Algebra of sets (Set identities) ①

① Idempotent law

(i) $A \cup A = A$ (ii) $A \cap A = A$

(ii) Associative law

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

③ Commutative law

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

④ Distributive law

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

⑤ Identity laws

(i) $A \cup \phi = A$ | (i) $A \cap \phi = \phi$

(ii) $A \cup U = U$ | (ii) $A \cap U = A$

⑥ $(A')' = A$ Involution law

⑦ Complement laws

(i) $A \cup A' = U$ | (i) $A \cap A' = \phi$

(ii) $U' = \phi$ | (ii) $\phi' = U$

Demorgen's laws

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Proof $A \cup B = B \cup A$

Let x be an element of $A \cup B$

$x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$A \cup B \subseteq B \cup A$ — (1)

$x \in B \cup A \Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

$B \cup A \subseteq A \cup B$ — (2)

from (1) & (2)

$\boxed{A \cup B = B \cup A}$

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$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$x \in A \cup (B \cap C) \Rightarrow x \in A$ or $(x \in B$ & $C)$

$\Rightarrow x \in (A$ or $B)$ & $x \in (A$ or $C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ —— 1

$x \in (A \cup B) \cap (A \cup C) \Rightarrow$

$(x \in A$ or $x \in B)$ &

$(x \in A$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A \cup (B \cap C)$

$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ —— 2

from 1 & 2

$$\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$$

$\Rightarrow (A \cap B)' = A' \cup B'$

$x \in (A \cap B)' \Rightarrow x \notin (A \cap B)$

$\Rightarrow \cancel{x \notin A} + \cancel{x} \notin B$

$\Rightarrow x \in A' \text{ or } x \in B'$

$\Rightarrow x \in A' \cup B'$

$(A \cap B)' \subseteq A' \cup B'$ —— (1)

R.h.s

$A' \cup B'$

$x \in A' \cup B' \Rightarrow x \in A' \text{ or } x \in B'$

$\Rightarrow x \notin A$ ~~and~~ & $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)'$ —— $\cancel{(2)}$

$A' \cup B' \subseteq (A \cap B)'$ —— (2)

from (1) & (2)

$$\boxed{(A \cap B)' = A' \cup B'}$$

Proved