# COMP 341 Intro to AI
# Decision Networks


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# Recap

- Uncertainty
  - The real world is uncertain to an agent!
  - Use probabilistic models for representation – Joint Distribution
- Bayesian Networks
  - An intuitive way of representing uncertainty with local conditional distributions
- Inference in BNs: $P(X_q|x_{e_1}, \ldots x_{e_k})$

  *Stuff you care about* *Stuff you already know*
- Exact Inference: Enumeration
- Approximate Inference: Sampling

# Making Decisions

- What do we do with the outcome of an inference query?
- Would you take an umbrella when:
    - No info
    - It is cloudy
    - Forecast says rain
    - It is cloudy and forecast says rain
- Model it as a BN
- Ask the query
- Do inference
- Is the calculated probability enough?

# Maximum Expected Utility

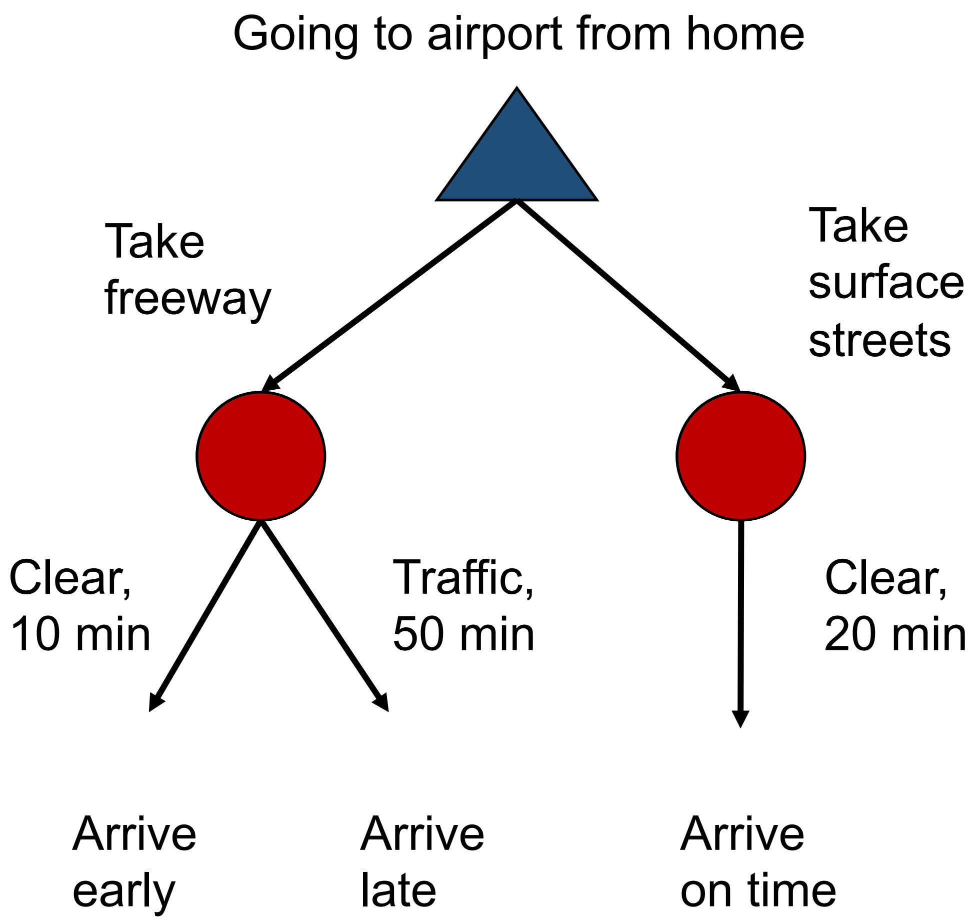
- A rational agent chooses the actions to maximize its expected utility, given its knowledge

- Questions:
  - Where do utilities come from?
  - What do utilities represent?
  - Why expected utility?

# Utilities and Unknown Outcomes

- One way has a chance to be better or worse
- How to decide?
- Which would you pick if you are catching a flight?
- Which if you are picking up a friend?

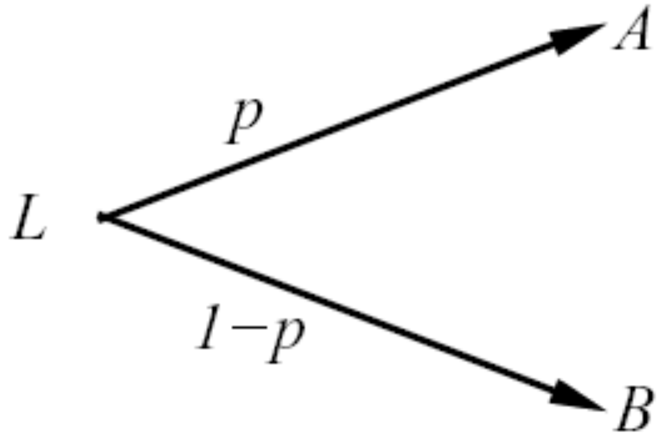
Assigning relative value to outcomes = Utilities



# Preferences

- An agent chooses among:
  - Prizes: $A$, $B$, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ (1-p), B]$$




- Notation:

$A \succ B$ $\quad$ $A$ preferred over $B$

$A \sim B$ $\quad$ indifference between $A$ and $B$

$A \succeq B$ $\quad$ $B$ not preferred over $A$

# Rational Preferences

- We want some constraints on preferences before we call them rational: Axioms of Rationality

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p \; [p, A; \; 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A; \; 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; \; 1-p, B] \succeq [q, A; \; 1-q, B])$$

Do you think human decision making satisfy these?

# MEU Principle

- Theorem: Rational preferences imply behavior describable as maximization of expected utility
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succeq B$$

$$U([p_1, S_1; \;\ldots\; ;\; p_n, S_n]) = \textstyle\sum_i p_i U(S_i)$$

- Maximum expected likelihood (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe

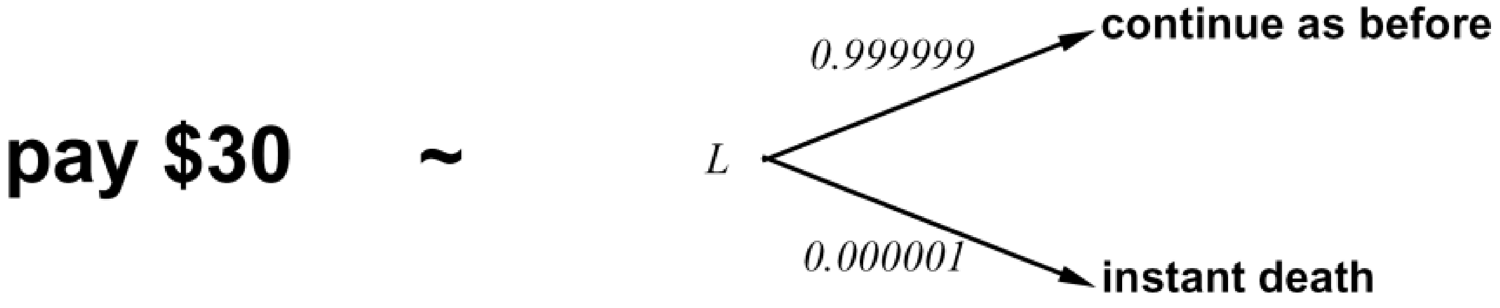
# Utility Scales

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

# Human Utilities

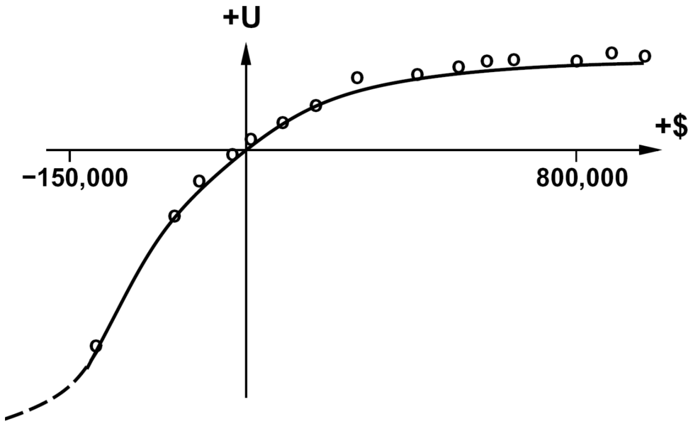
- Utilities map states to real numbers. Which numbers?
- E.g., Insurance: How much would you pay to avoid risk i.e., avoid the lottery all together?



# Money

- Money <u>does not</u> behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery $L = [p, \$X;\ (1-p), \$Y]$
  - The expected monetary value: $EMV(L) = pX + (1-p)Y$
  - $U(L) = pU(\$X) + (1-p)U(\$Y)$
  - Typically, $U(L) < U(\,EMV(L)\,)$ (the for humans is less than the monetary value) why?
  - In this sense, people are risk-averse
  - When deep in debt, we are risk-prone ("sunken fish swims sideways")

- Utility curve: for what probability p am I indifferent between:
  - Some sure outcome x
  - A lottery $[p, \$M;\ (1-p), \$0]$, M large



# Example: Insurance

- Consider the lottery $[0.5, \$1000;\;\; 0.5, \$0]$
- What is its expected monetary value?
  - $500
- What is its certainty equivalent?
  - Monetary value acceptable in lieu of lottery
  - $400 for most people

- Difference of $100 is the insurance premium
  - There’s an insurance industry because people will pay to reduce their risk
  - If everyone were risk-neutral, insurance would not be feasible!

# Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8,\$4k;  0.2,\$0]
  - B: [1.0,\$3k;  0.0,\$0]

  - C: [0.2,\$4k;  0.8,\$0]
  - D: [0.25,\$3k;  0.75,\$0]

- Most people prefer B > A, C > D
- But if U(\$0) = 0, then
  - B > A $\Rightarrow$ U(\$3k) > 0.8 U(\$4k)
  - C > D $\Rightarrow$ 0.8 U(\$4k) > U(\$3k)

# Another Example

- Would you bet $100 for a single coin flip?
- Would you bet $10 for each coin flip for 10 flips?

- Some explanations:
  - Humans are irrational
  - Value for humans and monetary value are not linearly related (utility curve)
  - Human utility is risked based and take the uncertainty into account (analogous to the above)

# Back to AI: Making Decisions

- What do we do with the outcome of an inference query?
- Would you take an umbrella when:
  - No info
  - It is cloudy
  - Forecast says rain
  - It is cloudy and forecast says rain
- Model it as a BN
- Ask the query
- Do inference
- Is the calculated probability enough?

# Our Definition of AI

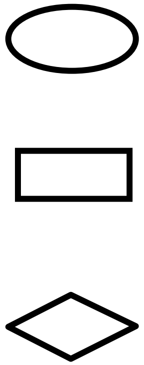
- "Science of making **rational agents**": A **rational** agent selects actions that maximize its (expected) utility
- Utilities in the umbrella example:

| **Real Weather** | **Umbrella Decision** | **Utility** |
|---|---|---|
| rain | take | |
| rain | leave | |
| sunny | take | |
| sunny | leave | |

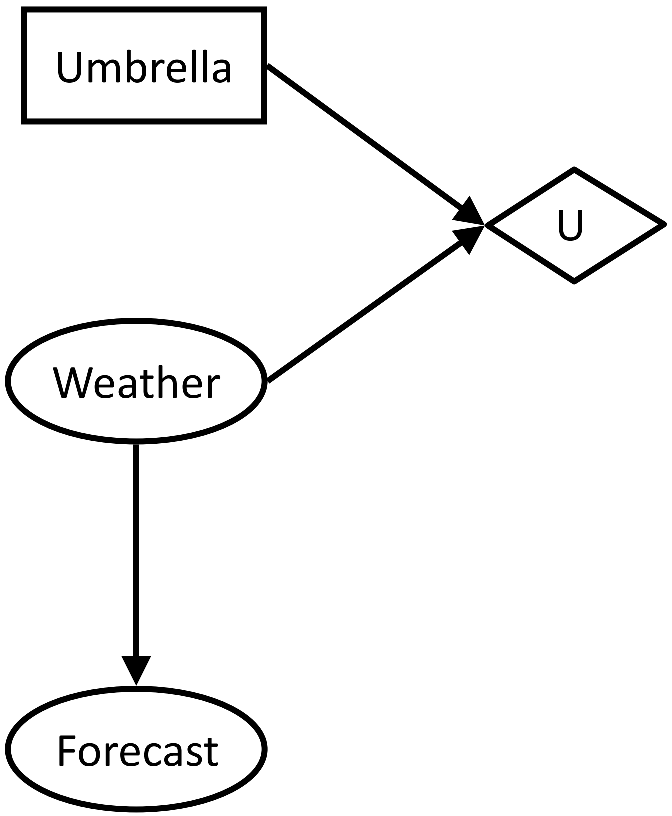
- Expected utility: probability of the outcome X utility of the outcome
- Question: How to put utilities into Bayesian Networks?

# Decision Networks: Representation

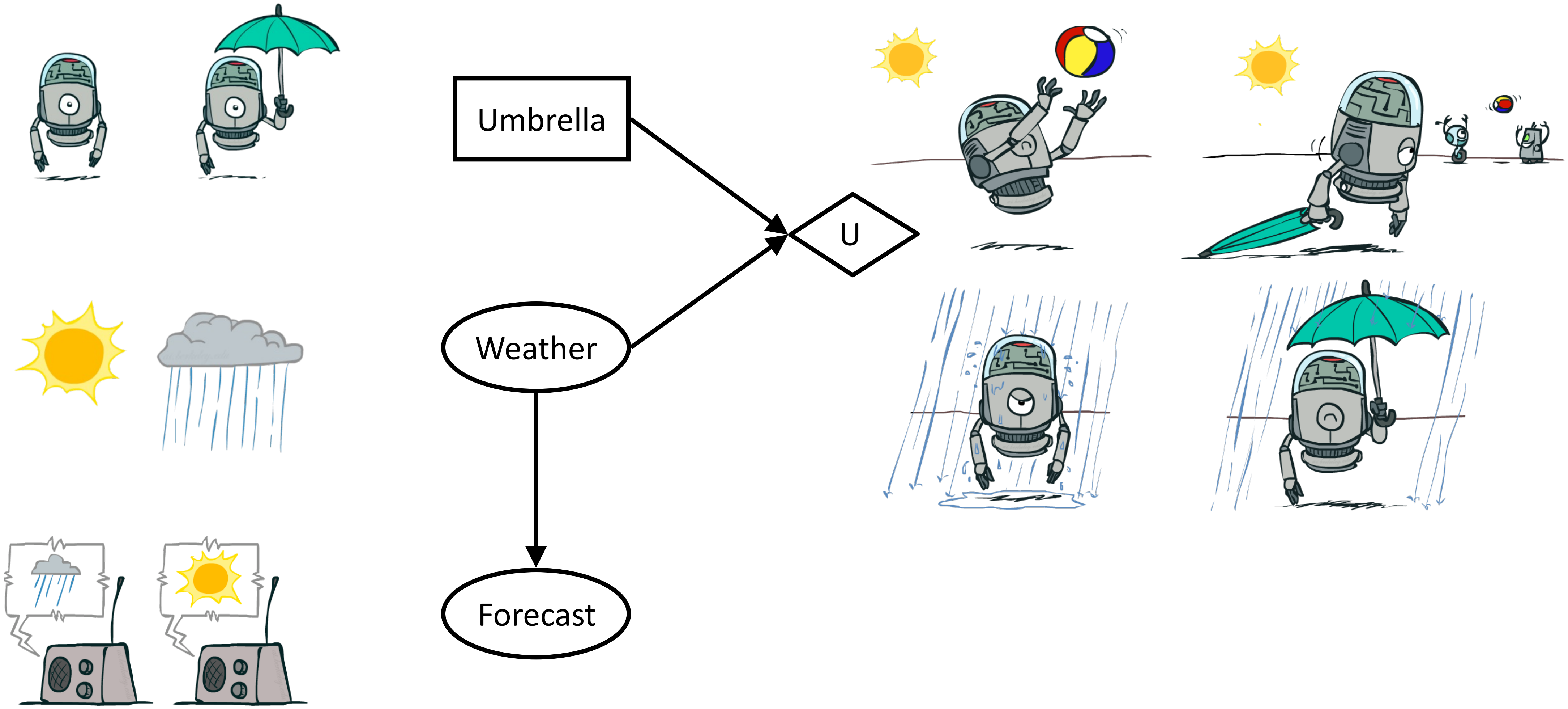
- New node types in the BNs:
  - In addition to regular nodes
  - Actions (rectangles, cannot have parents can be parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)



- Info in the nodes:
  - CPTs
  - Available Action List
  - Utility Table



# Decision Networks



# Decision Networks: Expected Utility

Chose actions to "Maximize Expected Utility"

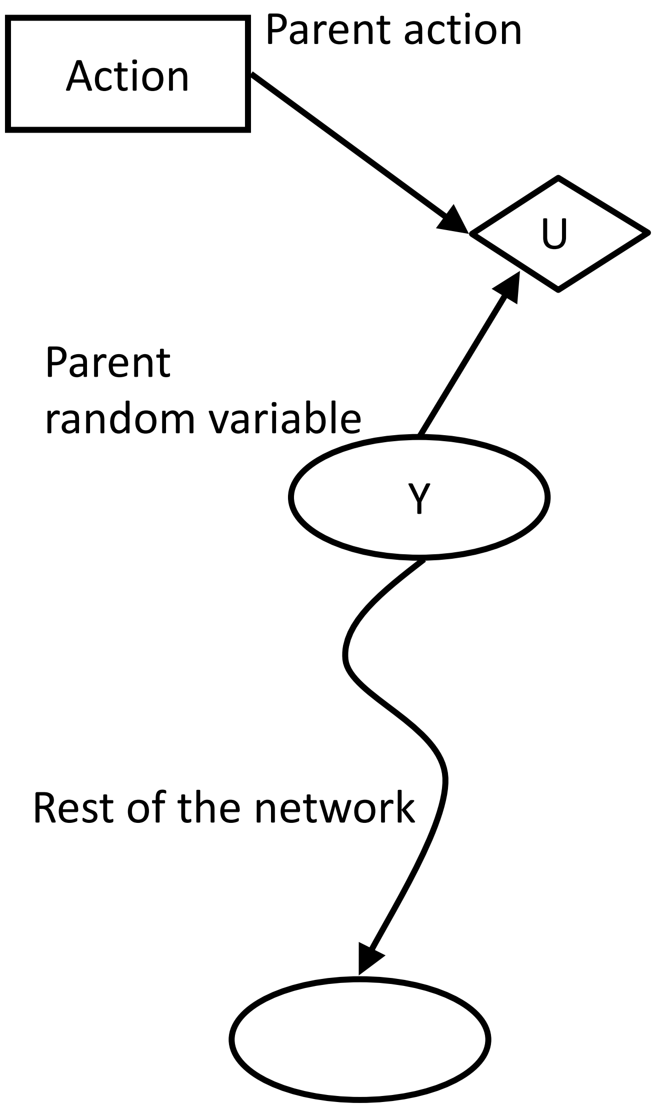
- Expected Utility (EU) of an action

$$EU(action|evidence) = \sum_{y \in Y} P(y_i|evidence)U(y_i, action)$$

- How to calculate $P(y_i|evidence)$?
  - Any inference method we have seen so far!
- Maximum Expected Utility (MEU)

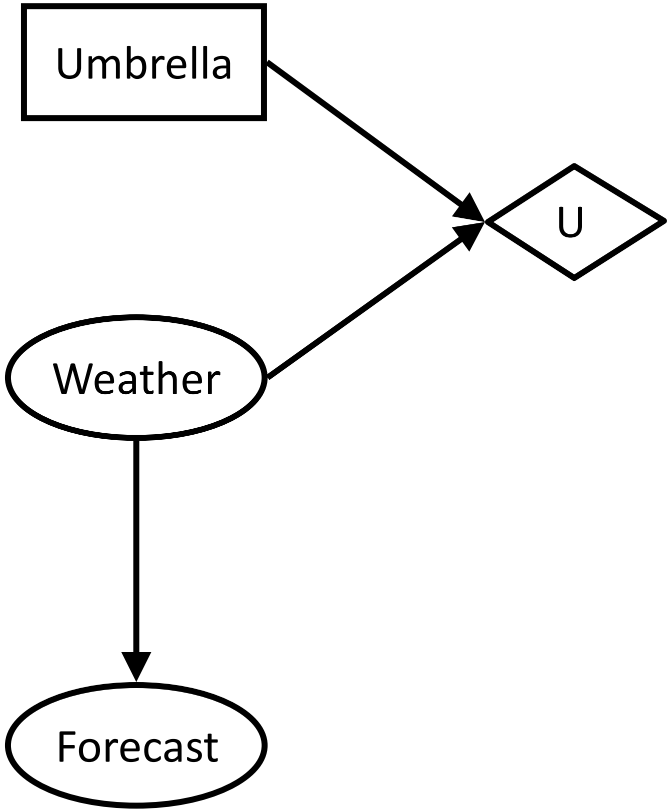
$$MEU(evidence) = \max_a(EU(a|evidence))$$

- Selected Action is $\operatorname{argmax}_a(EU(a|evidence))$



# Decision Networks

- Action selection
  - Instantiate all evidence
  - Calculate posterior for all parents of utility node, given the evidence (inference part)
  - Set action node(s) each possible way
  - Calculate expected utility for each action
  - Choose the maximizing action



# Decision Networks

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_{w} P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

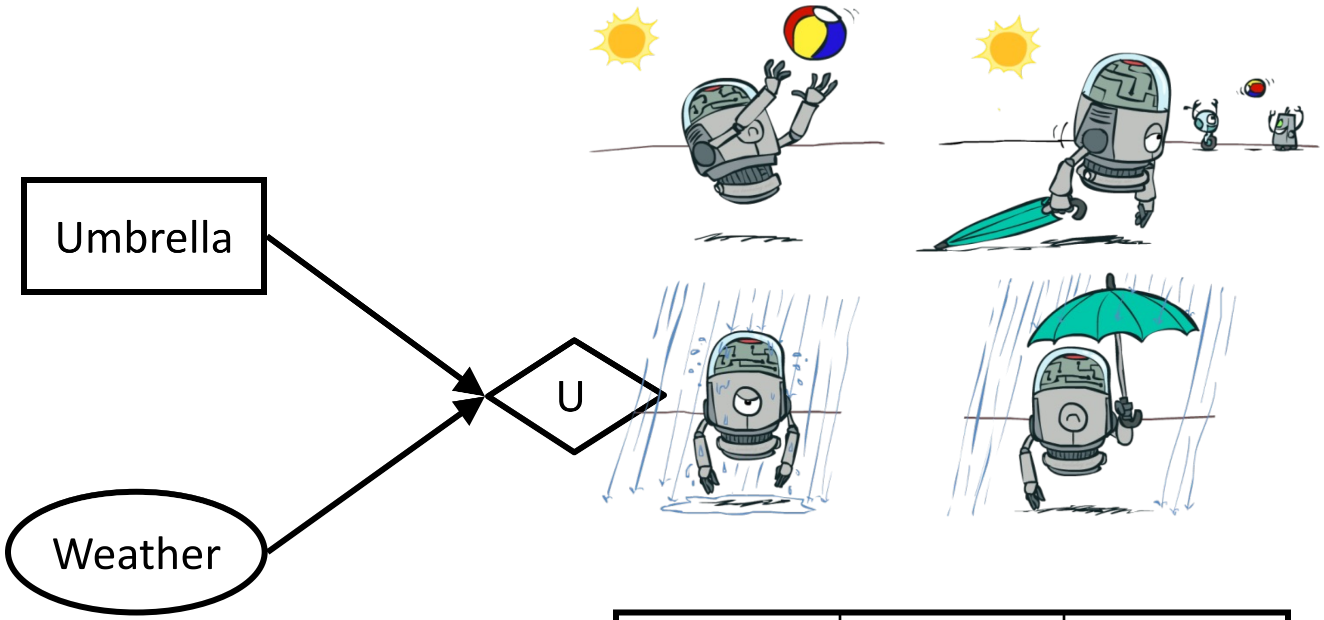
Umbrella = take

$$\mathrm{EU}(\mathrm{take}) = \sum_{w} P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\emptyset) = \max_{a} \mathrm{EU}(a) = 70$$

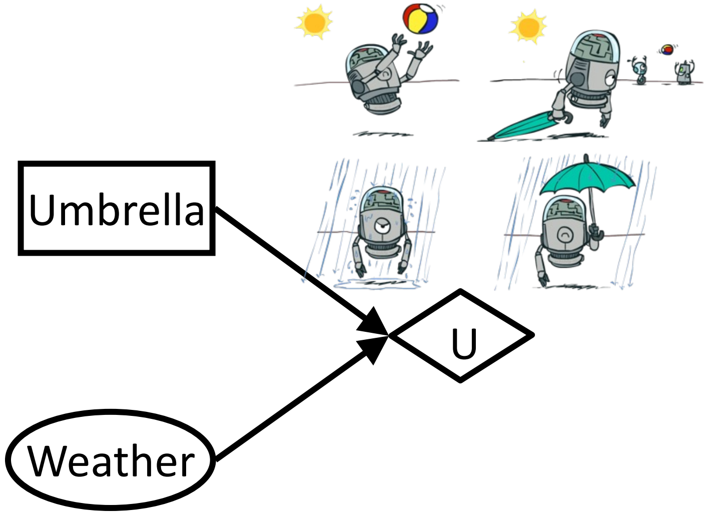


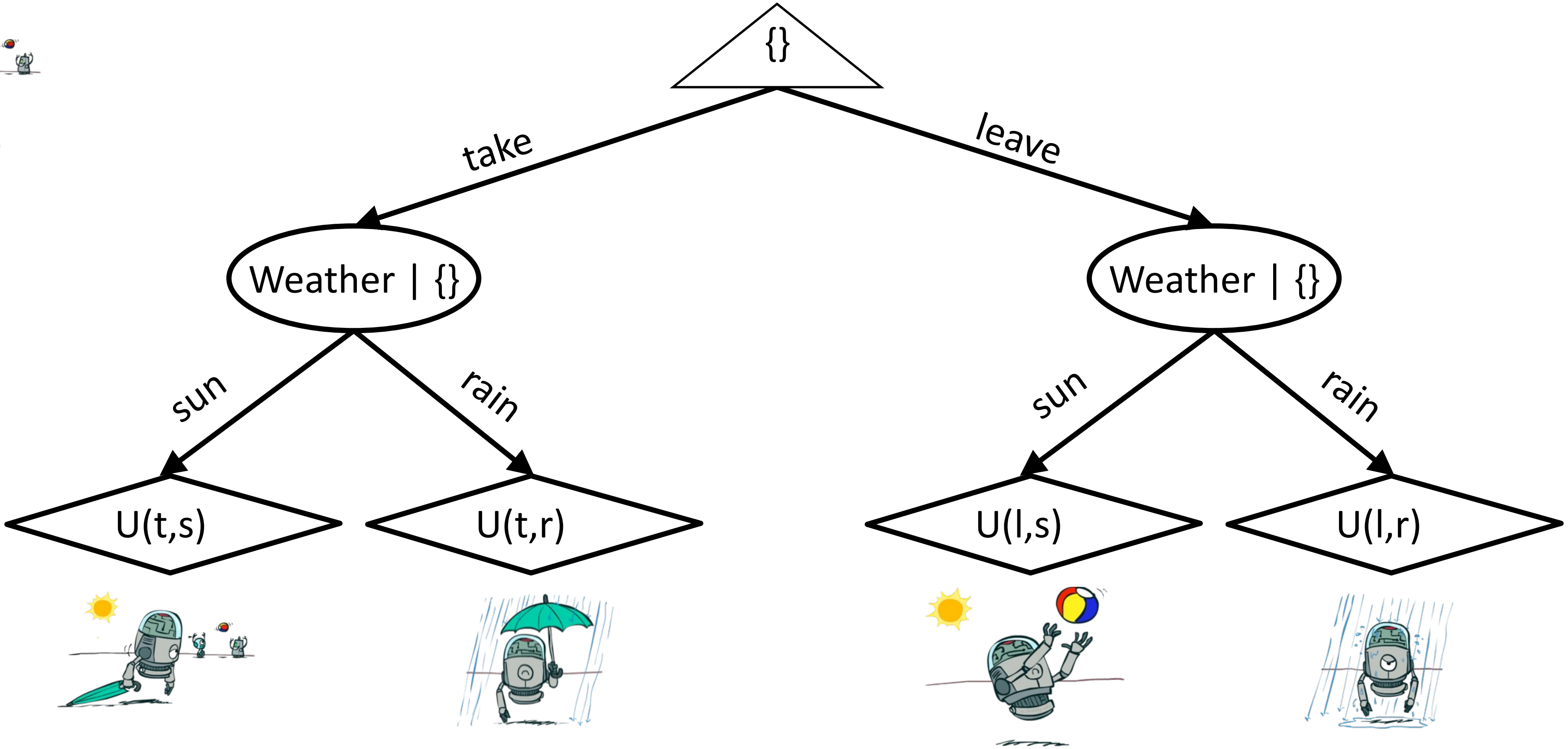

| W | P(W) |
|---|---|
| sun | 0.7 |
| rain | 0.3 |

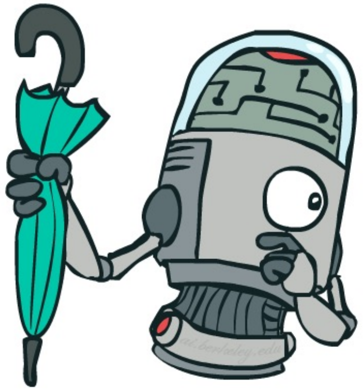
| A | W | U(A,W) |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |

What if W=rain?

# Decisions as Outcome Trees



- Almost exactly like expectimax
- What's changed?

# Example

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_{w} P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

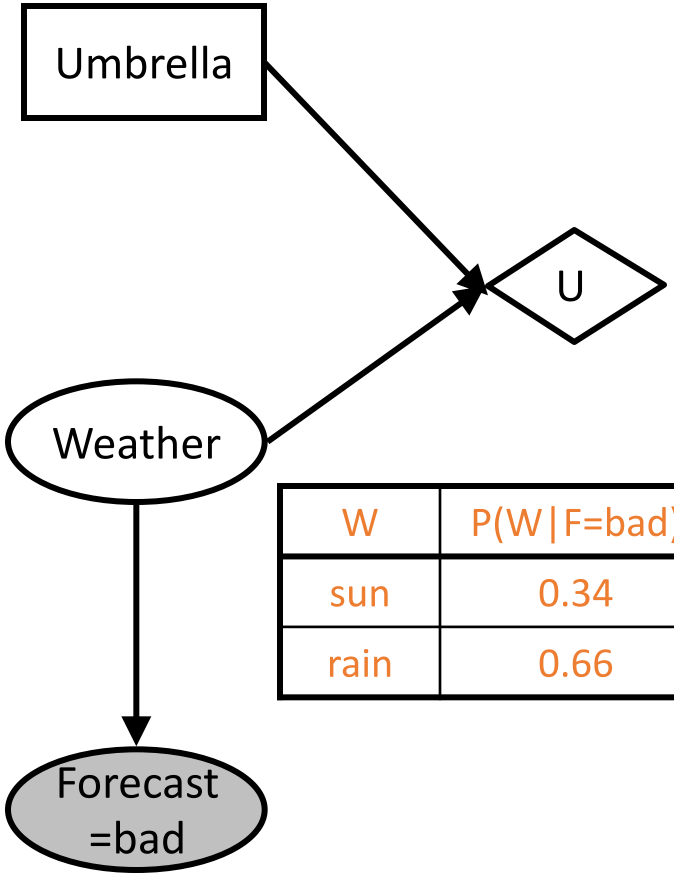
Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_{w} P(w|\mathrm{bad})U(\mathrm{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

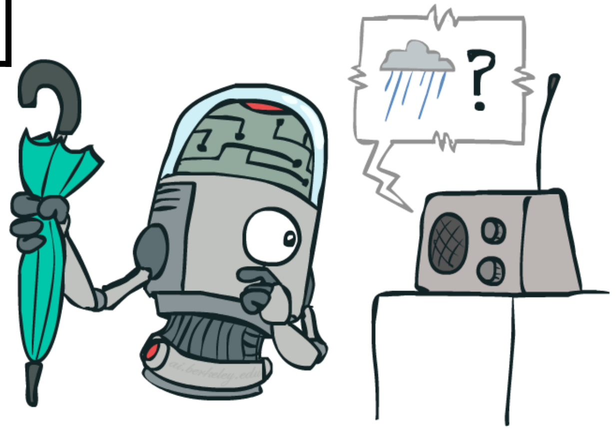
Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_{a} \mathrm{EU}(a|\mathrm{bad}) = 53$$

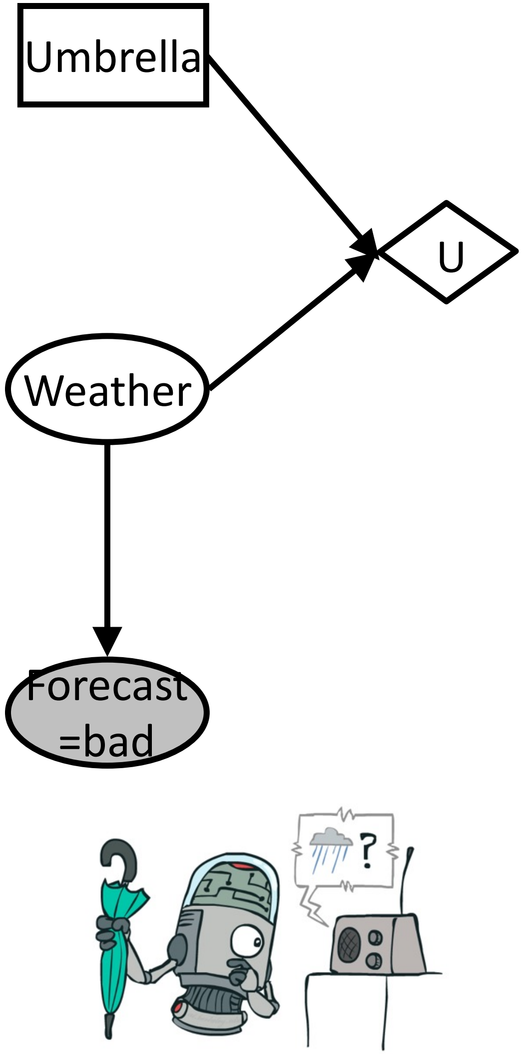


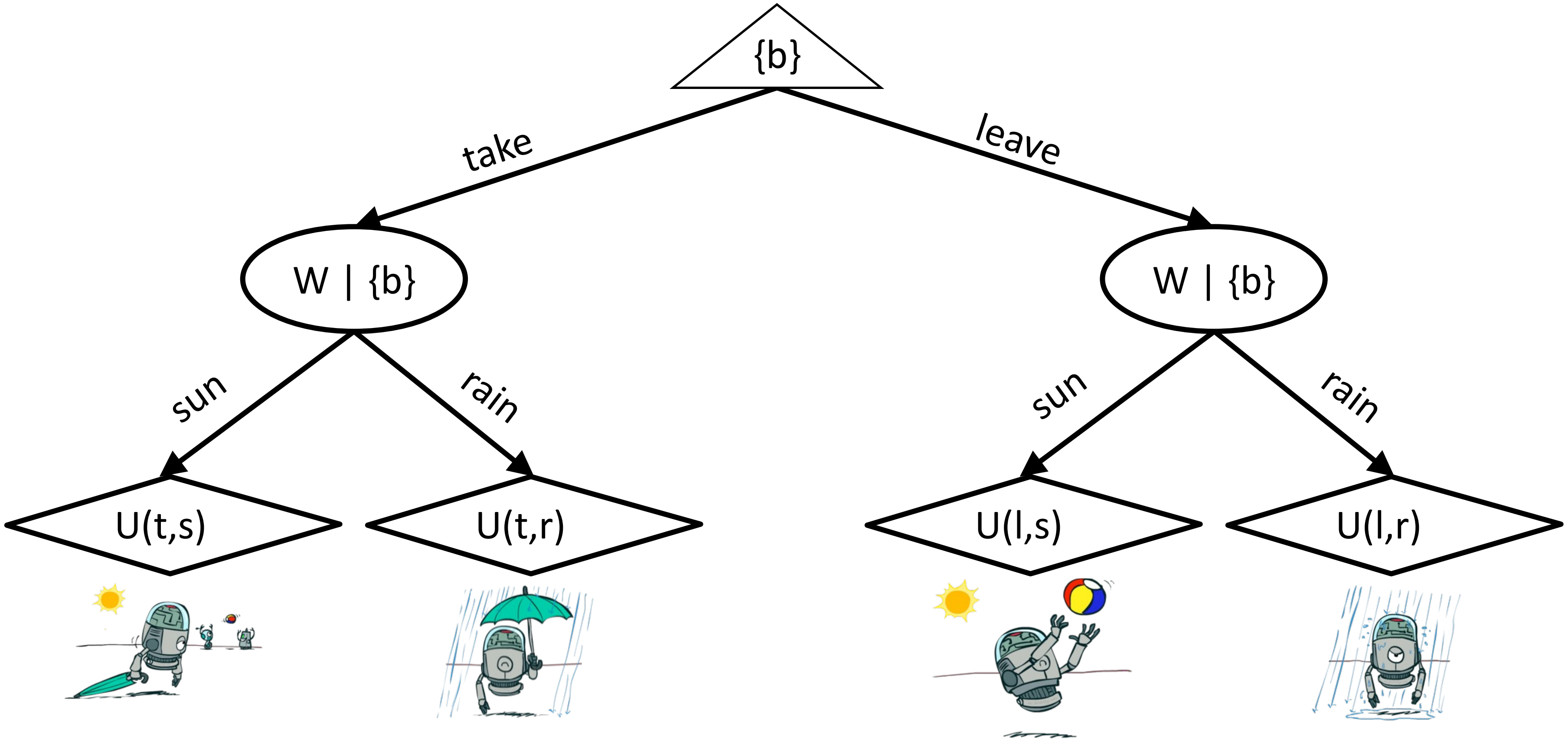

| A | W | U(A,W) |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |

| W | P(W\|F=bad) |
|---|---|
| sun | 0.34 |
| rain | 0.66 |

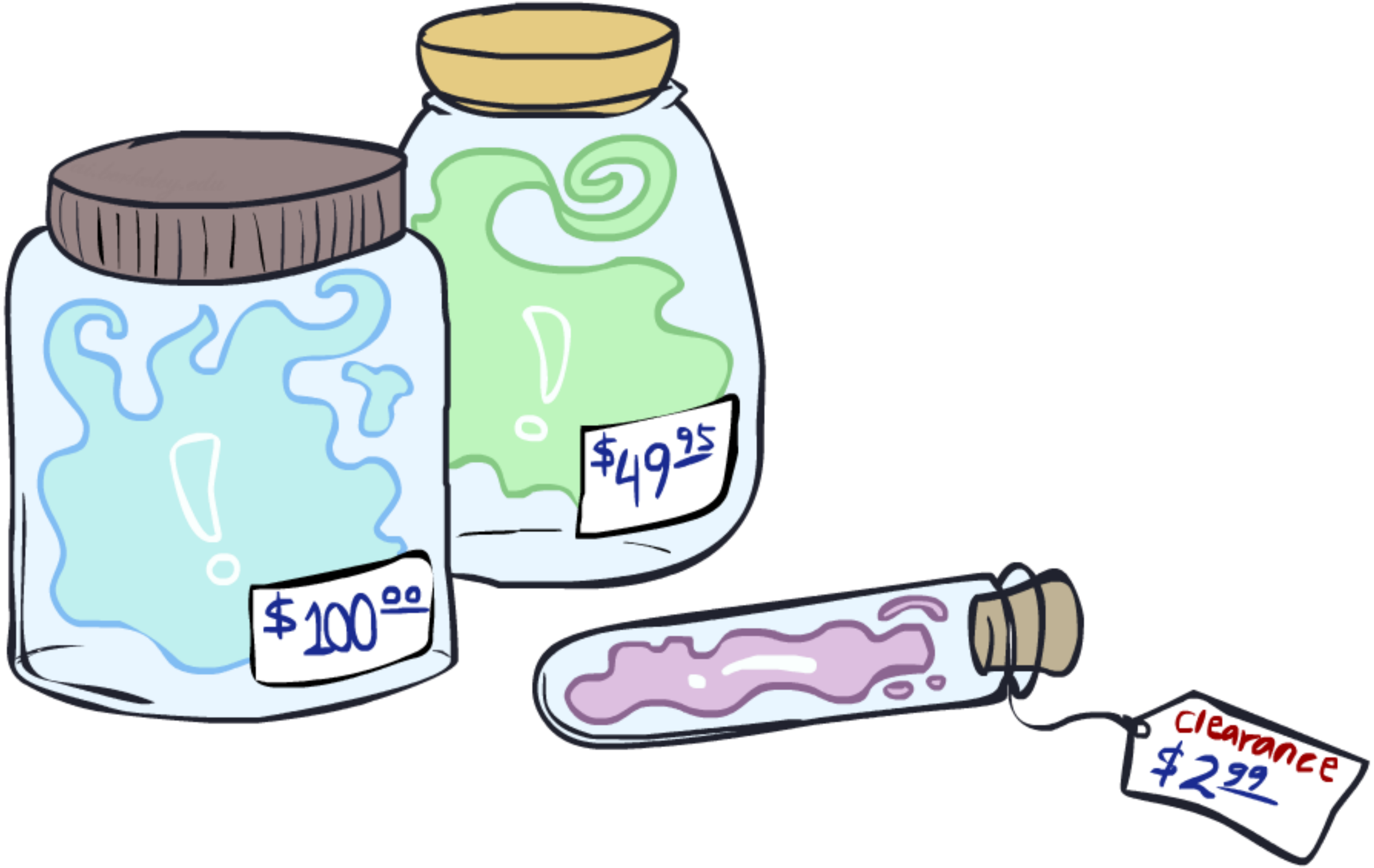


# Decisions as Outcome Trees



# Value of Information



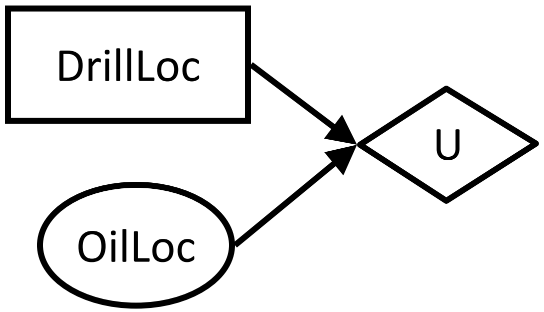
# Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

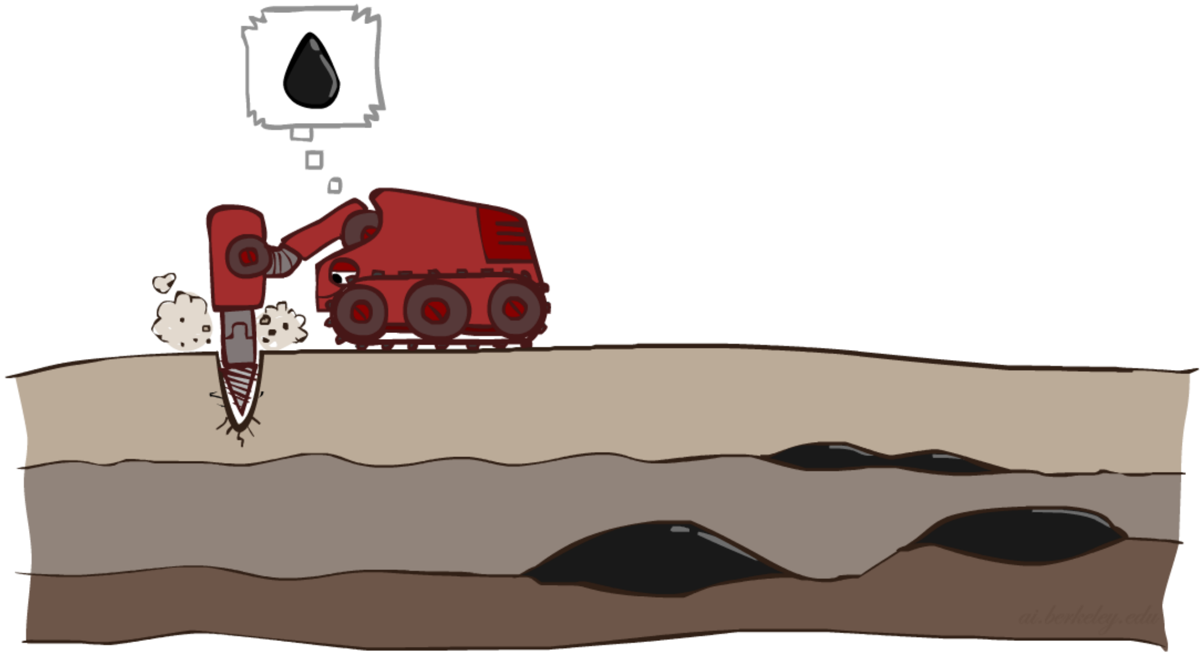
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b", prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

| O | P |
|---|---|
| a | 1/2 |
| b | 1/2 |




| D | O | U |
|---|---|---|
| a | a | k |
| a | b | 0 |
| b | a | 0 |
| b | b | k |



# VPI Example

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

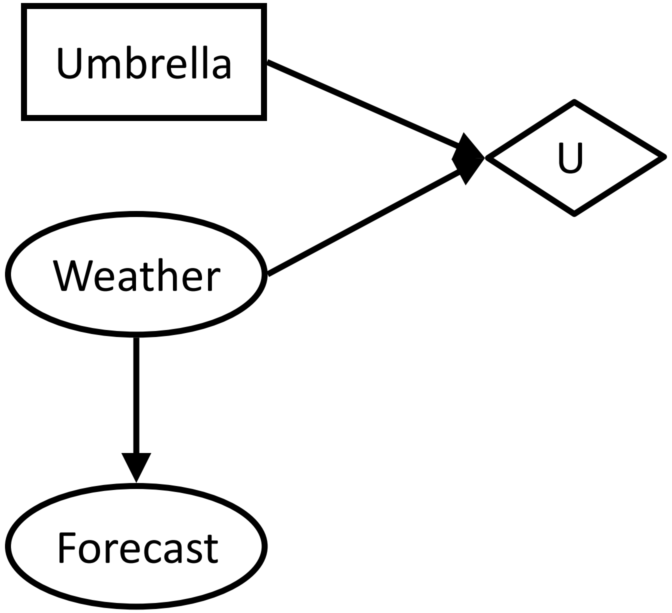
Forecast distribution

| F | P(F) |
|---|---|
| good | 0.59 |
| bad | 0.41 |

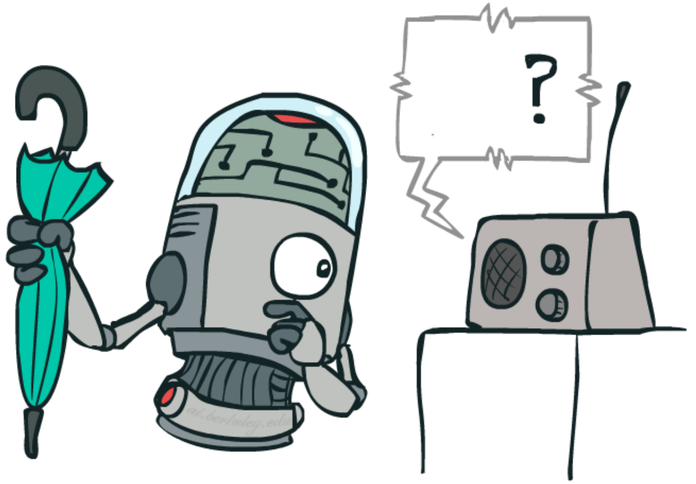
$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\mathrm{VPI}(E'|e) = \left( \sum_{e'} P(e'|e) \mathrm{MEU}(e, e') \right) - \mathrm{MEU}(e)$$




| A | W | U |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |



# Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Assume we see that E' = e'. Value if we act then:

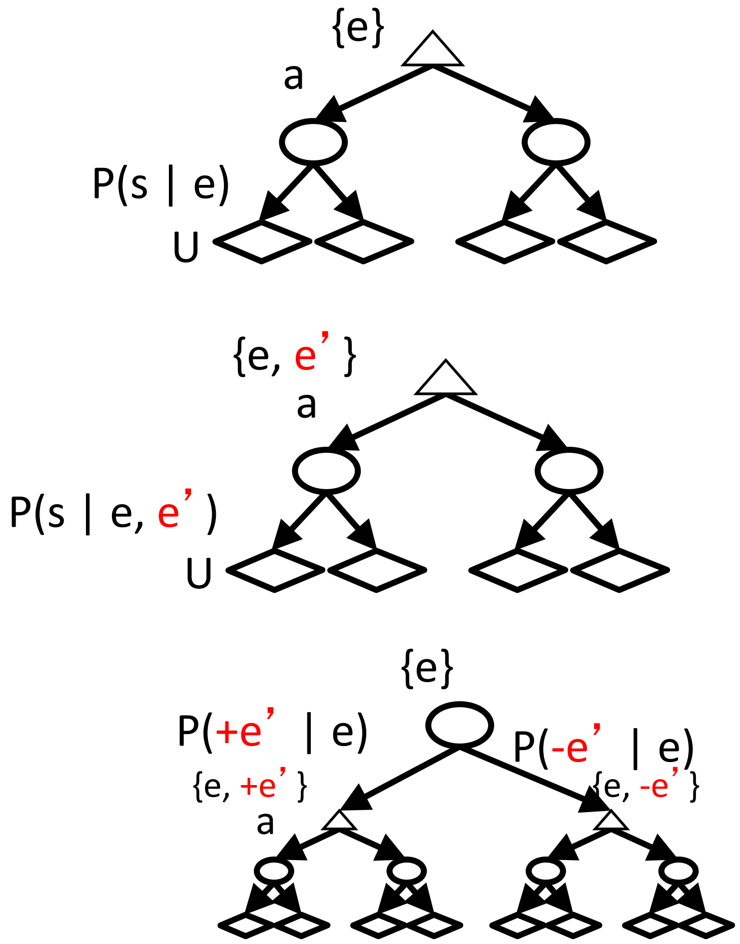
$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

- BUT E' is a random variable whose value is unknown, so we don't know what e' will be
- Expected value if E' is revealed and then we act:

$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: **how much MEU goes up by revealing E' first then acting, over acting now**

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$



# What do we need to “Infer”?

$$\text{MEU}(e) = \max_a \sum_s \boxed{P(s|e)}\, U(s,a)$$

$$\text{MEU}(e,e') = \max_a \sum_s \boxed{P(s|e,e')}\, U(s,a)$$

$$\text{MEU}(e,E') = \sum_{e'} \boxed{P(e'|e)}\, \text{MEU}(e,e')$$

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$

Careful, we need all of the highlighted distributions to calculate VPI!

# VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

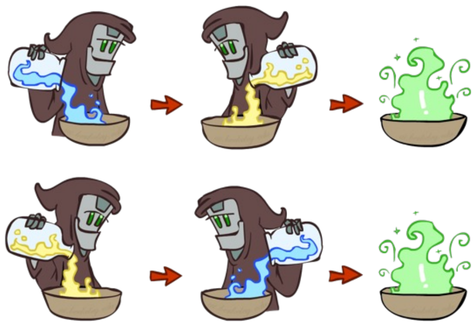

- Nonadditive

  (think of observing $E_j$ twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

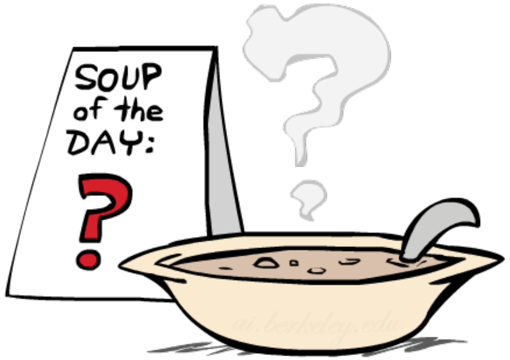
- Order-independent

$$\text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j)$$
$$= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)$$

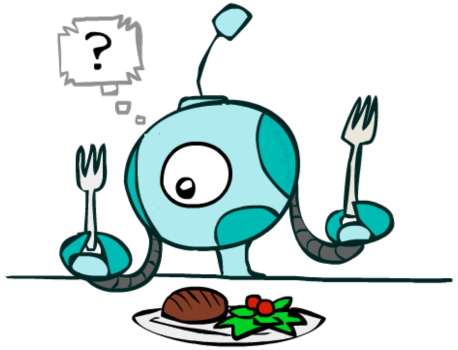


# Quick VPI Questions

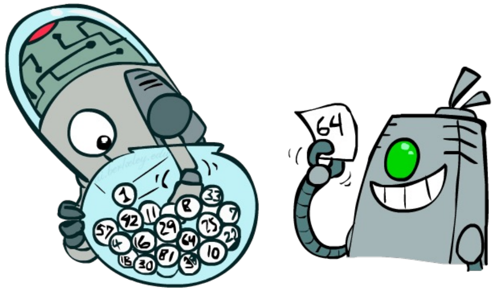
- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?




- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?




- You're playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



# Value of Imperfect Information?

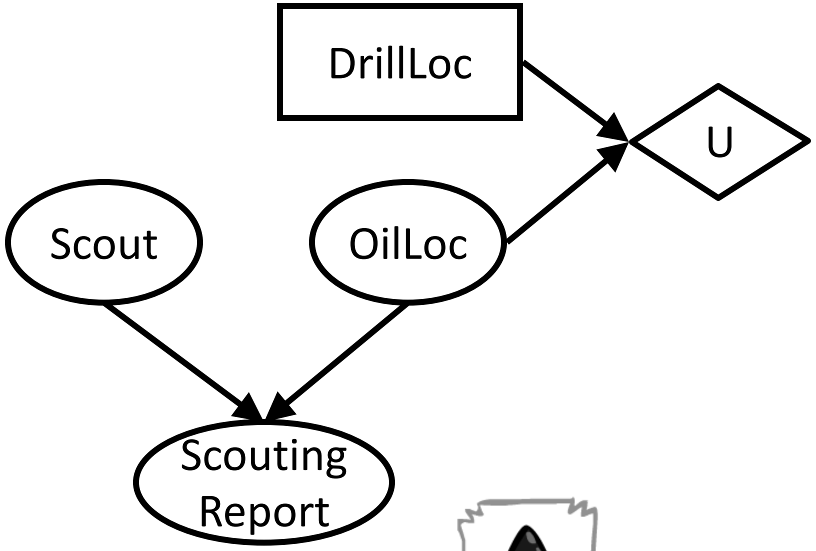
- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

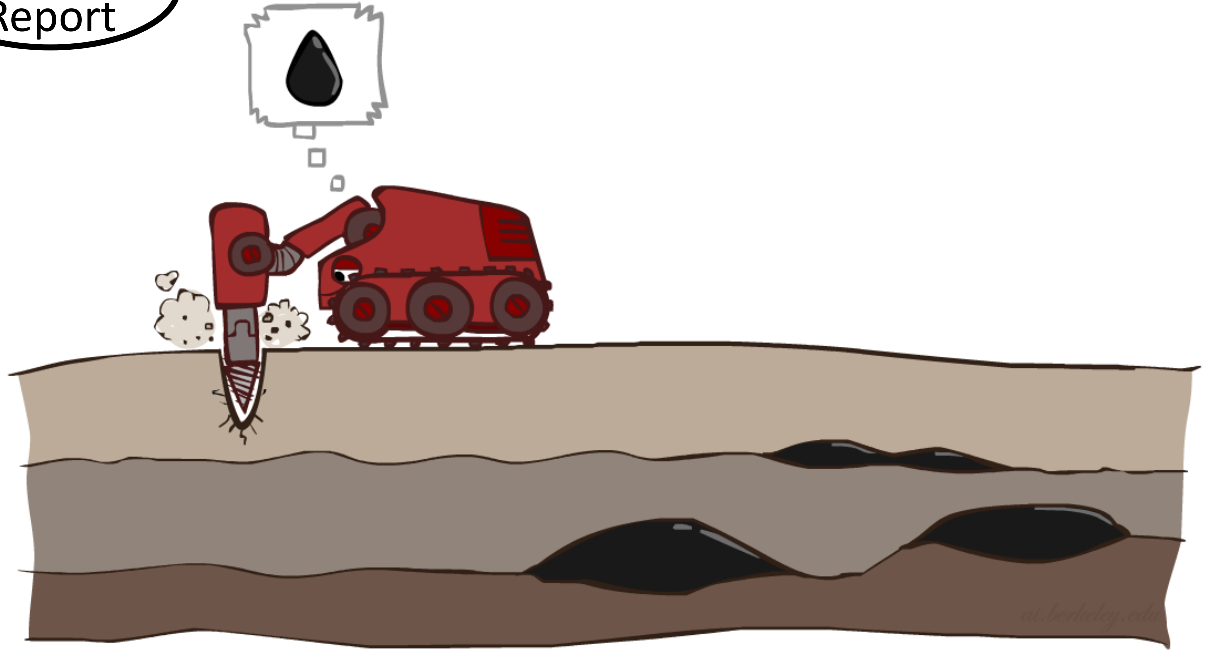
# VPI Question

- VPI(OilLoc) ?
  - k/2
- VPI(ScoutingReport) ?
  - Depends on probabilities
- VPI(Scout) ?
  - 0! (Scout and OilLoc are indep)
- VPI(Scout | ScoutingReport) ?
  - Non-zero as Scout and OilLoc are not cond. indep. given the report
- Generally:

  If $\quad \text{Parents}(U) \perp\!\!\!\perp Z \mid \text{CurrentEvidence}$

  Then $\quad \text{VPI}(Z \mid \text{CurrentEvidence}) = 0$



# Additional Notes

- Action nodes as parents to variable nodes
  - Treat as evidence when going over actions
- Utility nodes having multiple random variable parents
  - Calculate the posterior of parents to calculate the EU
- Utility nodes having multiple action parents
  - Instantiate all possible action combinations and max wrt these combinations
- Multiple utility nodes
  - Separate actions: Treat them individually
  - Overlapping actions: Max over the sum of EUs