POSET

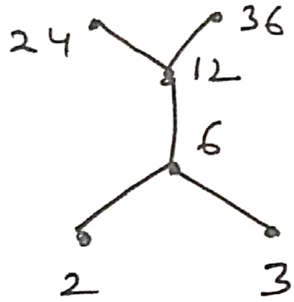
A Partial ordering $\leq$ on a Poset P Can be represented by a diagram known as Hass diagram

→ In such diagram we represent each element by circle or by a dot.

→ Any Two comparable elements are joined by lines in such a way that $a \leq b$. Then a line below below to non comparable elements are not Joined.

Thus There will not be any horizontal line in The diagram of Poset.

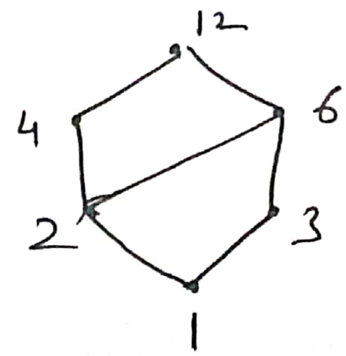
Ex Let $X = \{2, 3, 6, 12, 24, 36\}$ and relation $\leq$ be such that $x \leq y$ if x/y. Draw the Hass diagram of $(X, '/')$.



Q Draw the Hass diagram of $(S, '/')$ where $S = \{3, 5, 7\}$

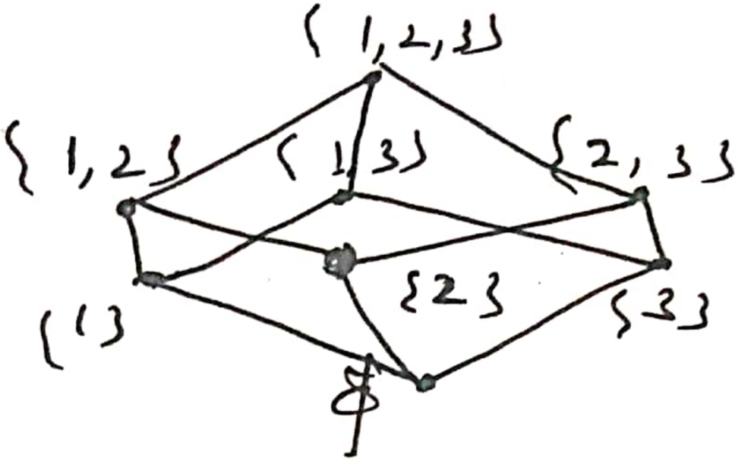
Q Draw the Hass diagram of Poset $(D_{12}, /)$ where D_{12} = set of factor of 12.

Sol $D_{12} = \{1, 2, 3, 4, 6, 12\}$

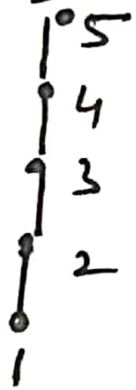


Q Draw the Hass diagram of Poset $(P(S), \subseteq)$ where $S = \{1, 2, 3\}$

$P(S) = \{ \phi, \{1\} \{2\} \{3\} \{1,2\} (1,3) \{2,3\} \{1,2,3\} \}$



Q Let $P = \{1, 2, 3, 4, 5\}$ and $\leq$ be the relation less than or equal to. Draw the Hass diagram of $(P, \leq)$



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Recursive function

A recursive function consist of 2 steps:-

Basic step (i) Specify the value of the function at initial value

(ii) give a rule of for finding its values at an integer from its values at smaller integers.
Recursive step

for $n > 0$, define $f(n)$ in term of $f(0), f(1)$ --- -- $f(n-1)$)

Ex **fibonacci sequence**

(a) if $n=0$ or $n=1$ Then $f_n = n \Rightarrow f_0 = 0, f_1 = 1$

(b) if $n > 1$ Then $f_n = f_{n-2} + f_{n-1}$

Here The base value are 0 and 1.

$$f_2 = f_0 + f_1 = 0+1 = 1$$

$$f_3 = f_1 + f_2 = 1+1 = 2$$

$$f_4 = f_2 + f_3 = 1+2 = 3$$

$$f_5 = f_3 + f_4 = 2+3 = 5$$

$\vdots$

0, 1, 1, 2, 3, 5, 8, 13 - - - - -

Ex find $f(1)$, $f(2)$, $f(3)$ and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$, and $f(n+1) = f(n) + 2$ for $n = 0, 1, 2, - - - -$

Sol

$$f(n+1) = f(n) + 2$$

$$f(1) = f(0+1) = f(0) + 2 = 1+2 = 3$$

$$f(2) = f(1+1) = f(1) + 2 = 3+2 = 5$$

$$f(3) = f(2+1) = f(2) + 2 = 5+2 = 7$$

$$f(4) = f(3+1) = f(3) + 2 = 7+2 = 9$$

Q Suppose that f is defined recursively by $f(0) = 3$

$f(n+1) = 2f(n) + 3$

find $f(1)$, $f(2)$ & $f(3)$ & $f(4)$.

Sol

$f(n+1) = 2f(n)+3, \quad f(0) = 3$

$f(0+1) = 2f(0) + 3 = 2\times 3 + 3 = 9$

$f(1+1) = 2f(1) + 3 = 2\times 9 + 3 = 21$

$f(2+1) = 2f(2) + 3 = 2\times 21 + 3 = 45$

$f(3+1) = 2f(3) + 3 = 2\times 45 + 3 = 93$

Q Give a recursive definition of $\sum_{k=0}^{n} a_k$

Sol The first part of the recursive define is

$$\sum_{k=0}^{0} a_k = a_0$$

The second part is

$$\sum_{k=0}^{n+1} a_k = a_0 + a_1 + a_2 + \cdots + a_n + a_{n+1}$$

$$\boxed{\sum_{k=0}^{n+1} a_k = \sum_{a=0}^{n} a_k + a_{n+1}}$$

Q obtain recursive definition for the function $f(n) = a_n$ where $a_n = 5n$

Sol given $f(n) = a_n = 5n$

$f(0) = a_0 = 0$

$f(1) = a_1 = 5$

$f(2) = a_2 = 10$

$f(3) = a_3 = 5\times 3 = 15$

We can rewrite these a

$a_0 = 0$ and

$\boxed{a_n = a_{n-1} + 5}$

for $n \geq 1$

Q. find recursive definition of the given function
$f(n) = a_n$ where $a_n = 6^n$

Sol given $f(n) = a_n = 6^n$

$f(0) = a_0 = 6^0 = 1$

$f(1) = a_1 = 6^1 = 6$

$f(2) = a_2 = 6^2 = 36$

$f(3) = a_3 = 6^3 = 216$

$f(4) = a_4 = 6^4 = 1296$

$\vdots$

$a_0 = 1$

and $\boxed{a_{n+1} = 6 \cdot a_n}$ for $n \geq 0$

Warshall's Algorithm

Q Using warshall's Algorithm find transitive closure of the relation $R = \{(1,4)\ (2,1)\ (2,3)\ (3,1)\ (3,4)\ (4,3)\}$ on set = A $A = \{1, 2, 3, 4\}$

Sol Given $R = \{(1,4), (2,1), (2,3), (3,1)\ (3,4), (4,3)\}$

$$M_R = R_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

I iteration

I st col & 1st Row

C R C×R

Position (2,3) {4} {(2,4) (3,4)}

$$P_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

II nd iteration for P_1

2nd col & 2nd Row

C R C×R

Position {0} {1, 3, 4} { } No change

$$P_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

III rd Iteration for P_2

3rd col & 3rd row

C R

Position {2,4} {1,4}

C×R = {(2,1) (2,4) (4,1), (4,4)}

$$P_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{vmatrix} \end{array}$$

IV th Iteration for P_3

$P_4 =$ 4th col & 4th Row

C R C×R

{1, 2, 3, 4} {1, 3, 4}

(1,1) (1,3) (1,4) (2,1) (2,3) (2,4) (3,1) (3,3) (3,4) (4,1) (4,3) (4,4)

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$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 1 \end{array}$$

Transitive Closure $R^{+} = \{ (1,1)\ (1,3)\ (1,4)$
$(2,1)\ (2,3)\ (2,4)$
$(3,1)\ (3,3)\ (3,4)$
$(4,1)\ (4,3)\ (4,4) \}$

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Partial order Relation

A relation on a set A is called a Partial order if it is,

(i) Reflexive i.e. $(a,a) \in R \ \forall \ a \in A$

(ii) Anti-symmetric i.e if $(a,b) \in R, (b,a) \in R \Rightarrow \boxed{a=b}$

$(a,b) \in R, (b,a) \notin R \quad \forall \ a,b \in A$

(iii) Transitive i.e.

$(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \ \forall \ a,b,c \in A$

Ex Let $A = \{1,2,3,4\}$

$R_1 = \{(1,1), (2,2) (3,3), (4,4)\}$ R ✓ A ✓ T ✓

$R_2 = \phi$ { R × A T

$R_3 = A \times A$ R A T ×

$R_4 = \{(1,1)(2,2)(3,3)(4,4), (1,2)(2,3)(1,3)\}$ R ✓ A ✓ T ✓

Sol A × =

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

Partial order set (POSET) $(\underset{\text{set}}{A}, \leq)$ → define a relation

if A is any non-empty set and R is a Partial order relation on a set A, then the order pair (A, R) is Called Partial order set or POSET.

Ex

The greatest than or equal ($\geq$) relation is a Partial ordering on Z, the set of integers

Sol Z → set of integer

Reflexive: since $a \geq a \ \forall \ a \in Z$,
i.e $\geq$ is reflexive

Antisymmetric: since $a \geq b$, $b \geq a \Rightarrow \boxed{a = b}$,
i.e. $\geq$ is Antisymmetric
$5 \geq 5$

Transitive: since $a \geq b$, $b \geq c \Rightarrow a \geq c$
i.e. $5 \geq 2$, $2 \geq 1$
$\Rightarrow 5 \geq 1$
i.e. $\geq$ is transitive

Hence:. $\geq$ is a Partial order relation on Z and
$(Z; \geq)$ is a Poset.

Q if 'A' is a set of Real No. then $[A, \leq]$ is a Poset,

A = Real NO

Reflexive $1.5 \leq 1.5$

Antisymmetric $(\underbrace{1 \leq 2}_{\text{exists}})$ $(2 \leq 1)$ does not exist ✗

Transitive = $(\underset{a}{1} \leq \underset{b}{2})$ & $\underset{b}{2} \leq \underset{c}{3} \Rightarrow \underset{a}{1} \leq \underset{c}{3}$
∴ Transitive Property hold,