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Set Theory

Set:- A well defined collection of distinct objects is called a set.
The objects in a set can be numbers, people, letters, rivers, countries etc.
The objects in a set are called elements or members of the set.

EX:-
1. The set of numbers 2, 4, 6, 8, 10
2. The set of letters a, e, i, o, u.

Note -
Sets are usually denoted by capital letters A, B, C ... or X, Y, Z ... and the elements are denoted by a, b, c, ... or x, y, z, ...

Standard notations for certain type of sets -

1. Natural Numbers $\mathbb{N} := \{1, 2, 3, \dots\}$
2. Integers $\mathbb{Z} := \{\dots -2, -1, 0, 1, 2, \dots\}$
3. Rational Numbers $\mathbb{Q} := \{\frac{a}{b} \mid a, b \text{ are Integers}, b \neq 0\}$
4. Irrational Numbers $\mathbb{Q}^c$ $\mathbb{I} := \{$ a non rational number such as $\sqrt{2}, e, \pi, e^{\pi}, \dots\}$
5. Real Numbers $\mathbb{R} := \{$ a rational or an irr. no $\}$
6. complex numbers $\mathbb{C} := \{a + ib \mid a, b \in \mathbb{R}, i = \sqrt{-1}\}$

Also $\mathbb{Z}^+ \rightarrow$ set of positive integers
$\mathbb{Q}^* \rightarrow$ set of non-zero rational numbers

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$2N = E \rightarrow$ Set of even Integers

$D_n \rightarrow$ set of divisors of a positive Integer n.

Cardinality or cardinal number of a set :→

The number of elements in a set A is called the cardinality or cardinal number of the set. It is denoted by $n(A)$ or $|A|$.

Equivalent set:- A and B are said to be equivalent sets if $|A| = |B|$.

Ex:-
if $S = \{3, 5, 7, 9, 11\}$
Then $|S| = 5$.

Empty set / Null set :—
A set having no element is called empty set. It is denoted by ϕ or $\{\}$.

eg → 1. The set of Integers b/w $\frac{9}{4}$ and $\frac{11}{4}$

2. The set of Integers less than 4 and greater than 8.

Finite and Infinite sets :-
A set is called finite if it has n distinct elements. otherwise set is called Infinite.

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(#) **Subset:-** if every element of the set A is also an element of the set B then A is called subset of B. denoted by $A \subseteq B$ which is read as A is contained in B.
i.e if $x \in A \Rightarrow x \in B$

Proper subset:- when $A \subseteq B$ and the set B has an element that is not in A. Then A is called proper subset of B and is denoted by $A \subset B$.

Equal Sets:- Two sets A and B are equal
i.e $A = B$ iff $A \subseteq B$ and $B \subseteq A$

Comparability:- Two sets A and B are said to be comparable if $A \subset B$ or $B \subset A$ or $A = B$.

(#) **Power Set:→** Let X be a non-empty set. The set of all subsets of X is called the power set of the set X.
It is denoted by $\mathcal{P}(X)$.
Since $\phi \subseteq X$ and $X \subseteq X$, so $\phi, X \in \mathcal{P}(X)$.
Particularly, $\mathcal{P}(\phi) = \{\phi\}$.

Number of elements in Power Set:-

Let a set A has n elements. Then its Power set $\mathcal{P}(A)$ has $\underline{\underline{2^n}}$ elements

Proof:- For $0 \leq k \leq n$, there are ${}^nC_k$ ways to select a subset having k elements of the set A

Therefore, in Total, Power Set $P(A)$ contains

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_{n-1} + {}^nC_n = 2^n$$

Sets. as asserted.

Example:-1 Find the Power set of the Sets
(i.) $\{a\}$, ii) $\{a, b\}$ (iii) $\{a, b, c\}$

Solⁿ:-

i.) let $A = \{a\}$
So $P(A) = \{\phi, \{a\}\}$

ii.) let $B = \{a, b\}$
So $P(B) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

iii.) let $C = \{a, b, c\}$
so $P(C) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$.

Example:-2 Find the Power set of the Sets
i.) $A = \{\phi, \{\phi\}\}$
ii.) $B = \{a, \{a\}\}$

Solⁿ:-

i.) Power set of A
$P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

ii.) Power set of B
$P(B) = \{\phi, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$

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Operations on Set :→

1. Union of Sets :- The union of two sets A and B is defined by
$A \cup B = \{x : x \in A \text{ or } x \in B\}$.

2. Intersection of sets :- The Intersection of two sets A and B is defined by
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

3. Difference of sets :- The difference of two sets A and B is defined by
$A - B = \{x : x \in A, x \notin B\}$.

4. Complement of a set :- The complement of a set A is the set of elements which do not belong to A denoted by A', A^c
$A^c = U - A$
where U is the Universal set.

5. Symmetric difference of two sets :-

Symmetric difference of two sets A and B is given by

$A \oplus B = A \Delta B = (A-B) \cup (B-A)$
or $A \vee B = \{x \in X \mid x \in (A \cup B) - (A \cap B)\}$
where $X \neq \phi$, $A, B \in P(X)$.

Note –
$A \Delta B = B \Delta A$ $\quad A \Delta A = \phi$
$A \Delta \phi = A$, $A \Delta X = A'$, $A \Delta A' = X$

Ques. Let $U = \{1, 2, \ldots, 8, 9\}$ be a universal set and the six sets given by
$A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$,
$C = \{5, 6, 7, 8, 9\}$, $D = \{1, 3, 5, 7, 9\}$
$E = \{2, 4, 6, 8\}$, $F = \{1, 5, 9\}$ Then
Find the sets
(a.) $A \cup B$, $D \cup F$ (b.) B^c, D^c, E^c
(c.) $A - B$, $D - E$, $E - D$ (d.) $C \cup D$, $E \cup F$, $A \cup B$
(e.) $(A \cup C) - B$, $(B \cup C) - A$.

Soln:-

(a.) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$
$D \cup F = \{1, 3, 5, 7, 9\}$

(b.) $B^c = \{1, 2, 3, 8, 9\}$, $D^c = \{2, 4, 6, 8\}$
$E^c = \{1, 3, 5, 7, 9\}$

(c.) $A - B = \{1, 2, 3\}$
$D - E = \{1, 3, 5, 7, 9\}$
$E - D = \{2, 4, 6, 8\}$

(d.) $C \cup D = \{1, 3, 6, 8\}$
$E \cup F = \{1, 2, 4, 5, 6, 8, 9\}$
$A \cup B = \{1, 2, 3, 6, 7\}$

(e.) $(A \cup C) = \{1, 2, 3, 4, 6, 7, 8, 9\}$
So $(A \cup C) - B = \{1, 2, 3, 8, 9\}$
$B \cup C = \{4, 8, 9\}$
So $(B \cup C) - A = \{6, 7, 8, 9\}$.

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Laws of Algebra of Sets :- Let $A, B, C \in \mathcal{P}(X)$

1. Idempotent Laws : a.) $A \cup A = A$
   b.) $A \cap A = A$
2. Associative Laws : a.) $(A \cup B) \cup C = A \cup (B \cup C)$
   $(A \cap B) \cap C = A \cap (B \cap C)$
3. commutative Laws : a.) $A \cup B = B \cup A$
   b.) $A \cap B = B \cap A$
4. Distributive Laws : a.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b.) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. Identity Laws :
   a.) $A \cup \phi = A$, $A \cup U = U$
   b.) $A \cap \phi = \phi$, $A \cap U = A$
6. Involution law :
   $(A')' = A$
7. complement laws : a.) $A \cup A' = U$, $A \cap A' = \phi$
   b.) $U' = \phi$, $\phi' = U$
8. De Morgan's law : a.) $(A \cap B)' = A' \cup B'$
   b.) $(A \cup B)' = A' \cap B'$
9. a.) $A - (B \cup C) = (A - B) \cap (A - C)$
   b.) $A - (B \cap C) = (A - B) \cup (A - C)$
10. Absorption law : a.) $A \cup (A \cap B) = A$
    b.) $A \cap (A \cup B) = A$
11. $A \cap (B - C) = (A \cap B) - (A \cap C)$
12. a.) $A - B = A \cap B'$
    b.) $B - A = B \cap A'$

Note:- a.) $x \in A$ and $x \in B \Leftrightarrow x \in A \cap B$
$x \in A$ or $x \in B \Leftrightarrow x \in A \cup B$
$x \notin A$ and $x \notin B \Leftrightarrow x \notin A \cup B$
$x \notin A$ or $x \notin B \Leftrightarrow x \notin A \cap B$

Ques.1. Prove that if $A \subseteq B$ then $A \cup B = B$.

Soln:-

We know that $B \subseteq A \cup B$ —(1)

Now we have to show $A \cup B \subseteq B$

let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ as $A \subseteq B$

So $A \cup B \subseteq B$ —(2)

Hence (1) and (2) $\Rightarrow$

$$A \cup B = B$$

Ques.2. Prove that if $A \subseteq B$ then $A \cap B = A$.

Soln:-

We know that $A \cap B \subseteq A$ —(1)

Now we have to show $A \subseteq A \cap B$

let $x \in A \Rightarrow x \in B$ as $A \subseteq B$

$\Rightarrow x \in A \cap B$

i.e $A \subseteq A \cap B$ —(2)

Hence from (1) and (2),

$$\boxed{A = A \cap B}$$

Ques.3 Prove that $A - B = A \cap B^c$.

Let $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in B^c$

$\Leftrightarrow x \in A \cap B^c$

Hence $A - B = A \cap B^c$.

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Ques. 4. Let A, B, C be three non-empty sets. Prove the following Identities:

1. $(A-C) \cap (C-B) = \phi$
2. $A-(B \cap C) = (A-B) \cup (A-C)$
3. $(A^c \cup B^c)^c \cup (A^c \cup B)^c = A$.

Sol^n:-

(1.) For, if $(A-C) \cap (C-B)$ is non-empty let $x \in (A-C) \cap (C-B)$

$\Rightarrow x \in (A-C)$ and $x \in (C-B)$

However, the two conditions can not hold simultaneously because

$x \in A-C \Rightarrow x \notin C$

and $x \in C-B \Rightarrow x \in C$

which is a contradiction.

Therefore,

$$(A-C) \cap (C-B) = \phi$$

(2.) we know that $X - Y = X \cap Y^c$

Therefore, we have

$$\begin{aligned} A-(B \cap C) &= A \cap (B \cap C)^c \\ &= A \cap (B^c \cup C^c) \quad \text{(by De Morgan law)} \\ &= (A \cap B^c) \cup (A \cap C^c) \quad \text{(by Distributive law)} \\ &= (A-B) \cup (A-C) \end{aligned}$$

(3.)

$$\begin{aligned} (A^c \cup B^c)^c \cup (A^c \cup B)^c &= (A \cap B) \cup (A \cap B^c) \quad \text{De Morgan's law \& Involution law} \\ &= A \cap (B \cup B^c) \quad \text{Distributive law} \\ &= A \cap U \quad \text{by complement law} \\ &= \underline{\underline{A}} \end{aligned}$$

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Some Important Results :—

1. No. of elements belonging to A or B or both $= n(A \cup B)$
2. No. of elements not belonging to either of the two $= n(A \cup B)'$
3. No. of elements belonging to both A and B $= n(A \cap B)$
4. Inclusion - Exclusion Principle -

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

5. $n(A-B) = n(A) - n(A \cap B)$ = no. of elements belonging to A only.
6. $n(B-A) = n(B) - n(A \cap B)$ = no. of elements belonging to B only
7. $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$
8. $n(A' \cup B') = n(A') + n(B') - n(A' \cap B')$
9. No. of elements Neither belonging to A nor B
$n(A' \cap B') = n(S) - n(A \cup B)$.

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Ex:-1. if $n(U) = 60$, $n(A) = 27$, $n(B) = 19$, $n(A \cup B)' = 31$. Then find
(i) $n(A \cap B)$ (ii) $n(A - B)$

Sol^n:-

given $n(A) = 27$, $n(B) = 19$, $n(A \cup B)' = 31$
we know $n(A \cup B)' = n(A' \cap B')$
$= n(U) - n(A \cup B)$
$\Rightarrow n(A \cup B) = n(U) - n(A \cup B)'$
$= 60 - 31$
$= 29$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(by Incl. excl. fn.)

$\Rightarrow$ i) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 27 + 19 - 29$
$= 27 - 10$
$= \underline{\underline{17}}$

ii) $n(A - B) = n(A) - n(A \cap B)$
$= 27 - 17$
$= \underline{\underline{10}}$

Ex:-2. It is found that out of 100 Students, 18 Can drive neither a Scooter nor a car, while 25 Can drive both these and 55 of them Can drive a Scooter. How many can drive car.

Sol^n:- Let A and B denote the set of students who can drive Scooter and car respectively.
according, $n(U) - n(A \cup B) = 18$
$100 - n(A \cup B) = 18 \Rightarrow n(A \cup B) = 82$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$82 = 55 + n(B) - 25$
giving $n(B) = \underline{\underline{52}}$

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Cartesian Product of sets :-

Ordered Pair :- An ordered pair, generally denoted by (a, b), is a pair of elements a and b of two sets, which is ordered in sense that $(a, b) \neq (b, a)$ when $a \neq b$ and a and b are known as first and second coordinates in (a, b).

Cartesian Product of two sets :-

Let A and B two non-empty sets. The Cartesian Product of A and B, denoted by $A \times B$, is the set of ordered pairs given by

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

For Example -

Let $A = \{a, b\}$ and $B = \{1, 2, 3\}$

Then we have

$$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$$

Note :- $A \times B \neq B \times A$

More generally if A and B are finite sets given by $A = \{a_1, \ldots, a_m\}$ and $B = \{b_1, \ldots, b_n\}$

Then the set $A \times B$ contains mn elements

$$|A \times B| = |A| \times |B| = mn$$

The Product set $A \times B$ in matrix form

	b_1	b_2	$\cdots$	b_n
a_1	(a_1, b_1)	(a_2, b_2)	$\cdots$	(a_1, b_n)
a_2	(a_2, b_1)	(a_2, b_2)	$\cdots$	(a_2, b_n)
$\vdots$	$\vdots$			
a_m	(a_m, b_1)	(a_m, b_2)	$\cdots$	(a_m, b_n)

Properties of cartesian Product

For the four sets A, B, C and D

1. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
2. $(A - B) \times C = (A \times C) - (B \times C)$
3. $(A \cup B) \times C = (A \times C) \cup (B \times C)$
4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Ques. Let A, B, C be three non empty sets, Prove that

1. $A \cap C = B \cap C$ and $A \cup C = B \cup C \Rightarrow A = B$
2. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
3. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof:-

1.) we have,

$A = A \cap (A \cup C)$ by absorption

$= A \cap (B \cup C)$ by given condition

$= (A \cap B) \cup (A \cap C)$ by distributivity

$= (A \cap B) \cup (B \cap C)$ by given condition

$= (A \cap B) \cup (C \cap B)$ by commutativity

$= (A \cup C) \cap B$ by distributivity

$= (B \cup C) \cap B$ by given condition

$= (C \cup B) \cap B$ by commutativity

$= B$ by absorption

Hence

$A \cup C = B \cup C$ and $A \cap C = B \cap C \Rightarrow A = B$

2.) $A \times (B \cup C) = \{(x, y) \mid x \in A \text{ and } (y \in B \cup C)\}$

$= \{(x, y) \mid x \in A \text{ and } (y \in B \text{ or } y \in C)\}$

$= \{(x, y) \mid (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)\}$

$= \{(x, y) \mid (x, y) \in A \times B \text{ or } (x, y) \in A \times C\}$

$= (A \times B) \cup (A \times C)$

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3.) $A \times (B \cap C) = \{(x,y) \mid x \in A \text{ and } y \in (B \cap C)\}$
$= \{(x,y) \mid x \in A \text{ and } (y \in B \text{ and } y \in C)\}$
$= \{(x,y) \mid (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)\}$
$= \{(x,y) \mid (x,y) \in A \times B \text{ and } (x,y) \in A \times C\}$
$= (A \times B) \cap (A \times C)$

Ques. if $A = \{1,4\}$, $B = \{4,3\}$, $C = \{3,6\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Soln:-
$B \cup C = \{4,3,6\}$
So $A \times (B \cup C) = \{1,4\} \times \{4,3,6\}$
$= \{(1,4), (1,3), (1,6), (4,4), (4,3), (4,6)\}$
$A \times B = \{1,4\} \times \{4,3\}$
$= \{(1,4), (1,3), (4,4), (4,3)\}$
$A \times C = \{1,4\} \times \{3,6\}$
$= \{(1,3), (1,6), (4,3), (4,6)\}$
So $(A \times B) \cup (A \times C)$
$= \{(1,4), (1,3), (1,6), (4,4), (4,3), (4,6)\}$
Hence
$A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Ques. if $A = \{1,3,5,7\}$, $B = \{2,3,5,8\}$
Then find $(A \cap B) \times (B - A)$

Soln:- $A \cap B = \{3,5\}$, $B - A = \{2,8\}$
So $(A \cap B) \times (B - A)$
$= \{(3,2), (5,2), (3,8), (5,8)\}$.

Relation or Binary Relation :-

Let A and B be non-empty sets. Then any subset R of the cartesian product $A \times B$ is called a relation from A to B and is denoted by R.

$\therefore$ R is a relation from A to B $\Rightarrow R \subseteq A \times B$

and

$$R = \{(x, y) \mid x \in A, y \in B \text{ and } xRy\}$$

where

xRy denotes that x is related to y by R

$x \not R y$ denotes that x is not related to y

Ex:- Let $A = \{1, 2, 5\}$ and $B = \{2, 4\}$ be two given sets. Find out the Relation from A to B defined by "is less than".

Soln:-

Given $A = \{1, 2, 5\}$

$B = \{2, 4\}$

$\therefore A \times B = \{(1,2), (1,4), (2,2), (2,4), (5,2), (5,4)\}$

Then

$R = \{(1,2), (1,4), (2,4)\}$

Domain of R :- The set of first coordinates of every element of R

Domain $R = \{x \in A : (x, y) \in R \text{ for some } y \in B\}$

Range of R :- The set of second coordinates of every element of R.

Range $R = \{y \in B : (x, y) \in R \text{ for some } x \in A\}$

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Ex:- Let $A = \{2, 3, 5\}$ and $B = \{2, 4, 6, 10\}$ and $R = \{(2,2), (2,4), (2,6), (2,10), (3,6), (5,10)\}$

Then Dom $R = \{2, 3, 5\}$
Range $R = \{2, 4, 6, 10\}$

Total number of distinct Relation from A to B

$$= 2^{|A \times B|} = 2^{|A| \cdot |B|} = 2^{mn}$$

where $m = |A|$
$n = |B|$.

Relation on a Set :-

A Relation R from a set A to the set A itself is called a Relation on A.
i.e $R \subseteq A \times A$
$R = \{(x, y) : x, y \in A \text{ and } xRy\}$

Ex:- Let $A = \{1, 2, 3\}$ find the Relation on A defined by $\leq$.

Solⁿ:-
$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$
$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

Then Relation $\leq$ on A is given by

$R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$.

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Identity Relation :-

The Identity Relation I_A on a set A is defined by

$$I_A = \{(x,y) : x, y \in A \text{ and } x = y\}$$
$$= \{(x,x) : x \in A\}$$

Ex:-

Let $A = \{a, b, c\}$

$$I_A = \{(a,a), (b,b), (c,c)\}$$

Inverse Relation :-

Let R be any Relation from a set A to B. The Inverse of R, R^{-1} is the Relation from B to A defined by

$$R^{-1} = \{(y,x) : (x,y) \in R\}$$

Consequently

$$xRy = yR^{-1}x$$

Ex:- Let $A = \{2, 3, 5\}$ and $B = \{6, 8, 10\}$ and Relation R is defined by "x divides y" Then

$$R = \{(2,6), (2,8), (2,10), (3,6), (5,10)\}$$

and

$$R^{-1} = \{(6,2), (8,2), (10,2), (6,3), (10,5)\}$$

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Operations on Relation :-

1.) **Complement of a Relation :-**

Let R be a Relation from set A to B. Then the complement of Relation R denoted by $\bar{R}$ or R' is a Relation from A to B such that

$$\bar{R} = \{(a,b) : (a,b) \notin R\}$$

Ex:- Let R be Relation from A to B where
$A = \{1, 2, 3\}$ $B = \{8, 9\}$
and $R = \{(1,8), (2,8), (1,9), (3,9)\}$
Find the complement of R.
we have

$$A \times B = \{(1,8), (1,9), (2,8), (2,9), (3,8), (3,9)\}$$

$$\bar{R} = \{(2,9), (3,8)\}, \cancel{(3,9)}$$

2.) **Intersection of Relations :-**

The Intersection of two Relations R and S is defined by

$$R \cap S = \{(x,y) : xRy \text{ and } xSy\}$$

3. **Union of Relation :-**

The Union of two Relations R and S is defined by

$$R \cup S = \{(x,y) : xRy \text{ or } xSy\}$$

Ex:- Let $R_1 = \{(1,1), (2,2), (3,3), (4,4), (3,4), (4,3)\}$
& $R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$

Then
$R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (4,4), (3,4), (4,3), (1,2), (2,1)\}$

$R_1 \cap R_2 = \{(1,1), (2,2), (3,3), (4,4)\}$

Properties of Relations :-

1.) Reflexive Relation :-

A relation R on a set A is Reflexive if aRa for every $a \in A$
i.e $(a,a) \in R \ \forall a \in A$.

Ex:- if $R_1 = \{(1,1), (2,2), (2,3), (3,3)\}$ be a Relation on a set $A = \{1,2,3\}$ Then R_1 is Reflexive since aRa for every $a \in A$.

2.) Irreflexive :-

A relation R on a set A is Irreflexive if $a \not R a$ for all $a \in A$
i.e $(a,a) \notin R$ for all $a \in A$.

Ex:- The Relation $R_2 = \{(1,2), (1,3), (2,1), (2,3)\}$ on $A = \{1,2,3\}$ is irreflexive.

3. Symmetric Relation :-

A Relation R on a set A is Symmetric if $aRb \Rightarrow bRa$,
i.e whenever $(a,b) \in R \Rightarrow (b,a) \in R$, $\forall a, b \in A$

Ex:-
Sol: Relation $R_3 = \{(1,1), (1,2), (1,3), (2,2), (2,1), (3,1)\}$ on A is symmetric

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4. <u>Asymmetric Relation:-</u>
A Relation R on a set A is asymmetric if $aRb \Rightarrow b\not{R}a$
i.e whenever $(a,b) \in R \Rightarrow (b,a) \notin R$ for $a \neq b$.

<u>Ex:-</u> The Relation $R_1 = \{(1,1), (1,2), (2,3), (3,1)\}$ on $A = \{1,2,3\}$ is an asymmetric Relation.

5. <u>Antisymmetric Relation:-</u>
A Relation R on a set A is antisymmetric if aRb and $bRa \Rightarrow a = b \ \forall a,b \in A$
i.e $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b \ \forall a,b \in A$

<u>Ex:-</u> The Relation $R = \{(1,2), (2,2), (2,3)\}$ on Set $A = \{1,2,3\}$ is an Antisymmetric Relation.

6. <u>Transitive Relation:-</u>
A Relation R on a set A is transitive if aRb and $bRc \Rightarrow aRc$
i.e whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$.

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Equivalence Relation :-

A Relation R on a set A is called an equivalence Relation or RST relation if it is reflexive, symmetric and Transitive

i.e
R is an equivalence Relation on A if It has the following three Properties:

1. $(a, a) \in R$ for all $a \in A$ (Reflexive)
2. $(a, b) \in R \Rightarrow (b, a) \in R$ (Symmetric)
3. (a, b) and $(b, c) \in R \Rightarrow (a, c) \in R$ (Transitive)

Ex:- if R be a Relation in the set of Integer Z defined by
$R = \{(x, y) : x \in Z, y \in Z, (x-y) \text{ is divisible by } 6\}$
Then Prove that R is an equivalence Relation.

Solⁿ:- i.) Reflexive - Let $x \in Z$
Then $x - x = 0$ which is divisible by 6
Therefore $xRx \quad \forall x \in Z$
Hence R is Reflexive

ii.) Symmetric :- Let $x, y \in Z$
Then $xRy \Rightarrow x - y$ is divisible by 6
$\Rightarrow -(x-y)$ is divisible by 6
$\Rightarrow (y-x)$ is divisible by 6
$\Rightarrow yRx$

Hence R is Symmetric.

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iii) <u>Transitive</u> :- Let $x, y, z \in \mathbb{Z}$

Then xRy and $yRz \Rightarrow$ x is divisible by y and y is divisible by z

$\Rightarrow x-y$ is divisible by 6 and $y-z$ is divisible by 6

$\Rightarrow [(x-y)+(y-z)]$ is divisible by 6

$\Rightarrow x-z$ is divisible by 6

$\Rightarrow xRz$

Hence, R is Transitive

Thus R is an equivalence Relation.

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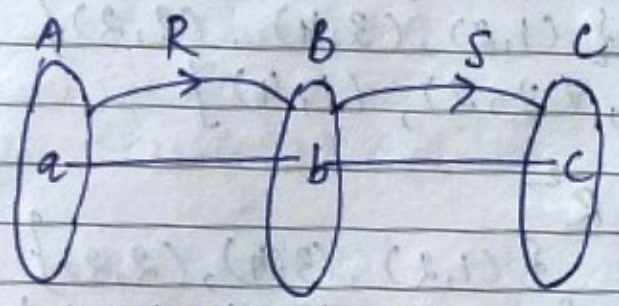
(#) <u>Composite Relation</u> :-

Let A, B and C be three non-empty sets. Suppose R be a relation from A to B and S be a relation from B to C. Then the composite relation of the R and S is a relation from A to C and is denoted by $S \circ R$, and defined as

$$S \circ R = \{(a, c) : \exists \text{ an element } b \in B \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$$



If $(a, b) \in R$
$(b, c) \in S$ Then $(a, c) \in S \circ R$

Note:- $R \circ R = R^2$, $R^2 \circ R = R^3$

Ex:- Let $A = \{1, 2, 3\}$, $B = \{p, q, r\}$, $C = \{x, y, z\}$ and let

$$R = \{(1, p), (1, r), (2, q), (3, q)\}$$

and $S = \{(p, y), (q, x), (r, z)\}$ Then compute $S \circ R$.

Soln:-

$$S \circ R = \{(1, y), (1, z), (2, x), (3, x)\}.$$

Ex:- Let $R = \{(1,2), (3,4), (2,2)\}$
and $S = \{(4,2), (2,5), (3,1), (1,3)\}$
Find SoR, RoS, RoR, $Ro(SoR)$, $(RoS)oR$

Sol^{n}:-

i.) SoR :-

$R = \{(1,2), (3,4), (2,2)\}$

$S = \{(4,2), (2,5), (3,1), (1,3)\}$

$\therefore SoR = \{(1,5), (3,2), (2,5)\}$

ii.) RoS :-

$S = \{(4,2), (2,5), (3,1), (1,3)\}$

$R = \{(1,2), (3,4), (2,2)\}$

$\therefore RoS = \{(4,2), (3,2), (1,4)\}$

iii.) $RoR = R^2$

$R = \{(1,2), (3,4), (2,2)\}$

$R = \{(1,2), (3,4), (2,2)\}$

$RoR = R^2 = \{(1,2), (2,2)\}$

iv.) $Ro(SoR) \rightarrow$

$SoR = \{(1,5), (3,2), (2,5)\}$

$R = \{(1,2), (3,4), (2,2)\}$

$Ro(SoR) = \{(3,2)\}$

v.) $(RoS)oR \rightarrow$

$R = \{(1,2), (3,4), (2,2)\}$

$RoS = \{(4,2), (3,2), (1,4)\}$

$(RoS)oR = \{(3,2)\}$

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Reversal law in composite relation →

Theorem:- Let R be a relation from the set A to the set B and S be a relation from the set B to set C, Then

$$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$$

Proof:-

let $(c, a) \in (S \circ R)^{-1}$

$\Leftrightarrow (a, c) \in (S \circ R) \quad \forall a \in A, c \in C$

$\Leftrightarrow \exists$ an element b s.t

$(a, b) \in R$ and $(b, c) \in S$

$\Leftrightarrow (b, a) \in R^{-1}$ and $(c, b) \in S^{-1}$

$\Leftrightarrow (c, b) \in S^{-1}$ and $(b, a) \in R^{-1}$

$\Leftrightarrow (c, a) \in R^{-1} \circ S^{-1}$.

$\therefore (c, a) \in (S \circ R)^{-1} \Leftrightarrow (c, a) \in R^{-1} \circ S^{-1}$

Thus $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

Order of Relation: →

1.) Partial order Relation
2.) Total order Relation.

(1.) Partial order Relation →

A relation R on a set A is called partial order relation if R is

1.) Reflexive:- $(a, a) \in R, \forall a \in A$

i.e $aRa \quad \forall a \in A$

2.) Antisymmetric:-

$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall a, b \in A$

i.e aRb & $bRa \Rightarrow a = b$

3.) <u>Transitive</u>:- $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

i.e aRb & $bRc \Rightarrow aRc$

<u>POSET</u> or <u>Partial ordered Set</u>

The set A together with a partially order relation R on the set A i.e (A,R) is called Partial order set or POSET.

<u>Ex:</u>- Show that the Relation $R = \{(x,y) : x \geq y\}$ where $x, y \in I^+$ the set of Positive Integers, is a partial order Relation.

<u>Sol^n</u>:-

Let $x, y, z \in I^+$
and given $R = \{(x,y) : x \geq y\}$.

i.) <u>Reflexive</u>:- Let $x \in I^+$
Since $x \geq x \Rightarrow (x,x) \in R, \forall x \in I^+$
$\Rightarrow xRx, \forall x \in I^+$
$\Rightarrow R$ is Reflexive

ii.) <u>Anti-Symmetric</u>:- Let $x, y \in I^+$
Now $(x,y) \in R$ and $(y,x) \in R$
$\Rightarrow x \geq y$ and $y \geq x$
$\Rightarrow x = y$
$\Rightarrow (x,y) \in R$ and $(y,x) \in R \Rightarrow x = y$
$\therefore R$ is antisymmetric

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3.) Transitive :- Let $x, y, z \in I^+$
let $(x, y) \in R$ and $(y, z) \in R$
$\Rightarrow x \geq y$ and $y \geq z$
$\Rightarrow x \geq z$
$\Rightarrow (x, z) \in R$
$\therefore (x, y) \in R$ & $(y, z) \in R$
$\Rightarrow (x, z) \in R$.

$\therefore$ R is Transitive
Hence R is a Partial order Reat^n on I^+
Thus (I^+, R) is a POSET.

(#) Total order Relation :-

Consider the Relation R on the set A.
if $\forall a, b \in A$, we have
either $(a, b) \in R$
or $(b, a) \in R$
or $a = b$
Then R is called total order relation on A.

Ex:- Show that the Relation '<' (less than) defined on N, the set of +ve Integers is neither an equivalence relation nor partially ordered relation but is a Total order relation.

Sol^n:-
Let $a, b \in N$
$R = \{(a, b) : a < b\}$
1.) Reflexive :- let $a \in N$
Since $a \not< a$
$\Rightarrow (a, a) \notin R \Rightarrow a \not R a$
$\Rightarrow$ R is not reflexive

∴ The Relation '<' is not Reflexive & Hence it is neither an equivalence relation nor the partial order relation.

But

as $\forall\, a, b \in N$

we have

either $a < b$ $(a,b) \in R$

or $b < a$ $(b,a) \in R$

or $a = b$

So, the Relation is a total ordered relation.

(#) composition of relations SOR by using matrix of Relations. →

<u>Matrix Representation of Relation</u> :—

Let R be the Relation from set A to set B where

$$A = \{a_1, a_2, \ldots, a_m\}$$
$$B = \{b_1, b_2, \ldots, b_n\}$$

be finite sets having m and n elements resp. Then R can be represented by mn matrix and defined as

$$M_R = [m_{ij}]_{m \times n}$$
$$= \begin{cases} 1, & \text{if } aRb \text{ i.e } (a,b) \in R \\ 0, & \text{if } a \not R b \text{ i.e } (a,b) \notin R \end{cases}$$

The matrix M_R is called matrix of Relation R.

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Ex:-1 Let R be the Relation from set $A = \{1,3,4\}$ to itself and defined by

$$R = \{(1,1), (1,3), (3,3), (4,4)\}$$

then find relation matrix.

Sol^n:-

$$M_R = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex:-2.

Let $A = \{1,2,3,4,8\}$, $B = \{1,4,6,9\}$
Let aRb iff $a|b$ i.e a divides b
Find the Relation matrix M_R.

Sol^n:-

Here $R = \{(1,1), (1,4), (1,6), (1,9), (2,4), (2,6), (3,6), (3,9), (4,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 6 & 9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Ex:- Let $P = \{2,3,4,5\}$. consider the Relation R and S defined by

$R = \{(2,2), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5), (5,3)\}$

$S = \{(2,3), (2,5), (3,4), (3,5), (4,2), (4,5), (5,2), (5,5)\}$.

Find the matrices of the above relations
Use matrix to find the composition relation of R and S.

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Solⁿ:- The matrices of the relation R and S are

$$M_R = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

and

$$M_S = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The composite Relation of R and S is SoR

So Multiplying the matrix M_S with M_R to obtain the matrix $M_S \times M_R$

$$M_S \times M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 \end{bmatrix} \end{array}$$

The non-zero enteries in the matrix $M_S \times M_R$ tell the elements related in composition relation SoR of R and S.

Hence

$SoR = \{(2,3), (2,4), (2,5), (3,3), (3,5), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5)\}$

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Total no. of Relations :-

Let A be a set with n elements i.e $|A| = n$
Then

1.) Total no. of Relations $= 2^{n^2}$

2.) Total no. of Reflexive relation $= 2^{n(n-1)}$ or 2^{n^2-n}

3.) Total no. of Irreflexive relation $= 2^{n^2-n}$

4.) Total no. of Symmetric relation $= 2^{\frac{n(n+1)}{2}}$

5.) Total no. of Asymmetric relation $= 3^{\frac{n(n-1)}{2}}$

6.) Total no. of Antisymmetric relation $= 2^n \cdot 3^{\frac{n(n-1)}{2}}$

7.) Total no. of both Reflexive and Symmetric $= 2^{\frac{1}{2}(n^2-n)}$

Digraph | Directed graph of Relation R →

If A is finite set and R is a Relation on A, then we can also represent R pictorially as

1. Draw a small circle for each element of A and label the circle with corresponding elements of A. These circles are called vertices or nodes.
2. Draw an arrow called an edge from vertex a_i to $a_j \Leftrightarrow a_i R a_j$.

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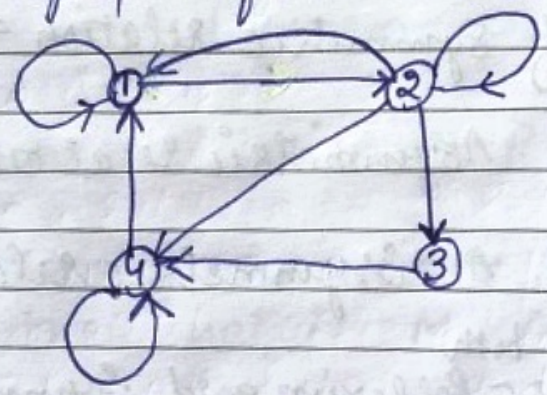
The Resulting pictorial Representation of R is called a Directed graph or Digraph of R.

Ex:- Let $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (2,4), (3,4), (4,1), (4,4)\}$
Construct the diagraph of R.

Soln:-
The digraph of R is



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Properties of Relation by Digraph

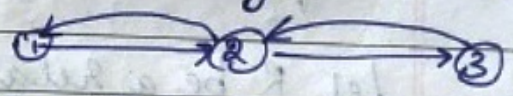
1.) **Reflexive:-** A Relation R is reflexive if there is a loop at every vertex of the directed graph.

e.g→

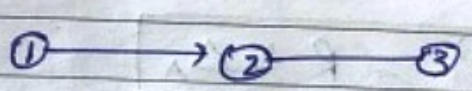
R:

Then R is Reflexive.

2.) **Symmetric:-** A Relation is symmetric if for every edge b/w distinct vertices in its digraph. There is an edge in the opposite direction.

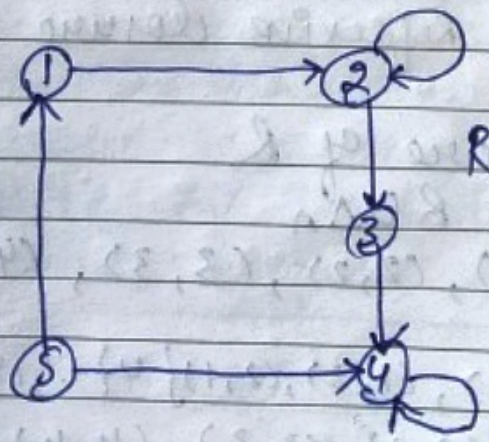


3.) **Transitive:-** A Relation is Transitive if there is an edge from vertex a to vertex b and from vertex b to vertex c.



Relation from diagraph:-

Find the Relation by the following figures.



R = {(1,2), (2,2), (2,3), (3,4), (4,4), (5,1), (5,4)}

Closure of Relations :→

Let R be a Relation on a Set A.
R may or may not have some Property P, such as reflexivity, symmetry or transitivity. If we add some pairs then we have the desired Property. The Smallest relation S on A that contains R and Possess the desired Property P is called closure of R with respect to that Property.

1. Reflexive closure :—

Let R be a relation on a Set A.
The Reflexive closure $R^{(r)}$ of a Relation R is the smallest reflexive relation that contains R as a Subset. and is given by

$$\boxed{R^{(r)} = R \cup \Delta_A}$$

where $\Delta_A = \{(a,a) : a \in A\}$ is the diagonal Relation on A.

Ex:-

Let $A = \{1,2,3,4\}$ and
$R = \{(1,1), (2,2), (1,2), (2,1)\}$
Then find the Reflexive closure of R.

Sol:-

Reflexive closure of R
$$R^{(r)} = R \cup \Delta_A$$
$$\Delta_A = \{(1,1), (2,2), (3,3), (4,4)\}$$

So $R^{(r)} = \{(1,1),(2,2),(1,2),(2,1)\} \cup \{(1,1),(2,2),(3,3),(4,4)\}$
$= \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1)\}$

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2. Symmetric closure :-

Let R be a Relation on a set A, which is not symmetric. The symmetric closure $R^{(s)}$ is the smallest symmetric relation that contains R as a subset. and is given by

$$\boxed{R^{(s)} = R \cup R^{-1}}$$

where $R^{-1} = \{(y, x) : (x, y) \in R\}$ be the Inverse of R.

Ex:-

If $R = \{(1,2), (2,1), (2,2), (3,1), (4,3)\}$ be a Relation on $S = \{1,2,3,4\}$. Find the symmetric closure of R.

Solⁿ:-

The symmetric closure of R can be found by taking the union of R and R^{-1}.

Now $R^{-1} = \{(2,1), (1,2), (2,2), (1,3), (3,4)\}$

So $R^{(s)} = R \cup R^{-1}$ $\{(2,1), (1,2), (2,2), (3,1), (1,3), (3,4), (4,3)\}$

$= \{(1,2), (2,1), (2,2), (3,1), (4,3)\} \cup \{(2,1), (1,2), (2,2), (1,3), (3,4)\}$

3. Transitive closure :-

The Relation obtained by adding the least number of ordered pairs to ensure transitivity is called the transitive closure of the Relatⁿ R. and is given by

$$\boxed{R^* = R \cup R^2 \cup \cdots R^n}$$

where $n = |A|$ & R be a Relatⁿ on A.

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Ex:- Let $A = \{1, 2, 3, 4\}$
and $R = \{(1,2), (2,3), (3,4)\}$
be a Relation on A. Find transitive closure R^*
using composition of R.

Sol^n:- given $R = \{(1,2), (2,3), (3,4)\}$

→ Now $R^2 = R \circ R$
$R = \{(1,2), (2,3), (3,4)\}$
$R = \{(1,2), (2,3), (3,4)\}$
So $R^2 = \{(1,3), (2,4)\}$

→ $R^3 = R^2 \circ R$
$R = \{(1,2), (2,3), (3,4)\}$
$R^2 = \{(1,3), (2,4)\}$
So $R^3 = \{(1,4)\}$

→ $R^4 = R^3 \circ R$
$R = \{(1,2), (2,3), (3,4)\}$
$R^3 = \{(1,4)\}$
So $R^4 = \phi$

Hence the transitive closure of R

$R^* = R \cup R^2 \cup R^3 \cup R^4$
$= \{(1,2), (2,3), (3,4)\} \cup \{(1,3), (2,4)\}$
$\cup \{(1,4)\} \cup \phi$

$= \{(1,2), (2,3), (3,4), (1,3), (2,4),$
$(1,4)\}$

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Warshall's Algorithm :→

An efficient method for computing the transitive closure of a Relation is known as Warshall's algorithm named after Stephen Warshall.

Step 1:- Let n be the number of elements in a given set A.
To find transitive closure of Relation R on A, maximum n warshall sets can be find
$W_0, W_1, \ldots, W_n$ where $W_0 = M_R$
and W_n is the required matrix to find transitive closure.

Step 2:- Procedure to compute W_K from W_{K-1}.

1. copy 1 entries from W_{K-1} in W_K with same position.
2. In K^{th} row and K^{th} coloum of W_{K-1}, See K^{th} coloum and check the position of 1 and put below p_i.
   See K^{th} row and check the position of 1 and put below q_i.
3. mark entries in W_K as 1 for (p_i, q_i) of W_K if there are not already 1.

Step 3:- Stop when W_n is obtained and it is the required transitive closure

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Ex:-1. Let $A = \{4, 6, 8, 10\}$ & $R = \{(4,4), (4,10), (6,6), (6,8), (8,10)\}$ is a Relation on set A. Determine the transitive closure of R using warshall's algorithm

Sol^n- Given $A = \{4, 6, 8, 10\}$ and $R = \{(4,4), (4,10), (6,6), (6,8), (8,10)\}$

$$M_R = \begin{array}{c} \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now we have $|A| = n = 4$
Thus we have to find warshall's sets W_0, W_1, W_2, W_3, W_4.
Here

$$W_0 = M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. To compute W_1 - Transfer all 1's from W_0 to W_1
The locations of non-zero entries in C_1 :- $P_i = 1$
The locations of non-zero entries in R_1 :- $q_i = 1, 4$

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1,1), (1,4)$$

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2. To compute W_2 – Transfer all 1's from W_1 to W_2

Positions of the non zero entries in C_2 : $p_i = 2$
Positions of non zero entries in R_2 : $q_i = 2, 3$

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2,2), (2,3)$$

3. To compute W_3 :- Transfer all 1's from W_2 to W_3

Positions of non zero entries in C_3 : $p_i = 2$
Positions of non zero entries in R_3 : $q_i =$

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2,4)$$

4. To compute W_4 :- Transfer all 1's from W_3 to W_4
Positions of non-zero entries in C_4 : $p_i = 1, 2, 3$
Positions of non-zero entries in R_4 : q_i —
No New pairs formes

So $W_4 = W_3$

$$= \begin{array}{c} \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore$ $R^* = \{(4,4), (4,10), (6,6), (6,8), (6,10), (8,10)\}$

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Ex:-2. Let $A = \{1, 2, 3, 4\}$ & $R = \{(1, 2), (2, 3), (3, 4)\}$ be a relation on A. Find transitive closure R^* by warshall's Algorithm.

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Here $|A| = 4$

So we have to find warshall's sets W_0, W_1, W_2, W_3, W_4.

$R = \{(1, 2), (2, 3), (3, 4)\}$

$$\therefore M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}_{4\times4}$$

Now $W_0 = M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

1.) $W_1 \rightarrow$

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C_1	R_1
p_i	q_i
ϕ	2

No new pair formed

2.) $W_2 \rightarrow$

$$W_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C_2	R_2
p_i	q_i
1	3

(1, 3)

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3.) $W_3 \rightarrow$

$$W_3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C_3	R_3
p_i	q_i
1	4
2	

(1, 4), (2, 4)

4.) $W_4 \rightarrow$

$$W_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C_4	R_4
p_i	q_i
1	ϕ
2	
3	

No new pairs formed

Hence the Required transitive closure is

$$R^* = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$$