Lecture 8 – Review
Data Abstraction
Interfaces & Representation

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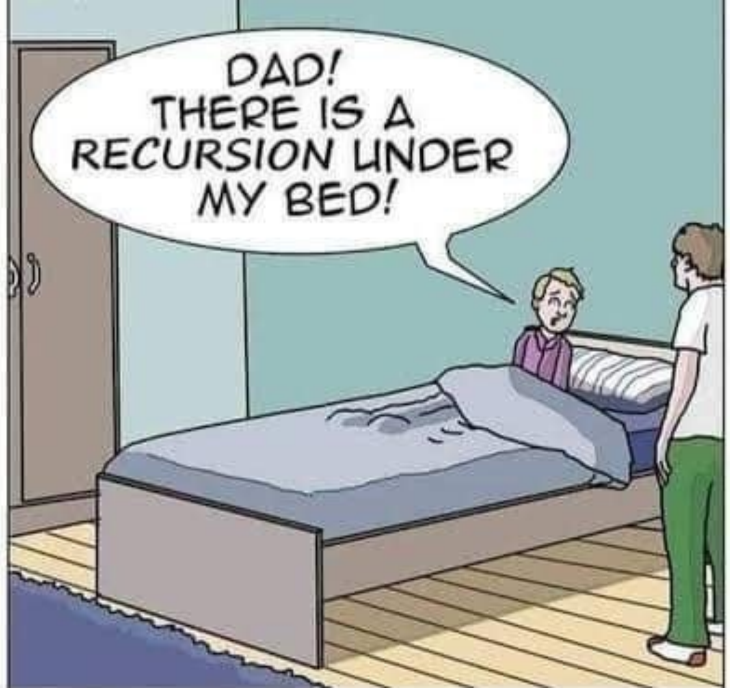
DAD! THERE IS A RECURSION UNDER MY BED!


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Lecture Nuggets

- A handful of key concepts in programming languages
  - Value
  - Abstraction
  - Interface
  - Representation
  - Implementation
- May have many implementations for an interface
- Representation of a value may take different forms
- The environment allows us to store variable value pairs

Interface vs. Implementation

- Teasing out the “interface” and the “implementation”
  - I don’t care how you manage it, but I’ll be happy as long as…
  - The particular way in which I accomplish my goal is by…

Representation vs. Value

Natural Numbers

$\lceil v \rceil$ "the representation of data v."

$$(\texttt{zero}) = \lceil 0 \rceil$$

$$(\texttt{is-zero?}\ \lceil n \rceil) = \begin{cases} \texttt{\#t} & n = 0 \\ \texttt{\#f} & n \neq 0 \end{cases}$$

$$(\texttt{successor}\ \lceil n \rceil) = \lceil n+1 \rceil \quad (n \geq 0)$$

$$(\texttt{predecessor}\ \lceil n+1 \rceil) = \lceil n \rceil \quad (n \geq 0)$$

Procedures manipulating the new data type

- How do we implement **`plus`**

```
(define plus
  (lambda (x y)
    (if (is-zero? x)
      y
      (successor (plus (predecessor x) y)))))
```

- Accomplish all you would like to accomplish through the **interface**
- And… $(\texttt{plus}\ \lceil x \rceil\ \lceil y \rceil) = \lceil x + y \rceil$

Back to Natural Numbers

- Constructors
- Observers

$$
\begin{aligned}
&(\texttt{zero}) = \lceil 0 \rceil \\
&(\texttt{is-zero?}\ \lceil n \rceil) = \begin{cases} \texttt{\#t} & n = 0 \\ \texttt{\#f} & n \neq 0 \end{cases} \\
&(\texttt{successor}\ \lceil n \rceil) = \lceil n+1 \rceil \quad (n \geq 0) \\
&(\texttt{predecessor}\ \lceil n+1 \rceil) = \lceil n \rceil \quad (n \geq 0)
\end{aligned}
$$

Back to Natural Numbers

- Constructors
- Observers

constructor

$(\text{zero}) = \lceil 0 \rceil$ → representation of zero in my implementation

observer

$(\text{is-zero?}\ \lceil n \rceil) = \begin{cases} \#t & n = 0 \\ \#f & n \neq 0 \end{cases}$ manipulate the data and gives information

constructor

$(\text{succesor}\ \lceil n \rceil) = \lceil n+1 \rceil \quad (n \geq 0)$

$(\text{predecessor}\ \lceil n+1 \rceil) = \lceil n \rceil \quad (n \geq 0)$

Digdem Yildiz

implementing plus:

```
(define plus
  (lambda (x y)
    (if (is-zero? x)
      y
      (successor (plus (predecessor x) y)))))
```

x-1+y

x-1

x-1+y+1 → "x+y"

$$(\texttt{plus}\ \lceil x \rceil\ \lceil y \rceil) = \lceil x+y \rceil,$$

Ceren Tarim

Implementation of Natural Numbers

- Unary representation
  - Use `#t`'s to represent numbers

$$\lceil 0 \rceil = \texttt{()}$$
$$\lceil n+1 \rceil = \texttt{(\#t . } \lceil n \rceil \texttt{)}$$

  - Scheme implementation

```
(define zero (lambda () '()))
(define is-zero? (lambda (n) (null? n)))
(define successor (lambda (n) (cons #t n)))
(define predecessor (lambda (n) (cdr n)))
```

Another implementation

- Scheme number representation
  - Use scheme numbers to represent numbers
  - Scheme implementation

```
(define zero (lambda () 0))
(define is-zero? (lambda (n) (zero? n)))
(define successor (lambda (n) (+ n 1)))
(define predecessor (lambda (n) (- n 1)))
```

- Data Structure Representation
- Procedural Representation

- Environment: Function that maps variables to values

- Interface

$$(\text{empty-env}) = \lceil \emptyset \rceil$$

$$(\text{apply-env } \lceil f \rceil \text{ var}) = f(\text{var})$$

$$(\text{extend-env var } v \lceil f \rceil) = \lceil g \rceil, \text{ where } g(\text{var}_1) = \begin{cases} v & \text{if } \text{var}_1 = \text{var} \\ f(\text{var}_1) & \text{otherwise} \end{cases}$$

The environment interface

Environment interface:

$$
\begin{array}{lll}
(\texttt{empty-env}) & = \lceil \emptyset \rceil & \\
(\texttt{apply-env}\ \lceil f \rceil\ var) & = f(var) & \text{→ get the value} \\
(\texttt{extend-env}\ var\ v\ \lceil f \rceil) & = \lceil g \rceil, & \text{→ rep. of another environment} \\
& \text{where } g(var_1) = \begin{cases} v & \text{if } var_1 = var \\ f(var_1) & \text{otherwise} \end{cases} &
\end{array}
$$

Implementation:

Grammar →

$$
\begin{aligned}
Env &= \texttt{(empty-env)} \mid \texttt{(extend-env } Var\ SchemeVal\ Env\texttt{)}\\
Var &= Sym
\end{aligned}
$$

empty-env : $() \rightarrow Env$

```
(define empty-env
  (lambda () (list 'empty-env)))
```

extend-env : $Var \times SchemeVal \times Env \rightarrow Env$

```
(define extend-env
  (lambda (var val env)
    (list 'extend-env var val env)))
```

apply-env : $Env \times Var \rightarrow SchemeVal$

```
(define apply-env
  (lambda (env search-var)
    (cond
      ((eqv? (car env) 'empty-env)
       (report-no-binding-found search-var))
      ((eqv? (car env) 'extend-env)
       (let ((saved-var (cadr env))
             (saved-val (caddr env))
             (saved-env (cadddr env)))
         (if (eqv? search-var saved-var)
           saved-val
           (apply-env saved-env search-var))))
      (else
        (report-invalid-env env)))))
```

Lecture 9
Representation Strategies for Data Types

T. METIN SEZGIN

Lecture Nuggets

- We can represent data types using data structures
- We can represent data types using procedures
- Use the environment as an example
- We can automate mundane data type definitions

Nugget

The environment allows us to store variable value pairs

Representation strategies

- Two strategies
  - Data Structure Representation
  - Procedural Representation
- Test case
  - Environment
    - Function that maps variables to values
      - List, function, hashtable…
  - Start with the interface
  - Introduce implementation

The Environment Interface

- Environment
  - Function that maps variables to values

$\{(var_1, val_1), \ldots, (var_n, val_n)\}$

- The interface

$$
\begin{aligned}
&\texttt{(empty-env)} && = \lceil \emptyset \rceil \\
&\texttt{(apply-env } \lceil f \rceil \ var\texttt{)} && = f(var) \\
&\texttt{(extend-env } var \ v \ \lceil f \rceil\texttt{)} && = \lceil g \rceil, \\
& && \text{where } g(var_1) = \begin{cases} v & \text{if } var_1 = var \\ f(var_1) & \text{otherwise} \end{cases}
\end{aligned}
$$

Data Structure Representation

- The interface
  - Constructors
  - Observers

$$
\begin{aligned}
&\texttt{(empty-env)} &&= \lceil \emptyset \rceil \\
&\texttt{(apply-env } \lceil f \rceil \ \mathit{var}\texttt{)} &&= f(\mathit{var}) \\
&\texttt{(extend-env } \mathit{var}\ v\ \lceil f \rceil\texttt{)} &&= \lceil g \rceil, \\
& &&\text{where } g(\mathit{var}_1) = \begin{cases} v & \text{if } \mathit{var}_1 = \mathit{var} \\ f(\mathit{var}_1) & \text{otherwise} \end{cases}
\end{aligned}
$$

- For example

```
(define e
  (extend-env 'd 6
    (extend-env 'y 8
      (extend-env 'x 7
        (extend-env 'y 14
          (empty-env))))))
```

$e(\mathrm{d}) = 6, e(\mathrm{x}) = 7, e(\mathrm{y}) = 8$

- The grammar

$$
\begin{aligned}
\mathit{Env\text{-}exp} &::= \texttt{(empty-env)} \\
&::= \texttt{(extend-env } \mathit{Identifier}\ \ \mathit{Scheme\text{-}value}\ \ \mathit{Env\text{-}exp}\texttt{)}
\end{aligned}
$$

Implementation

Env = `(empty-env)` | `(extend-env` *Var SchemeVal Env*`)`
Var = *Sym*

Implementation

Env = (empty-env) | (extend-env *Var* *SchemeVal* *Env*)
Var = *Sym*

empty-env : () → *Env*

```
(define empty-env
  (lambda () (list 'empty-env)))
```

extend-env : *Var* × *SchemeVal* × *Env* → *Env*

```
(define extend-env
  (lambda (var val env)
    (list 'extend-env var val env)))
```

apply-env : *Env* × *Var* → *SchemeVal*

```
(define apply-env
  (lambda (env search-var)
    (cond
      ((eqv? (car env) 'empty-env)
       (report-no-binding-found search-var))
      ((eqv? (car env) 'extend-env)
       (let ((saved-var (cadr env))
             (saved-val (caddr env))
             (saved-env (cadddr env)))
         (if (eqv? search-var saved-var)
           saved-val
           (apply-env saved-env search-var))))
      (else
        (report-invalid-env env)))))
```

Implementation

Env = (empty-env) | (extend-env *Var* *SchemeVal* *Env*)
Var = *Sym*

empty-env : () → *Env*

```
(define empty-env
  (lambda () (list 'empty-env)))
```

extend-env : *Var* × *SchemeVal* × *Env* → *Env*

```
(define extend-env
  (lambda (var val env)
    (list 'extend-env var val env)))
```

apply-env : *Env* × *Var* → *SchemeVal*

```
(define apply-env
  (lambda (env search-var)
    (cond
      ((eqv? (car env) 'empty-env)
       (report-no-binding-found search-var))
      ((eqv? (car env) 'extend-env)
       (let ((saved-var (cadr env))
             (saved-val (caddr env))
             (saved-env (cadddr env)))
         (if (eqv? search-var saved-var)
           saved-val
           (apply-env saved-env search-var))))
      (else
        (report-invalid-env env)))))
```

```
(define e
  (extend-env 'd 6
    (extend-env 'y 8
      (extend-env 'x 7
        (extend-env 'y 14
          (empty-env))))))
```

$e(\mathrm{d}) = 6, e(\mathrm{x}) = 7, e(\mathrm{y}) = 8$

Env-exp ::= (empty-env)
::= (extend-env *Identifier* *Scheme-value* *Env-exp*)

Nugget

We can represent data types using procedures

Procedural Representation

$Env = Var \rightarrow SchemeVal$

empty-env : $() \rightarrow Env$

```
(define empty-env
  (lambda ()
    (lambda (search-var)
      (report-no-binding-found search-var))))
```

extend-env : $Var \times SchemeVal \times Env \rightarrow Env$

```
(define extend-env
  (lambda (saved-var saved-val saved-env)
    (lambda (search-var)
      (if (eqv? search-var saved-var)
        saved-val
        (apply-env saved-env search-var)))))
```

apply-env : $Env \times Var \rightarrow SchemeVal$

```
(define apply-env
  (lambda (env search-var)
    (env search-var)))
```

Nugget

We can automate mundane data type definitions

(Racket is a powerful language that will simplify life for us)

Implementation

Env = (empty-env) | (extend-env *Var* *SchemeVal* *Env*)
Var = *Sym*

empty-env : () → *Env*

```
(define empty-env
  (lambda () (list 'empty-env)))
```

extend-env : *Var* × *SchemeVal* × *Env* → *Env*

```
(define extend-env
  (lambda (var val env)
    (list 'extend-env var val env)))
```

apply-env : *Env* × *Var* → *SchemeVal*

```
(define apply-env
  (lambda (env search-var)
    (cond
      ((eqv? (car env) 'empty-env)
       (report-no-binding-found search-var))
      ((eqv? (car env) 'extend-env)
       (let ((saved-var (cadr env))
             (saved-val (caddr env))
             (saved-env (cadddr env)))
         (if (eqv? search-var saved-var)
           saved-val
           (apply-env saved-env search-var))))
      (else
        (report-invalid-env env)))))
```

```
(define e
  (extend-env 'd 6
    (extend-env 'y 8
      (extend-env 'x 7
        (extend-env 'y 14
          (empty-env))))))
```

$e(\mathrm{d}) = 6, e(\mathrm{x}) = 7, e(\mathrm{y}) = 8$

Env-exp ::= (empty-env)
::= (extend-env *Identifier* *Scheme-value* *Env-exp*)

The general form of **define-datatype**

```
(define-datatype environment environment?
  (empty-env)
  (extend-env
    (bvar symbol?)
    (bval expval?)
    (saved-env environment?))
  (extend-env-rec
    (id symbol?)
    (bvar symbol?)
    (body expression?)
    (saved-env environment?)))
```

(define-datatype *type-name* *type-predicate-name*
  $\{$ (*variant-name* $\{$ (*field-name* *predicate*) $\}^*$) $\}^+$)

Example uses of `define-datatype`

- Lets define a "triple" structure using racket

Depending on how you look at it, **Racket** is

- a *programming language*—a dialect of Lisp and a descendant of Scheme;

> See Dialects of Racket and Scheme for more information on other dialects of Lisp and how they relate to Racket.

- a *family* of programming languages—variants of Racket, and more; or
- a set of *tools*—for using a family of programming languages.

Where there is no room for confusion, we use simply *Racket*.

Racket's main tools are

- **racket**, the core compiler, interpreter, and run-time system;
- **DrRacket**, the programming environment; and
- **raco**, a command-line tool for executing **Racket commands** that install packages, build libraries, and more.

Example uses of `define-datatype`

$$S\text{-}list ::= (\{S\text{-}exp\}^*)$$
$$S\text{-}exp ::= Symbol \mid S\text{-}list$$

```
(define-datatype s-list s-list?
  (empty-s-list)
  (non-empty-s-list
    (first s-exp?)
    (rest s-list?)))

(define-datatype s-exp s-exp?
  (symbol-s-exp
    (sym symbol?))
  (s-list-s-exp
    (slst s-list?)))
```

Nugget

We can represent any data structure easily using define-datatype