Spectral Properties of Bounded Linear ${\it Operators}^1$

Spectral	Space			Operator		
Properties	NLS	Banach	Hilbert	$T = T^*$	$T \ge 0$	T Compact
$\rho(T)$ open						
$\sigma(T) \subset \overline{B_{\ T\ }(0)}$						
$\sigma(T)$ compact						
$\lambda \in \rho(T) \Leftrightarrow T_{\lambda} \text{ bdd below}$						
$\sigma_r(T) = \varnothing$						
$\sigma_p(T) \subset \sigma(T) \subset [r, R] \subset \mathbb{R}$						
$ T = \max\{ r , R \}$						
$r, R \in \sigma(T)$						
$\sigma(T) \ge 0$						
$\exists ! \ T^{1/2} \ge 0$						
$\sigma_p(T)$ countable, can only $\to 0$						
$\lambda \neq 0 \Rightarrow \dim N(T_{\lambda}) < \infty$						
$0 \neq \lambda \in \sigma(T) \Rightarrow \lambda \in \sigma_p(T)$						
\exists ON basis of e-vectors, $Tx = \sum_{\alpha} \lambda_{\alpha} \langle x, u_{\alpha} \rangle u_{\alpha}$						
$\exists ! \text{ compact } T^{1/2} \geq 0$						

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