Lecture 04
Structures and Patterns in Functional Programming

T. METIN SEZGIN

Announcements

1. Reading SICP 1.2 (pages 31-50)
2. Etutor 1 due on Sunday midnight
3. Attend your PSes
4. 3 People per group

Lecture 3 – Review
Functional Programming

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Lecture Nuggets

- Lambda expressions creates procedures
  - Formal parameters
  - Body
  - Procedures allow creating abstractions
- We can solve problems by creating functions
- The substitution model is a good mental model of an interpreter

Controlling the process

```
(define sqrt
    (lambda (x)
        (sqrt-loop 1.0 x)))


(define sqrt-loop (lambda G X)
   (if (close-enuf? G X)
       G
       (sqrt-loop (improve G X) X ) ) )
```

Nugget

The substitution model is a good mental model of an interpreter

Iterative and Recursive versions of fact

```scheme
;; RECURSIVE

(define (fact-r x)
  (if (= x 0) 1 (* x (fact-r (- x 1)))))


;; ITERATIVE
(define (fact-i x)
  (fact-i-helper 1 1 x))

(define fact-i-helper
  (lambda (product counter n)
    (if(> counter n)
       product
       (fact-i-helper (* product counter) (+ counter 1) n))))
```

```scheme
(define fact(lambda (n)
  (if (= n 1)1(* n (fact (- n 1))))))

(fact 3)
(if (= 3 1) 1 (* 3 (fact (- 3 1))))
(if #f 1 (* 3 (fact (- 3 1))))
(* 3 (fact (- 3 1)))
(* 3 (fact 2))
(* 3 (if (= 2 1) 1 (* 2 (fact (- 2 1)))))
(* 3 (if #f 1 (* 2 (fact (- 2 1)))))
(* 3 (* 2 (fact (- 2 1))))
(* 3 (* 2 (fact 1)))
(* 3 (* 2 (if (= 1 1) 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 (if #t 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 1))
(* 3 2)
6
```

Iterative algorithm to compute 4! as a table

- In this table:
    - One column for each piece of information used
    - One row for each step

first row handles `0!` cleanly

product	counter	N
1	1	4
1	2	4
2	3	4
6	4	4
24	5	4

product * counter

counter +1

answer

- The last row is the one where counter > n
- The answer is in the product column of the last row

Iterative factorial in scheme

- (define ifact (lambda (n) (ifact-helper 1 1 n)))

initial
row of table

(define ifact-helper (lambda (product counter n)

(if (> counter n)

product

compute next row of table

(ifact-helper (* product counter) (+ counter 1) n))))

answer is in product column of last row

at last row when counter > n

Partial trace for `(ifact 4)`

```
(define ifact-helper (lambda (product count n)
      (if (> count n) product
          (ifact-helper (* product count)
                        (+ count 1) n))))
```

```
(ifact 4)
(ifact-helper 1 1 4)
(if (> 1 4) 1 (ifact-helper (* 1 1) (+ 1 1) 4))
(ifact-helper 1 2 4)
(if (> 2 4) 1 (ifact-helper (* 1 2) (+ 2 1) 4))
(ifact-helper 2 3 4)
(if (> 3 4) 2 (ifact-helper (* 2 3) (+ 3 1) 4))
(ifact-helper 6 4 4)
(if (> 4 4) 6 (ifact-helper (* 6 4) (+ 4 1) 4))
(ifact-helper 24 5 4)
(if (> 5 4) 24 (ifact-helper (* 24 5) (+ 5 1) 4))
24
```

Iterative = no pending operations when procedure calls itself

- Recursive factorial:

```
(define fact (lambda (n)
    (if (= n 1) 1
        (* n (fact (- n 1)) )
    )))
```

pending operation

- `(fact 4)`

```
(* 4 (fact 3))
(* 4 (* 3 (fact 2)))
(* 4 (* 3 (* 2 (fact 1))))
```

- Pending ops make the expression grow continuosly

Iterative = no pending operations

- Iterative factorial:

```
(define ifact-helper (lambda (product count n)
     (if (> count n) product
        (ifact-helper (* product count)
                      (+ count 1) n))))
```

- `(ifact-helper 1 1 4)`
  `(ifact-helper 1 2 4)`
  `(ifact-helper 2 3 4)`
  `(ifact-helper 6 4 4)`
  `(ifact-helper 24 5 4)`

no pending operations

- Fixed size because no pending operations

Lecture notes

λ expressions create procedures
- formal parameters
- body
- procedures allow creating abstractions

problems get solved -> func.

Substitution model -> model of an interpeter.

```
(define fact (lambda (x) ---- etc. ))
(define (fact x) ( etc. ——— ))
```

OR

- Zeynep Aydin

Lecture notes

- Samet Biber

Lecture notes

Factorial Example

- Tail Recursion :

```
(define fact (lambda (x)
    (if (= n 1) 1 → base case
        (* n (fact (- n 1))))))
```

pending operation is this multiplication

- Iterative

```
(define (fact-i x)
    (fact-i-helper 1 1 x))

(define fact-i-helper
    (lambda (product counter n)
        (if (> counter n) product
            (fact-i-helper (* product counter) (+ counter 1) n))))
```

$\underbrace{\texttt{(* product counter)}}_{\text{next product}}$ $\underbrace{\texttt{(+ counter 1)}}_{\text{next counter}}$

→ no pending operation

- Mete Erdogan

Lecture notes

→ lambda : procedure

parameters body

(lambda (x) (* x x))

to process something multiply it by itself

○ Ceren Tarim

Lecture notes

Scheme Basics

- Rules for evaluation

1. If self evaluating, return value
2. If a name, return value associated with name in environment.
3. If a special form, do something special.
4. If a combination, then
   a. Evaluate all of the subexpressions of combination (in any order)
   b. apply the operator to the values of the operands (arguments) and return result

Digdem Yildiz

Lecture notes

General form of recursive algorithms

```
(define fact
  (lambda (n)
    (if (= n 1)   ; test for base case
        1         ; base case
        (* n (fact (- n 1)))  ; recursive
                                case
)))
```

- Design recursive → decompose the problem
  - → identify non-decomposable (smallest) problems

○ Digdem Yildiz

Lecture 04
Structures and Patterns in Functional Programming

T. METIN SEZGIN

Lecture Nuggets

- Order of growth matters
- Support for compound data allows data abstraction
  - Pairs
  - Lists
  - Others
- Two main patterns when dealing with lists
  - Consing up – to build
  - Cdring down – to process

Nugget

Order of growth matters

Orders of growth of processes

- Suppose n is a parameter that measures the size of a problem
- Let R(n) be the amount of resources needed to compute a procedure of size n.
- We say R(n) has order of growth Θ(f(n)) if there are constants k_1 and k_2 such that $k_1 f(n) <= R(n) <= k_2 f(n)$ for large n
- Two common resources are space, measured by the number of deferred operations, and time, measured by the number of primitive steps.

Examples of orders of growth

- FACT
  - Space Θ`(n)` – linear
  - Time Θ`(n)` – linear

- IFACT
  - Space Θ`(1)` – constant
  - Time Θ`(n)` – linear

Nugget

Support for compound data allows data abstraction

Language Elements

- Primitives
  - prim. data: numbers, strings, booleans
  - primitive procedures
- Means of Combination
  - procedure application
  - compound data (today)
- Means of Abstraction
  - naming
  - compound procedures
    - block structure
    - higher order procedures (next time)
  - conventional interfaces – lists (today)
  - data abstraction

Compound data

- Need a way of gluing data elements together into a unit that can be treated as a simple data element
- Need ways of getting the pieces back out

- Need a contract between the “glue” and the “unglue”

- Ideally want the result of this “gluing” to have the property of **closure:**
    - “the result obtained by creating a compound data structure can itself be treated as a primitive object and thus be input to the creation of another compound object”

Pairs (cons cells)

- **`(cons <x-exp> <y-exp>) ==> <P>`**

  - Where **`<x-exp>`** evaluates to a value **`<x-val>`**, and **`<y-exp>`** evaluates to a value **`<y-val>`**

  - **Returns a** pair **`<P>`** whose car-part is **`<x-val>`** and whose cdr-part is **`<y-val>`**

- **`(car <P>) ==> <x-val>`**

  - Returns the car-part of the pair **`<P>`**

- **`(cdr <P>) ==> <y-val>`**

  - Returns the cdr-part of the pair **`<P>`**

Compound Data

- Treat a PAIR as a single unit:
  - Can pass a pair as argument
  - Can return a pair as a value

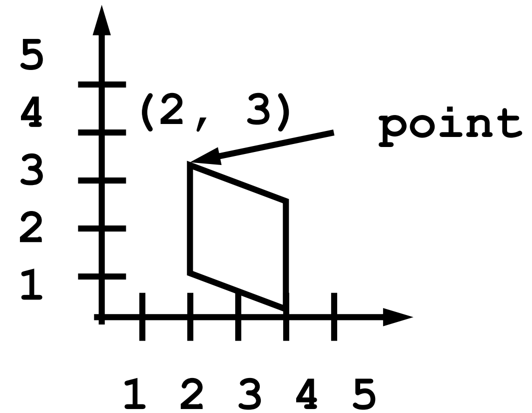
```
(define (make-point x y)
  (cons x y))

(define (point-x point)
  (car point))

(define (point-y point)
  (cdr point))

(define (make-seg pt1 pt2)
  (cons pt1 pt2))

(define (start-point seg)
   (car seg))
```



Pair Abstraction

- Constructor

```
; cons: A,B -> A X B
; cons: A,B -> Pair<A,B>
  (cons <x> <y>) ==> <P>
```

- Accessors

```
; car: Pair<A,B> -> A
  (car <P>) ==> <x>
; cdr: Pair<A,B> -> B
  (cdr <P>) ==> <y>
```

- Predicate

```
; pair? anytype -> boolean
  (pair? <z>)
   ==> #t if <z> evaluates to a pair, else #f
```

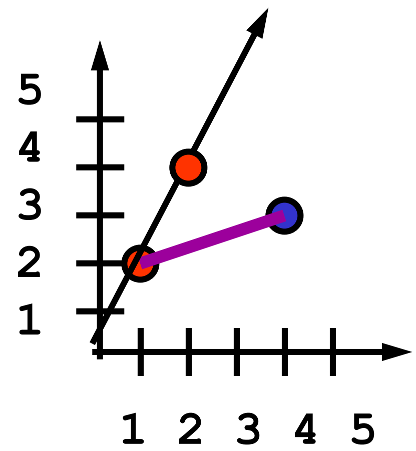
Pair abstraction

- Note how there exists a contract between the constructor and the selectors:
  - *(car (cons <a> <b>))* ➔ <a>
  - *(cdr (cons <a> <b>))* ➔ <b>
- Note how pairs have the property of closure – we can use the result of a pair as an element of a new pair:
  - *(cons (cons 1 2) 3)*

Using pair abstractions to build procedures

- Here are some data abstractions

```
(define p1 (make-point 1 2))
(define p2 (make-point 4 3))
(define s1 (make-seg p1 p2))
```




```
(define stretch-point
  (lambda (pt scale)
    (make-point (* scale (point-x pt))
                (* scale (point-y pt)))))
(stretch-point p1 2) ➔ (2 . 4)
p1 ➔ (1 . 2)
```

Grouping together larger collections

- Suppose we want to group together a set of points. Here is one way

```
(cons (cons (cons (cons p1 p2)
                  (cons p3 p4))
            (cons (cons p5 p6)
                  (cons p7 p8)))
      p9)
```

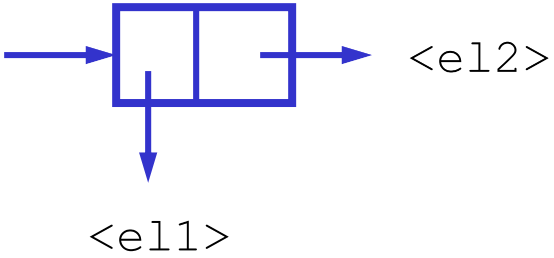
- **UGH!!** How do we get out the parts to manipulate them?

Conventional interfaces -- lists

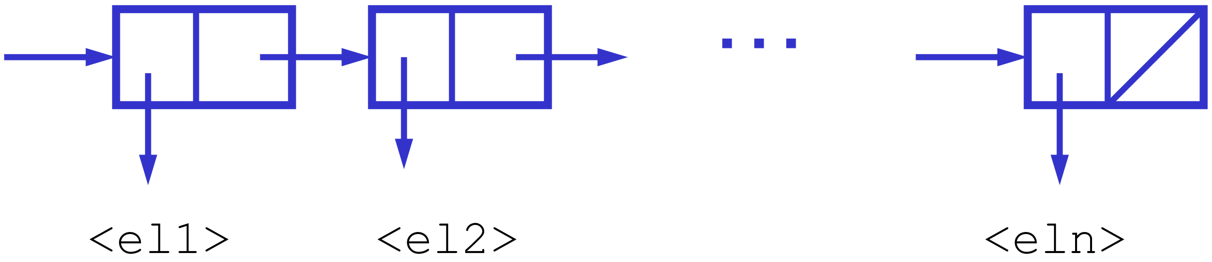
- A list is a data object that can hold an arbitrary number of ordered items.
- More formally, a list is a sequence of pairs with the following properties:
  - Car-part of a pair in sequence – holds an item
  - Cdr-part of a pair in sequence – holds a pointer to rest of list
  - Empty-list **`nil`** – signals no more pairs, or end of list
- Note that lists are closed under operations of **`cons`** and **`cdr`**.

Conventional Interfaces - Lists

`(cons <el1> <el2>)`



`(list <el1> <el2> ... <eln>)`



Predicate

`(null? <z>)`

`==> #t` if **`<z>`** evaluates to empty list

… to be really careful

- For today we are going to create different constructors and selectors for a list
  - **`(define first car)`**
  - **`(define rest cdr)`**
  - **`(define adjoin cons)`**
- Note how these abstractions inherit closure from the underlying abstractions!

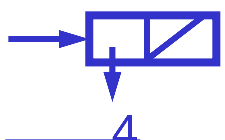
Nugget

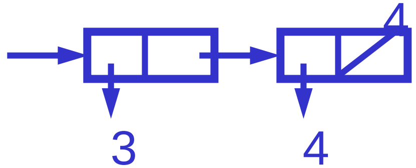
Two patterns for dealing with lists

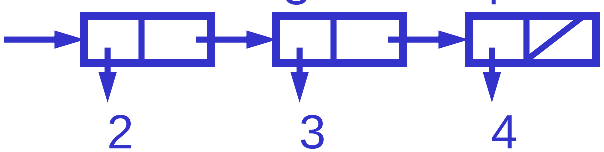
Common Pattern #1: cons'ing up a list

```
(define (enumerate-interval from to)
  (if (> from to)
      nil
      (adjoin from
              (enumerate-interval
                (+ 1 from)
                to))))
```

```
(e-i 2 4)
(if (> 2 4) nil (adjoin 2 (e-i (+ 1 2) 4)))
(if #f nil (adjoin 2 (e-i 3 4)))
(adjoin 2 (e-i 3 4))
(adjoin 2 (adjoin 3 (e-i 4 4)))
(adjoin 2 (adjoin 3 (adjoin 4 (e-i 5 4))))
(adjoin 2 (adjoin 3 (adjoin 4 nil)))
```

(adjoin 2 (adjoin 3 ))

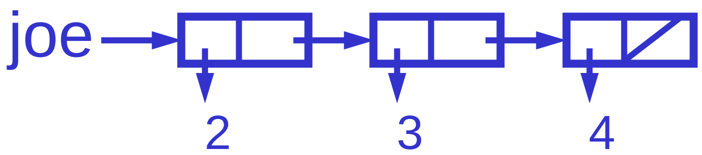
(adjoin 2 )



==> (2 3 4)

Common Pattern #2: cdr'ing down a list

```
(define (list-ref lst n)
  (if (= n 0)
      (first lst)
      (list-ref (rest lst)
                (- n 1))))
```




```
(list-ref joe 1)
```

```
(define (length lst)
  (if (null? lst)
      0
      (+ 1 (length (rest lst)))))
```