Dynamic Delta Hedging Strategy in Commodities Markets

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Abstract

This project explores the implementation of a dynamic hedging strategy in the commodities market using the Black 76 model. The strategy involves continuously adjusting the hedge position to maintain a delta-neutral portfolio, thereby reducing risk and stabilizing returns. By recalculating delta and adjusting hedge positions at each time step, we aim to mitigate the volatility of commodities prices. The effectiveness of this strategy is evaluated against unhedged positions based on performance metrics such as total returns, annualized returns, mean returns, standard deviation, and the Sharpe ratio. Our findings provide valuable insights into optimizing risk management and return stabilization in commodities trading.

Disclaimer

This is in no way investment advice; it's merely a formalization of results from some independent research I conducted. This work is in no way related to any company or organization. If you decide to trade based on this, just know you're taking advice from someone with a mere math undergrad and limited trading experience.

Introduction

Hedging is an essential risk management strategy employed in financial markets to mitigate potential losses arising from adverse price movements. In commodities markets, where prices are often volatile and influenced by a multitude of factors such as geopolitical events, supply-demand dynamics, and macroeconomic indicators, hedging becomes particularly crucial. This paper focuses on implementing a **dynamic hedging strategy** in the commodities market, specifically targeting metals.

Dynamic hedging involves continuously adjusting the hedge position to maintain a delta-neutral portfolio, thus reducing risk and stabilizing returns [1]. The Black 76 model, an extension of the Black-Scholes model tailored for pricing options on futures, is employed to compute the necessary Greeks, with a particular focus on delta (Δ). By recalculating delta at each time step and adjusting hedge positions accordingly, we aim to achieve more precise risk management and ideally more stable and favorable returns over time.

The period of analysis spans from July 1, 2023, to July 1, 2024. The metals included in this analysis are as follows:

Ticker	Name
GC=F	Gold
SI=F	Silver
HG=F	Copper
PL=F	Platinum
PA=F	Palladium
ALI=F	Aluminum
ZN=F	Zinc

Table 1: Metals Tickers

Ticker	Name
ZC=F	Corn
ZS=F	Soybeans
ZW=F	Wheat
LE=F	Live Cattle
HE=F	Lean Hogs
KC=F	Coffee
SB=F	Sugar

Table 2: Agriculture Tickers

Ticker	Name
CL=F	Crude Oil
NG=F	Natural Gas
RB=F	Gasoline
HO=F	Heating Oil
BZ=F	Brent Crude
QG=F	Propane

Table 3: Energy Tickers

These commodities are chosen due to their significant role in various industrial applications, investment portfolios, and their liquidity in the commodities markets. The study evaluates the dynamic hedging strategy's effectiveness by comparing the performance of hedged positions against unhedged positions based on metrics such as total returns, annualized returns, mean returns, standard deviation, and the Sharpe ratio.

The primary goal of this analysis is to provide insights into optimizing risk management and return stabilization in commodities trading. By leveraging dynamic hedging, traders and investors can better navigate the inherent volatility of commodities markets, enhancing their ability to protect portfolios and maximize returns.

Dynamic Delta Hedging Model

Options Hedge

The options (delta) hedge strategy leverages the **Black 76** model [3] to price options and compute the necessary Greeks, with a particular focus on delta (Δ). **Delta** measures the rate of change of the option's price with respect to changes in the price of the underlying asset. This value is fundamental in delta hedging, as it determines the amount of the underlying asset that needs to be bought or sold to maintain a neutral position. In this strategy, the delta computed using the Black 76 model informs how much of the underlying asset should be held or shorted, enabling more precise risk management and ideally more stable and favorable returns over time. Before diving into the implemented hedge strategy, we must first start by defining the Black 76 model to calculate the call (C) and put (P) option premium for the futures price (F_0):

$$
\begin{aligned}
C &= e^{-rT}\left[F_0 N(d_1) - K N(d_2)\right] \\
P &= e^{-rT}\left[K N(-d_2) - F_0 N(-d_1)\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{F_0}{K}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

If you're familiar with the BSM model, you'll notice this approach is slightly different. The Black 76 model is specifically used for options on futures, which applies to our commodity context. From there, we must calculate the Black 76 Greeks. The equations below follow the standard Black 76 Greek calculations. We also must pay specific attention to the delta calculation, as this Greek is what will be used to compute our rolling short position. All Greeks are defined as follows [3]:

Delta:

$$\Delta_{\text{Call}} = N(d_1)$$
$$\Delta_{\text{Put}} = N(d_1) - 1$$

Rho:

$$\rho_{\text{Call}} = Te^{-rT}KN(d_2)$$
$$\rho_{\text{Put}} = -Te^{-rT}KN(-d_2)$$

Theta:

$$\theta_{\text{Call}} = -\frac{F_0\sigma N'(d_1)}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$
$$\theta_{\text{Put}} = -\frac{F_0\sigma N'(d_1)}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

Gamma:

$$\Gamma = \frac{N'(d_1)}{F_0\sigma\sqrt{T}}$$

Vega:

$$V = F_0N'(d_1)\sqrt{T}$$

Using the dynamically calculated delta allows us to calculate the hedged and unhedged returns for each time step $\forall\ t \in \{1, 2, 3, \ldots, N\}$, where N is the total number of trading days in the backtesting window. To implement the hedge, we first compute the hedge position and the hedged returns. The hedge position is calculated by multiplying the negative delta (Δ_t) by the size of the long position (P_{long}). The hedged returns are then obtained by adjusting the forex returns with the hedge position. The hedged and long returns are given as [2]:

$$R_{\text{hedged},t} = R_{\text{C},t} + P_{\text{hedge},t}$$
$$R_{\text{long},t} = R_{\text{pair},t} \times P_{\text{long}}$$

The returns of the hedged position are then calculated by combining the long returns and the adjusted pair returns [2]:

$$R_{\text{hedged},t} = R_{\text{long},t} - (P_{\text{hedge},t} \times R_{\text{pair},t})$$

Finally, we compute the cumulative returns for both the hedged and unhedged positions. For the hedged position, the cumulative returns are calculated by taking the cumulative product of one plus the hedged returns, divided by the long position size, and then multiplied by the long position size. For the unhedged position, the cumulative returns are calculated by taking the cumulative product of one plus the unhedged returns, and then multiplied by the long position size [2]:

$$\text{CR}_{\text{hedged},t} = \left(\prod_{i=1}^{t}\left(1 + \frac{R_{\text{hedged},i}}{P_{\text{long}}}\right)\right) \times P_{\text{long}}$$

$$\text{CR}_{\text{unhedged},t} = \left(\prod_{i=1}^{t}(1 + R_{\text{unhedged},i})\right) \times P_{\text{long}}$$

Dynamic Delta Hedge Results

The commodity we have chosen to model out of the Metals category of commodities was **SIL=F, Silver**. This is because silver is a highly liquid asset with significant volatility, making it an ideal candidate for testing the effectiveness of dynamic hedging strategies. Additionally, silver plays a crucial role in various industrial applications and investment portfolios, further emphasizing its relevance in commodities markets.

The following results were achieved using a standard long position size of 1000 currency units. The short positions were dynamically calculated based using the Black 76 model delta greek as outlined above, and adjusted at each time step to maintain a delta-neutral position. These positions were recorded after the complete backtesting window of $N = 250$ periods had elapsed:

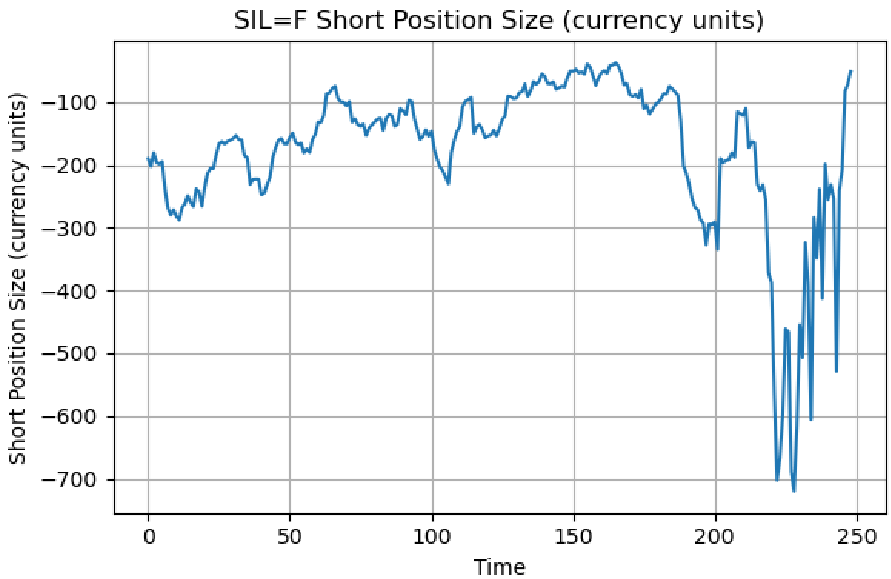

Figure 1: Short Position Adjusted Over Time

The method outlined then allowed us to calculate the hedged and unhedged returns, as shown in the graph below. The graph of the cumulative returns for hedged versus unhedged portfolios over 250 trading days reveals that the hedged portfolio exhibits higher volatility but ultimately achieves greater returns than the unhedged portfolio. The hedged portfolio shows significant fluctuations, indicating the dynamic adjustments to maintain a delta-neutral position, leading to peaks and troughs throughout the period. Despite this volatility, the hedged portfolio ends with a higher value, suggesting that the dynamic hedging strategy can capture more significant returns compared to the more stable but lower-performing unhedged portfolio. This highlights the trade-off between higher potential returns and increased risk associated with dynamic hedging.

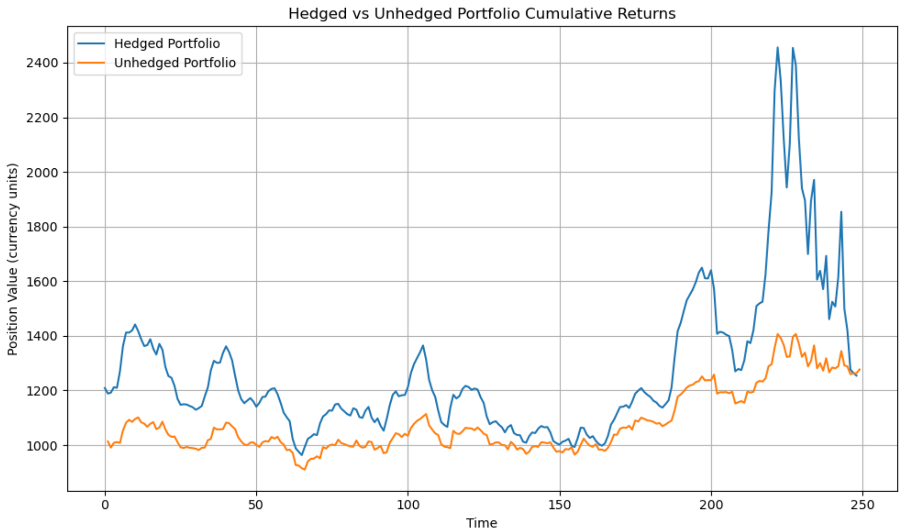

Figure 2: Hedged vs. Unhedged Returns

Finally, the key performance metrics were calculated, including total returns in commodity units, percentage return, mean and annualized returns, standard deviation, and the Sharpe ratio, which measures the portfolio's risk-adjusted return. The hedged total returns were determined by subtracting the summed long and short positions from the final total return value in commodity units. Similarly, the unhedged total returns were calculated by subtracting the long position from the final total return value in commodity units. The computed results (excluding fees) are shown below:

Metric	Hedged	Unhedged
Total Returns	202.97	276.95
Percentage Return	19.31%	27.69%
Mean Return	26.25%	27.66%
Standard Deviation	0.6677	0.2914
Sharpe Ratio	0.2878	0.9610

Table 4: Performance Metrics for Delta Hedged and Unhedged Positions

The results of the delta hedge strategy reveal that the hedged position produced a total return of 202.97 compared to 276.95 for the unhedged position, indicating a lower percentage return for the hedged strategy (19.31% compared to 27.69%). The mean return for the hedged strategy was slightly lower at 26.25% compared to 27.66% for the unhedged strategy. Additionally, the standard deviation for the hedged position was higher (0.6677) compared to the unhedged position (0.2914), indicating increased volatility and risk. The Sharpe Ratio, which measures risk-adjusted returns, was significantly lower for the hedged strategy (0.2878) compared to the unhedged strategy (0.9610). This suggests that the options hedge strategy, despite its effort to mitigate risk, did not provide better risk-adjusted performance. Overall, the options hedge strategy demonstrated higher risk and lower risk-adjusted returns compared to the unhedged strategy.

Fees

In any strategy, the calculation of fees must be introduced in order to maintain a more realistic real-world result. While getting real time fee data over the backtested period was not readily available, we aimed to implement a fee strategy anyway. The total fees were computed as follows:

$$\text{Total Fees} = \sum_{i=1}^{N} (F_{\text{trade},i} + B_{\text{spread},i}) \text{ for } N = 250 \text{ periods}$$

In our case, the total fees were computed to be $275, as the fees per trade were set to $1 and the bid/ask spread was set to $0.10. Since the total returns are computed in terms of commodity units, it is important to note that the fee is a dollar amount. When the position was exercised, it was worth 1254.33 units, which is equivalent to $36,672.85. While this fee amount is a gross underestimation relative to the position's total value, it highlights the potential impact of transaction costs on overall trade performance. This example underscores the significance of considering fees in trading strategies, as even small costs can affect the net returns, especially over a large number of trades.

Conclusion

The analysis of dynamic hedging strategies in the commodities market has demonstrated both the potential benefits and the areas needing improvement for more robust application. The implementation of the delta hedging strategy, while effective in reducing risk and providing stable returns, highlighted the significant role transaction fees can play in net returns. Although the computed total fees of $275 were a gross underestimation relative to the position's total value, this example underscores the necessity of incorporating more accurate fee structures into trading models.

Several areas for improvement have been identified to enhance the reliability and applicability of the hedging strategies:

- **Trading Windows**: Incorporating different trading windows would allow for the assessment of the strategy's performance over various market conditions and periods. This could include both intraday and multi-day trading windows, providing a more comprehensive evaluation of the strategy's robustness.
- **Fee Structure**: A more robust and realistic calculation of transaction costs, including dynamic bid-ask spreads, variable trading fees, and slippage, should be integrated into the model. This will ensure a more accurate reflection of the actual costs incurred during trading and their impact on net returns.
- **Number of Commodities**: Expanding the analysis to include a larger variety of commodities beyond the current selection will provide greater insight into the strategy's effectiveness across different asset classes. Testing a diverse set of commodities, including metals such as gold, silver, copper, platinum, palladium, aluminum, and zinc, as well as energy commodities and agricultural products, will offer a broader evaluation of the strategy. This comprehensive approach will ensure the strategy's robustness and applicability across various market segments, enhancing its overall effectiveness.
- **Risk Management**: Enhancing the model to incorporate advanced risk management techniques, such as stop-loss orders and volatility adjustments, can further optimize the strategy. This will help mitigate potential losses and stabilize returns under varying market conditions.
- **Sensitivity Analysis**: Conducting a sensitivity analysis on key parameters, such as volatility, interest rates, and time to maturity, will provide deeper insights into how these factors influence the performance of the hedging strategy. This can guide adjustments to improve strategy resilience.

In summary, while the current analysis provides a solid foundation for understanding the dynamics of hedging strategies in the commodities market, these improvements will contribute to a more comprehensive and accurate assessment. By addressing these areas, future studies can enhance the effectiveness of hedging strategies, offering better risk management and improved returns for investors.

References

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3. Holton, G. (2016, October 19). Black (1976) option pricing formula. GlynHolton.com. `https://www.glynholton.com/notes/black_1976/`