Math 103: Introduction to Abstract Mathematics, Fall 2024

Topics Covered in the Lectures

Lecture Number	Date	Content	Corresponding Reading material
1	Oct. 08	General structure of mathematical theories: Definition of object of study, axioms, theorems (lemmas, propositions, corollaries, fundamental theorems), Fermat's last theorem, conjectures, undecidable statements & Gödel's incompleteness theorem; Euclid's axioms and the discovery of non-Euclidean geometries; General structure of scientific theories: Description of objects of study, postulates (laws of nature), predictions, experimental tests & differences with mathematical theories; 4-color problem and the need for a precise mathematical language	Textbook 1 (A First Course in Abstract Mathematics): Pages 1-6
2	Oct. 10	A crude concept of a set, statements, their truth value, some basic mathematical symbols, predicates, qualifiers, negation of a statement, truth tables, negation of a qualified predicate, compound statements: disjunctions and conjunctions	Textbook 1: Pages 6-17
3	Oct. 15	Implications, basic properties of disjunctions, conjunctions, implications, and logical equivalence, negation of basic compound statements, tautologies and contradictions, propositional calculus	Textbook 1: Pages 17-31
4	Oct. 17	Theorem types and proof methods: Existence, uniqueness, and classification theorems; proving implications: Trivial, direct, contrapositive, and deductive proofs; Proof by cases	Textbook 1: pages 35-41 & 45-46; Textbook 2 (Mathematical Proofs): Pages 87-110
5	Oct. 24	Proof by contradiction, great common divisor (g.c.d.) of two integers, well-ordering principle and the proof of existence of g.c.d., relatively prime integers, Lemma: Rational numbers are ratios of relatively prime integers, Theorem: Square root of 2 is not rational. Proof by induction: Basic idea, induction axiom, principle of mathematical induction	Textbook 1: Pages 41-45 & 47-49; Textbook 2: Pages 319-320
6	Oct. 31	Examples of applications of induction, complete induction, recursive definitions, characterization theorems	Textbook 1: Pages 50-60; Textbook 2: Pages 164-177
7	Nov. 05	Set Theory: General introduction, axioms of extensionality, specification, and existence, equal sets, empty set, subsets, power set axiom, Russel's paradox, pairing axiom and unordered pairs, intersection of sets	Textbook 1: Pages 65-71; Textbook 2: Pages 60-66

8	Nov. 07	Basic properties of intersection of sets, Universal sets, intersection of sets having a universal set, disjoint sets, difference of two sets, complement of a set, Union axiom and the union of sets	Textbook 1: Pages 71-80; Textbook 2: Pages 67-73
	Nov. 08	**Midterm Exam 1**	
9	Nov. 12	Basic properties of the union and intersection of collections of sets, ordered pairs and the Cartesian product of two and finitely many sets; Successor of a set and natural numbers	Textbook 1: Pages 80-88
10	Nov. 14	Inductive sets and Infinity Axiom, basic properties of inductive sets, construction of the set of natural numbers as the intersection of inductive subsets of a given inductive set and the proof of its uniqueness, the definition of addition, multiplication, and inequalities of pairs of natural numbers; Relations as subsets of Cartesian product of two sets	Textbook 1: Pages 88-95 and Supplementary matrix: "Natural Numbers"
11	Nov. 19	Relations, the image and inverse image of subsets under a relation, the domain and range of a relation, equality of two relations, identity relation	Textbook 1: Pages 95-101
12	Nov. 19	Restriction and extensions of a relation, image and inverse image of the intersection and union of a collection of sets under a relation, reflexive, symmetric, antisymmetric, and transitive relations, the inverse of a relation, composition of two relations and its domain	Textbook 1: Pages 101-106
13	Nov. 26	Non-commutativity and associativity of composition of relations, inverse of the composition of two relations, composing a relation with its inverse, reflextive, symmetric, antisymmetric, and transitive relations	Textbook 1: Pages 107-110
14	Nov. 26	Partitions of a set, equivalence relations, equivalence classes and the quotient of a set by an equivalence relation, finding all possible equivalence relations on the set {1,2,3}	Textbook 1: Pages 110-116
15	Dec. 03	Partial ordering relations, total ordering, posets, graphical representations of posets; maximal, minimal, greatest, and least elements of a poset; upper and lower bounds, supremum, and infimum of a subset of a poset	Textbook 1: Pages 116-125
16	Dec. 05	Supremum property, chains of a poset, statement of Zorn's lemma, Well-ordered posets and Well-Ordering Principle, well-ordering of natural numbers, application to division algorithm; Functions: Well-defined relations, notation, image and inverse image of sets under functions, domain and range of a function, equal functions	Textbook 1: Pages 125-141

17	Dec. 10	Everywhere-defined, one-to-one, and onto functions, bijections, examples. Restriction and extensions of functions, inclusion maps, construction of a bijection mapping disjoint union of two sets to the disjoint union of two sets. Image and inverse image of intersection of a collection of sets and the complement of a set under a function	Textbook 1: Pages 142-147
18	Dec. 12	Image and inverse image of intersection of a collection of sets and the complement of a set under a 1-to-1function; composition of functions and its domain, composition of everywhere-defined, one-to-one, and onto functions, composition of bijections, invertible functions	Textbook 1: Pages 147-155
		Midterm Exam 2	
19	Dec. 17	Inverse of invertible functions is invertible, inverse of a bijection is a bijection; Transpositions of $I_n:=\{1,2,\ldots,n\}$; Theorem: Transpositions of I_n are bijections; Inverse of every Transpositions of I_n is itself. Trivial extensions of Transpositions of I_n , Permutations of $I_n:=\{1,2,\ldots,n\}$, Theorem: A function $f: I_n \to I_n$ is a bijection if and only if it is a permutation of I_n, Non-existence of everywhere-defined and 1-to-1 functions $f: I_n \to A$ for proper subsets of I_n; Non-existence of onto functions $f: A \to I_n$ when A is a proper subset of I_n	Textbook 1: Pages 155-161
20	Dec. 19	n=m as the necessary and sufficient condition for the existence of bijections $f: I_n \to I_m$, Equivalent sets, Finite sets, order of a finite set, finiteness of subsets of a finite set	Textbook 1: Pages 161 & 177-182
21	Dec. 24	Finiteness of the intersection of two finite sets, finiteness and order of the union of two finite sets, Cartesian product of two finite sets and its order, characterization of finite sets with different order, power set of finite sets and its order	Textbook 1: Pages 182-187
22	Dec. 26	Sequences, sequences with distinct terms, subsequences; Infinite sets: Definition, examples, characterization theorems, Cartesian product of finite and infinite sets, Countably infinite sets: Definition, examples, characterization theorems for countably infinite sets	Textbook 1: Pages 162-165 & 187-192
23	Jan. 02		
		Midterm Exam 3	
24	Jan. 07		

25	Jan. 07		
26	Jan. 09		

Note: The pages from the textbook listed above may not include some of the material covered in the lectures.