An Introduction to Quantum Chemistry and Potential Energy Surfaces

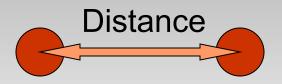
Benjamin G. Levine

This Week's Lecture

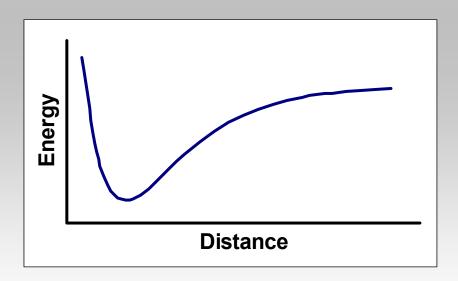
- Potential energy surfaces
 - What are they?
 - What are they good for?
 - How do we use them to solve chemical problems?

What do quantum chemists do?

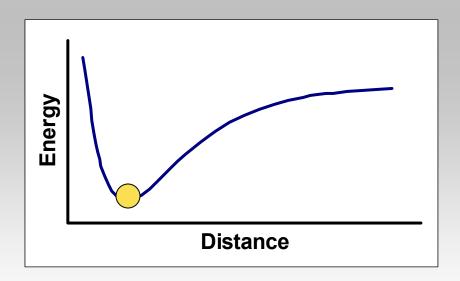
- Approximate the electronic structures of molecules
- Extract useful chemical information from them

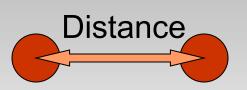


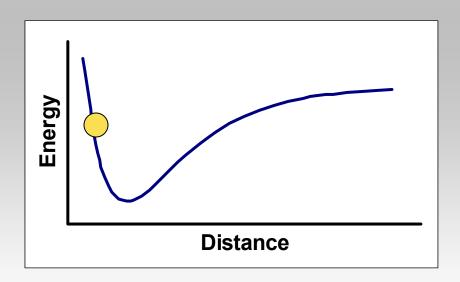




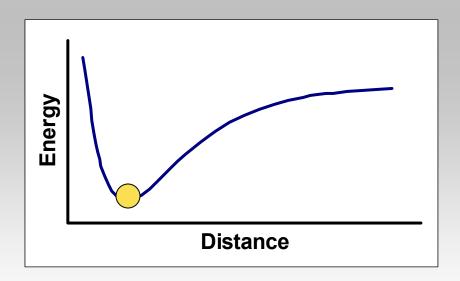


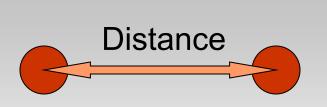


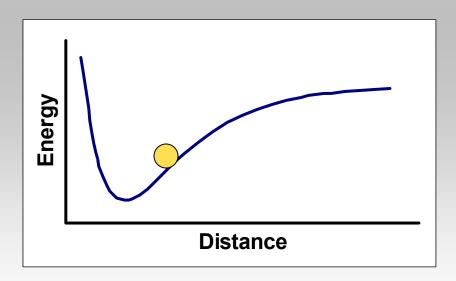


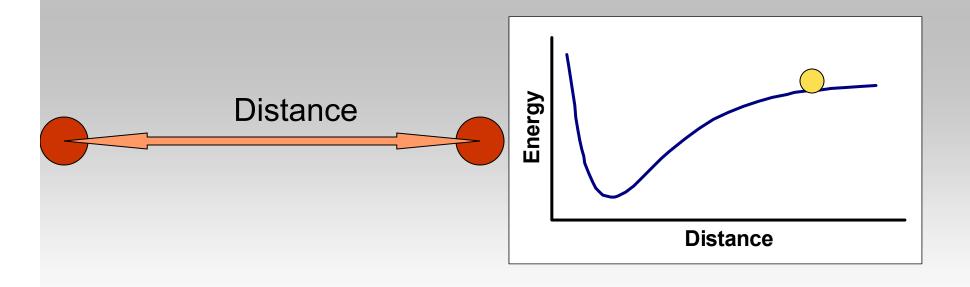




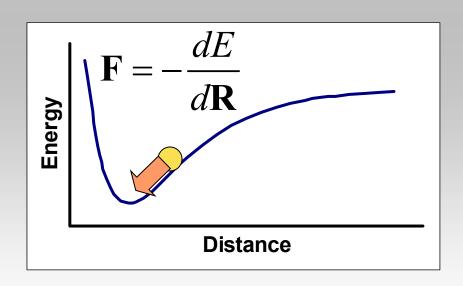


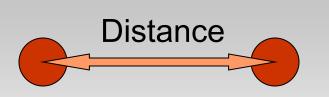


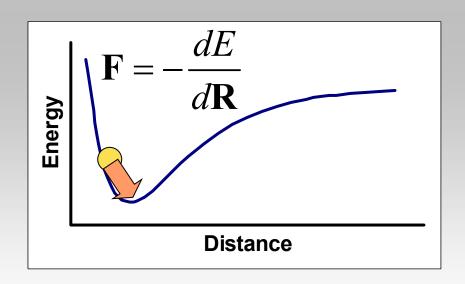




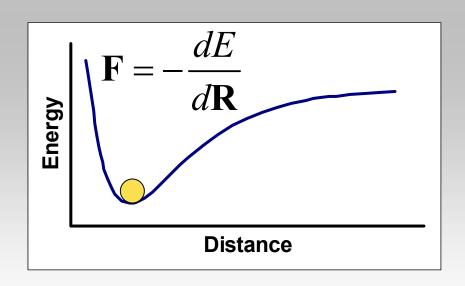












Using the PES

- Many important chemical question can be rephrased in terms of the PES
- We rarely calculate the whole PES, but instead explore and describe it in a variety of ways
- The PES can be calculated in many ways which vary in
 - Accuracy
 - Computational cost
 - The portions of the PES which they are capable of describing

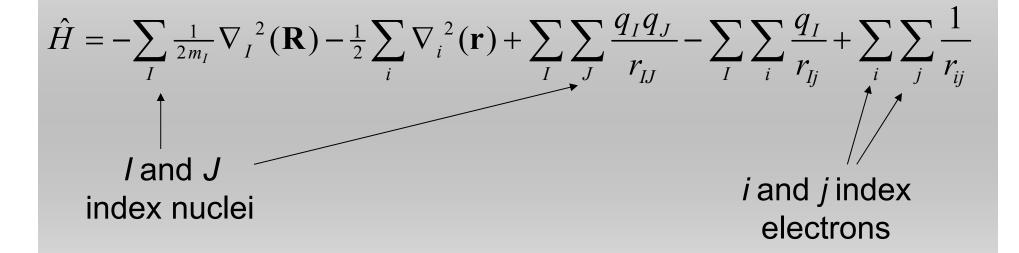
The PES idea comes from the approximate solution of the full molecular time-independent Schrodinger equation

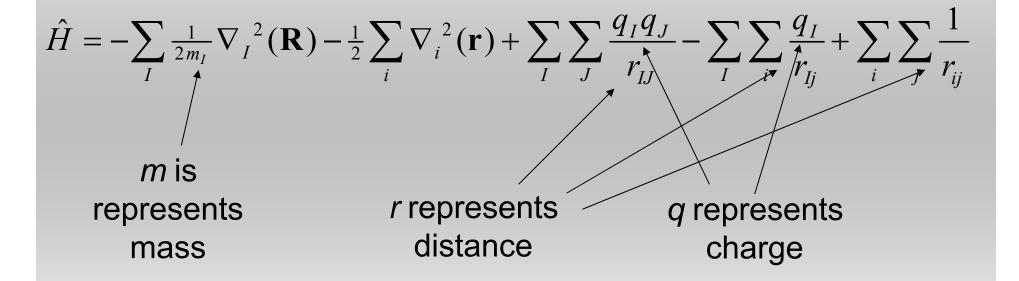
$$\hat{H}\Psi(\mathbf{R},\mathbf{r}) = E\Psi(\mathbf{R},\mathbf{r})$$

The PES idea comes from the approximate solution of the full molecular time-independent Schrodinger equation

$$\hat{H}\Psi(\mathbf{R},\mathbf{r}) = E\Psi(\mathbf{R},\mathbf{r})$$
Nuclear Electronic coordinates coordinates

$$\hat{H} = -\sum_{I} \frac{1}{2m_{I}} \nabla_{I}^{2}(\mathbf{R}) - \frac{1}{2} \sum_{i} \nabla_{i}^{2}(\mathbf{r}) + \sum_{I} \sum_{J} \frac{q_{I}q_{J}}{r_{IJ}} - \sum_{I} \sum_{i} \frac{q_{I}}{r_{Ii}} + \sum_{i} \sum_{J} \frac{1}{r_{ii}}$$





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Nuclear
Kinetic
Energy

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$$\uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow$$
Nuclear
Kinetic
Energy
Electronic
Nuclear
Repulsion
Repulsion
Repulsion

The molecular Hamiltonian is

$$\hat{H} = -\sum_{I} \frac{1}{2m_{I}} \nabla_{I}^{2}(\mathbf{R}) - \frac{1}{2} \sum_{i} \nabla_{i}^{2}(\mathbf{r}) + \sum_{I} \sum_{J} \frac{q_{I}q_{J}}{r_{IJ}} - \sum_{I} \sum_{i} \frac{q_{I}}{r_{Ij}} + \sum_{i} \sum_{J} \frac{1}{r_{ij}}$$

Atomic Units

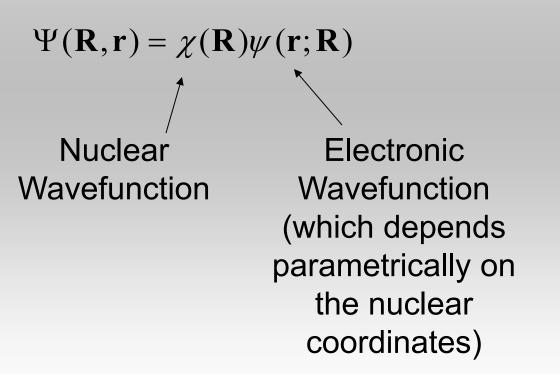
$$\hbar = 1$$

$$e = 1$$

$$m_e = 1$$

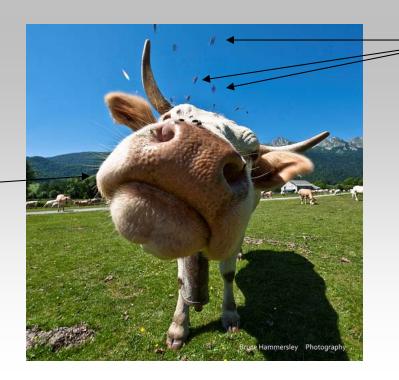
$$1/(4\pi\varepsilon_0) = 1$$

Separate the nuclear and electronic problems (Born-Oppenheimer Approx.)



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$$\Psi(\mathbf{R}, \mathbf{r}) = \chi(\mathbf{R}) \psi(\mathbf{r}; \mathbf{R})$$



Fast Electrons

Slow Nucleus

Separate the nuclear and electronic problems (Born-Oppenheimer Approx.)

$$\Psi(\mathbf{R},\mathbf{r}) = \chi(\mathbf{R})\psi(\mathbf{r};\mathbf{R})$$

$$\hat{H}_{elec}\psi(\mathbf{r};\mathbf{R}) = V(\mathbf{R})\psi(\mathbf{r};\mathbf{R})$$

$$\hat{H}_{elec} = -\frac{1}{2} \sum_{i} \nabla_{i}^{2}(\mathbf{r}) + \sum_{I} \sum_{J} \frac{q_{I}q_{J}}{r_{IJ}} - \sum_{I} \sum_{j} \frac{q_{I}}{r_{Ij}} + \sum_{i} \sum_{j} \frac{1}{r_{ij}}$$

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 $V(\mathbf{R})$ is the (adiabatic) potential energy surface

Separate the nuclear and electronic problems (Born-Oppenheimer Approx.)

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 $V(\mathbf{R})$ is the (adiabatic) potential energy surface

This is the electronic structure problem

Separate the nuclear and electronic problems (Born-Oppenheimer Approx.)

$$\Psi(\mathbf{R},\mathbf{r}) = \chi(\mathbf{R})\psi(\mathbf{r};\mathbf{R})$$

$$\hat{H}_{elec}\psi(\mathbf{r};\mathbf{R}) = V(\mathbf{R})\psi(\mathbf{r};\mathbf{R})$$

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$$\hat{H}_{nuc}\chi(\mathbf{R}) = E\chi(\mathbf{R})$$

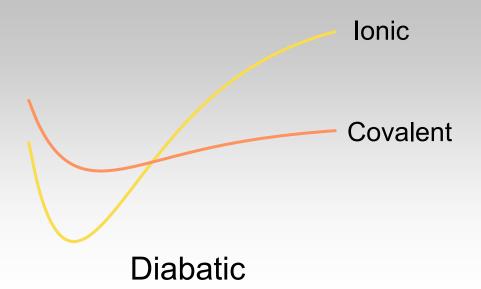
$$\hat{H}_{nuc} = -\sum_{I} \frac{1}{2m_{I}} \nabla_{I}^{2}(\mathbf{R}) + V(\mathbf{R})$$

- The Born-Oppenheimer Approximation (BOA) neglects couplings between electronic and nuclear motions
- These missing terms are called nonadiabatic or vibronic coupling terms
- The BOA is a good approximation when electronic states are not near each other in energy (ground states of stable molecules with simple electronic structures)
- Electronic states **are** near each other in energy sometimes:
 - Some transition states (nonadiabatic reaction dynamics)
 - Excited states/photochemistry
 - Jahn-Teller systems

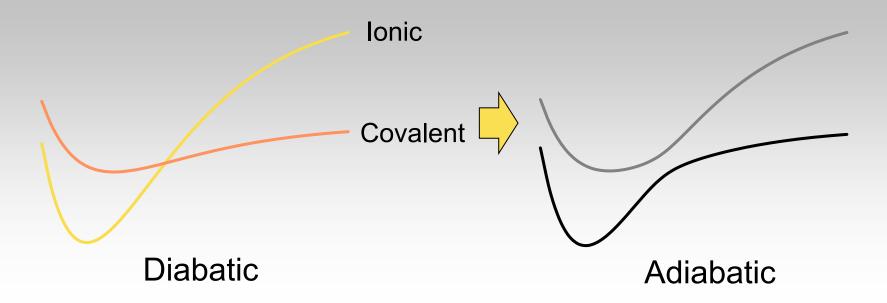
The BOA is valid for the majority of problems you see in the literature and in this course

- Adiabatic PES are the solution to the electronic Schrodinger equation
- Diabatic PES are defined according to state character (e.g. covalent vs ionic)

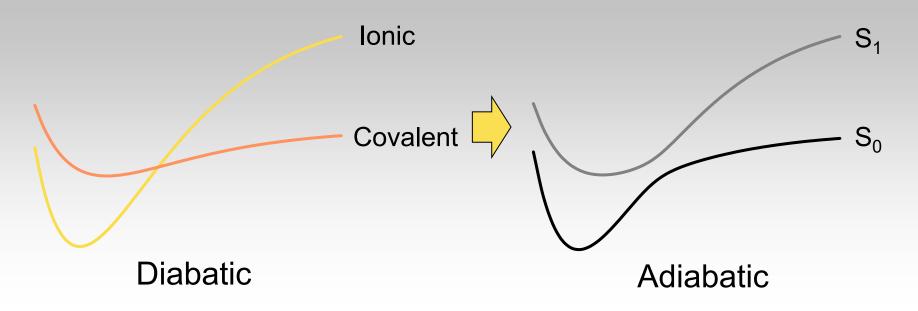
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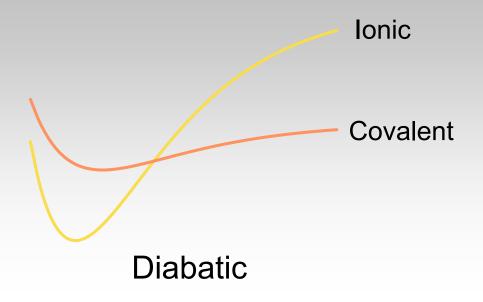


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Diabatic PES

- Diabats are conceptually convenient
- There is no unique way to define diabats
- Diabats can be very strongly coupled over a broad range of configuration space



The PES is many-dimensional

- Each atom is a point in a 3-D space the PES is 3N dimensional (where N is the number of atoms)
- The molecular geometry (configuration, structure) can be represented many ways
 - Cartesian coordinates
 - Internal Coordinates (e.g. Z-matrix)

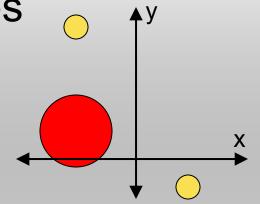
Cartesian Coordinates

Also known as XYZ coordinates

```
O -0.464 0.177 0.0

H -0.464 1.137 0.0

H 0.441 -0.143 0.0
```



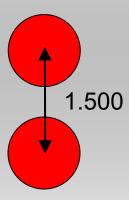
Used in various common storage formats:
 .xyz, .pdb

Define bond lengths, angles, dihedrals

 \cap

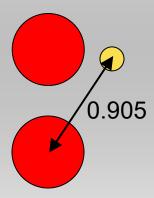
Define bond lengths, angles, dihedrals

O O, 1, 1.500

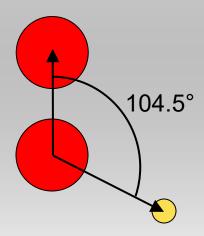


Define bond lengths, angles, dihedrals

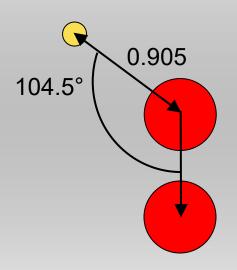
O O, 1, 1.500 H, 1, 0.905

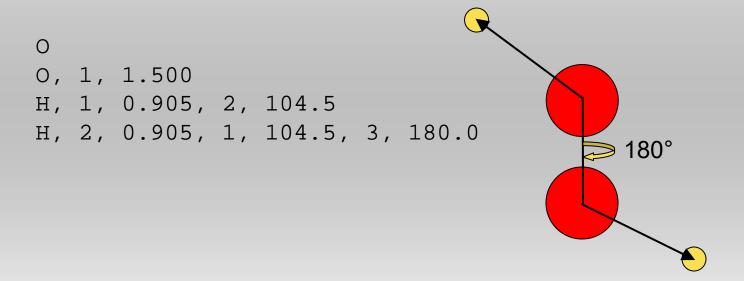


```
O, 1, 1.500
H, 1, 0.905, 2, 104.5
```



```
O
O, 1, 1.500
H, 1, 0.905, 2, 104.5
H, 2, 0.905, 1, 104.5
```

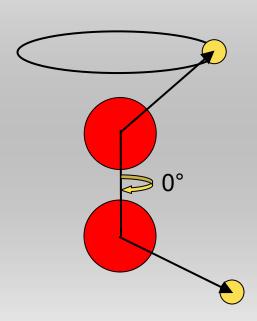


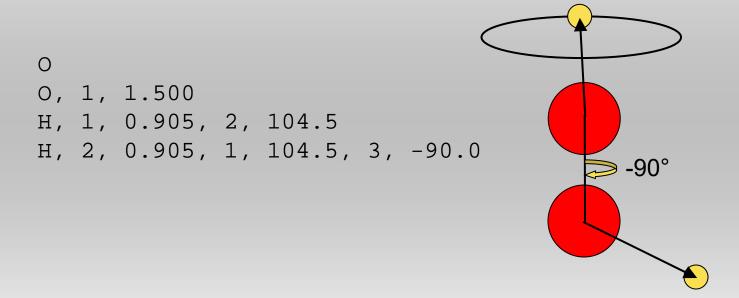


```
O
O, 1, 1.500
H, 1, 0.905, 2, 104.5
H, 2, 0.905, 1, 104.5, 3, 180.0
```

```
O
O, 1, 1.500
H, 1, 0.905, 2, 104.5
H, 2, 0.905, 1, 104.5, 3, 90.0
```

```
O
O, 1, 1.500
H, 1, 0.905, 2, 104.5
H, 2, 0.905, 1, 104.5, 3, 0.0
```



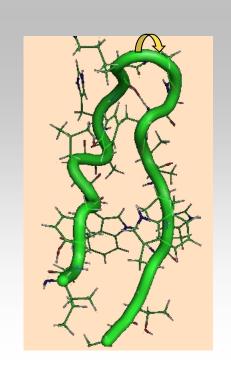


Advantages

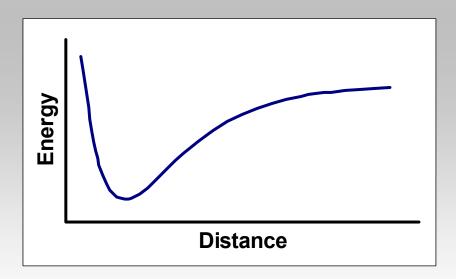
- Chemically intuitive
- Non-redundant (eliminates translation, rotation)

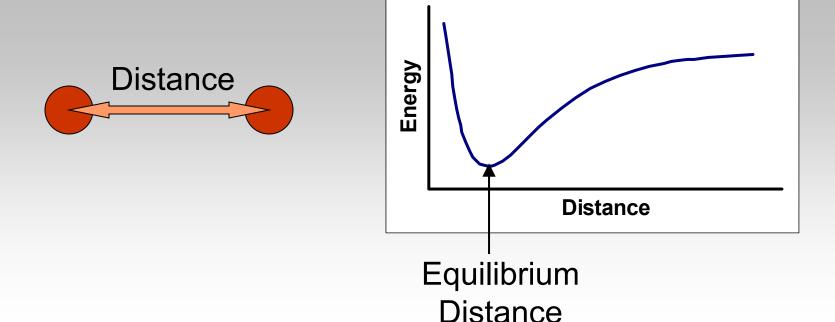
Disadvantages

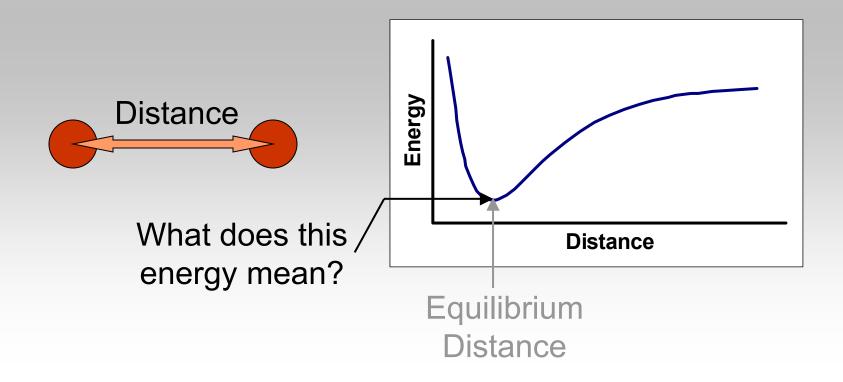
- For large or cyclic molecules the geometry can be very sensitive to small changes in the parameters
- Equations of motion/geometry optimization are more nature in Cartesians.

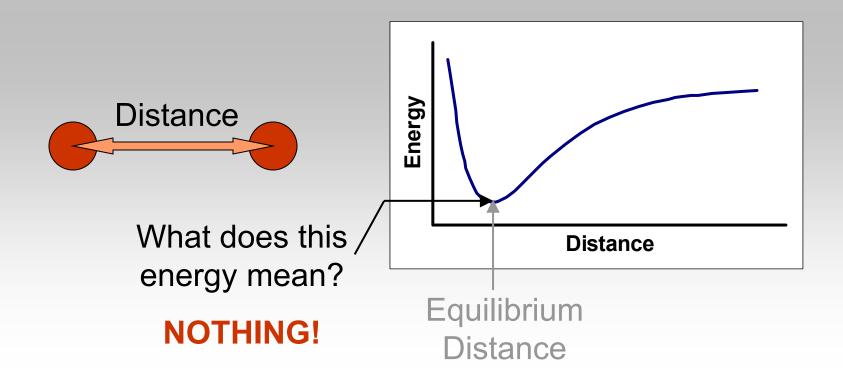


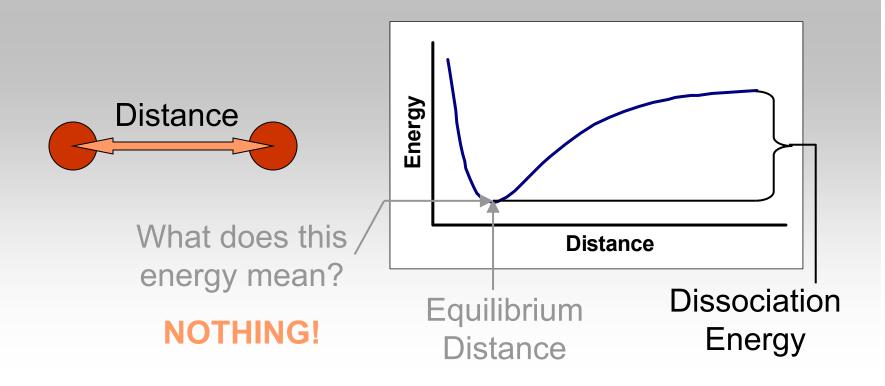




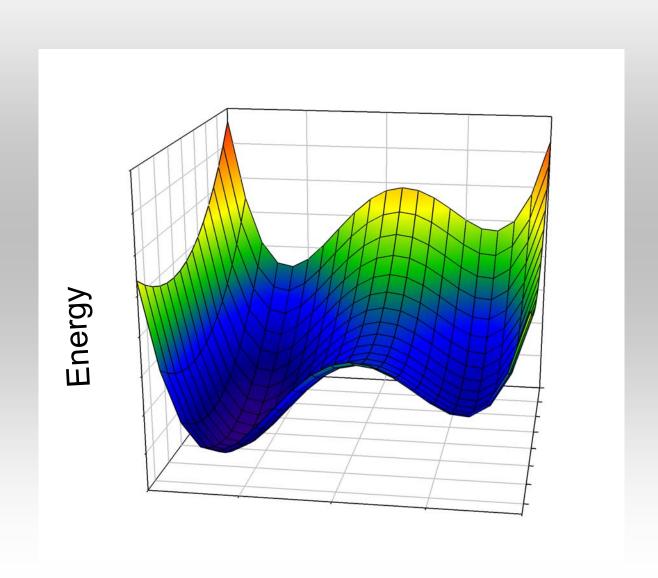


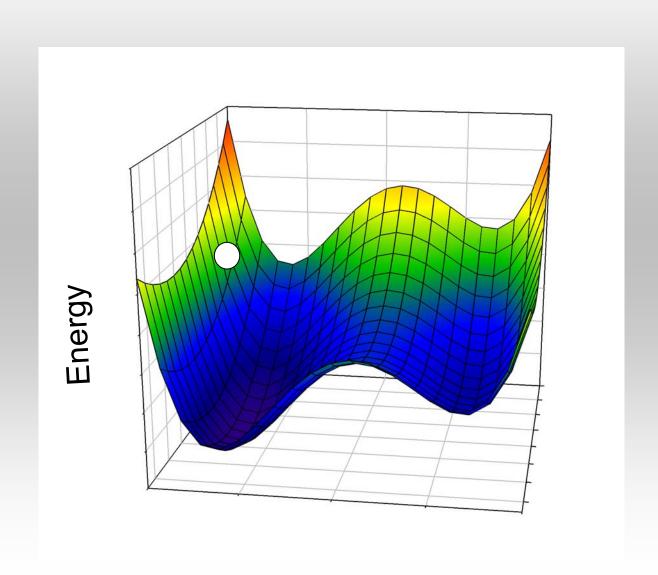


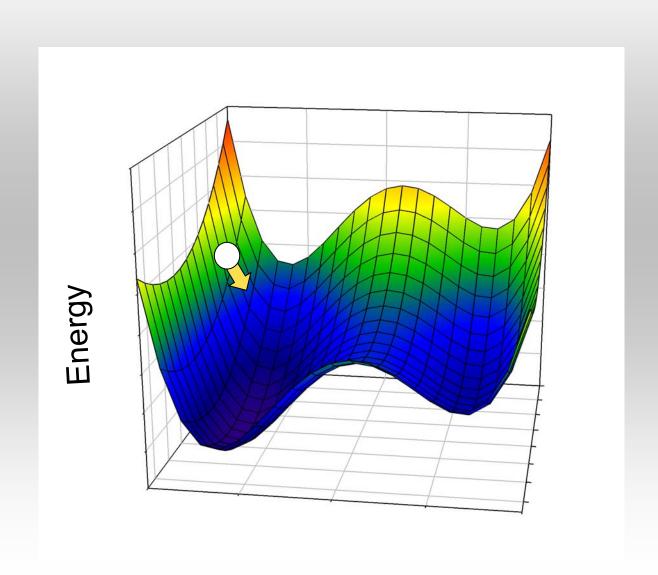


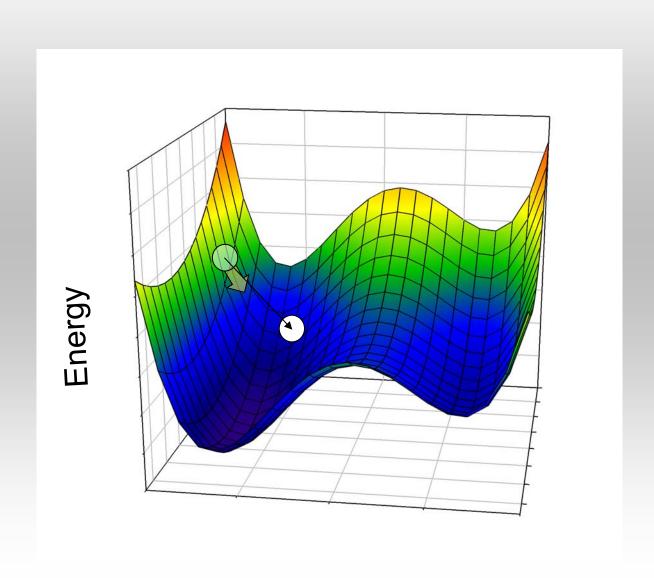


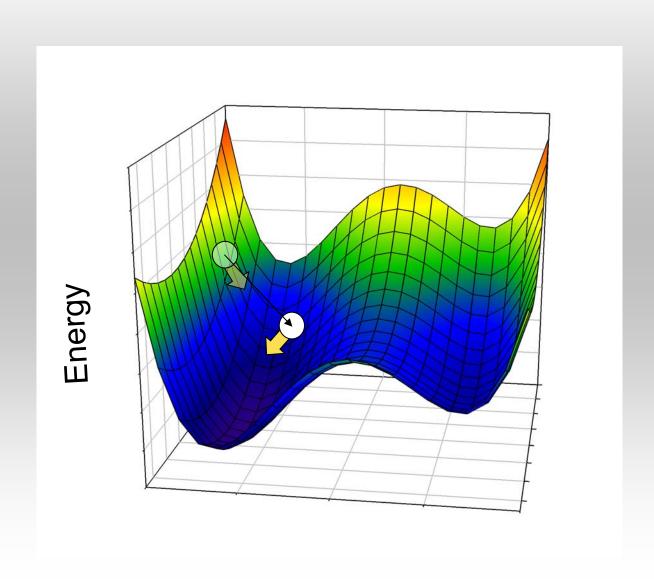
- In many-dimensional systems we use geometry optimization algorithms
- Iterative process
 - Calculate gradient of the PES (AKA forces)
 - Update geometry
 - Check for convergence
 - Repeat if necessary

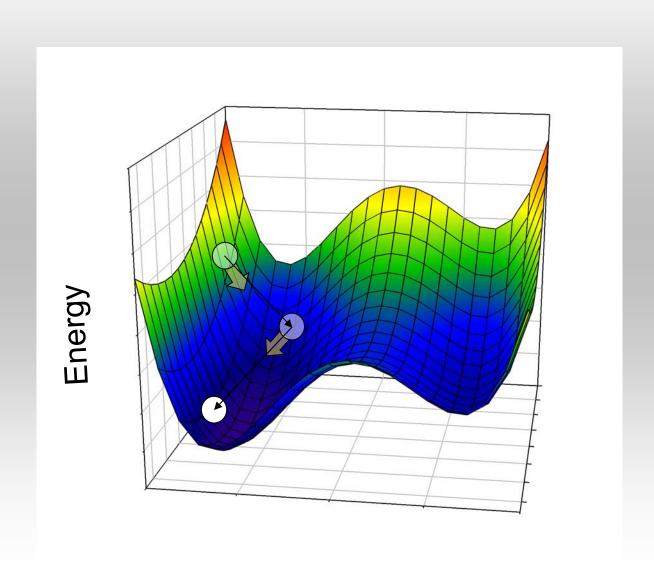


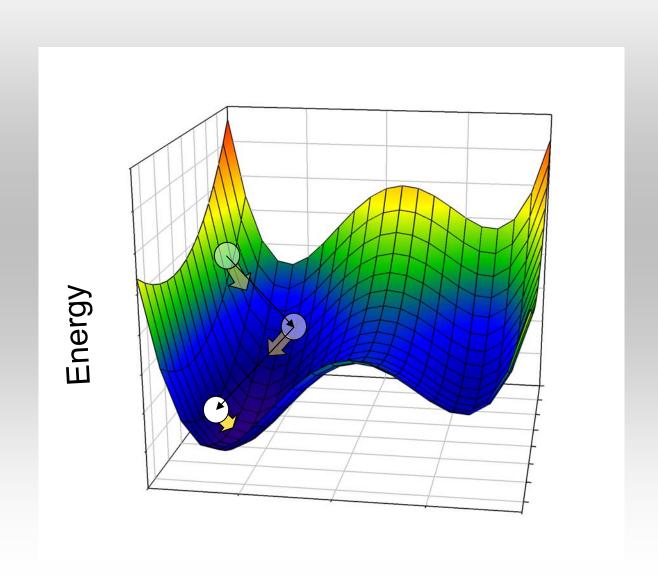


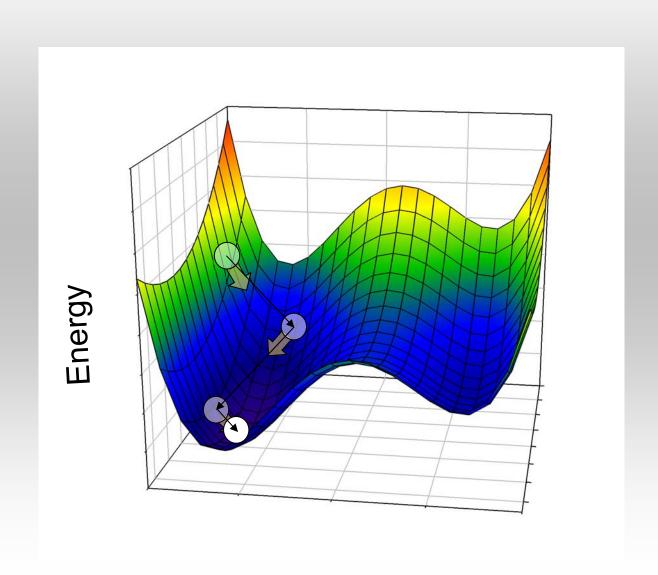










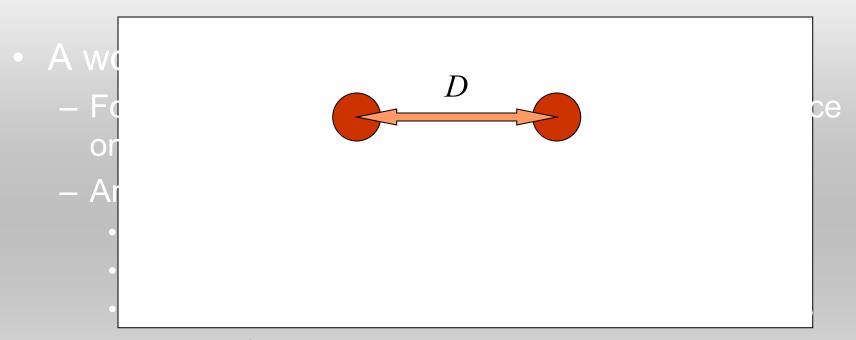


- Convergence criteria
 - The gradient (force) is below a threshold
 - The change in geometry from the previous step is below a threshold
 - The change in energy from the previous step is below a threshold

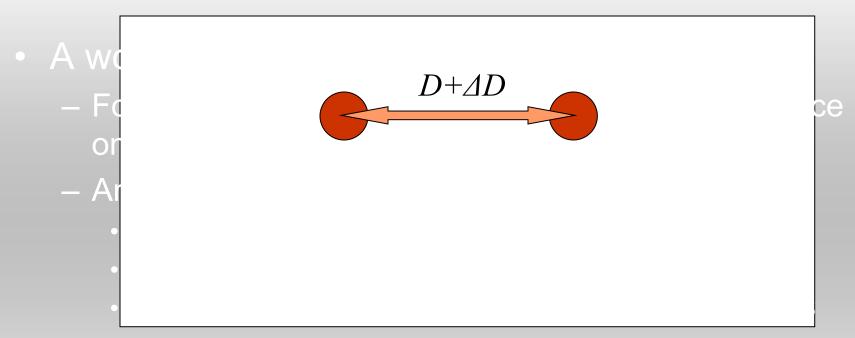
A word on forces

- Forces are a 3N-dimensional vector (x, y, and z force on each atom)
- Analytical forces
 - Actually differentiate
 - (Computational) cost is similar to calculating energy
 - Available for most simpler methods of calculating the PES

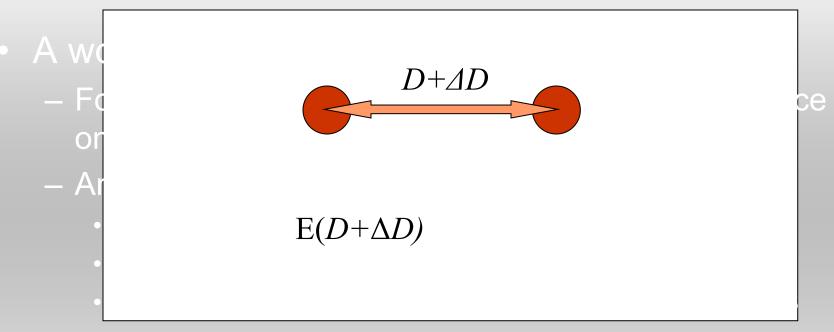
- Compute derivative by finite difference
- Cost is at least 3N times the cost of calculating the energy
- Available for all methods of calculating the PES



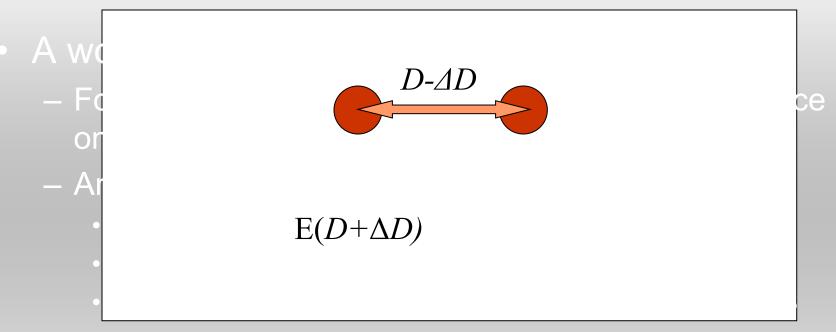
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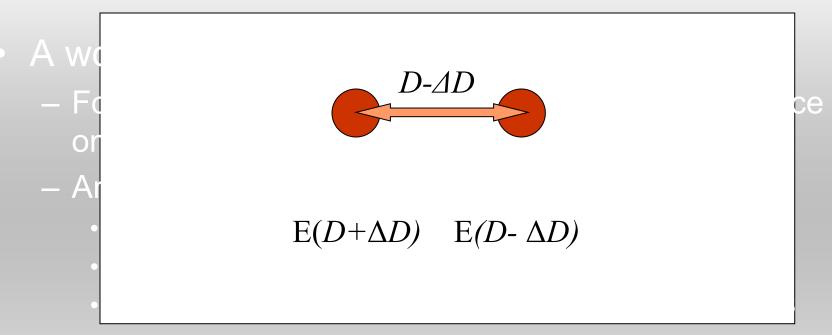
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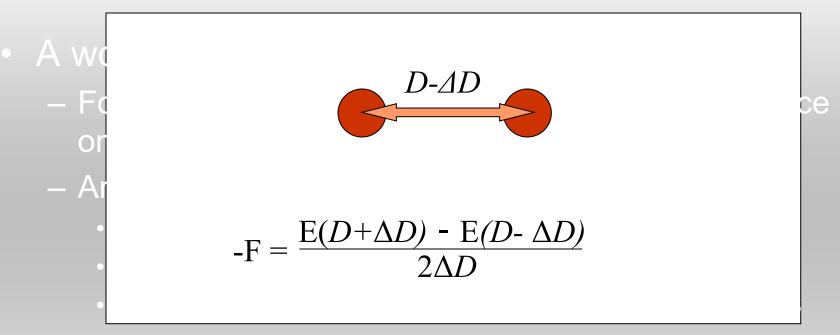
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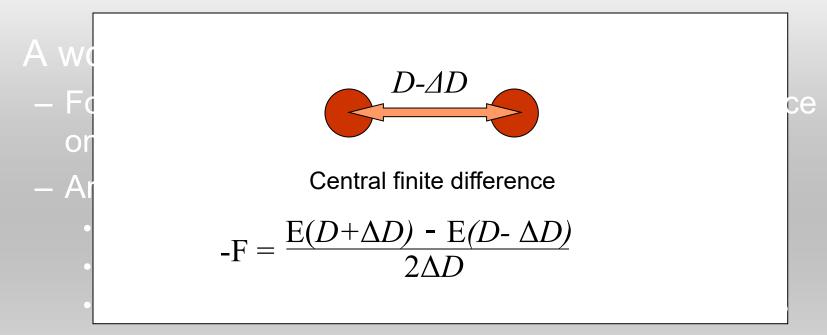
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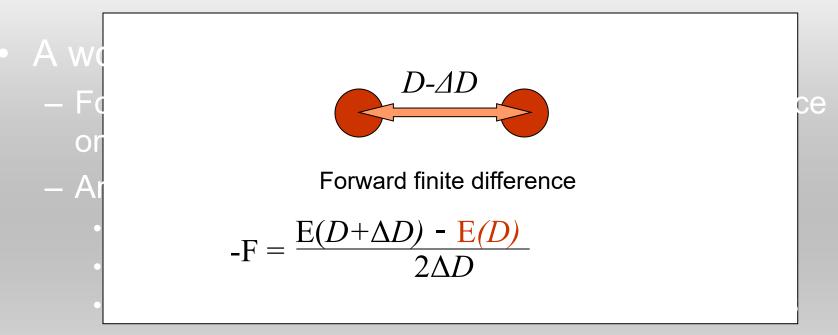


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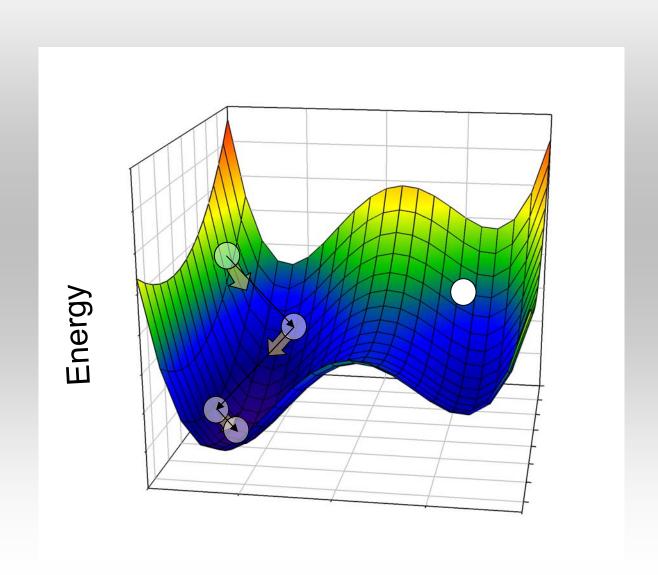
Numerical forces

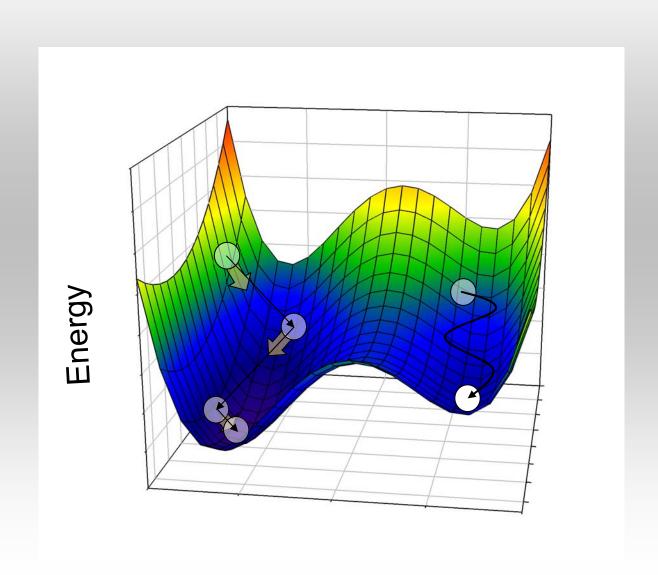
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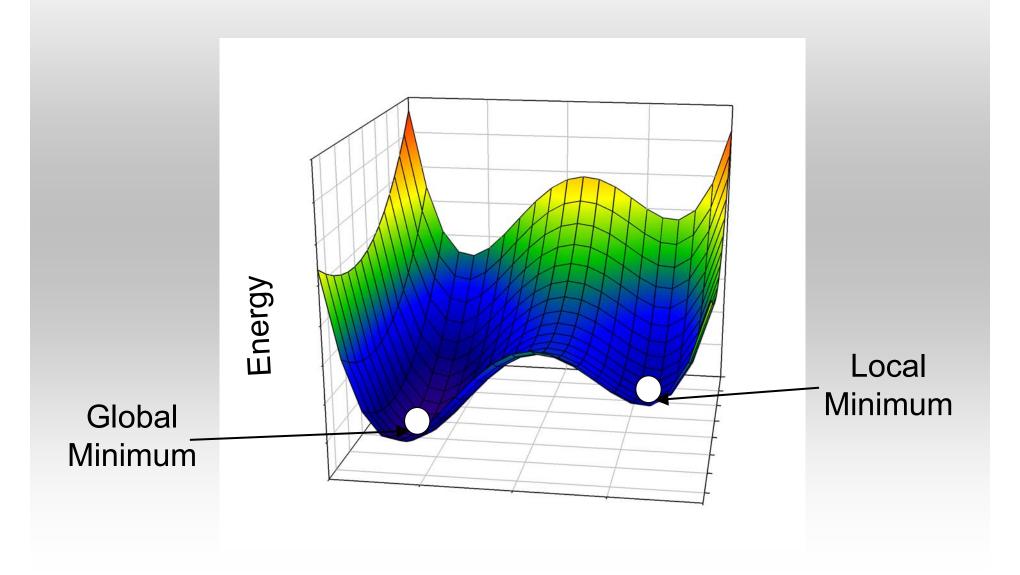


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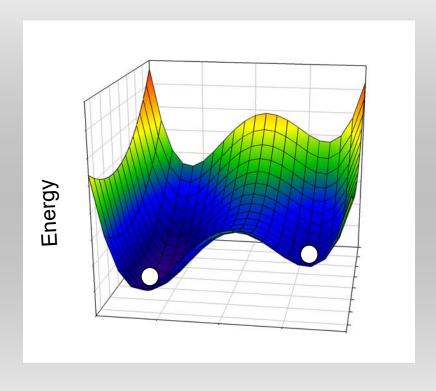
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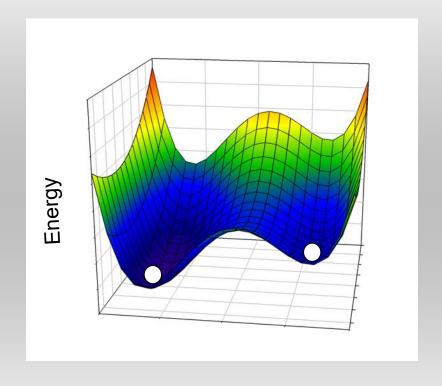




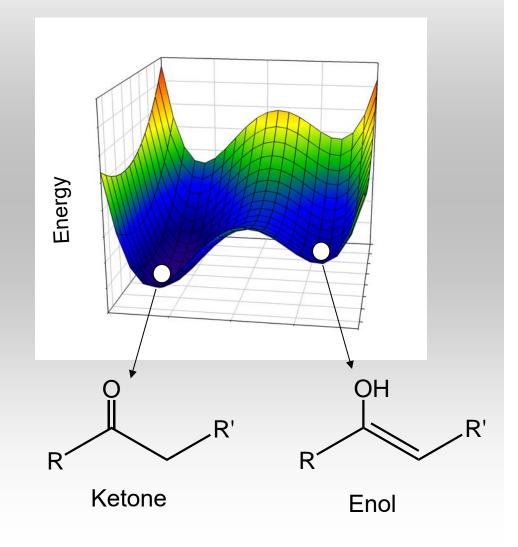
 How do you know you've found the global minimum?



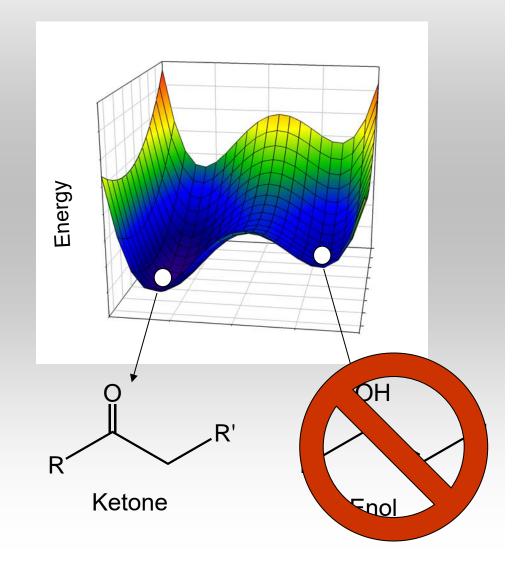
- How do you know you've found the global minimum?
- Which minimum do you want?



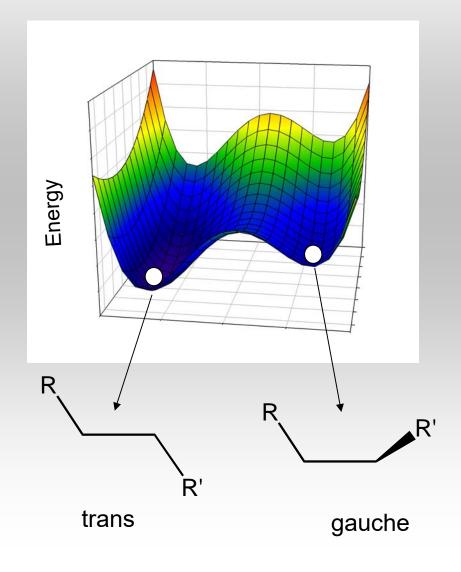
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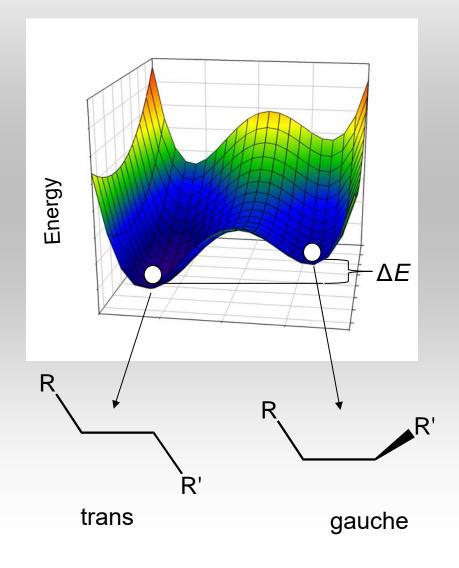
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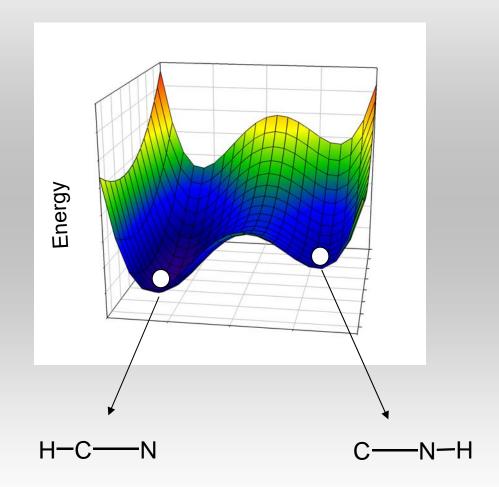
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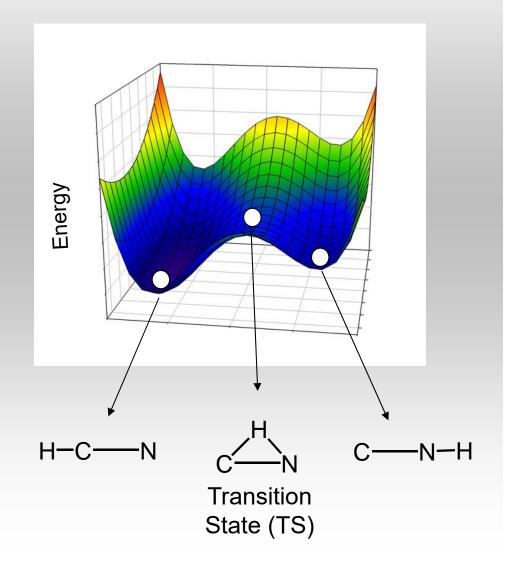
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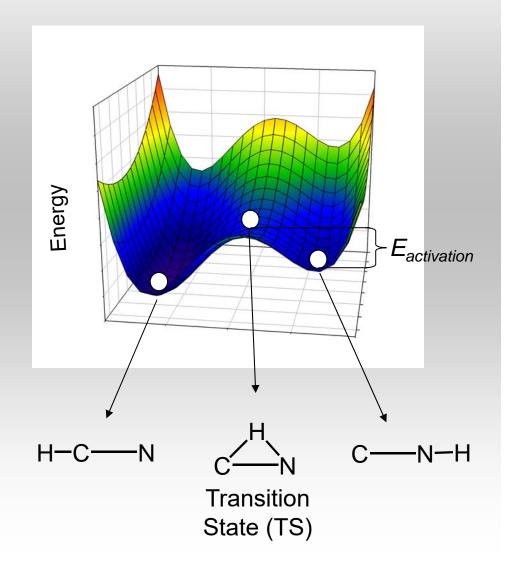
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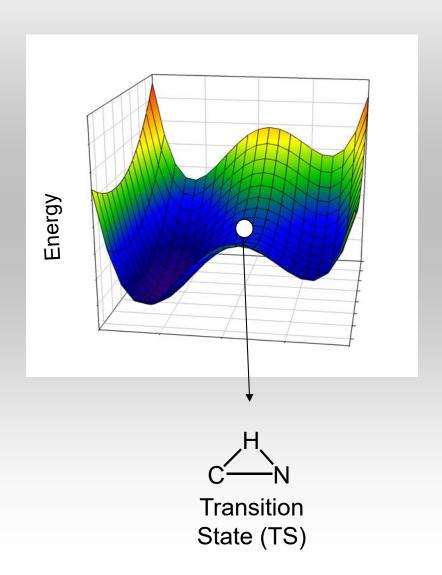
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A Good Transition State is *Really*Hard to Find

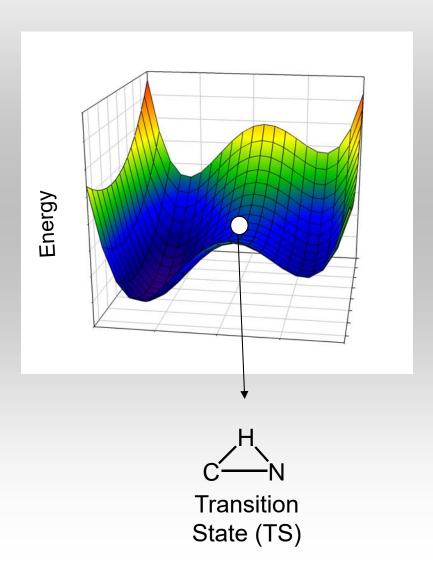
A TS

- is defined as the highest point along the lowest energy path between reactant and product
- is a stable point (F=0)
- has a single
 "imaginary frequency"
 (one normal mode in which the second derivative is negative)

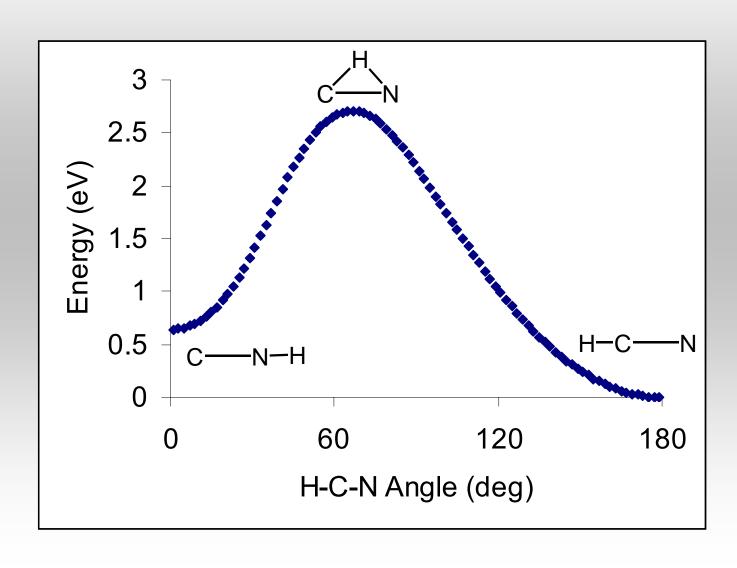


A Good Transition State is *Really*Hard to Find

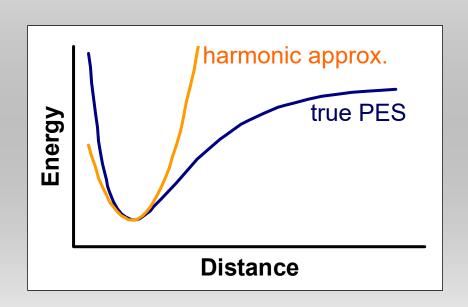
- A TS can be found
 - using one of the TS
 optimization algorithms
 available in most quantum
 chemistry codes
 - by a series of constrained optimizations with only a single reaction coordinate is constrained



A Good Transition State is *Really*Hard to Find



Calculating Vibrational Frequencies



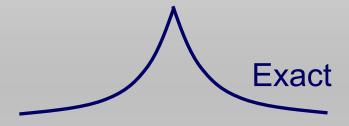
 Calculate the second derivative matrix (Hessian):

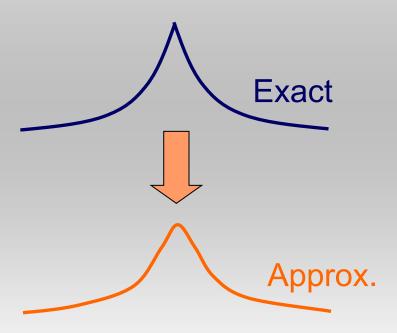
$$\frac{d^2E}{d\mathbf{R}^2}$$

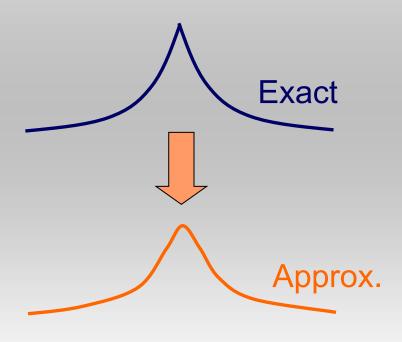
at the minimum

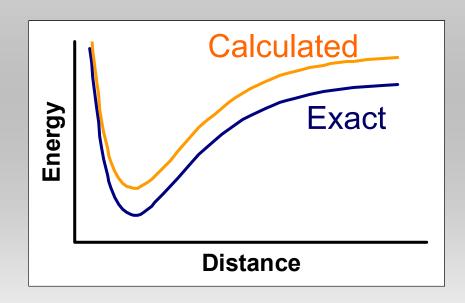
- Diagonalize mass weighted hessian to get "normal modes"
- Calculate frequencies according to the well known solution of the harmonic oscillator problem

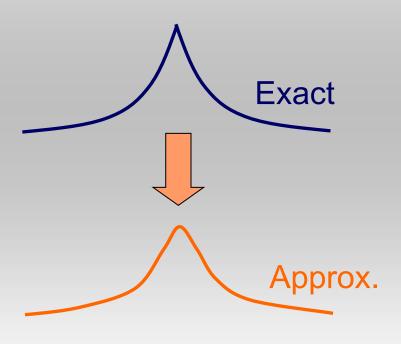
- Two types of errors
 - Constant error an error that is the same for all points on the PES
 - Non-parallelity error an error that varies from point to point of the PES

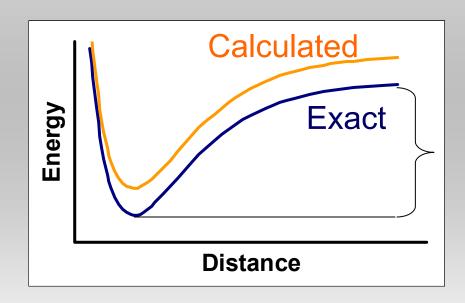


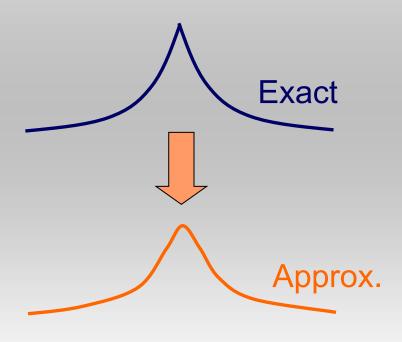


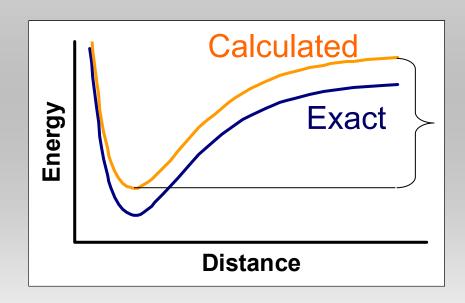




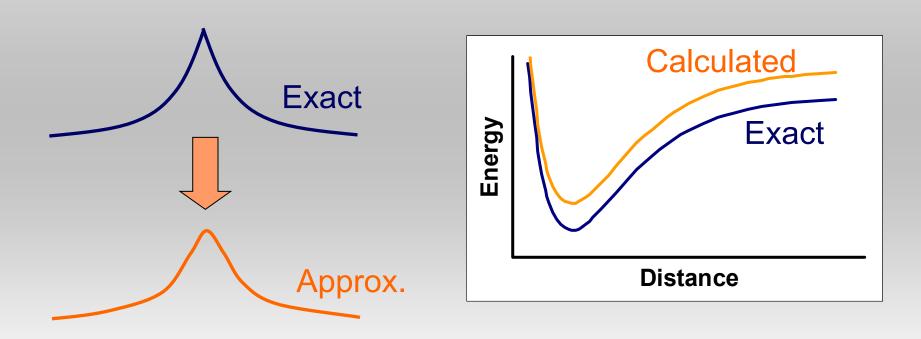






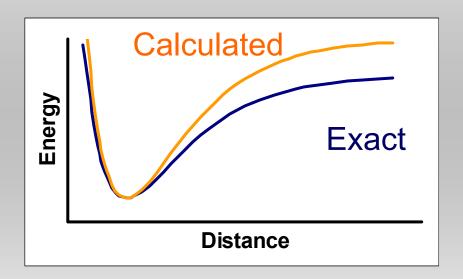


Constant Error

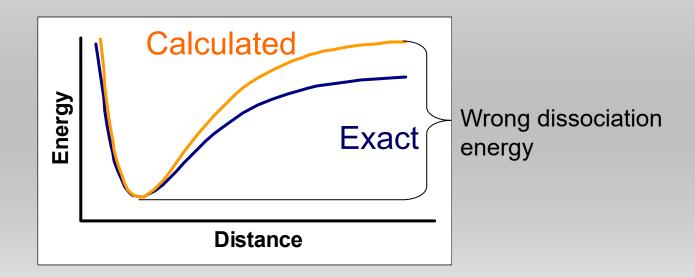


Constant errors in energy usually don't matter

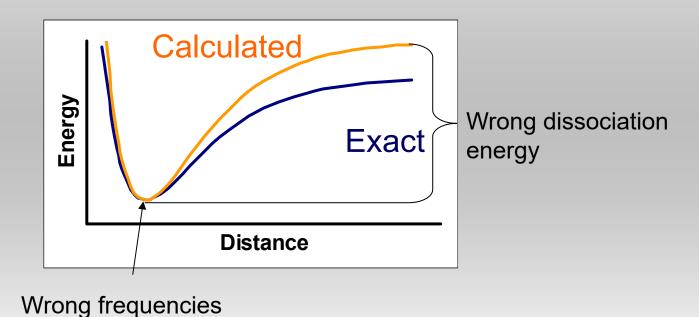
Non-parallelity Error



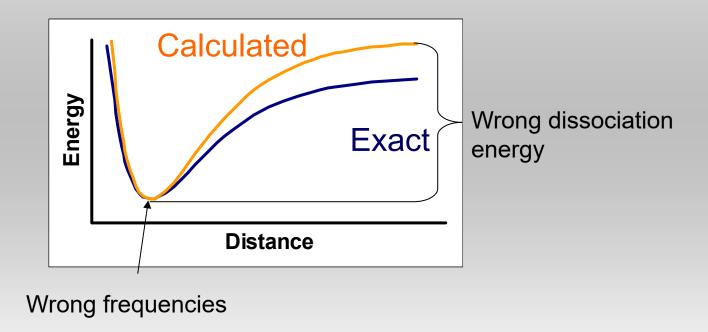
Non-parallelity Error



Non-parallelity Error

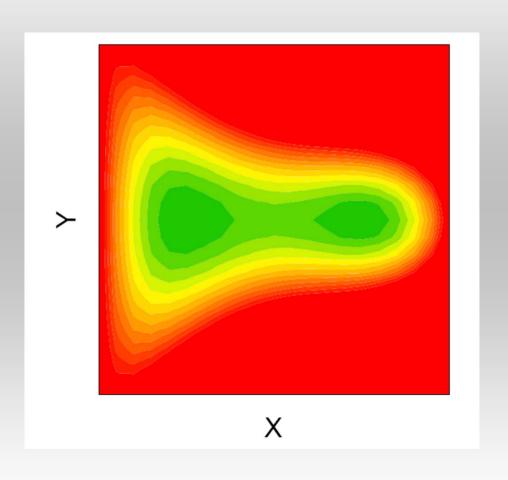


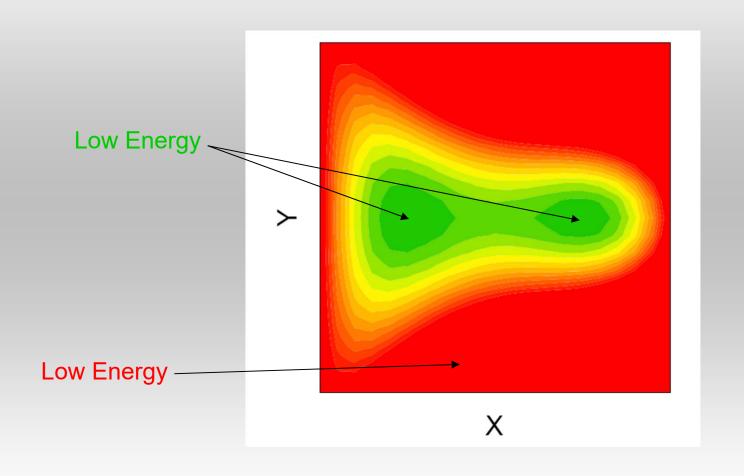
Non-parallelity Error

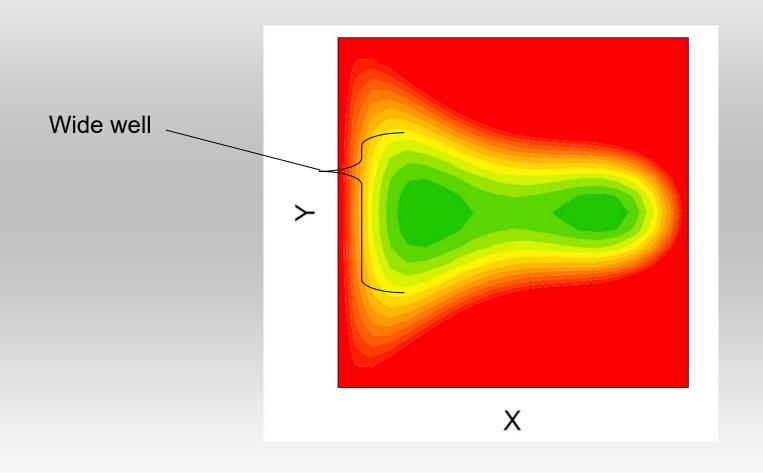


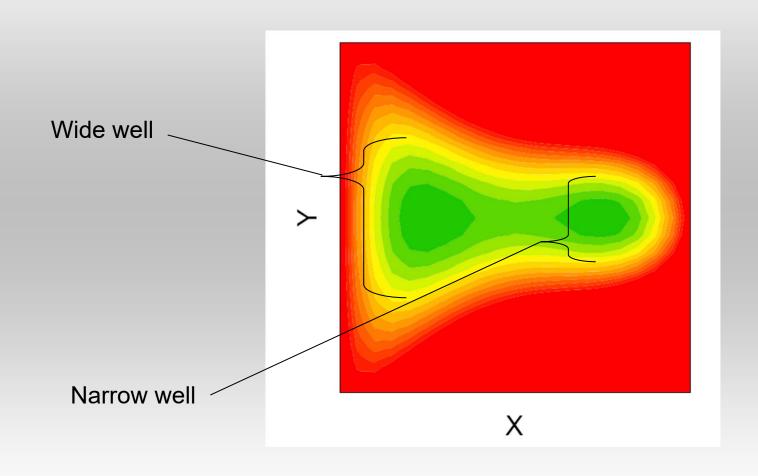
Non-parallelity errors matter!

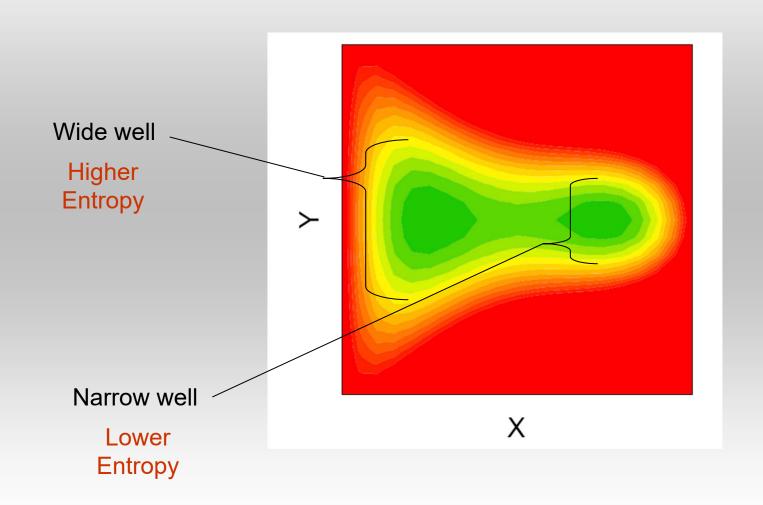
- Free energies, not energies, determine the probability of being in a specific state at finite temperature
- The energy of a molecule at a specific geometry is an energy, NOT a free energy!
- Some additional work is required to get a free energy
 - estimate from frequencies
 - RRKM theory estimate reaction rates from frequencies at the reactant and transition state
 - molecular dynamics

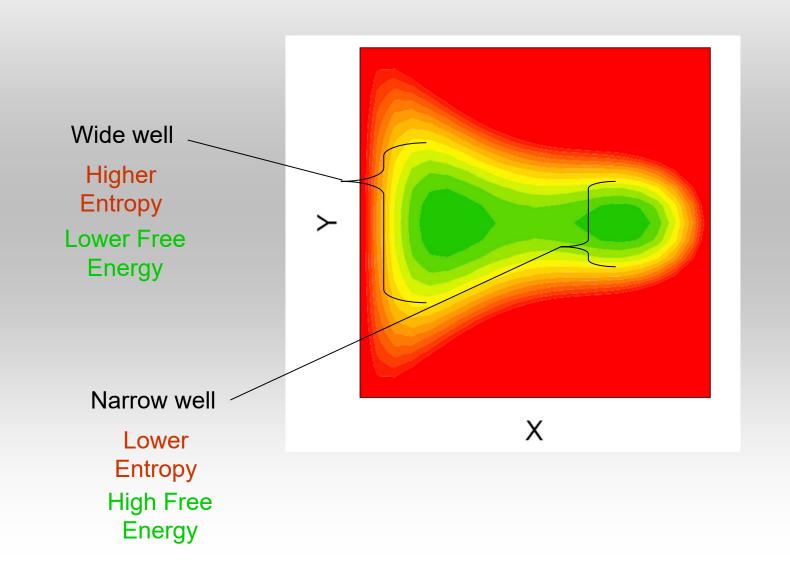




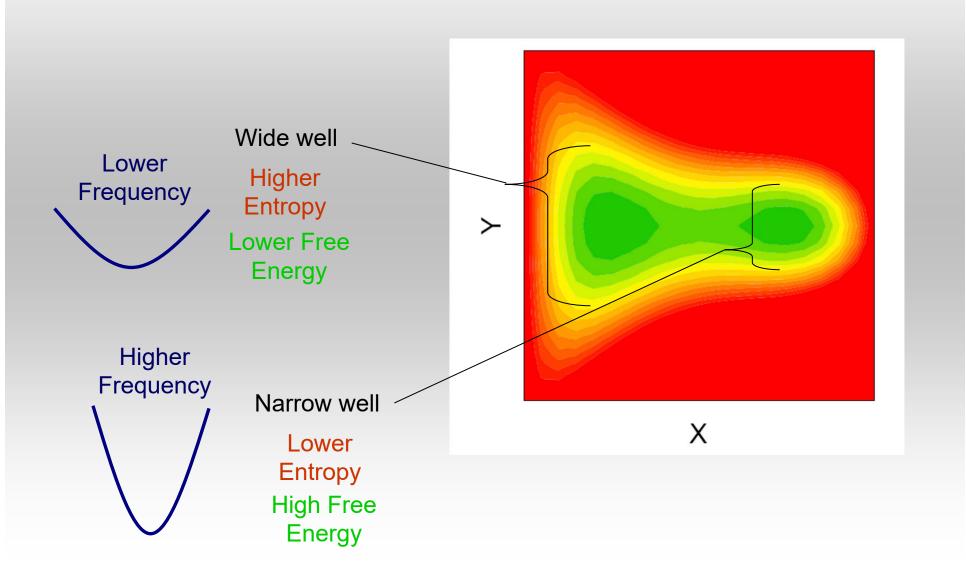








Energy v. Free Energy

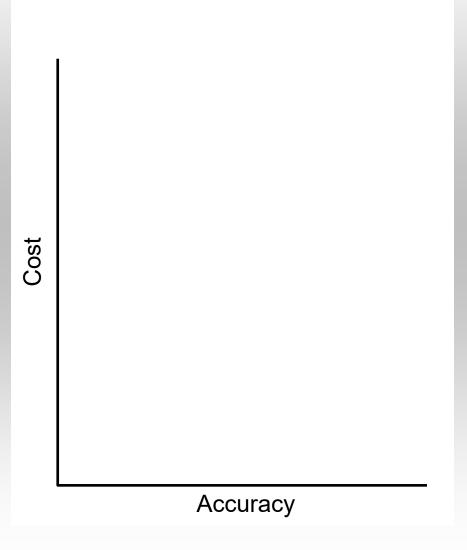


The PES isn't everything

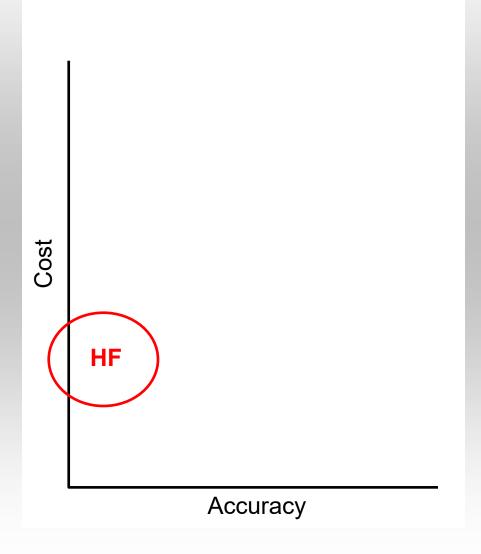
- Electronic properties can be calculated from the wavefunction
 - Dipole, quadrupole... moments
 - Atomic charges
 - Polarizabilities
 - IR intensities
 - NMR chemical shifts

Summary

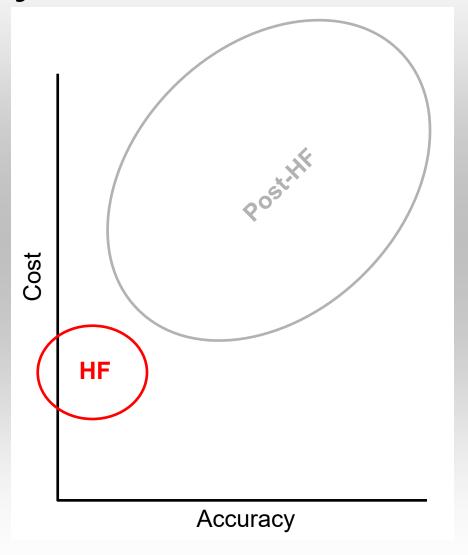
- The PES is the energy as a function of the location of the nuclei
- Stationary points (F=0) on the PES are representative of stable structures and transition states
- Non-parallelity error in the PES is bad; error cancellation is your friend
- We can calculate electronic properties from the wavefunction



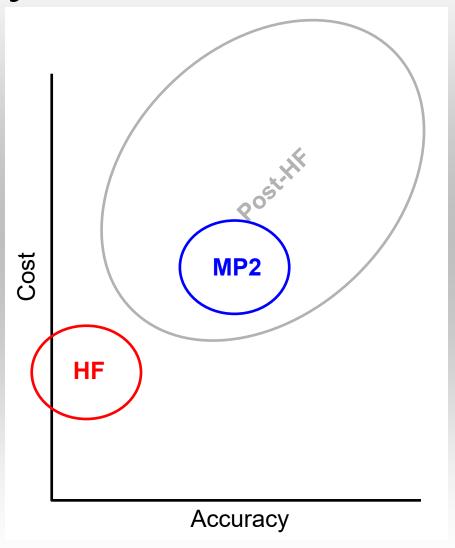
Hartree-Fock (HF)



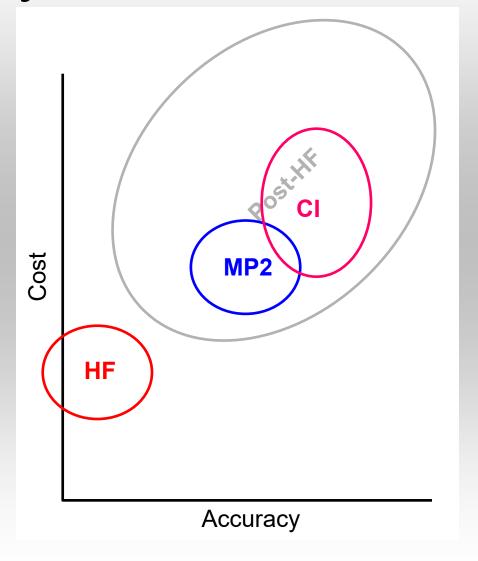
- Hartree-Fock (HF)
- Post-HF



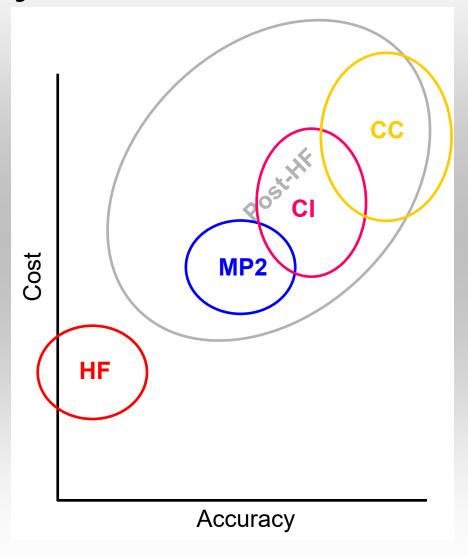
- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)



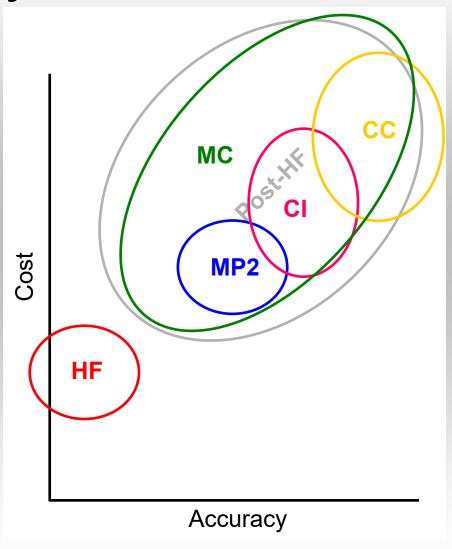
- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)
 - Configuration Interaction (CI)



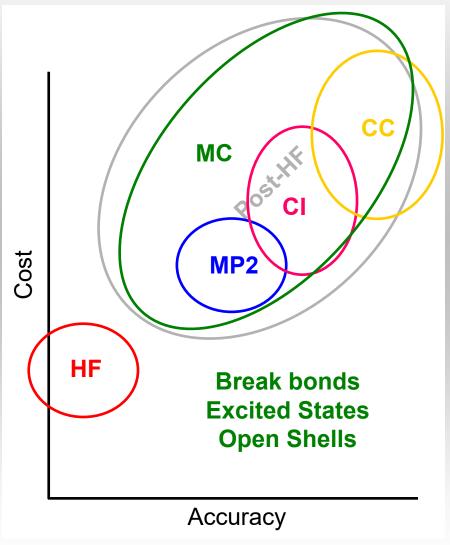
- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)
 - Configuration Interaction (CI)
 - Coupled Cluster (CC)



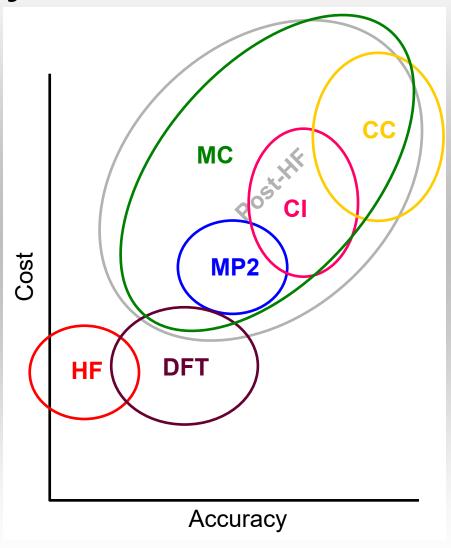
- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)
 - Configuration Interaction (CI)
 - Coupled Cluster (CC)
 - Multiconfigurational Methods (MC)



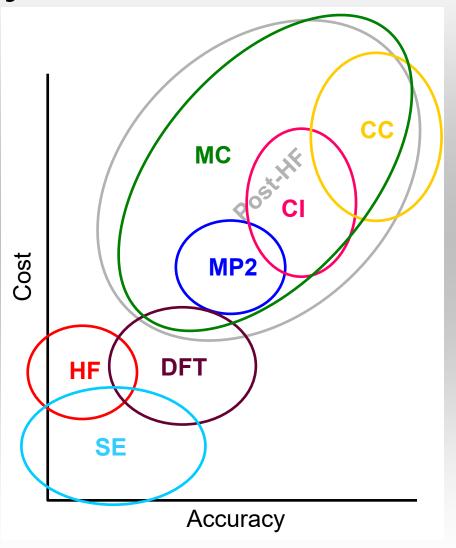
- Hartree-Fock (HF)
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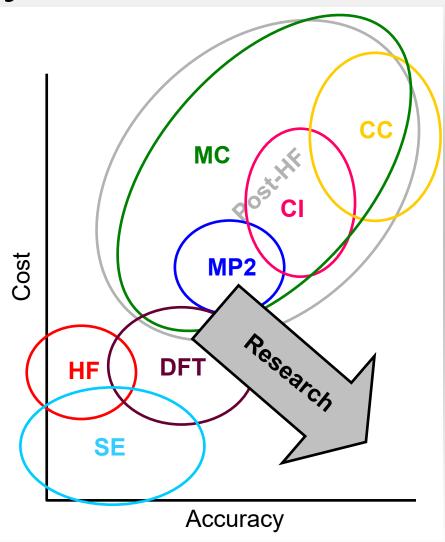
- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)
 - Configuration Interaction (CI)
 - Coupled Cluster (CC)
 - Multiconfigurational Methods (MC)
- Density Functional Theory (DFT)



- Hartree-Fock (HF)
- Post-HF
 - Moller-Plesset Perturb.Theory (MP2)
 - Configuration Interaction (CI)
 - Coupled Cluster (CC)
 - Multiconfigurational Methods (MC)
- Density Functional Theory (DFT)
- Semiempirical (SE)



- Hartree-Fock (HF)
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 - Multiconfigurational Methods (MC)
- Density Functional Theory (DFT)
- Semiempirical (SE)



Coming Up Next

- Linear Algebra Review
- Superposition principle
- Variational Principle
- Basis Sets and the relationship between matrices and operators
- Hartree-Fock Approximation
 - What is it?
 - How is it implemented in the computer?
 - When does it fail and why?