Any unnamed values for the CEV are certainly possible, including negative values (hypernormality) and values exceeding 1 (hyperlognormality).

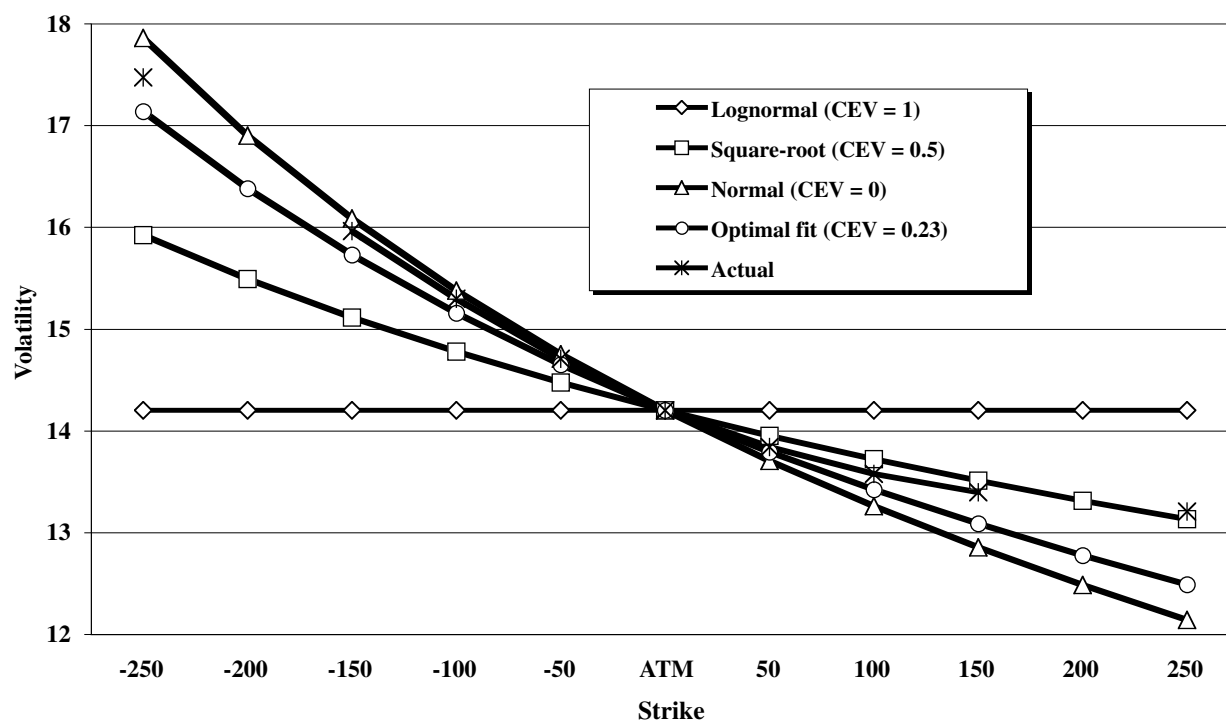
Blyth and Uglum [1999] propose a simple method of recovering the most suitable CEV constant by just looking at the observed swaption volatility skew. They argue that, if a swap forward rate satisfies the random process (1), the skew should have the approximate form:

$$\frac{\sigma_K}{\sigma_F} \approx \left( \frac{F}{K} \right)^{\frac{1-\gamma}{2}} \quad (2)$$

where  $\sigma_K$  and  $\sigma_F$  are the Black (i.e., proportional) volatilities for the actual strike ( $K$ ) and the at-the-money strike ( $F$ ),  $F$  is today's forward rate, and  $K$  is the swaption strike.

## EXHIBIT 3

### Implied Volatility Skew on 5-Year-into-10-Year Swap



\*Source for actual volatility history: Bank of America; volatility for 200 ITM/OTM was not quoted.

Let us analyze the same set of CEV special values, 0, 1.0, and 0.5. If  $\gamma = 1.0$ , there will be no skew at all:  $\sigma_K \equiv \sigma_F$  for any strike  $K$ . This is the Black-Scholes case. For  $\gamma = 0$ , the skew has a functional form of inverse square root. For  $\gamma = 0.5$ , it will have the shape of an inverse fourth-degree root. It is worth mentioning here that each inverse root function is a convex one, so the theoretical skew should not be deemed a straight line (except when  $\gamma = 1$ ). In fact, it should not be confused with a more aggressive convex volatility smile that may or may not be present in addition to the skew.<sup>2</sup>

The object of our study—the five-year into ten-year swaption (5-into-10)—was selected with modeling volatility of mortgage rates and valuation of the prepayment option in mind. Exhibit 3 depicts five skew lines plotted for three named CEVs, the actual volatility observations averaged from January 1998 through May 2002, and the optimal fit line ( $\gamma = 0.23$ ) for the same period. The best CEV is therefore found to be generally between the normal case (HW model) and the square root case (SG or CIR models). It is also seen that low-struck options are traded with a close-to-normal volatility, while high-struck

options are traded with a square root volatility. This phenomenon may be a combination of a slight theoretical smile and the broker commission demand.

Exhibit 4 illustrates historical month-by-month skew, suggesting that the normalization effect ( $\gamma \approx 0$ ) has been observed since the beginning of 2001.

This CEV analysis unambiguously rejects lognormality and reveals a more suitable model. Although the best-fit CEV constant varies somewhat, any volatility model between the normal one and the square root seems to be a decent choice. Because of its analytical tractability and the recent CEV trend, we focus on the HW model as the chief alternative to the BDT or BK models.

## VOLATILITY INDEX

It is useful to design a market volatility index—a single number reflecting the overall level of swaption volatility deemed relevant to the mortgage market. This gives us a convenient way to communicate with practitioners and compare models; it can also serve as a risk assessment tool.

We designate a family of at-the-money swaptions

and, assuming no mean reversion, optimize for the single short rate volatility constant  $\sigma$  (volatility index) best matching the swaptions' volatility surface, on average. This volatility index is model-specific; unlike some other volatility indexes (such as the Lehman Brothers indexes), it is not a simple average of swaption volatilities. The internal analytics of each model (exact or approximate) are used to translate the short rate volatility constant into swaption volatilities used for calibration.

Note that this constant-volatility zero mean reversion framework is used only to derive the volatility index. It is not recommended for actual option-adjusted spread valuation, where we strongly prefer the accuracy gained by optimizing a time-dependent volatility function  $\sigma(t)$  and a mean reversion constant.

Exhibit 5 depicts the history of three volatility indexes (sigmas) computed from the beginning of 2000 for the Hull-White normal model, the Black-Karasinski lognormal model, and the squared Gaussian model. Each index is calibrated to the same family of equally weighted ATM swaptions deemed relevant to the mortgage market: options on the two-year and the ten-year underlying swaps with expirations ranging from six months to ten years. We add for comparison a line for the seven-year rate level, and scale all four lines so that they start at 1.0.

Exhibit 5 confirms a spectacular normalization of the

market; the volatility index constructed for the Hull-White model has gradually become the most stable one. For example, the swap rate plunged a good 60% between January 2000 and June 2003, but the absolute volatility index barely changed. The two other models have produced volatility indexes that follow the rate level but in the opposite direction (the lognormal model does by far the worst job).

Interestingly enough, the squared Gaussian index was stable for most of 2003 and could handle the rate plunge to a new historical low (2.9%) in June 2003. This confirms that a square root volatility functional pattern may outperform others when rates are very low.

These findings are consistent with the swaption skew measures we have discussed. This is not a coincidence at all. People who set the market for ATM swaptions are the same ones who trade out-of- and in-the-money options.

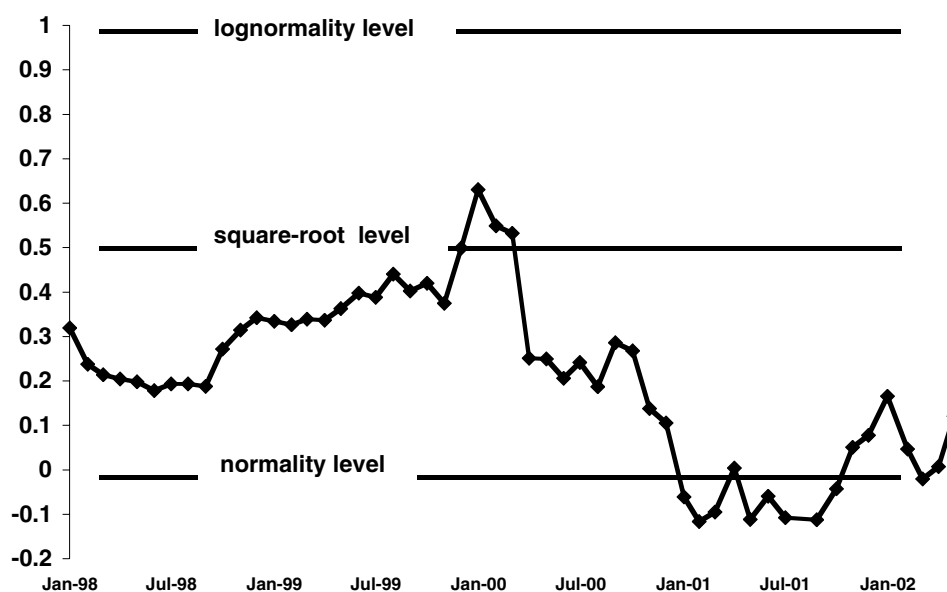
## OTHER PROBLEMS WITH LOGNORMALITY

Although the Black, Derman, and Toy [1990] and the Black and Karasinski [1991] models have been the bread and butter of option traders since they were developed, a full-scale implementation required for good mortgage analytics is not a simple task.

Short rate lognormal models are not analytically

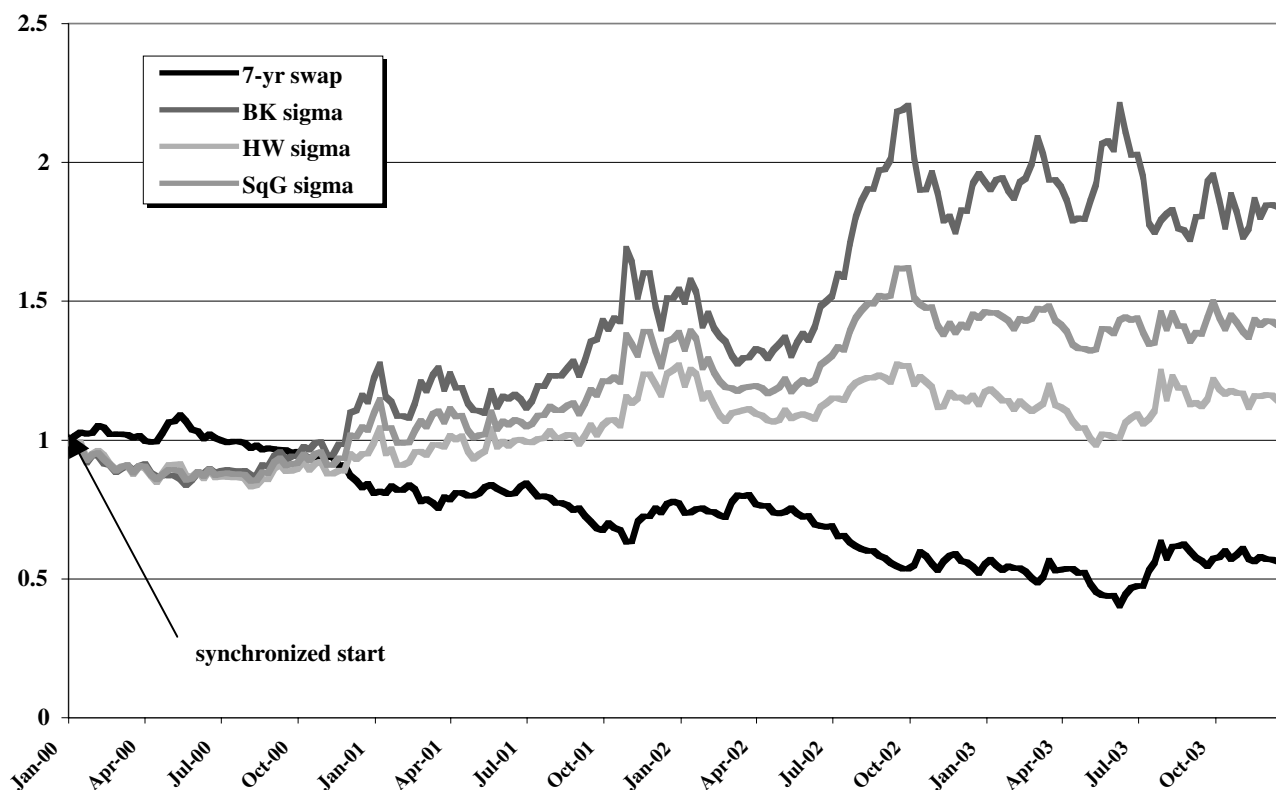
## EXHIBIT 4

### Historical CEV Values



## EXHIBIT 5

### What Volatility Index is Most Stable?



tractable. For example, a Monte Carlo simulation, or any other forward sampling method employed as the primary mortgage pricing tool will simulate only the short rate process on its own. Analytics that would map this process into long rate dynamics (a mortgage rate) simply do not exist and need time-consuming numerical replacements.

Relative (Black) volatility is used for quotation only; it is merely a price-volatility conversion tool, not requiring any acceptance of the Black-Scholes model. As we have seen, volatility changes drastically with the level of rates, and therefore with the expiration of traded options. One constant number cannot describe the entire universe of swaptions deemed relevant for mortgage pricing. The BK model with constant volatility cannot be recommended, in view of a steep yield curve and a sharply inverse proportional volatility term structure.

Contrary to common belief, long rates in the BDT and the BK models are not lognormal, and generally are less volatile than short rates, even in the absence of mean reversion. This may be an unpleasant surprise for those

who think a 20% short rate volatility plugged into the model results in a 20% swaption volatility. Therefore, model calibration to the mortgage-relevant options (not the options on the short rate) can be complicated.

### THE HULL-WHITE MODEL: AN OVERVIEW

The short rate in the HW [1990] model is driven by a linear stochastic differential equation, which is a special case of the CEV Equation (1):

$$dr = a(t)(\theta(t) - r)dt + \sigma(t)dz \quad (3)$$

where  $a(t)$  denotes mean reversion, and  $\sigma(t)$  stands for volatility; both can be time-dependent. Function  $\theta(t)$  is sometimes referred to as arbitrage-free drift. That is, by selecting a proper  $\theta(t)$ , we can match any observed yield curve.

Since (3) is a linear differential equation disturbed by normally distributed Brownian motion, its output, the short rate process, will also be normally distributed.

Negative rates are not precluded. Although this fact is well known (but never met with enthusiasm among practitioners), there are many advantages in the model that make up for this drawback.

The model is analytically tractable. For example, the arbitrage-free function  $\theta(t)$  is expressed analytically through a given forward curve. The average zero-coupon rates and their standard deviations are also known for any maturity and any forward time. (Many derivations of the HW and other Gaussian models can be found in Levin [1998].)

Any long zero-coupon rate  $r_T$  of arbitrary maturity  $T$  is proven also to be normally distributed and linear in the short rate; volatilities are related as

$$\frac{\text{Long-Rate Volatility}}{\text{Short-Rate Volatility}} = \frac{1 - e^{-aT}}{aT} \equiv B_T \quad (4)$$

at any time  $t$ .

Function  $B_T$  of maturity  $T$  plays an important role in the HW model. It allows for calibrating the volatility function  $\sigma(t)$  to the option market. If mean reversion is positive, then  $B_T < 1$ , and the model allows for quasi-parallel shocks, with rate deviations gradually depressed

along the curve. This feature agrees with the behavior of absolute implied volatility for traded swaptions; it generally falls with the swap maturity. This observation therefore helps us calibrate mean reversion in the model.

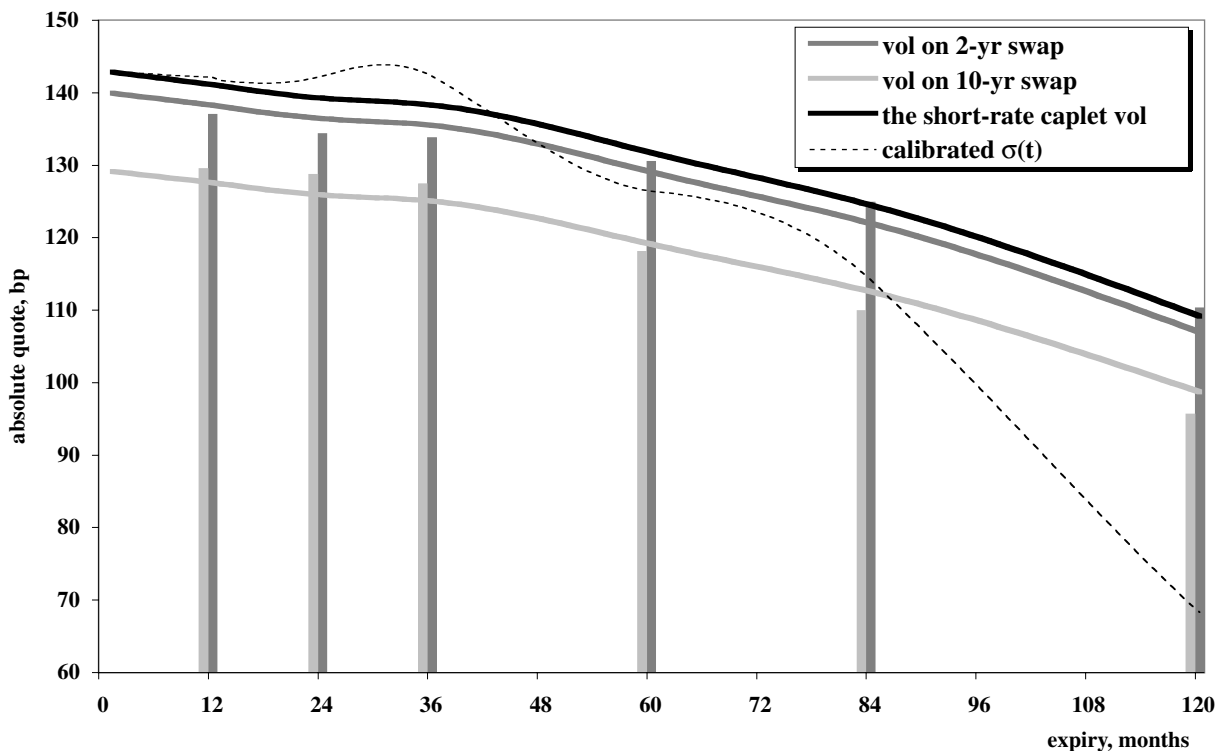
If  $a = 0$ , function  $B_T$  becomes identical to 1.0, regardless of maturity  $T$ . This important special case, called the Ho-Lee [1986] model, allows for a pure parallel change in the entire zero-coupon curve (every point moves by the same amount). Such an opportunity can be advantageous for standardized risk measurement tests. No other model allows parallel shocks to be mathematically consistent with its internal analytics.

### Calibration to ATM Swaptions

Because the standard deviation of any zero-coupon rate can be found explicitly for any bond maturity and any forward time, it can be directly compared with quoted Black volatility. Although market swaps are coupon-bearing instruments, zero-coupon volatility analysis remains quite accurate within the maturity range deemed relevant for the mortgage market (up to ten years). Whether the model operates with a time-dependent volatility function

## EXHIBIT 6

### Calibrated Volatility Term Structure—May 2002



## EXHIBIT 7

### Comparative Calibration for the HW Model—5/13/2002

Volatility, bp (best fit)	Mean Reversion (%)	Accuracy (bp)*
123.1 (constant)	Set to zero	12.4
145.9 (constant)	3.36 (best fit)	6.8
Time-Dependent	Set to zero	7.8
Time-Dependent	2.05 (best fit)	3.6

\* Effective RMSE measured across the volatility matrix.

$\sigma(t)$  or a constant volatility parameter,  $\sigma(t) \equiv \sigma = \text{const}$ , it can be optimized to approximate the volatility matrix of traded ATM swaptions. This calibration procedure includes finding the best mean-reversion parameter  $a$  in the sense discussed above.

Exhibit 6 plots the calibration results using a series of ATM options on the two-year swap and on the ten-year swap as the input. The bars for six different expirations show known volatility quotes converted into the absolute form (i.e., the relative quote multiplied by the forward rate).<sup>3</sup>

The overall effective error of calibration is just 3.6 basis points of absolute volatility, as measured across the swaption matrix. The mean-reversion parameter was restricted to be a constant; its best value is found as  $a = 2.05\%$ . For some particular applications (like standardized risk tests), one may prefer a zero mean reversion, or a constant volatility parameter. These restrictions generally reduce the calibration accuracy as shown in Exhibit 7.

Calibrated  $\sigma(t)$  in Exhibit 6 is rather responsive to the slope and the shape of the input volatility structure. It falls sharply beyond the six-year horizon, perhaps as a result of market perception about current versus long-term volatility. During the stormy market of the first half of 2002, short-dated options were indeed traded at unprecedented absolute volatility levels of 125–140 basis points, well above their long-term averages. This was not the case in the calm August of 1998, just prior to the Russian crisis (*Exhibit 8*).

#### Issues Related to Caps

We have demonstrated that the HW model can be calibrated to traded swaptions. Would it be even easier to use caps? After all, the function we seek,  $\sigma(t)$ , is the short rate volatility function, and derivatives on short rates (LIBOR) seem to be good candidates to examine volatility.

Although many market participants perceive that caps and swaptions trade in unison, they may overlook an important modeling difference—the jumps. Long swaps are chiefly diffusive, and a model disturbed by a Brownian motion [like  $z(t)$  in Equation (1)] makes sense. Short rates combine continuous diffusion (small day-after-day changes) with sudden regulatory corrections.

It is mathematically not very difficult to add jumps to diffusion (Merton did it in 1976), but the equivalent volatility term structure will become hump-shaped. Under a jump or a jump-diffusion disturbance, the short-dated Black volatilities come up considerably suppressed.

Exhibit 9 compares market volatilities for traded caps (solid bars) with volatilities of caps derived from the swaption-fitted HW [1990] model. The model drastically overstates short-dated cap volatilities, in both absolute and relative terms. As the cap's maturity extends, swaptions and caps seem to converge. Can the cap (rather than the swaption) volatility structure be plugged into a mortgage pricing system? Perhaps so, if the system's interest rate model maintains the jump-diffusion setting. As developers of mortgage analytical systems traditionally do not do this, the blind use of caps will understate volatility and therefore the prepayment option value.<sup>4</sup>

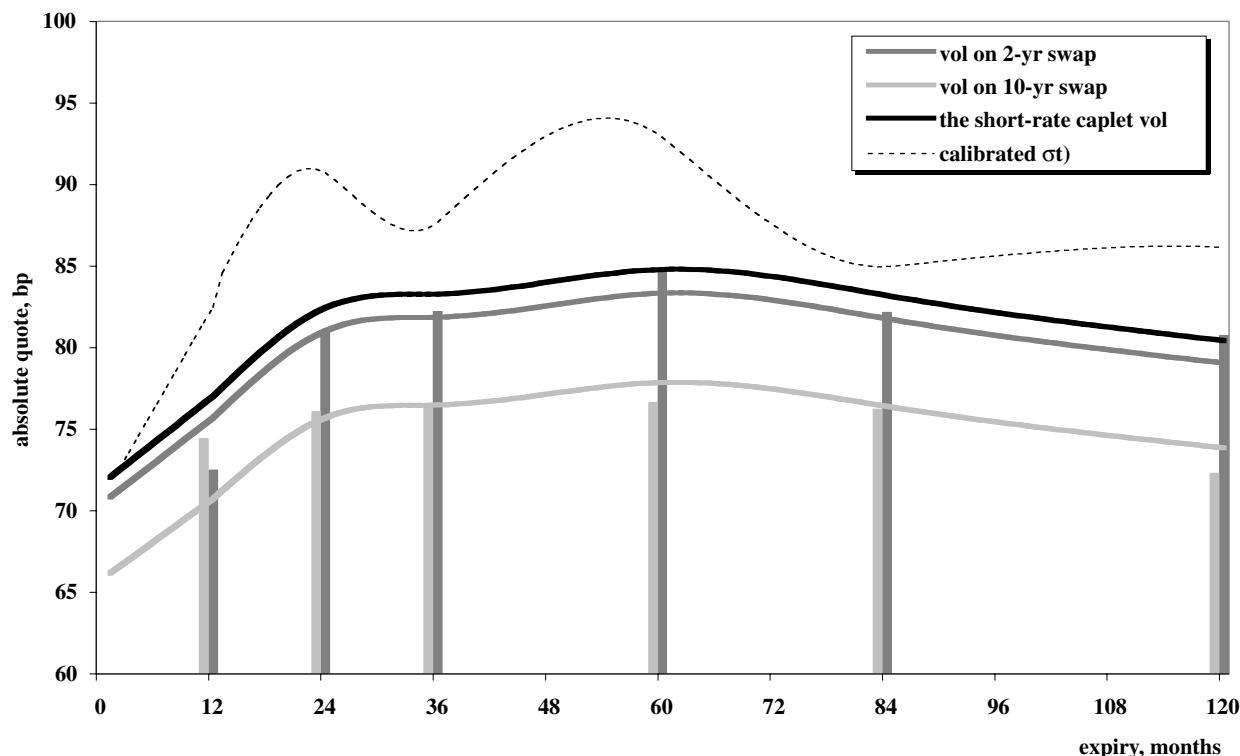
We prefer using the swaption market for benchmarking volatility, especially for fixed-rate mortgages. Valuation of adjustable-rate mortgages may need additional attention in view of embedded reset caps.

#### MBS VALUATION AND RISK MANAGEMENT IMPLICATIONS

Let us assume that the normal HW [1990] model and the lognormal BK [1991] model are independently calibrated to the ATM swaptions. They should value ATM swaptions identically, but the volatility skew of the

## EXHIBIT 8

### Calibrated Volatility Term Structure—August 1998



normal model (curve  $\gamma = 0$  in Exhibit 3) is quite unlike the flat one for the lognormal model ( $\gamma = 1$ ). This means the two models will value any option other than those employed for calibration differently.

Since embedded mortgage options (prepayment, ARM caps and floors, clean-up calls) are spread over time and instruments, changing from the BK model to the HW model will generally result in a change of values. Even more important, the interest rate sensitivity measures will change visibly—as a direct result of different volatility specifications. Under the BK model, every up move in rates proportionally inflates the absolute volatility, thereby reducing the modeled value of the mortgage-backed security. This can be considered an indirect (via volatility) interest rate effect, artificially extending the effective duration of mortgages.

Exhibit 10 shows a 0.4-year duration reduction when moving from the BK model to the HW model, for the current-coupon agency MBS. This may considerably requantify the delta-hedging needs in secondary marketing and MBS portfolio management.

The table in Exhibit 10 summarizes comparative valuation results for 30-year fixed-rate agencies obtained

under the three different term structure models. In each case the short rate volatility function is calibrated to ATM swaptions.

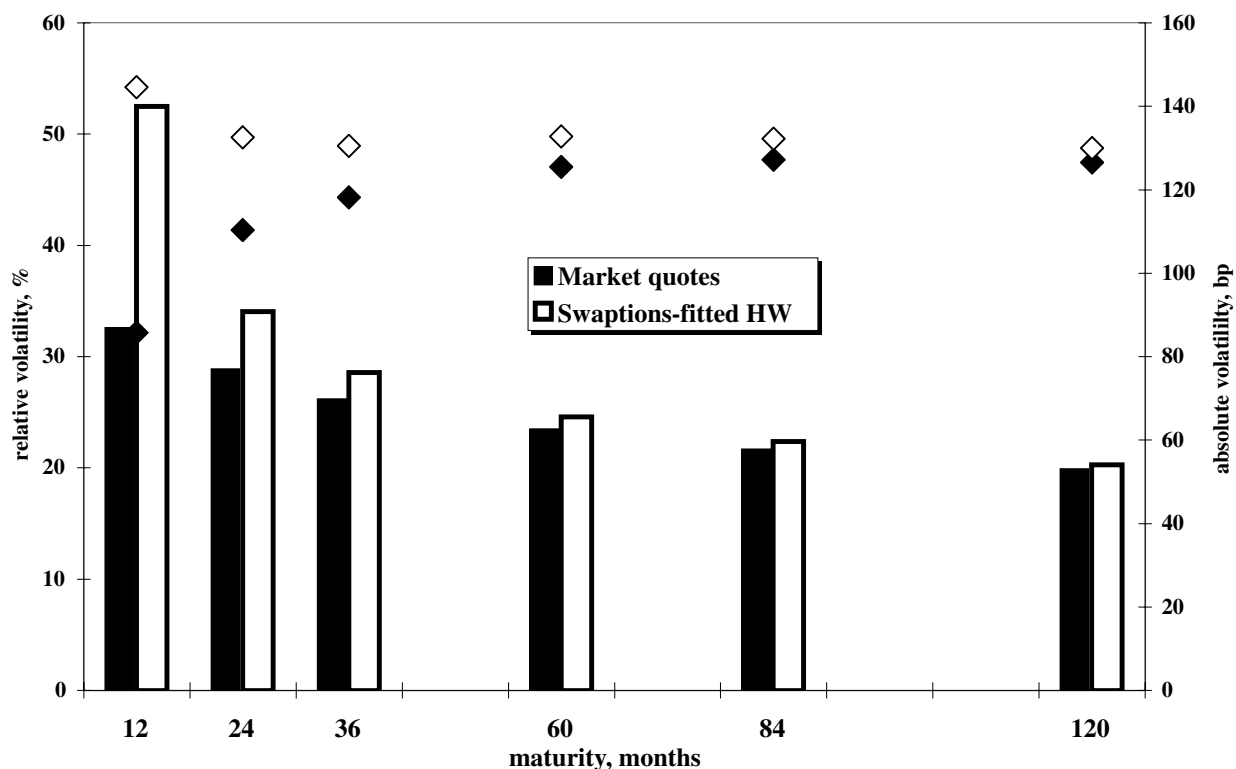
As one would expect, cuspy mortgages located at the center of refinancing curve (“at-the-money” FNCL7) are valued in a very close OAS range by all three models. When the prepayment option is out of the money (the discount sector), this option will be triggered in a falling rate environment. This sector therefore looks relatively rich under the HW model, while the premium sector benefits from using this model. As one could expect, the SG model produces valuation results that are between results of the HW and the BK models.

Although most mortgage instruments will look shorter under the HW model, there are some notable exceptions. As we point out above, the primary divergence of HW from BK is found in differing volatility models. Since mortgage interest-only (IO) classes and mortgage servicing rights (MSR) have drastically changing convexity profiles, they will also have unsteady exposures to volatility, i.e., vega.

For example, vega is typically positive for an IO taken from a premium pool (case 1), negative for that

## EXHIBIT 9

Implied Cap Pricing—May 2002



stripped off a discount pool (case 2), and about zero when the pool's rate is at the center of the refinancing curve (cuspy premium, case 3). Therefore, the BK model will generally overstate the rate sensitivity for case 1, understate it for case 2, and be close to the HW model in case 3 (see Exhibit 11). Keep in mind that an IO value, contrary to a regular MBS, increases with rates. Curiously enough, the value of an IO stripped off the current-coupon pool is always found to be higher under the HW model than under the BK model, for all rate moves.

These interesting findings, although affecting valuation and delta-hedging, do not contradict what is well known; MBS stripped derivatives and MSRs are influenced greatly by prepayments and slightly by interest rate models, provided that models are calibrated to the same set of volatility benchmarks. The latter constraint is critical. As Exhibit 11 shows, the static (zero-volatility) valuation profile differs considerably from the option-adjusted one. Buetow, Hanke, and Fabozzi [2001] provide a good reminder to practitioners who may underestimate the importance of model calibration.

Can two different rate models be used for risk man-

agement: one for the assets, and another for the hedge? Suppose a mortgage desk uses the BK model, while the swaps desk trades with a skew. Unless the position is made vega-neutral, differing volatility specifications in the models may considerably reduce hedge efficiency.

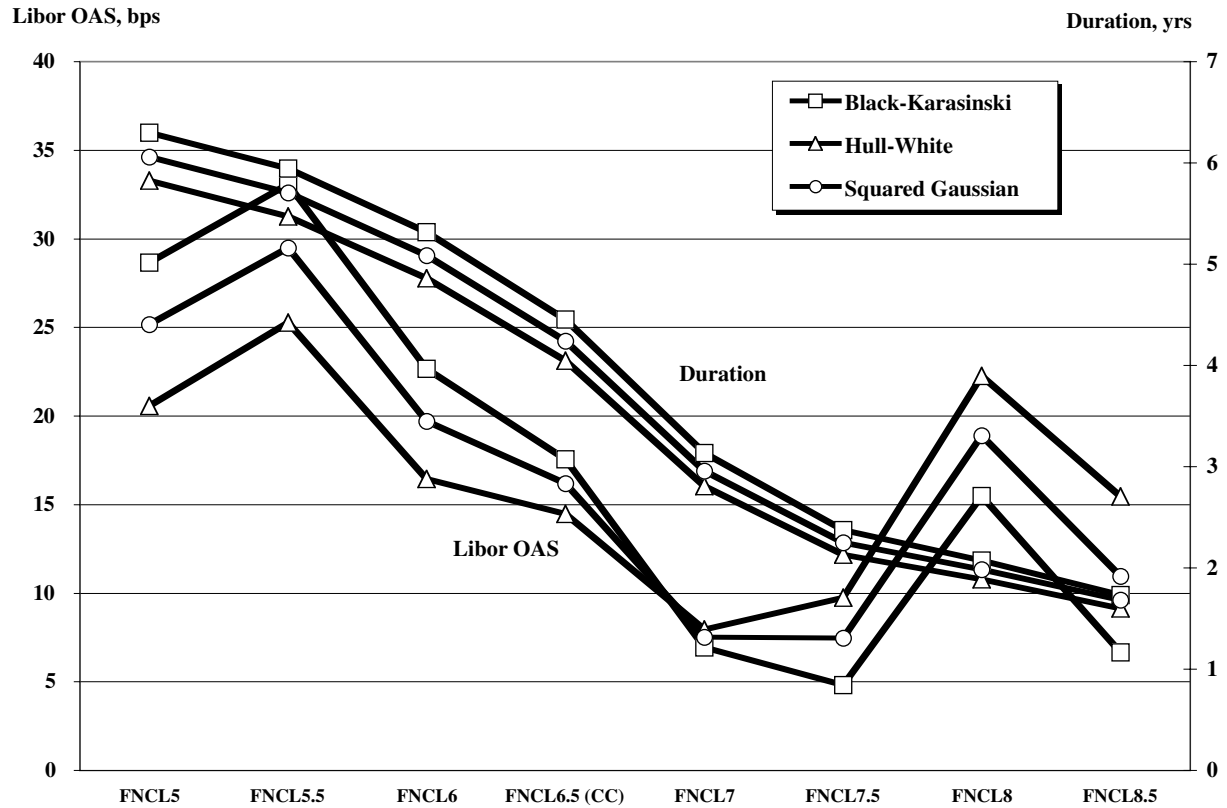
### NEGATIVE RATES

Knowing that interest rates have never been negative in U.S. history, we should question what detrimental effects might occur upon use of the Hull-White [1990] model. Some have tried to estimate the probability of getting negative rates in the model; this approach typically creates needless concern. Indeed, the odds of such an event are far from infinitesimal, but how badly can that damage the value of an MBS?

Answering this question may be simpler than it sounds. Consider a LIBOR floor struck at zero. This non-existent derivative will have a sure zero practical value, but not under the HW model. We have priced such a hypothetical instrument using assumptions that inflate its value: market as of May 2002 (low rates, high volatil-

## EXHIBIT 10

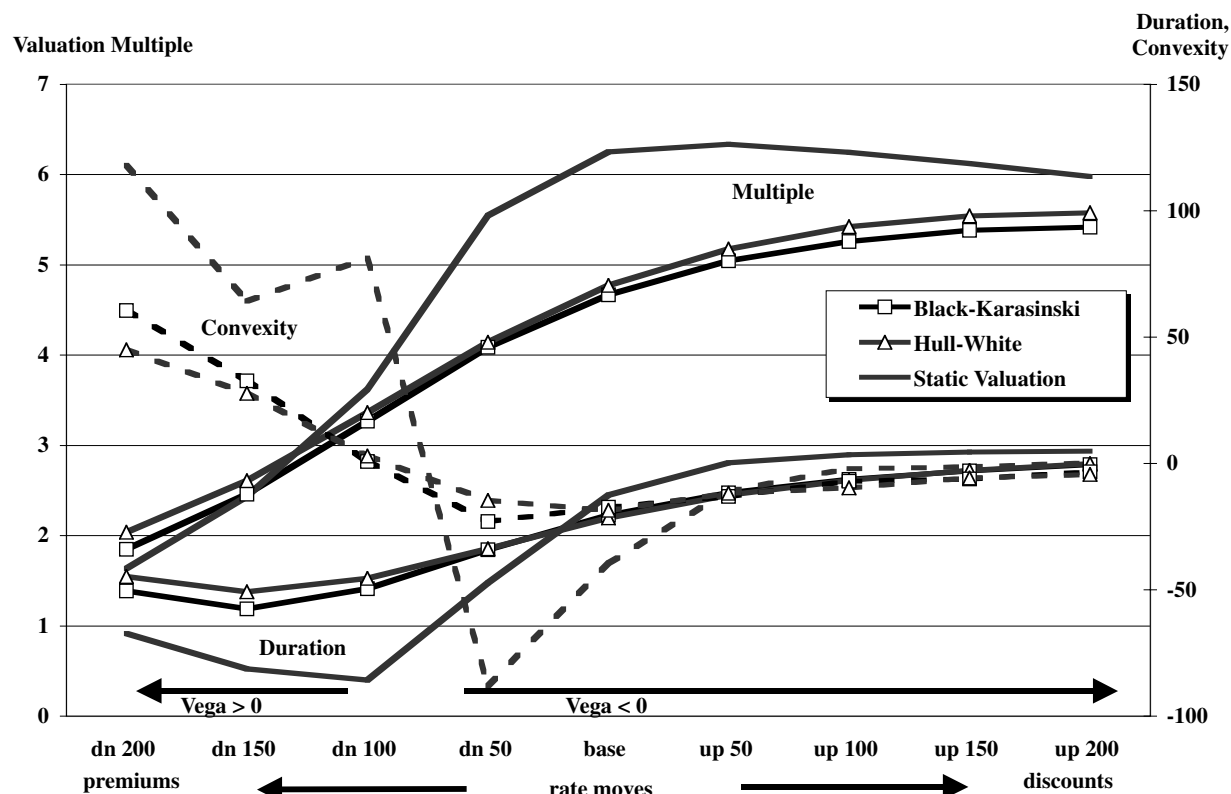
LIBOR OAS and Duration Profiles for New FNCLs—May 13, 2002



	LIBOR OAS			Effective Duration		
	HW	SG	BK	HW	SG	BK
GNSF5	10.5	15.7	19.6	6.13	6.38	6.66
GNSF5.5	15.3	20.1	24.0	5.75	6.00	6.28
GNSF6	4.3	8.0	11.1	5.15	5.41	5.68
GNSF6.5 (CC)	6.4	8.4	10.0	4.28	4.50	4.74
GNSF7	5.9	5.7	5.1	3.20	3.38	3.56
GNSF7.5	7.0	5.1	2.8	2.29	2.43	2.55
GNSF8	21.6	18.2	14.6	2.02	2.14	2.22
GNSF8.5	28.4	24.0	19.6	1.77	1.87	1.92
FNCL5	20.6	25.2	28.7	5.82	6.06	6.30
FNCL5.5	25.3	29.5	33.1	5.47	5.70	5.94
FNCL6	16.4	19.7	22.6	4.86	5.08	5.31
FNCL6.5 (CC)	14.5	16.2	17.6	4.04	4.24	4.45
FNCL7	7.9	7.5	6.9	2.80	2.95	3.13
FNCL7.5	9.7	7.4	4.8	2.13	2.25	2.37
FNCL8	22.2	18.9	15.5	1.89	1.98	2.07
FNCL8.5	15.4	11.0	6.6	1.60	1.68	1.73

## EXHIBIT 11

### Valuation Results for IO on Current-Coupon FNCL Pool



ity), with model mean reversion set to zero. The volatility function  $\sigma(t)$  is calibrated and extrapolated beyond the ten-year expiration.

The value of a zero-struck floor is found to be insignificant for the average life range relevant to mortgage pricing (up to ten years). Thus, for the ten-year non-amortizing floor, the value is 7 basis points, which is equivalent to a 0.8 basis point error in the OAS. The error grows with the horizon; the 30-year floor is priced at 35 basis points, which would lead to 2.5 basis points of spurious OAS.

We can conclude that the Hull-White [1990] model is rather harmless. It will not lead to sizable mispricing even in the worst mortgage-irrelevant case. This conclusion, however, certainly merits periodic review.

## ENDNOTES

This article comes from several Andrew Davidson Co. publications. It has greatly benefited from joint research with Andrew Davidson (many views are as much his as the author's)

and from the insightful comments of James Barrett, Robert Landauer, and Yung Lim. The volatility skew data used in the analysis are courtesy of Craig Lindemann and Krystn Paternostro. The author thanks Jay DeLong and Steven Heller who incorporated the new library of interest rate models into the OAS products. Initial AD&Co. internal publication would not have been possible without the diligent production efforts of Ilda Pozhegu.

All interest rate models discussed in the paper are included in AD&Co's Vector™ suite of analytical models. They can operate with time-dependent or constant volatility calibrated to an arbitrary family of ATM swaptions.

<sup>1</sup>The CEV model is analyzed in many published sources, including Wilmott [1998] and James and Webber [2000].

<sup>2</sup>This smile can be explained by the jumpy nature of the underlying rate. For example, LIBOR caps are traded with a considerable volatility smile because these rates are subject to regulatory interference. Swap rates are much more diffusive than jumpy, so swaption smiles are much less pronounced.

<sup>3</sup>This method of conversion is more accurate in matching option values under the lognormal and normal versions of Black-Scholes than the formal variance match.

<sup>4</sup>A recent complex three-factor jump-diffusion development by Chan et al. [2003] is a rare exception.

## REFERENCES

- Black, F., E. Derman, and W. Toy. "A One-Factor Model of Interest Rates and its Application to Treasury Bond Options." *Financial Analysts Journal*, January/February 1990, pp. 33-39.
- Black, F., and P. Karasinski. "Bond and Option Pricing when Short-Term Rates are Lognormal." *Financial Analysts Journal*, July/August 1991, pp. 52-59.
- Blyth, S., and J. Uglum. "Rates of Skew." *Risk*, July 1999, pp. 61-63.
- Buetow, G., B. Hanke, and F. Fabozzi. "Impact of Different Interest Rate Models on Bond Value Measures." *The Journal of Fixed Income*, December 2001, pp. 41-53.
- Chan, Y. K., R. Bhattacharjee, R. Russell, and M. Teytel. "A New Term-Structure Model Based on the Federal Funds Target." *Mortgage Securities*, Citigroup, May 2003.
- Cox, J.C., J.E. Ingersoll, and S.A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (March 1985), pp. 385-408.
- Ho, T.S.Y., and S.-B. Lee. "Term Structure Movements and Pricing Interest Rate Contingent Claims." *Journal of Finance*, 41 (December 1986), pp. 1011-1129.
- Hull, J. *Options Futures, and other Derivative Securities*, 4th ed. Englewood Cliffs, NJ: Prentice-Hall, 2002.
- Hull, J., and A. White. "Pricing Interest Rate Derivative Securities." *Review of Financial Studies*, 3 (1990), pp. 573-592.
- James, J., and N. Webber. *Interest Rate Modeling: Financial Engineering*. New York: John Wiley & Sons, 2000.
- Levin, A. "Deriving Closed-Form Solutions for Gaussian Pricing Models: A Systematic Time-Domain Approach." *International Journal of Theoretical and Applied Finance*, 1 (1998), pp. 349-376.
- Merton, R. "Option Pricing when the Underlying Stock Returns are Discontinuous." *Journal of Financial Economics*, 5 (1976), pp. 125-144.
- Wilmott, P. *Derivatives*. New York: John Wiley & Sons, 1998.
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