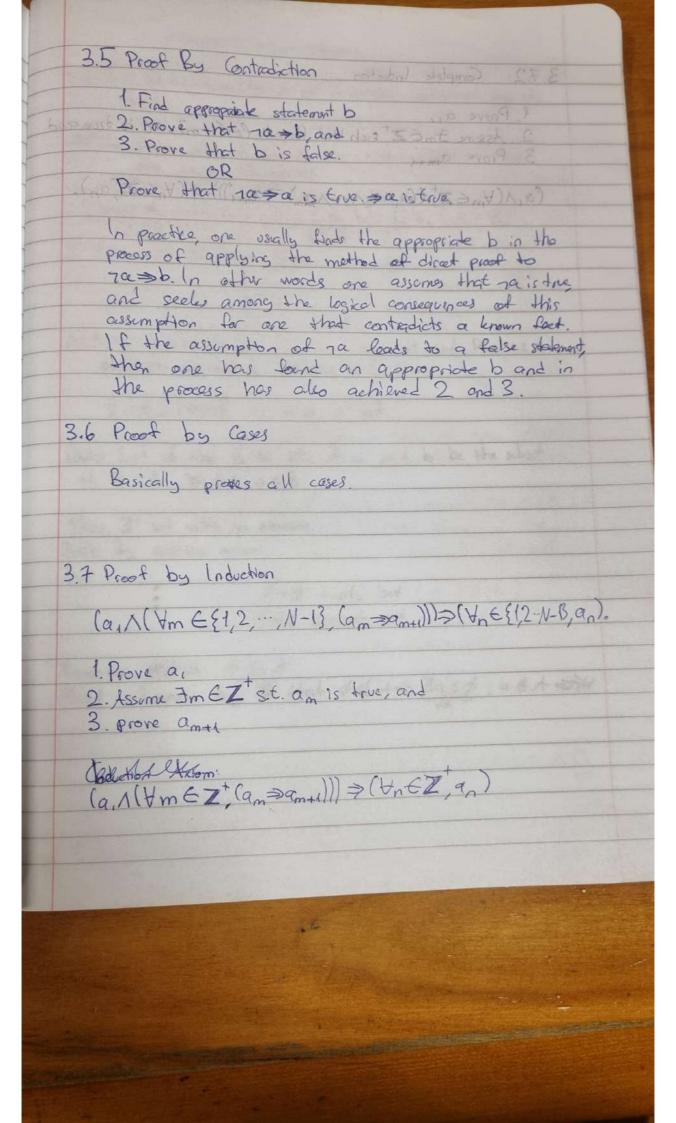
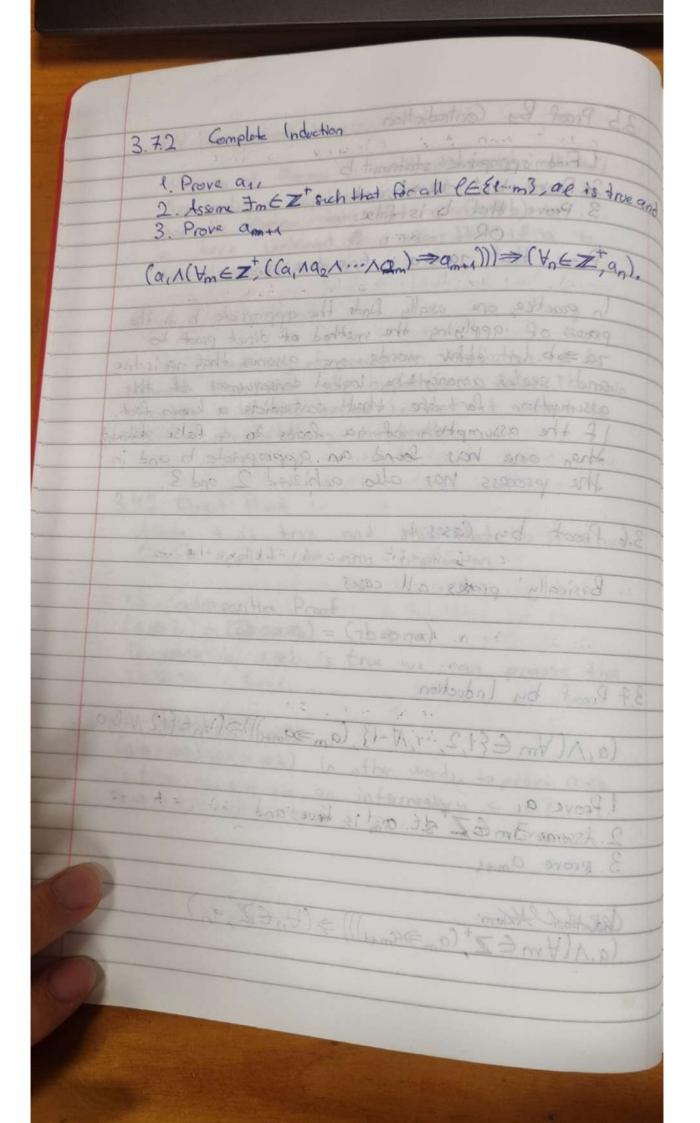
Notes CH3) thoosen Types and Proof Methods 3.1 Existence theorems Either one constructs an example of the object in Shows that if it did not exist a known fout would be falsified -> Proof by contradiction 3.2 Uniqueness Theorems theorems starting the uniqueness of a mathematical object X whose existence has already been established To prove a uniquenes theorem one takes an arbitrary mathematical object Y that satisfies the conditions dealing X and shows that X=Y. 3.3 Classification Theorems Once the existence and and non-uniqueness of a methempotal object are established one may pose the question of its classification (sted) = (design) = (des) to adaptish as by is true we can pre

3.4 Proving Implications (24) Majority of theorem in mathematics are implications thes involve a hisporthesis a and a conclusion by thes involve a response that $c = (a \Rightarrow b)$ is and the theorem. A c means that if a is drue, then so is b. to his to the to 3.4.1 Trivial Proof the easisest way of proving an implication is to show that its hypothesis is false, the main problem with this method is that it cannot be applied to abit ray implication 3.4.2 Pirect Proof Assume a is true and show that by true May be applied to any implication. 3.4.3 Contra positive Proof (a ⇒ b) = (Que (2006) = (1b ⇒ 7a) to establish a > b is true we can proone that 76=>70 is true 3.4.4 Peductive Proof (a⇒c⇒b) ⇒ (a⇒b) in other words, to prove a⇒b is true, one can use an intermediary a s.t (a >c) 1 (c >b) is true 11 1:





Definitions: Definition 3.1.1 Let m and n be integer then m is said to divide n, if there exists an integer k such that n=km, iolo, the following statement holds. IkEZ, n=km. In this case, m is said to be a divisor of n, and n is called a multiple of m. mlh Definition 3.5.1 Let m and n be integers, then an integer that divides both m and n is said to be their common divisor. The largest of which is called the greatest common divisor. We denote it by "god(m,n)". Definition 3.5.2 Two integers in and in (not both 200) are said to be relatively prime if gcd (m,n)=1.i.e. the only possible integer that durides mand n is 1. We can express this as VKEZ+ (klm /kln) > (k=1) Definition 3,8.1 Let n E Z and sik EZ. Then is said to be congruent to k modulon it; and k have the same remainder.

Theorems (Lemmas Propositions Corollaries Included) 178 mil theorem 3.2.1 Every integer has a divisor? Theorem 3.2.1 The only natural number in sorts tying the following condition is O.

Vj EN, n+j=j. Proposition 3.4.1 Let m, n, p be integers. If m divides n and n divides py then m divides prier (mln Anlp) > mlp Proposition 3.4.2 For every integer n, if nº is even, then so is n. 21n² => 21n to the total to me of the total theorem 3.41 Let n E L' and C1, C2: Cn, C1 be statements. then the following is a tautology t:=((0 \$ 6,0 \$ 0 \$ 0 \$ 0)) \$ (0 \$ 6))) & moreout theorem 3.5.1 Letimo and n be integer such that at least one of them is nonzero then in and no have a greatest common divisor that is greater than or equal to 1. (millionally released) morning Zandise That Ine Zand Trefoun Lemma 3.5.1 Every rational can be expressed as the radio of two relatively prime integers, i.e. Yr CQ, ∃p, q EZ (q ≠0) ∧ (r== q) ∧ (g.cd(p,q)=1). theorem 3.5.2 12 is not a rational number. Let Yo 62 to be a statement then for cars labour 6 agration (,0E, p.6, pe, p) = 0

Axiom 3.7.1 (Induction tolom) Let S be a colletter of positive integers (a subset of Z+)- If 165 and Vm EZ+ ((m65) > ((m41)65)) then Sincludes all possible integer, i.e. S= 2+ Theorem 3.7.1 (Poinciple Of Mathematical Indiction) Let an be a statement for all n \(Z\f \) Suppose
that an and "Ym \(Z\f \) (am \(\angle \) are for all n \(Z\f \).

Then an is true for all n \(Z\f \). Proposition 3.7.1 Every possible integer can be written as the sim of distact powers of 2, i.e. Vn EZ+, Fl EZ, Fp, P2, Pe Pe N, st. P1 <P2 <Pe, and n=2 1+2 + 2 = 2 2 Pk Theorem 3.7.2 (Principle of Complete Induction) Let an be a statement for all n EZt. Suppose that a, and (Vm & Z , ((a, 1 a21 1am) > (antil) are true. Then an is true for all a nEZT have a prestest common distr Theorem (Division Algorithm) . A set loops so north Let n EZtandik EZ. Then I ! m EZ and I'r E [0.n-1] s.t. i=mn+r. n is called the remainder of the dussion theorem 3.8.1(9=+) 10+p) IDpgE DDAY Let n EZ+ and i, h EZ, then is is congruent to k modelon if and only if in divides july ton 21 at 128 most Theorem 3.8.2 Let $\forall n \in \mathbb{Z}^+$ and be a statement. Then for a consinteger & greater then 1, the following two standements are logically agriculent. bi= (a, (3) a2 (3) ... ax) (4:= (a, > a, ≥ ··a, ≥a,)