Implementation of static and dynamic semantics for a calculus with algebraic effects and handlers using PLT Redex

(Implementacja statycznej i dynamicznej semantyki rachunku z efektami algebraicznymi i ich obsługą z pomocą biblioteki PLT Redex)

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25 stycznia 2019

 ${\bf English\ abstract}$

Abstrakt w języku polskim

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Introduction

Algebraic effects [8] are an increasingly popular technique of structuring computational effects. They allow for seamless composition of multiple effects, while retaining (unlike monads) applicative style of programs. Coupled with handlers [9] which give programmers ability to interpret effects, they provide a great tool for abstracting over a set of operations which some program may perform and separating this interface from semantics of those operations defined as effect handlers.

As these features are highly desirable many calculi and languages have been developed in order to get them just right; most notable of them being: Koka[7] (featuring type inference, effect polymorphism with row-types and Javascript-like syntax), Links[4] (featuring ML-like syntax, row-typed effect polymorphism and ad-hoc effects) and Eff[1] with implicit effect checking and recent work on direct compilation to OCaml[5]. On more theoretical side, various approaches to semantics of algebraic effects can be spotted in literature, both in respect to type system and run-time semantics. Although most calculi use some form of row-types to implement tracking of effects there are differences in permitted shapes (at most one effect of given type or many effects), whether effects must be defined before use or not and how effects interact with polymorphism and abstraction. At run-time handlers can wrap the captured continuation (giving so-called deep handlers) or not (shallow handlers) and the very act of finding the right handler can be implemented in various ways, mainly depending on some constructs which skip handlers.

All this variety naturally invites Us to experiment with different features and components of a calculus. In this thesis I will build such a calculus, describing and justifying my choices and discussing the trade-offs I faced. In order to rapidly iterate on design and test the calculus, I decided to use PLT Redex library which allows for building language model with executable type system judgments and reduction relation. As such the other goal of my thesis was to assess viability of PLT Redex for development of bigger calculi. To briefly summarize my development, the calculus consists of:

- Curry style type system with ad-hoc effects in the style of *Links*, effect rows based on *Koka* and *lift* construct first shown in [2], implemented as unification based type inference algorithm.
- Executable reduction semantics most similar to system of [2].
- CEK style abstract machine with stack and meta-stack of handlers
- Language front-end which translates human-friendly programs to calculus terms, integrated with Racket environment, which allows for easy experimentation.

The rest of this thesis is structured as follows: in chapter 2 I describe the calculus in greater detail, in chapter 3 I discuss technicalities of implementation, in chapter 4 I summarize the process of integration with Racket environment and chapter 5 is user's manual. In the reminder of this chapter I introduce main topics of this thesis.

1.1 Algebraic effects and handlers

Algebraic effects and handlers are a language level framework which allow for coherent presentation, abstraction, composition and reasoning about computational effects. The key idea is to separate invocation of an effectful operation in an expression from the meaning of such operation. When one invokes an operation, current continuation (up to nearest handler) is captured and passed along with operation's argument to nearest handler. The handler in turn may execute arbitrary expression, using the continuation once, twice, returning a function which calls the continuation or simply ignoring it. This way many control structures can be modeled and generalized by algebraic effects and appropriate handlers. For example, the exceptions can be modeled using a single operation Throw and a handler which either returns the result when computation succeeded or returns default value, ignoring passed continuation.

```
handle e with
| Throw () r -> // return default value
| return x -> x
end
```

From the language design standpoint algebraic effects provide single implementation of various phenomena which may happen during execution of a program, for example mutable state, I/O, environment lookup, exceptions etc. in a sense that every effect is treated the same, the typing rules are defined for invocation of any operation, and handling of any operation. Similarly the operational semantics is also quite simple and succinct thanks to uniform treatment of various effects. This framework

is also extendable. With small additions if can handle built-in effects in addition to user-defined ones.

From the language user perspective algebraic effects provide means of abstraction over effects used in a program. Thanks to easy creation of new effects, one can define special purpose operations and their handlers to better represent domain specific problems while simultaneously using well known effects, defined in standard library. With effects being tracked by the type system, programmers can enforce purity or specific set of used effects at compile-time, or using effect polymorphism they can write reusable functions which abstract over effects which may happen. The separation of definition and implementation of effects allows for various interpretations of operations, similar to a technique of dependency-injection used for example during testing.

1.2 Type inference

Type inference is a technique of algorithmic reconstruction of types for various constructions used in a language. It allows programmers to write programs with no type annotations, which often feel redundant and obfuscate the meaning of a program. The most well known type system with inference is a system for ML family of languages - Haskell, OCaml, SML which infers the types with no annotations whatsoever. Formal type system defines grammar of types consisting of base types (int, bool etc.), type constructors (arrows, algebraic data types) and type variables. The typing rules require types which should be compatible (f.e. formal parameter and argument types) to unify. The key feature of this system is so called let-polymorphism - generalization of types of let-bound variables. This way code reuse can be accomplished without complicating the type system and compromising type safety. The basis of implementation of this system is first order unification algorithm, which syntactically decomposes types and builds a substitution from type variables to types.

1.3 Reduction semantics and abstract machines

Reduction semantics is a format for specifying dynamic semantics of a calculus in an operational style. The basic idea is to first define redexes - expressions which can be reduced, and contexts in which the reduction can happen. Taking λ -calculus with call-by-value reduction order as an example, the only redex is application of a function to value $(\lambda x.e)v$. The possible contexts are: empty context \square or evaluation of operator part of application Ee or evaluation of operand vE. With these possibilities in mind, we will define binary relation \longrightarrow which describes single step of reduction. Such relation can be thought of as a transition system, rewriting terms into simpler ones step by step. There usually are two approaches to definition of such relation:

```
v ::= (\lambda x e) \mid number

e ::= v \mid (e e) \mid x

x ::= variable-not-otherwise-mentioned

E ::= [] \mid (E e) \mid (v E)
```

Figure 1.1: λ -calculus abstract syntax

```
E[((\lambda \times e) \ v)] \longrightarrow E[\text{substitute}[e, \times, v]] \ [\beta]
```

Figure 1.2: λ -calculus reduction relation

- Definition of primitive reduction $(\lambda x.e)v \longrightarrow_p e\{v/x\}$ which operates only on redexes and giving it a closure via following inference rule: $\frac{e \longrightarrow_p e'}{E[e] \longrightarrow E[e']}$, which says that if we can primitively reduce some expression, than we can do it in any context.
- Or definition of \longrightarrow directly, with decomposition of terms on both sides: $E[(\lambda x.e)v] \longrightarrow E[e\{v/x\}]$

where the syntax $e\{v/x\}$ means term e with value v substituted for variable x, and E[e] means some context E with expression e inserted into the hole. For both approaches it is important, that any term can be uniquely decomposed into redex and context, because when it is the case, then the relation is deterministic and gives good basis for formulation of abstract machines, interpreters or transformations to some other intermediate representations.

Abstract machine is a mathematical construction, usually defined as a set of configurations with deterministic transformations, which are computationally simple. The goal for formulation of an abstract machine is to mechanize evaluation of terms while retaining semantics given in more abstract format, f.e. reduction semantics, with the correspondence being provable[3]. As an example I will show a CEK-machine for the λ -calculus defined earlier. The name CEK comes from C ommand, E nvironment and E ontinuation. The machine configuration is a triple (e, ρ, κ) where e is an expression which is decomposed or reduced, ρ is an environment mapping variables to values, and the last component E is a continuation stack, which determines what will happen with value, to which first component eventually reduces. Thanks to the environment we no longer have to explicitly perform substitution, leading to more machine friendly and efficient implementation. Given an initial state, the machine can then repeatedly apply transformation relation, either looping, arriving at a final value, or getting stuck.

1.4 PLT Redex

 λ -calculus reduction example

Figure 1.3: λ -calculus example reduction sequence

1.4. PLT REDEX

CEK-machine for λ -calculus

Figure 1.4: λ -calculus abstract machine

The calculus

The calculus implemented in this thesis is based on lambda calculus with call-by-value semantics. This choice follows other calculi which allow for computational effects, because fixed evaluation order is essential to obtaining sane program semantics. Inspired by Links [4] the operations are truly ad-hoc meaning that they don't have to be declared before usage. Moreover the calculus requires no type annotations whatsoever in spirit of ML family of languages while still tracking effects which occur in a program. The λ -calculus is extended with base types (Numeric and Boolean) with corresponding operations and literals, conditional expression and recursive functions. Effectful operations are invoked similarly to function calls (although they are a distinct syntactic category) and are handled using handle construct, they may also be lifted with lift syntactic form. Usually the operation will be handled by the closest handler with appropriate case, unless there is a lift for this operation in between, each one causing operation to skip one handler. Abstract syntax of the calculus is presented in Figure 2.1

2.1 Static semantics

The type system is based on Koka (Leijen's style of row types[6]), Links [4] (ad-hoc operations) and Biernacki et al. [2] (lift construct) systems. Initially I implemented a variant of System-F extended with row-types but it proved to be a bit of a mouthful to write even simplest programs. Moreover the Redex's facilities for automatic testing were not able to generate well typed terms, so some type inference was inevitable. To limit the amount of work I decided to present the calculus in Curry style, with typing relation inferring the type for unannotated terms. Building on well known foundations, types are inferred via first order unification. The system does not feature polymorphism in first class fashion, as there is no rule where types are generalized, but I believe it to be a straightforward addition, following the Koka [7] calculus. Still, after inferring type of an expression, we can see which unification variables are left abstract and could be generalized. There are two main features

```
b ::= true \mid false
     m ::= number
      v ::= m \mid b \mid (\lambda x e) \mid (\operatorname{rec} x x e)
 prim ::= + | - | * | == | <= | >=
      e ::= v | x | (e e) | (if e e e) | (fix e)
            | (op e) | (handle e hs ret) | (lift op e) | (prim e ...)
    hs ::= ((op_{!1} hexpr) ...)
hexpr ::= (x_{!\_1} x_{!\_1} e)
    ret ::= (return x e)
      h ::= (op \ hexpr)
      t ::= Int | Bool | (t \rightarrow row \ t) \ | \ (t \Rightarrow t) \ | \ row \ | \ a
  row ::= (op \ t \ row) \mid a \mid \cdot
      x ::= (variable-prefix v:)
      a ::= (variable-prefix t:)
    op ::= (variable-prefix op:)
      \Gamma ::= (x t \Gamma) \mid \cdot
      E ::= [] | (E e) | (v E) | (prim E e) | (prim v E) | (if E e e)
            |(op E)| (handle E hs ret) | (lift op E)
      S ::= (a \ t \ S) \mid \cdot
  N, n ::= natural
   SN ::= (S N)
```

Figure 2.1: Abstract syntax

differentiating this system from Koka's; firstly effects need not be defined before use, their signature is inferred as with any other construction; secondly the system is algorithmic, with rules explicitly encoding a recursive function which can infer the type of an expression.

2.1.1 Row-types

Row types as described in [6]

2.1.2 Type inference

The judgment $\Gamma \mid [S_1 N_1] \vdash e : t! row \mid [S_2 N_2]$ asserts that in typing context Γ under type substitution S_1 , with name supply state N_1 expression e has type t with effects row under type substitution S_2 and with name supply state S_2 . Algorithmically this judgment infers a type and effect row, and calculates new substitution, given typing environment, current substitution and an expression. As in ML languages only simple types can be inferred, along with effect rows.

Base rules for constants and variable lookup are straightforward, each introducing fresh effect row variable. To check λ expression, we first introduce fresh type variable, and then check the body in extended environment. The arrow gets annotated with effects which may occur during evaluation of the body and the λ abstraction itself is returned with fresh effect row. The recursive function checking is similar to normal functions. First variable denotes function itself, while second

```
fresh-row[N_1, row, N_2]
                                                                            \Gamma \mid [S N_1] \vdash b : Bool ! row \mid [S N_2]
                                                                                                  fresh-row[N_1, row, N_2]
                                                                             \Gamma \mid [S N_1] \vdash m : Int ! row \mid [S N_2]
                                                                    lookup[\Gamma, x, t] fresh-row[N_1, row, N_2]
                                                                                \Gamma \mid [S N_1] \vdash x : t ! row \mid [S N_2]
                                                           fresh-var[N_1, t_1, N_2] fresh-row[N_2, row_2, N_3]
                                                                         (x t_1 \Gamma) \mid [S_1 N_3] \vdash e : t_2 ! row_1 \mid SN
                                                     \Gamma \mid [S_1 N_1] \vdash (\lambda \times e) : (t_1 \rightarrow row_1 t_2) ! row_2 \mid SN
                                       fresh-arr[N_1, t_1, ->, row_1, t_2, N_2]
                                                                                                                                                    fresh-row[N_2, row_2, N_3]
                                           (x_f(t_1 \to row_1 t_2) (x_a t_1 \Gamma)) | [S_1 N_3] \vdash e : t ! row | SN_1
                                                                    SN_1 row_1 \sim row SN_2 SN_2 t_2 \sim t SN_3
                                             \Gamma \mid [S_1 N_1] \vdash (\text{rec } x_f x_a e) : (t_1 \rightarrow row_1 t_2) ! row_2 \mid SN_3
                   \Gamma \mid SN_1 \vdash e_1 : t_a \mid row_a \mid SN_2 unify-arr [SN_2, t_a, t_1, -\rangle, row_1, t_2, SN_3]
                                                                                     \Gamma \mid SN_3 \vdash e_2 : t_3 ! row_2 \mid SN_4
                       SN_4 t_1 \sim t_3 SN_5 SN_5 row_1 \sim row_2 SN_6 SN_6 row_1 \sim row_a SN_7
                                                                             \Gamma \mid SN_1 \vdash (e_1 e_2) : t_2 ! row_2 \mid SN_7
                                                            check-prim[\Gamma, SN_1, prim, (e ...), t, row, SN_2]
                                                                        \Gamma \mid SN_1 \vdash (prim \ e \ ...) : t ! row \mid SN_2
                                   \Gamma \mid SN_1 \vdash e_{cond} : t_{cond} ! row_{cond} \mid SN_2 \qquad SN_2 t_{cond} \sim \mathsf{Bool} \ SN_3
             \Gamma \mid SN_3 \vdash e_{\textit{then}} : t_{\textit{then}} ! \; row_{\textit{then}} \mid SN_4 \qquad \Gamma \mid SN_4 \vdash e_{\textit{else}} : t_{\textit{else}} ! \; row_{\textit{else}} \mid SN_5 
     SN_5 t_{then} \sim t_{else} SN_6 SN_6 row_{cond} \sim row_{then} SN_7 SN_7 row_{then} \sim row_{else} SN_8
                                                       \Gamma \mid SN_1 \vdash (\text{if } e_{cond} \ e_{then} \ e_{else}) : t_{then} ! \ row_{then} \mid SN_8
\Gamma \mid SN_1 \vdash e : t_1 ! row_1 \mid [S_1 N_1] fresh-row [N_1, row_2, N_2]
                                                                                                                                                                                                           fresh-var[N_2, t_2, N_3]
                                                                    [S_1 N_3] (op (t_1 \Rightarrow t_2) row_2) ~ row_1 SN_2
                                                                              \Gamma \mid SN_1 \vdash (op \ e) : t_2 ! row_1 \mid SN_2
                                               \Gamma \mid SN_1 \vdash e : t \mid row \mid [S_1 N_1] fresh-var[N_1, a, N_2]
                                                            \Gamma \mid SN_1 \vdash (\text{lift op } e) : t ! (op a row) \mid [S_1 N_2]
                 \Gamma \mid SN_1 \vdash e : t_1 ! row_1 \mid SN_2 \quad (x t_1 \Gamma) \mid SN_2 \vdash e_{ret} : t_{ret} ! row_{ret} \mid SN_3
  infer-handlers \llbracket \Gamma, SN_3, t_{ret}, hs, row_{out}, row_{handled}, SN_4 \rrbracket SN_4 row_{out} \sim row_{ret} SN_5
                                                                                          SN_5 row_1 \sim row_{handled} SN_6
                                          \Gamma \mid SN_1 \vdash (\text{handle } e \ hs \ (\text{return } x \ e_{ret})) : t_{ret} ! \ row_{out} \mid SN_6
                                                                                         Figure 2.2: Type system
```

it's argument. Accordingly the environment gets extended with functional type $t_1 \to row_1 t_2$ to check the body of the function, and afterwards the result type of body t gets unified with the result type of function t_2 , same with effect row. Whole function, as it is a value, is returned with fresh effect row. The application requires expression at function position to be of functional type and parameter type to unify with argument type. All effect rows (from evaluation of function value, argument value and function body) must unify as well. The checking of primitive operation gets deferred to auxiliary judgment, which checks arity and argument types, returning result type and usually fresh effect row. Conditional expression requires the condition expression to be of type Bool and types of two branches to unify. As usual all effect rows must also unify. Operation invocation requires the effect row to contain operation op whose input type is the inferred type for e and output type being fresh. Operation lifting prepends fresh op to the effect row of e. Finally, to check handle expression we first infer the type of enclosed expression e, then in environment extended with e's type t_1 we infer return expression's type t_ret . Helper judgment infer-handlers returns the result effect row of handlers row_{out} and row marking handled effects $row_{handled}$ whose tail is the same as result's. By unifying result row with return row and handled row with row_1 we ensure that effects which may occur during handling of operations, at return and leftovers from the inner expression are all accounted for.

2.1.3 Effect handlers

2.2 Dynamic semantics

The dynamic semantics is defined in the format of contextual reduction semantics. With syntax of execution contexts in Figure 2.1 we see that they describe computation in call-by-value order, with expressions in applications (both normal and primitive) being evaluated left to right. All rules perform reduction in some context E leaving it unchanged. First rule describes standard β reduction via substitution of argument for variable in function body, while the second β reduction of recursive function, where first we substitute the function for function variable and then substitute the argument. Next rule deals with built-in primitive operations, delegating to auxiliary function prim-apply which pattern matches on prim and performs appropriate operation. Next two rules perform choice of correct branch in conditional expression depending on value of condition. The rule lift-compat returns value from lift expression, leaving it unchanged. The rule handle-return handles the case when inner expression of handle expression evaluates to a value, which means we have to evaluate return clause - by substituting the result value for x, and plugging this expression into evaluation context E. The last rule handle-op describes the behavior when an expression calls some operation. Before we can define it's behavior we must define notion of freeness. The judgment free[op, E, n] asserts that operation op is

```
E[((\lambda \times e) \ v)] \longrightarrow E[\text{substitute}[e, x, v]]
                                                                                                                                              [β-λ]
                   E[((\text{rec } x_f x_a \ e) \ v)] \longrightarrow E[\text{substitute}[\text{substitute}[e, x_f, (\text{rec } x_f x_a \ e)]], x_a, v]] \ [\beta\text{-rec}]
                        E[(prim \ v_1 \ v_2)] \longrightarrow E[prim-apply[prim, v_1, v_2]]
                                                                                                                                              [prim-op]
                       E[(\text{if true } e_1 \ e_2)] \longrightarrow E[e_1]
                                                                                                                                             [if-true]
                      E[(\text{if false } e_1 \ e_2)] \longrightarrow E[e_2]
                                                                                                                                             [if-false]
                             E[(\text{lift } op \ v)] \longrightarrow E[v]
                                                                                                                                              [lift-compat]
  E[(\text{handle } v \text{ } hs \text{ } (\text{return } x \text{ } e))] \longrightarrow E[\text{substitute}[e, x, v]]
                                                                                                                                              [handle-return]
E_1[(\text{handle } E_2[(op \ v)] \ hs \ ret)] \longrightarrow E_1[\text{substitute}[\text{substitute}[e, x_1, v]],
                                                                                                                                              [handle-op]
                                                                                  x_2, (\lambda v:z (handle E_2[v:z] hs ret))
                                    where free [op, E_{in}, 0], get-handler [op, hs, (x_1 x_2 e)], v:z fresh
```

Figure 2.3: Reduction relation

n free in evaluation context E, which means that it will be handled by n+1-st handler outside the context E [2]. With this definition in mind, we can see that to handle an operation we must find 0-free inner context E_2 which is directly surrounded by handle expression which has a case for op. Then we substitute value v for first variable of operation handler and the inner context E_2 surrounded with the very same handler (the continuation delimited by the handler) closed in a lambda for the second argument. This way the operation handler can resume the evaluation of expression which invoked the handled operation.

2.3 Abstract machine

Implementation

- 3.1 PLT Redex
- 3.2 Typing relation
- 3.3 Unification
- 3.4 Reduction relation
- 3.5 Automatic testing

The Racket environment

- 4.1 Front-end
- 4.2 Back-end

User's manual

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