Properties of Relation

(1) Reflexive Relation :→ $A = \{1, 2, 3\}$

$R_1 = \{ (1,1) (2,2) (3,3) (1,2), (2,3) \}$ for every $a \in A$, $(a, a) \in R_1$

* $R_2 = \{ (1,1) (1,2) (2,3) \}$ $2 \in A$ but $(2,2) \notin R_2$
$3 \in A$ but $(3,3) \notin R_2$

(2) Irreflexive Relation :→

$A = \{ (1,2) \}$ & $R = \{ (1,2), (2,1) \}$
$a \in A$, $(a, a) \notin R$

(3) NON-reflexive Relation :—

if R is neither reflexive nor irreflexive

$R = \{ (1,2) (2,3) (2,2) (3,1) \}$ on $A = \{1, 2, 3\}$

$_2R_2$ is true but $_1R_1$ & $_3R_3$ are false.

(4) Symmetric Relation $aR_b \Rightarrow bRa$

$A = \{1, 2, 3\}$
$R = \{ (1,2) (2,1) (1,3) (3,1) (2,3) (3,2) \underline{(2,2)(3,3)} \}$

NOTE → It may be in set

(5) Asymmetric Relation

$R = \{ (1,2) (1,3) (2,3) \underline{(1,1)} \}$ → It may be in a set

$aR_b \Rightarrow b \not R a$

(6) Transitive Relation

aR_b & $_bR_c$ then aR_c

if $A = \{1, 3, 5\}$, $R = \{ (1,3), (3,5), (1,5) \}$

Antisymmetric Relation →

aRb & $bRa \Rightarrow a = b \quad \forall\ a, b \in A$

$R = \{ (1,2)\ (3,2)\ (2,2) \}$ on $A = \{1, 2, 3\}$

$x \leq y$ & $y \leq x \Rightarrow x = y$

$(x, y) \in R$ & $(y, x) \in R \Rightarrow \boxed{x = y}$

Ex = $A = \{1, 2, 3\}$

$R = \{ \underline{(1,2)}\ \underline{(2,2)}\ (2,1) \}$ symmetric ✓ Antisymmetric ✗

self loop allowed

$R = \{ (1,1)\ (2,2)\ (2,1)\ (1,2)\ (1,3)\ (2,3) \}$ = Antisymmetric

Asymmetric → A relation R on a set A is said to be Asymmetric if $\forall\ a, b \in A$, $(a, b) \in R$, $(b, a) \notin R$

$A = (a, b, c)$

	a	b	c
a	aa	ab	ac
b	ba	bb	bc
c	ca	cb	cc

① ✓ $\{ (a,b)\ (b,c)\ (c,a) \}$

② ✗ $\{ (a,b), (b,c)\ (c,b) \}$

③ ✗ $\{ (a,a)\ (b,b), (c,c) \}$

④ ✗ $\{ (a,b)\ (b,a)\ (a,a)\ (b,b) \}$

⑤ ✓ ϕ

✗ ⑥ $A \times A$

✓ ⑦ $\{ (a,b)\ (a,c)\ (b,c) \}$

Anti symmetric Relation:-

A relation R on a set A is said to be anti symmetric if $\forall\, a, b \in A$, $(a,b) \in R$, $(b,a) \in R$ then $a = b$

$A = \{a, b, c\}$

$A \times A$

	a	b	c
a	aa	ab	ac
b	ba	bb	bc
c	ca	cb	cc

✓ 1. $\{(a,b)\ (b,c)\ (a,c)\}$

✓ 2. $\{(a,b)\ (a,a)\ (b,b)\}$

✓ 3. $\{(a,a)\ (b,b)\ (c,c)\}$

✗ 4. $\{(a,b)\ (b,a)\ (b,c)\ (c,c)\}$

✓ 5. ϕ

✗ 6. $A \times A$

✗ 7. $\{(a,b)\ (b,c)\ (a,c)\ (c,a)\ (a,a)\ (c,c)\}$

ab	ba	Anti symmetric
✗	✗	✓
✗	✓	✓
✓	✗	✓
✓	✓	✗

Note: 1) If (a,b) exist then (b,a) does not exist

2) diagonal elements can exist.

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