Comparative Analysis of Hedging Strategies in FX Markets

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Abstract

Hedging is an essential risk management strategy employed in financial markets to mitigate potential losses arising from adverse price movements. This paper compares four different hedging strategies in the Forex market: Direct Hedge, Correlated Hedge, Beta Hedge, and Options Hedge. The goal of this comparative analysis is to determine the most profitable strategy, while factoring in risk reduction, based on historical data and performance metrics. Our ultimate goal is to provide insights into the effectiveness of these hedging models, guiding investors in making informed decisions to protect their portfolios and enhance returns while reducing overall portfolio exposure in the Forex market. Our results indicate that the Correlated Hedge strategy was identified as the best approach for stable and risk-adjusted returns, while the Direct Hedge strategy provides the most returns.

Disclaimer

This is in no way investment advice; it's merely a formalization of results from some independent research I conducted. This work is in no way related to any company or organization. If you decide to trade based on this, just know you're taking advice from someone with a mere math undergrad and limited trading experience.

Introduction

Hedging is a crucial technique in risk management, especially in volatile markets such as FX. A hedging strategy involves choosing two opposite positions, in our case long and short, and using a computed hedging ratio to choose the optimal short position size based on a set long position size. The long position refers to buying and holding an asset, expecting its price to increase over time. The size of the long position is the amount of the asset you hold. The short position involves borrowing and selling an asset you do not own, expecting its price to decrease over time and then buying back the asset at a lower price, return it to the broker, and profit from the price difference. The overall goal of a hedging strategy is to offset losses in one position with gains in another [1]. This paper examines and compares four hedging strategies: **Direct Hedge, Correlated Hedge, Beta Hedge, and Options (Delta) Hedge**. We utilize historical data to backtest these strategies and evaluate their performance.

Assumptions

Each of the four models begins with the same assumptions in order to directly compare profitability and risks of the hedge strategy only:

- Python Library for Data Import: /yfinance

- FX pairs: EUR/USD, GBP/USD, USD/JPY, USD/CHF, AUD/USD, NZD/USD, USD/CAD, EUR/GBP, EUR/JPY, EUR/CHF, EUR/AUD, EUR/NZD, EUR/CAD, GBP/AUD, GBP/JPY, GBP/CHF, AUD/JPY, AUD/CHF, AUD/NZD, NZD/JPY, NZD/CHF, CAD/JPY, CAD/CHF, CHF/JPY
- Long position: 1000 units
- Backtesting period: July 1, 2023 - July 1, 2024

Hedging Models and Results

Direct Hedge

The first model we implement and analyze is the direct hedging model. While other models discussed in this comparison use slight differences to compute the hedge ratio, the direct method is considered to be the standard approach. The direct hedge involves taking a position in the forex market that directly offsets the risk of the primary position. After running through the entire model [1], we computed the most profitable pairing to be USD/JPY and CAD/CHF, so we proceed using this pairing.

We start by loading forex rate data for the 24 pairs in the "Assumptions" section above. From there, we iterate over the pairs to create a list of all possible pairings. Using the daily log computed returns of the underlying rates, we then compute the hedge ratio. The hedge ratio is computed by first finding the returns of pair 1 (R_1) and the returns of pair 2 (R_2). The returns are then used to fit a linear regression model, where the hedge ratio is extracted as the slope coefficient between the primary asset and the hedging asset [1]:

$$R_2 = \alpha + \text{Hedge Ratio} \times R_1 + \epsilon$$

The hedge ratio was computed to be 0.1533, and given the long position was set at 1000, the short position was set to -153.30. This indicates that for every 1000 units in the long position, you would take a short position of -153.30 units to hedge. The calculation of hedged returns is as follows [1]:

$$R_{\text{Hedged}} = R_{\text{Long}} - (\text{Hedge Ratio} \times R_{\text{Short}})$$

Finally, the cumulative returns for both the hedged and unhedged positions are computed by taking the cumulative product of one plus the respective returns, multiplied by the long position size [1]:

$$\text{CR}_{\text{Hedged}} = \left(\prod_{t=1}^{T} (1 + R_{\text{Hedged},t}) \right) \times P_{\text{Long}}$$

$$\text{CR}_{\text{Unhedged}} = \left(\prod_{t=1}^{T} (1 + R_{\text{Unhedged},t}) \right) \times P_{\text{Long}}$$

The hedged and unhedged cumulative returns form the results of the completed test, and are used to compute the results below.

Direct Hedge Results

Metric	Hedged	Unhedged
Total Returns	117.7046	112.8063
Percentage Return	11.77%	11.28%
Mean Return	1037.3707	1032.5118
Standard Deviation	33.5907	34.2198
Sharpe Ratio	0.4902	0.4790

Table 1: Performance Metrics for Direct Hedged and Unhedged Positions

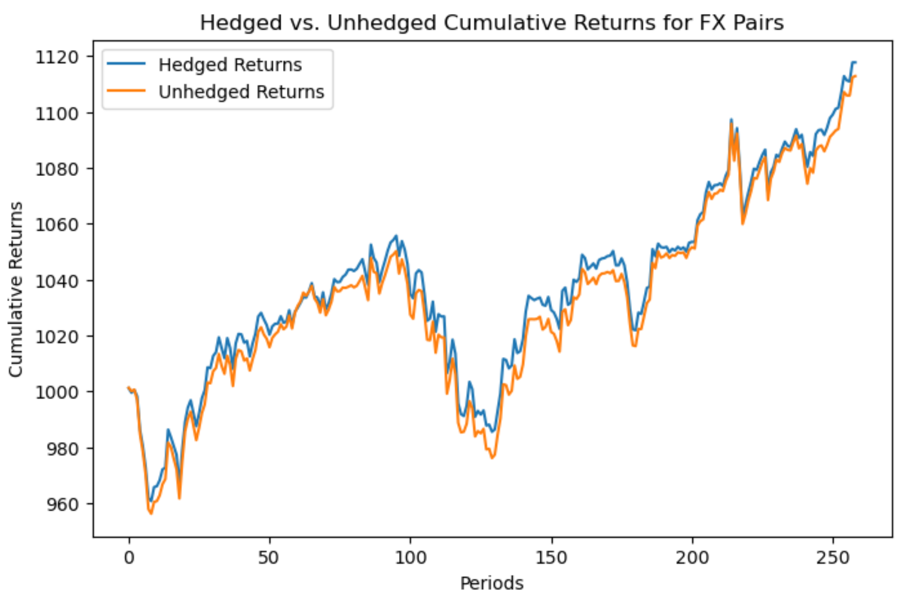

Figure 1: Graph of Direct Hedge Method Results

The results demonstrate that the hedged position produced a total return of 117.7046 compared to 112.8063 for the unhedged position. The hedged strategy exhibited a lower standard deviation (33.5907) compared to the unhedged position (34.2198), indicating reduced volatility and risk. Furthermore, the Sharpe Ratio, which measures risk-adjusted returns, was higher for the hedged strategy (0.4902) compared to the unhedged strategy (0.4790). This suggests that the direct hedge strategy offers better risk-adjusted performance, providing a more stable return profile. Overall, while both strategies show comparable performance, the direct hedge strategy offers a more favorable risk-return trade-off, making it a valuable approach for investors seeking to manage forex market risks effectively.

Correlated Hedge

The correlated hedging strategy leverages the historical correlation between two forex assets to mitigate risk and enhance returns. By computing a hedge ratio, the strategy determines the sizes of the long and short positions in the base and correlated currency pairs, respectively. The primary objective is to offset potential losses in the long position with gains in the short position based on their correlated price movements. Unlike traditional pairs trading, which typically assumes mean reversion, this strategy does not rely on the assets reverting to their historical mean. Instead, it focuses on the relative movements driven by their correlation. This method provides a dynamic and flexible approach to risk management, allowing traders to adjust their positions according to the changing correlations in the market. The effectiveness of the correlated hedge is demonstrated through performance metrics, which often show that hedged cumulative returns can outperform unhedged returns. By exploiting the correlation between the two assets, this strategy reduces risk exposure and aims to achieve more stable and favorable returns over time.

Once the historical stock data is loaded in, the correlation is computed between each pair, and the correlation values are then stacked. This allows us to filter out any correlation values which equal 1 (same pairs) and then find the two pairs with the highest correlation value. In this case, the two most correlated pairs were EUR/JPY and GBP/JPY with a computed correlation of 0.977174. The correlation is extremely close to 1, implying that as EUR/JPY increases, GBP/JPY decreases nearly proportionally.

The hedge ratio is computed by first finding the returns of Pair 1 and Pair 2. The returns are then used to fit a linear regression model, where the hedge ratio is extracted as the slope coefficient between the primary asset and the hedging asset. The hedge ratio was computed to be 0.9148, and given the long position was set at 1000, the short position was set to -914.78. This indicates that for every 1000 units in the long position, you would take a short position of -914.78 units to hedge. The linear regression model is given as:

$$R_2 = \alpha + \text{Hedge Ratio} \times R_1 + \epsilon$$

Next, we must calculate the hedged and unhedged returns. We start by computing the short position which requires multiplying the negative hedge ratio by the long position size. We then compute the returns for the long position which are equal to the returns of Pair 1, while the returns for the short position are calculated as the negative hedge ratio multiplied by the returns for Pair 2 [1].

$$R_{\text{long}} = R_1$$

$$R_{\text{short}} = -\text{Hedge Ratio} \times R_2$$

$$R_{\text{Hedged}} = R_{\text{Long}} + R_{\text{Short}}$$

Finally, the cumulative returns for both the hedged and unhedged positions are computed by taking the cumulative product of one plus the respective returns, multiplied by the long position size [1]:

$$\text{CR}_{\text{Hedged}} = \left(\prod_{t=1}^{T} (1 + R_{\text{Hedged},t}) \right) \times P_{\text{Long}}$$

$$\text{CR}_{\text{Unhedged}} = \left(\prod_{t=1}^{T} (1 + R_{\text{Unhedged},t}) \right) \times P_{\text{Long}}$$

The hedged and unhedged cumulative returns form the results of the completed test, and are used to compute the results below.

Correlated Hedge Results

Metric	Hedged	Unhedged
Total Returns	-6.2808	92.5317
Percentage Return	-0.63%	9.25%
Mean Return	1001.3410	1023.8302
Standard Deviation	6.5379	28.5993
Sharpe Ratio	2.4313	0.5683

Table 2: Performance Metrics for Correlated Hedged and Unhedged Positions

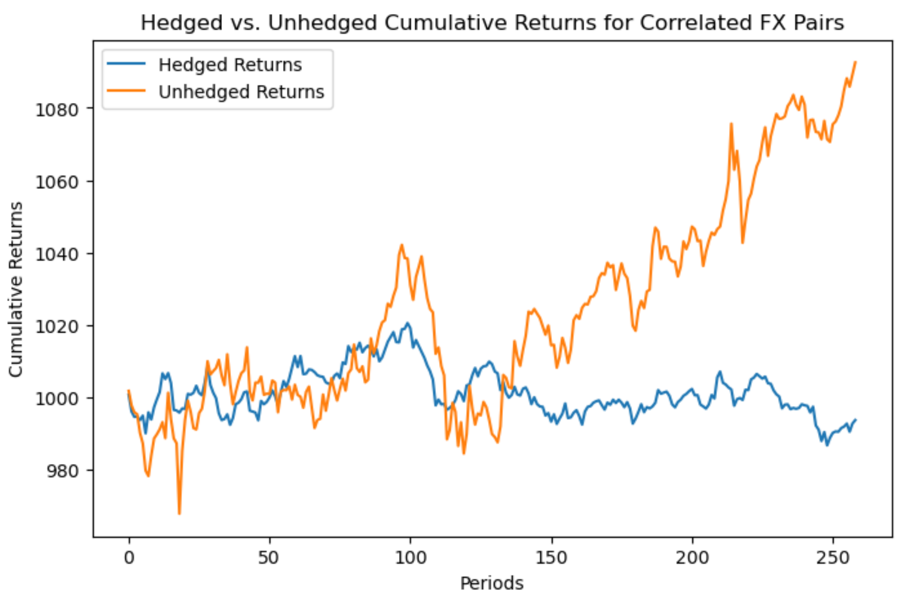

Figure 2: Graph of Correlated Hedge Method Results

The results show that the hedged position produced a total return of -6.2808 compared to 92.5317 for the unhedged position, indicating a higher percentage return for the unhedged strategy (9.25% vs. -0.63%). However, the hedged position had a much lower standard deviation (6.5379) compared to the unhedged position (28.5993), indicating reduced volatility and risk. The Sharpe Ratio was higher for the hedged strategy (2.4313) compared to the unhedged strategy (0.5683). This suggests that the correlated hedge strategy offers better risk-adjusted performance and provides a more stable return profile. In summary, while the correlated hedge strategy may not have yielded higher total returns in this specific case, it demonstrated a significantly lower risk profile and higher risk-adjusted returns.

Beta Hedge

The beta hedge is very similar to the direct and correlated hedge strategies, except this method uses the beta coefficient to determine the hedge ratio. Traditionally, the beta coefficient (β) is used to measure the sensitivity of an asset's returns relative to the market returns [2]. In the case of our model, we compute β using the linear regression model, which first involves selecting a base pair and a benchmark pair. The base pair is the primary currency pair or asset we used to hedge, while the benchmark pair is a secondary asset used as a reference. The returns of the base pair are analyzed relative to the benchmark pair to determine the beta coefficient. In the context of our analysis, we opted for the base pair to be USD/JPY and the benchmark to be EUR/USD. The beta indicates how sensitive USD/JPY returns are to movements in EUR/USD returns. The returns are then used to fit a linear regression model, where the hedge ratio is extracted as the slope coefficient (β) between the primary asset and the hedging asset. The beta is computed using the following regression model as 0.5929, with the short position as -592.91, after computing the following:

$$R_{\text{Base}} = \alpha + \beta \times R_{\text{Benchmark}} + \epsilon$$

It is important to note that in the case of our analysis, we began by choosing ten different base pairs to compare against a single benchmark. More advanced analysis would involve computing all possible combinations of pairs in the form [base pair, benchmark pair] from the list of 24 pairs provided in the "Assumptions" section. The total number of combinations was computed to be:

$$\text{Total combinations} = n \times (n-1) = 24 \times (24-1) = 24 \times 23 = 552$$

For the beta hedge strategy, considering all 552 pairings in the form of [base pair, benchmark pair] is ideal, as this allows each pair in the list to be both a base pair and a benchmark pair. This approach captures the full range of relationships between currency pairs, ensuring accurate hedge ratios through comprehensive analysis. By examining all possible pairings, we avoid bias and achieve a robust and reliable hedging strategy. However, upon completion of our test case of ten base pairs and single benchmark pair, we achieved a set of pairs that produced highly desirable results. Once our pairs were chosen, and beta was computed, this trading model requires the calculation of the hedged returns, which are calculated as follows:

$$\begin{aligned} R_{\text{Long}} &= R_{\text{Base}} \\ R_{\text{Short}} &= -\beta \times R_{\text{Benchmark}} \\ R_{\text{Hedged}} &= R_{\text{Long}} + R_{\text{Short}} \end{aligned}$$

Finally, the cumulative returns for both the hedged and unhedged positions are computed by taking the cumulative product of one plus the respective returns, multiplied by the long position size [1]:

$$\begin{aligned} \text{CR}_{\text{Hedged}} &= \left(\prod_{t=1}^{T} (1 + R_{\text{Hedged},t}) \right) \times P_{\text{Long}} \\ \text{CR}_{\text{Unhedged}} &= \left(\prod_{t=1}^{T} (1 + R_{\text{Unhedged},t}) \right) \times P_{\text{Long}} \end{aligned}$$

The hedged and unhedged cumulative returns form the results of the completed test, and are used to compute the results below.

Beta Hedge Results

Metric	Hedged	Unhedged
Total Returns	121.3483	112.8063
Percentage Return	12.13%	11.28%
Mean Return	1035.9468	1032.5118
Standard Deviation	38.8079	34.2198
Sharpe Ratio	0.4238	0.4790

Table 3: Performance Metrics for Beta Hedged and Unhedged Positions

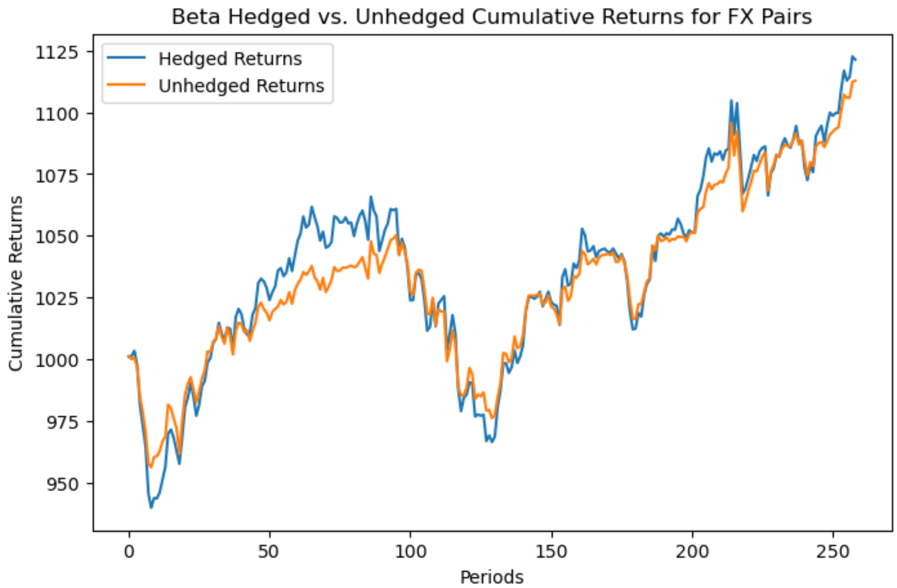

Figure 3: Graph of Beta Hedge Method Results

The results of the beta hedge strategy reveal a total return of 121.3483 for the hedged position compared to 112.8063 for the unhedged position, indicating a higher percentage return for the hedged strategy (12.13% compared to 11.28%). However, the standard deviation for the hedged position was higher (38.8079) compared to the unhedged position (34.2198), indicating slightly increased volatility and risk. The Sharpe Ratio was slightly lower for the hedged strategy (0.4238) compared to the unhedged strategy (0.4790). This suggests that, although the beta hedge strategy yielded higher total and percentage returns, it also came with marginally higher risk, resulting in a lower risk-adjusted performance. Overall, the beta hedge strategy provided higher absolute returns but at the cost of increased volatility.

Options Hedge

The options (delta) hedge strategy leverages the **Black-Scholes Merton (BSM)** [4] to price options and compute the necessary greeks, with a particular focus on delta (Δ). **Delta** measures the rate of change of the option's price with respect to changes in the price of the underlying asset. This value is fundamental in delta hedging, as it determines the amount of the underlying asset that needs to be bought or sold to maintain a neutral position. In this strategy, the delta computed using the BSM model informs how much of the underlying asset should be held or shorted, enabling more precise risk management and ideally more stable and favorable returns over time. Before diving into the implemented hedge strategy, we must first start by defining the BSM model to calculate the call (C) and put (P) option premium for the spot price (S_0):

$$C = S_0 e^{-r_f T} N(d_1) - e^{-rT} K N(d_2)$$
$$P = e^{-rT} K N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

where:

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

If you're familiar with the BSM model, you'll notice this model includes not just the domestic risk-free rate (r), but also the foreign risk-free rate (r_f). Suppose for the sake of example the underlying, S_0 (the value of the domestic currency per unit of the foreign currency) = 1.08 CHF/EUR, this means the annual domestic risk free rate applies to CHF and the foreign (carry) rate applies to EUR [4].

From there, we must calculate the BSM greeks. The equations below follow the standard BSM greek calculations, however, these are of course adjusted for the domestic and foreign risk free rates. We also must pay specific attention to the delta calculation, and due to the fact we have the possibility of an option premium being paid in units of the foreign currency, we must adjust the delta to reflect through the method of **premium-adjusted delta** [5]. All greeks adjusted for foreign and domestic currency units are defined as follows:
Delta:

$$
\begin{aligned}
\Delta_{\text{Call}} &= \exp(-r_f T) \cdot N(d_1) \\
\Delta_{\text{Put}} &= \exp(-r_f T) \cdot [N(d_1) - 1]
\end{aligned}
$$

Rho:

$$
\begin{aligned}
\rho_{\text{Call}} &= KTe^{-rT}N(d_2) \\
\rho_{\text{Put}} &= -KTe^{-rT}N(-d_2)
\end{aligned}
$$

Theta:

$$
\begin{aligned}
\theta_{\text{Call}} &= \left(-\frac{S_0\sigma e^{-r_f T}N'(d_1)}{2\sqrt{T}}\right) - rKe^{-rT}N(d_2) + r_f S_0 e^{-r_f T}N(d_1) \\
\theta_{\text{Put}} &= \left(-\frac{S_0\sigma e^{-r_f T}N'(d_1)}{2\sqrt{T}}\right) - rKe^{-rT}N(-d_2) + r_f S_0 e^{-r_f T}N(-d_1)
\end{aligned}
$$

Gamma:

$$
\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{t}}
$$

Vega:

$$
V = S_0 N'(d_1)\sqrt{t}
$$

Now in terms of our specific model, we created the entire project, and iterated over the list of FX pairs to determine the most profitable pair. The results concluded the most profitable pair to be USD/JPY, for a strike price set at ±2% for a call/put option and utilized the following input parameters:

Variable Name	Symbol	Value
Option Type	N/A	'call'
Spot Price	S_0	\$160.68
Strike Price	K	\$163.90
Volatility	σ	0.0843
Domestic Risk Free Rate	r	0.02
Foreign Risk Free Rate	r_f	0.01
Time to Expiry	T	1 year

Table 4: BSM Variable Input Parameters

We can then use the above inputs to calculate our option variables and greeks. Utilizing the above table of variables in our BSM model function produces the following output results:

Variable	Value
Option Price	4.6276
Delta	0.4658
Gamma	0.0291
Theta	-3.3236
Rho	70.2156
Vega	63.2928
Probability ITM (Call)	0.4705
Probability ITM (Put)	0.5295
Probability of Exercise (Call)	0.4371
Probability of Exercise (Put)	0.5629

Table 5: BSM Variable Output Parameters

Using the premium-adjusted delta allows us to calculate the hedged and unhedged returns. To implement the hedge, we first compute the hedge position and the hedged returns. The hedge position is calculated by multiplying the negative delta (Δ) by the size of the long position (P_{long}). The hedged returns are then obtained by adjusting the forex returns with the hedge position. The hedged and long returns are given as:

$$R_{\text{hedged}} = R_{\text{fx}} + P_{\text{hedge}}$$
$$R_{\text{long}} = R_{\text{pair}} \times P_{\text{long}}$$

The returns of the hedged position are then calculated by combining the long returns and the adjusted pair returns:

$$R_{\text{hedged}} = R_{\text{long}} - (P_{\text{hedge}} \times R_{\text{pair}})$$

Finally, we compute the cumulative returns for both the hedged and unhedged positions. For the hedged position, the cumulative returns are calculated by taking the cumulative product of one plus the hedged returns, divided by the long position size, and then multiplied by the long position size. For the unhedged position, the cumulative returns are calculated by taking the cumulative product of one plus the unhedged returns, and then multiplied by the long position size:

$$\text{CR}_{\text{hedged}} = \left(\prod_{t=1}^{T} \left(1 + \frac{R_{\text{hedged},t}}{P_{\text{long}}} \right) \right) \times P_{\text{long}}$$

$$\text{CR}_{\text{unhedged}} = \left(\prod_{t=1}^{T} (1 + R_{\text{unhedged},t}) \right) \times P_{\text{long}}$$

Options Hedge Results

Metric	Hedged	Unhedged
Total Returns	58.0344	108.4864
Percentage Return	5.973%	11.28%
Mean Return	1022.7909	1041.8966
Standard Deviation	13.2192	24.5886
Sharpe Ratio	1.2042	0.6597

Table 6: Performance Metrics for Delta Hedged and Unhedged Positions

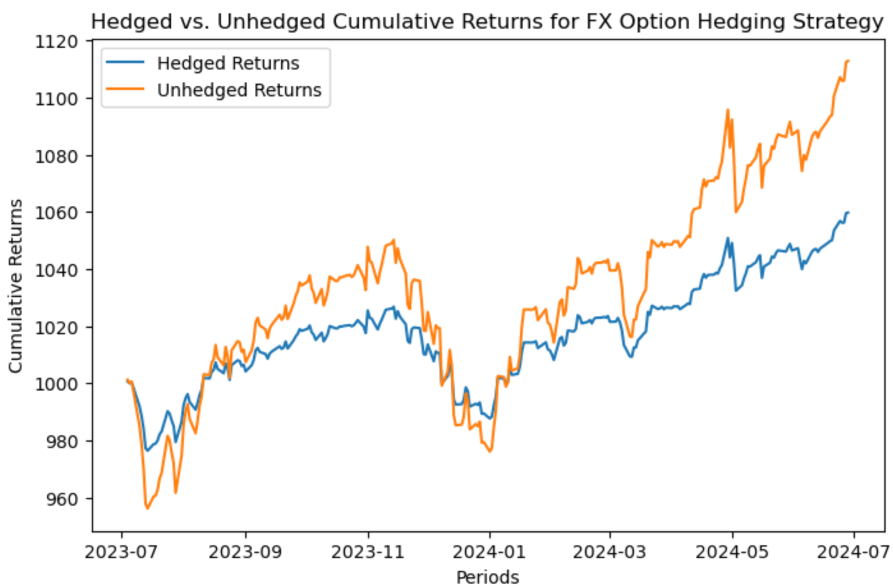

Figure 4: Graph of Options (Delta) Hedge Method Results

The results of the options hedge strategy reveal that the hedged position produced a total return of 58.0344 compared to 108.4864 for the unhedged position, indicating a lower percentage return for the hedged strategy (5.973% compared to 11.28%). However, the standard deviation for the hedged position was significantly lower (13.2192) compared to the unhedged position (24.5886), indicating reduced volatility and risk. The Sharpe Ratio, which measures risk-adjusted returns, was significantly higher for the hedged strategy (1.2042) compared to the unhedged strategy (0.6597). This suggests that the options hedge strategy, despite its lower total and percentage returns, offers better risk-adjusted performance, providing a more stable return profile. Overall, the options hedge strategy demonstrated a significantly lower risk profile and slightly higher risk-adjusted returns.

Conclusion

Upon comparing the four strategies, the Beta Hedge produced the highest total returns, but with significantly higher volatility. The Correlated Hedge, while yielding a negative total return, offered the best risk-adjusted performance, emphasizing stability over raw returns. The Delta Hedge also performed well in terms of risk-adjusted returns, striking a balance between stability and profitability. The Direct Hedge, though straightforward, provided moderate returns and risk-adjusted performance.

Best Strategy

The Correlated Hedge strategy was identified as the best approach for investors seeking stable and risk-adjusted returns, despite its lower total returns in this specific analysis. For those primarily driven by risk mitigation, the Correlated Hedge strategy provides the most stable return profile with significantly lower volatility.

For investors focused on maximizing total returns, the Direct Hedge strategy might be more suitable, as it demonstrated higher total and percentage returns in the analysis. However, this comes with higher volatility and risk.

Investors interested in both risk mitigation and total returns should consider the Beta Hedge strategy. This method balances risk and return by leveraging the beta coefficient to manage exposure, offering a reasonable compromise between stability and profitability.

Recommendations for Improvement

- **Dynamic Hedge Ratios**: Implementing dynamic hedge ratios that adjust based on market conditions could improve the effectiveness of each strategy.
- **Incorporating Machine Learning**: Using machine learning models to predict future correlations and volatility may enhance the accuracy of the hedging strategies.
- **Extended Historical Data**: Utilizing a longer historical data period could provide more robust insights and improve the reliability of the hedge ratios.
- **Diversification**: Combining multiple hedging strategies might offer a more balanced approach between increased returns and risk, leveraging the strengths of each method to achieve optimal risk-adjusted returns.

References

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