Heston Model Put Approximation Equation Derivation

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Put-Call Parity

The put-call parity equation for European options states that the price of a put option P is equal to the difference between the current stock price SS times the probability of exercise P_1, and the present value of the strike price K discounted to time T times the probability of non-exercise P_2, plus the present value of the strike price K discounted to time T minus the stock price S.

The put-call parity relationship is given by:

$$C + Ke^{-rT} = P + S$$

where:

- C is the price of the European call option.
- P is the price of the European put option.
- K is the strike price.
- r is the risk-free interest rate.
- T is the time to maturity.
- S is the current stock price.

Rearranging this equation to solve for the put option price P, we get:

$$P = C + Ke^{-rT} - S$$

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The call option price C under the Heston model is given by:

$$C(S, K, T) = SP_1 - Ke^{-rT}P_2$$

where the probabilities P_1 and P_2 are:

$$P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re\left(\frac{e^{-iu\ln K}\phi(u-i)}{iu\phi(-i)}\right) du$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re\left(\frac{e^{-iu\ln K}\phi(u)}{iu}\right) du$$

Substituting the Heston Model call function into the above Put-Call Parity function, and simplifying gives us:

$$\begin{aligned}
P &= C + Ke^{-rT} - S \\
&= (SP_1 - Ke^{-rT}P_2) + Ke^{-rT} - S \\
&= SP_1 - Ke^{-rT}P_2 + Ke^{-rT} - S \\
&= SP_1 - S + Ke^{-rT} - Ke^{-rT}P_2 \\
&= S(P_1 - 1) + Ke^{-rT}(1 - P_2)
\end{aligned}$$

Therefore, the simplified expression for the put option price P is:

$$P = S(P_1 - 1) + Ke^{-rT}(1 - P_2)$$