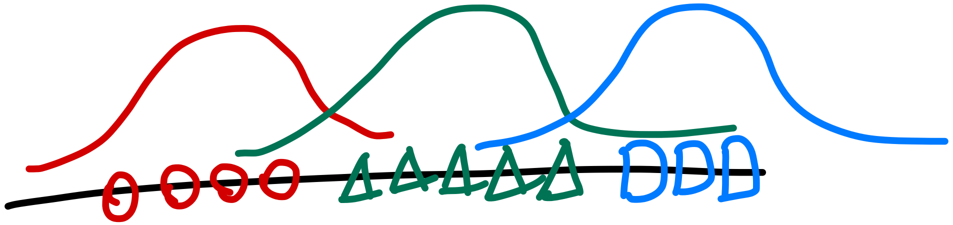
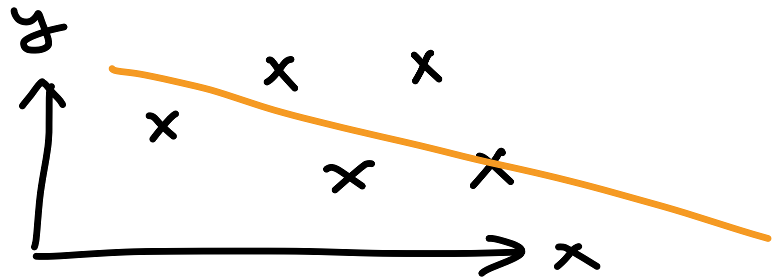
Multivariate methods





⇒ multiple measurements are taken from each point

$x_i \in \mathbb{R}^D$ $\qquad \mathbf{x}_i = [x_{i1}\ \ x_{i2}\ \ \cdots\ \ x_{iD}]^T$

i^{th} data point $\qquad$ ↳ first feature $\qquad$ → second feature $\qquad$ ↳ D^{th} feature

$y_i \Rightarrow$ class label $\qquad$ $y_i \Rightarrow$ target value.

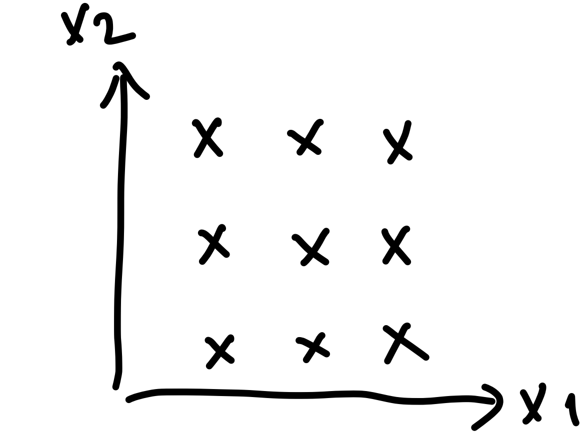
$$\mathcal{X} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{N} \qquad \mathbf{x}_i \in \mathbb{R}^D$$

$$y_i \in \{1, 2, \dots, K\}$$

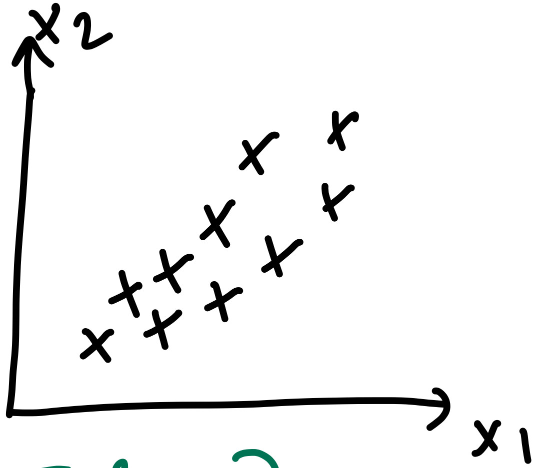
$$y_i \in \mathbb{R}^1$$

$$\mathbf{X} = \underbrace{\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ x_{21} & x_{22} & \cdots & x_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}}_{N \times D} \begin{array}{l} \rightarrow \mathbf{x}_1^T \\ \rightarrow \mathbf{x}_2^T \\ \\ \rightarrow \mathbf{x}_N^T \end{array}$$

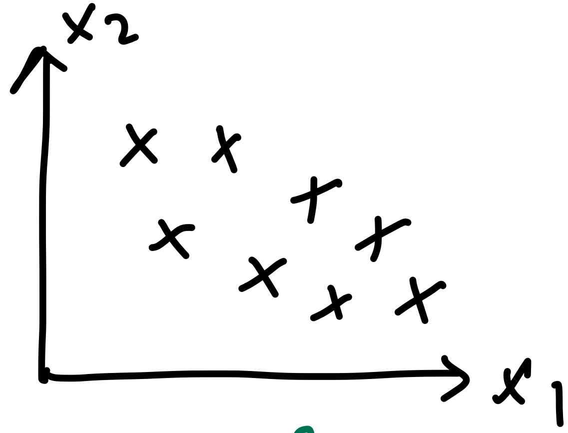
$$\mathbf{y} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}}_{N \times 1}$$



$$\begin{bmatrix}\sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2\end{bmatrix} \Rightarrow \begin{matrix}\sigma_{12} \cong 0 \\ \sigma_{21} \cong 0\end{matrix}$$



$$\begin{bmatrix}\sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2\end{bmatrix} \Rightarrow \sigma_{12} = \sigma_{21} > 0$$



$$\begin{bmatrix}\sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2\end{bmatrix} \Rightarrow \sigma_{12} = \sigma_{21} < 0$$

Univariate $\Rightarrow x \sim N(x; \mu, \sigma^2)$

multivariate $\Rightarrow \mathbf{x} \sim N(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$

mean vector

covariance matrix

$$\hat{\mu}_{1\times1} = \frac{\sum_{i=1}^{N} x_{i\,(1\times1)}}{\;}$$

$$\hat{\sigma}^2_{II\,(1\times1)} = \frac{\sum_{i=1}^{N} (x_i - \hat{\mu})^2_{1\times1}}{N}$$

$$\hat{\boldsymbol{\mu}}_{D\times1} = \frac{\sum_{i=1}^{N} \mathbf{x}_{i\,(D\times1)}}{N}$$

$$\hat{\boldsymbol{\Sigma}}_{D\times D} = \frac{\sum_{i=1}^{N} (\mathbf{x}_i - \hat{\boldsymbol{\mu}})_{D\times1} (\mathbf{x}_i - \hat{\boldsymbol{\mu}})^T_{1\times D}}{N}$$