Ideological Bias and Trust in Information Sources

Matthew Gentzkow* — *Stanford and NBER*
Michael B. Wong — *HKU*
Allen T. Zhang — *Harvard*

February 2023

Abstract

We study the role of endogenous trust in amplifying ideological bias. Agents in our model seek to learn a sequence of states using information from sources whose accuracy is *ex ante* uncertain. Agents learn these accuracies by comparing their own reasoning about the states based on introspection or direct experience to the sources' reports. Small biases in this reasoning can cause large ideological differences in the agents' trust in information sources and their beliefs about the states, and may lead agents to become overconfident in their own judgment. Disagreements can be similar in magnitude whether agents see only ideologically aligned sources or a diverse range of sources.

Keywords: Bias, trust, polarization, media bias
JEL: D83, D85, L82

*Email: gentzkow@stanford.edu, mbwong@hku.hk, allenzhangtl@gmail.com. We thank Daron Acemoglu, Isaiah Andrews, David Autor, Abhijit Banerjee, Ben Golub, Jesse Shapiro, Francesco Trebbi, David Ritzwoller, Ashesh Rambachan and seminar participants at UC Berkeley, Harvard, the University of Michigan, MIT, the University of Southern California, Stanford, and Stony Brook for helpful comments and suggestions. We also thank our predocs and research assistants for their dedication and support. We acknowledge funding from the Stanford Institute for Economic Policy Research (SIEPR), the Toulouse Network for Information Technology, the Knight Foundation, the Kuok Foundation, the National Science Foundation, Office of Naval Research Army Research Office MURI Grant W911NF-20-1-0252, and the Stanford Cyber Initiative. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Office or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation herein.

1 Introduction

Ideological divisions in society often seem intractable, with those on either side persistently disagreeing about objective facts. In recent years, for example, fervent debates over the validity of global warming, evolution, and vaccination have persisted long after the establishment of a scientific consensus (McIntyre 2018; Jerit and Zhao 2020). Partisans also disagree about which sources can be trusted to provide reliable information about such facts. In the United States, for instance, 75 percent of conservative Republicans say they trust news and information from Fox News, while 77 percent of liberal Democrats say they distrust it (Pew Research Center 2020). Such divisions have deepened even as new media technologies have made information more widely and cheaply available than ever before. The information age has, paradoxically, produced what has been dubbed a "post-truth" era (Keyes 2004).

Such patterns seem at odds with the prediction of many Bayesian (e.g., Blackwell and Dubins 1962) and non-Bayesian (e.g., DeGroot 1974) learning models in which widespread availability and distribution of information leads all agents' beliefs to converge to the truth. Many possible alternatives have been proposed. However, such accounts generally require that individuals have substantial psychological biases towards cognitive consistency or confirmation (e.g., Lord, Ross, and Lepper 1979; Cotton 1985; Rabin and Schrag 1999; Baliga, Hanany, and Klibanoff 2013) or have limited memory or attention (e.g., Fryer, Harms, and Jackson 2019; Che and Mierendorff 2019).

In this paper, we explore a different possibility, which is that rational Bayesian inference may magnify the influence of even small cognitive biases when agents are uncertain which sources they can trust. Building on insights by Acemoglu, Chernozhukov, and Yildiz (2016) and Sethi and Yildiz (2016), among others, we show that small biases may lead to substantial and persistent divergence in both trust in information sources and beliefs about facts, with partisans on each side trusting unreliable ideologically aligned sources more than accurate neutral sources, and also becoming overconfident in their own judgment. Consistent with evidence suggesting that the magnitude of selective exposure has generally been limited (Gentzkow and Shapiro 2011; Flaxman, Goel, and Rao 2016; Barberá 2020), these patterns arise whether agents selectively view only ideologically aligned sources or are exposed to a diverse range of sources. Increasing the number of available information sources in such a setting may deepen rather than mitigate ideological differences.

Agents in our model wish to learn about a sequence of unobserved states $\omega_t \sim N(0,1)$, which are drawn independently in each period t. We think of each period's ω_t as capturing a distinct item discussed in the news. In one period this might be the effectiveness of masks at stopping disease transmission, in the next period the extent of fraud in a recent election, in a third period the magnitude of global warming due to human activity, and so on. In each period, each agent i observes a normally distributed signal s_{jt} correlated with ω_t from one or more information sources j. We refer to the correlation between s_{jt} and ω_t as the *accuracy* of source j. We analyze two scenarios, one in which the agent observes exactly one source j in each period (she "single-homes" in the language of Rochet and Tirole 2003), and another in which she observes all sources j in each period (she "multi-homes").

To introduce a political dimension to the model, we assume that the issue in each period t is associated with an *ideological valence* r_t, which captures the way realizations of that issue map to the political arena. We think of r_t as the belief about ω_t that would be most favorable to conservatives and $-r_t$ as the belief that would be most favorable to liberals, with $r_t = 0$ representing the politically neutral position in each period. If ω_t is the effectiveness of masks, r_t would be negative (since the conservative position tends to be that masks are ineffective). If ω_t is the extent of fraud in the 2020 election, r_t would be positive (since the conservative position is that fraud was extensive). We allow the information sources j to have ideological *biases* in the sense that their errors may be correlated with r_t. The accuracies and biases of the sources are the main persistent state variables that the agent seeks to learn over time.

The final ingredient of our model is each agent's own *reasoning* about the state based on introspection or direct experience. We model this as an additional signal x_{it} capturing each agent's independent evaluation of the plausibility of different ω_t values. In the case of global warming, this might be based on local temperatures or weather events the agent has experienced directly and/or her understanding of the mechanisms of climate science. In the case of an election, this might be based on her observations of the procedures in place at polling places in her neighborhood and her evaluation of the difficulty of committing fraud. In the case of masks, it might be based on the agent's understanding of the way disease spreads and the mechanisms by which masks could stop it.

Our key assumption is that agents believe their own reasoning to be unbiased, even though this may not be true. In other words, x_{it} may be systematically correlated with r_t conditional on ω_t, but each agent's prior assigns probability one to the case where this correlation is zero.[1] This prior belief is what distinguishes x_{it} from the other information sources s_{jt} in the model. It means that even if an agent knows her own reasoning is noisy, and so has limited information about ω_t on its own, she is able to use it as a yardstick to determine the accuracy and biases of the s_{jt} and so ultimately to learn which sources she can trust. If in fact her reasoning *is* biased, the result will be distorted learning about the accuracy of information sources and, consequently, the states ω_t. Our main results characterize the form such distortions take and show that they can in some cases be large even when the magnitude of the agent's bias (i.e., the correlation between x_{it} and r_t conditional on ω_t) is small.

Both parts of our key assumption are strongly supported by evidence. Large literatures in psychology and behavioral economics establish mechanisms by which individuals' own reasoning about politically sensitive topics may be subject to ideological bias, including motivated reasoning, selective memory, and availability. The implication of such biases is that two agents who for exogenous reasons begin with different ideologies and who have access to the same direct experiences—observe the same observations of local weather events, say—might reach conclusions about ω_t that are systematically biased toward or away from r_t. Moreover, substantial evidence also suggests that individuals themselves are not aware of these biases or at a minimum significantly underestimate them.

Our formal results characterize the limiting distribution of each agent's beliefs about accuracies and states as $t \to \infty$. We show that their beliefs about the accuracies of information sources eventually converge to a limiting distribution, the mean of which we define to be the agent's asymptotic *trust* in the respective sources.

In a benchmark case in which an agent has no ideological bias and there is no uncertainty about the accuracy of her reasoning x_{it}, her trust is a distribution degenerate at the true accuracies of the information sources, and her asymptotic beliefs about ω_t are the same as if she knew the true data

[1] In an extension, we show that our main results extend to the case where agents entertain the possibility that their reasoning is biased, provided the magnitude of bias in the support of their priors is sufficiently small.

generating process. If the accuracy of the information sources is sufficiently high, her beliefs about ω_t are close to correct in each period.

Introducing small biases in an agent's reasoning changes the results of the benchmark case dramatically. An agent with a small conservative bias may come to trust right-leaning sources more than is warranted by their true accuracy, trust unbiased sources less than is warranted, and believe that left-leaning sources are perverse, in the sense that their signals are negatively correlated with the true state. She may become overconfident in the accuracy of her own reasoning (i.e., come to believe that x_t is more strongly correlated with ω_t than it really is). She will generally come to believe that the state ω_t is positively correlated with the ideological valence r_t, and thus begin any period in which she knows r_t with a conservatively biased prior. All of these effects may be large even if bias is small, provided that the true accuracy of an agent's reasoning is sufficiently low.

To see the intuition for the way small biases are amplified in our model, note that an agent will come to see source j as more accurate the greater the observed correlation between its report s_{jt} and her reasoning x_{it}. This correlation will be zero when s_{jt} is perfectly uncorrelated with the state and positive when the source is perfectly accurate. But when the reasoning x_{it} is noisy, the magnitude of the correlation will be small even in the perfectly accurate case. Small differences in observed correlation—such as those that might be induced by a small ideological bias—thus imply large differences in perceived accuracy. An agent who knows her own ability to discern the truth of global warming is limited will view even a weak correspondence between her own conclusions and the reports of an information source as highly diagnostic.

Distortions in the way agents learn about the informational environment can translate into substantial disagreements about the states ω_t. We first show how biases affect the accuracy of agents' posterior beliefs about ω_t. We then show how the magnitude of disagreement between different agents depends on the accuracy and bias of their reasoning, as well as on those of the observed sources. When agents all observe a common unbiased source, disagreement is generally small even when the agents themselves are biased. When biased sources are introduced to the market, even small biases on the part of agents can lead to large disagreement.

The final section of our main results considers how these findings differ under single and multi-homing. A common intuition is that divergent trust and polarization might be mitigated if agents were exposed to an ideologically diverse set of information sources. We show that it is possible for multi-homing to have beneficial effects consistent with this intuition, but also that this need not be the case. Multi-homing may leave trust and polarization unchanged, or even exacerbate them.

Two extensions explore the implications of ideological bias for media competition and political behavior. First, we endogenize the choice of bias by media outlets in a sequential positioning game. We find that media competition can lead to greater media bias as well as intensified disagreements among viewers. Second, we show mistrust of motives across ideological divides can arise when agents underestimate both their own and others' biases. Ideological bias in this case can intensify political conflict, leading to costlier battles for power.

Our work contributes to the theoretical literature on sources of persistent polarization. It is most related to other models in which agents must simultaneously learn about states of the world and the accuracy of sources that provide information about those states. Cheng and Hsiaw (2022) study a related model based on a different form of underlying bias—a misspecified learning rule in which agents do not integrate information about states and credibility correctly—and show that this can lead to persistent polarization. Heidhues, Kőszegi, and Strack (2019) study a model of misspecified learning in which the covariance of signals is uncertain and show that misspecified priors about own ability leads to more negative perceptions of out-groups. Neither of these papers develop mechanisms for amplification of small biases.[2]

Other related work links polarization to behavioral biases but does not consider endogenous learning about information sources.[3] Examples include models of confirmation bias (Rabin and Schrag 1999), ambiguity aversion (Baliga, Hanany, and Klibanoff 2013), limited memory (Fryer, Harms, and Jackson 2019), and limited attention (Che and Mierendorff 2019). Bowen, Dmitriev, and Galperti (2021) show that belief polarization can arise in a social network where agents have small misperceptions about the sharing behavior of their neighbors.

Our work also builds on insights from the broader literature on misspecified learning. Berk

[2]Liang and Mu (2020) study learning about information sources over time and, like us, focus on a setting where signals from one source can allow an agent to infer characteristics of other sources. Agents in their model are fully Bayesian, however, and their analysis focuses on failures of learning rather than persistent polarization or disagreement.

[3]A distinct literature attempts to explain the finding from a large number of experimental studies (e.g., Lord, Ross, and Lepper 1979) that the beliefs of subjects polarized after the presentation of new evidence using Bayesian models. Dixit and Weibull (2007) is an early theory paper that considers the possibility of belief polarization under Bayesian updating. Andreoni and Mylovanov (2012), Kondor (2012), Glaeser and Sunstein (2014), and Benoit and Dubra (2019) present models wherein initial differences in the interpretation of signals can generate polarization.

(1966) provides a general statement that beliefs need not converge in the long run under misspecified learning. We are particularly indebted to Acemoglu, Chernozhukov, and Yildiz (2016), who show that arbitrarily small differences in beliefs about the interpretation of signals can generate large disagreements about an underlying state. Heidhues, Kőszegi, and Strack (2018) study an overconfident agent who becomes misdirected away from the optimal action as she learns about a fundamental. Frick, Iijima, and Ishii (2020) study social learning with misspecification and show that small misspecification can lead to extreme failures of learning. Bohren and Hauser (2021) provide a general framework for the analysis of learning in misspecified models.[4]

Finally, our model is linked to the literature on media bias and competition (Mullainathan and Shleifer 2005; Gentzkow, Shapiro, and Stone 2016). The mechanism by which agents in our model come to trust like-minded sources is closely related to the one explored by Gentzkow and Shapiro (2006). That model is essentially static, however, and does not provide a mechanism by which diverging beliefs or trust can persist over time.[5]

The paper proceeds as follows. Section 2 describes the model. Section 3 presents results on trust and polarization. Section 4 analyzes ideology and perceived bias. Section 5 allows for overconfidence. Section 6 considers the multi-homing case. Section 7 presents extensions. Section 8 concludes.

2 Model

2.1 Setup

Each agent i learns about a sequence of unobservable states $\omega_t \sim N(0,1)$ over time periods $t = 1,2,\dots,T$. There are observable signals s_{jt} from information sources $j = 1,...,J$, such as media

[4]A different strand of this literature considers opinion dynamics in social networks with non-Bayesian learning rules, beginning with DeGroot (1974). DeMarzo, Vayanos, and Zwiebel (2003) and Golub and Jackson (2010, 2012) are state of the art models that characterize conditions under which disagreements may persist in groups. Another strand of this literature considers Bayesian learning, and begins with Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). In these models, Bayesian individuals only observe posterior beliefs or actions of other individuals, and may fail to learn underlying states because they are able only to recall or communicate coarse information.

[5]There is a large empirical literature on the relationship between media markets and political polarization (Glaeser and Ward 2006; McCarty, Poole, and Rosenthal 2006; Campante and Hojman 2013; Prior 2013). A growing number of papers provide experimental evidence on the link between trust in information sources and political beliefs (Levendusky 2013; Nisbet, Cooper, and Garrett 2015; Benedictis-Kessner et al. 2019; Thaler 2020; Jo 2020).

outlets or talkative neighbors. There is also a random variable x_{it}, which represents agent i's independent *reasoning* about ω_t based on logic, introspection, direct experience, or other factors unrelated to s_{jt}. The conservative position on ω_t (i.e., the realization of ω_t most consistent with a conservative platform) is the ideological valence r_t, which may or may not be observed by the agents. The true value of ω_t is not observed *ex post.*

In each period, each agent observes her own reasoning x_{it} followed by at least one signal s_{jt}. Our baseline analysis focuses on the *single-homing* case, where agents choose one source to observe in each period to minimize her cumulative expected loss over all periods. We later study the *multi-homing* case, where all agents observe all available sources in every period. After observing the signal(s) and her own reasoning, the agent chooses an action $d_{it} \in \mathbb{R}$. At the end of the game, the agent receives loss equal to $\sum_{t=1}^{T} (d_{it} - \omega_t)^2$.

Together, ω_t, r_t, x_{it}, and the J-vector s_t of s_{jt} are jointly normal and are drawn independently over time:

$$\begin{bmatrix} \omega_t \\ r_t \\ x_{it} \\ s_t \end{bmatrix} \sim N(0, \Omega_i)$$

where $\Omega_{i11} = 1$ and the other elements of Ω_i are free parameters.

We define the *accuracy* α_j of signal j to be the correlation of s_{jt} with ω_t. Note that this correlation is a sufficient statistic for the value of observing s_{jt} to an agent who knows Ω_i and seeks to learn about ω_t because we can rescale all of the variables to have variance one without changing the agent's posterior beliefs.[6] Accuracy will be high when the covariance of s_{jt} and ω_t is large and/or when the variance of the noise in s_{jt} orthogonal to ω_t is low.

We define the accuracy a_i of each agent's reasoning analogously to be the correlation of x_{it} with ω_t. We are primarily interested in the case where the value of a_i as small, so that the agents' ability to learn ω_t through independent reasoning alone is limited, and substantially less than what

[6]Scaling each element of $\begin{bmatrix} r_t \\ x_t \\ s_t \end{bmatrix}$ by a multiplicative constant does not change an agent's posterior distribution on ω_t given s_{jt}, provided the constant is known (as in the limit as $t \to \infty$). This scaling therefore cannot change the expected loss of an agent who observes s_{jt} and then chooses a decision $d \in \mathscr{D}$ to generate loss $L(d, \omega)$. If we scale each element by the inverse of their standard deviation we obtain a new model where Ω_i is the correlation matrix, all the variances are one, and the expected loss is the same.

she could potentially learn through media or other information sources s_{jt}.

To define bias, let $\tilde{r}_t$ denote the residual from a regression of r_t on ω_t. We will focus throughout on the case where in fact r_t is independent of ω_t, so $\tilde{r}_t = r_t$. However, for reasons that will become clear below, we will want to allow agents to entertain the possibility that ω_t and r_t are correlated. In the case where they are correlated, we would not want to say that a signal is biased if it is correlated with r_t solely through its correlation with ω_t. Thus, we define the *bias* β_j of signal j to be the correlation of s_{jt} with $\tilde{r}_t$. We define the bias b_i of x_{it} to be its correlation with $\tilde{r}_t$.

For ease of exposition, we say that a source j is *perfectly right-biased* if $\beta_j = 1$, *perfectly left-biased* if $\beta_j = -1$, and *perfectly accurate* if $\alpha_j = 1$. When b_i and β_j have the same sign, we say that agent i and source j are *like-minded.* When they have the opposite sign, we say agent i and source j are *opposite-minded.*

We further assume that $a_i > 0$, so that x_{it} is always positively correlated with ω_t, that x_{it} is independent of s_t conditional on ω_t and r_t, and that both of these restrictions are known by the agents. As shown in Remark 1 below, the conditional independence of x_t and s_t implies that the correlations between the observable variables, namely r_t, x_{it}, and s_t, is fully determined by $(a_i, b_i, \alpha, \beta, \gamma, \Sigma)$, where γ is the correlation of r_t with ω_t, $\Sigma = \text{corr}(s_t)$, and α and β denote the J-vectors of α_j and β_j respectively. The independence of ω_t and $\tilde{r}_t$ implies that $a_i^2 + b_i^2 \leq 1$ and $\alpha_j^2 + \beta_j^2 \leq 1$.

In the single homing case, each agent chooses a source j to observe in each period to minimize expected loss. This choice is akin to a multi-arm bandit problem. However, unlike the standard bandit problem, the payoffs from each period are not observed immediately, and the agent observes auxiliary information in the form of x_{it}. This renders standard solutions to the bandit problem inapplicable. To tractably capture the tradeoff between exploration and exploitation, we assume that all agents follow an ε-first strategy (Slivkins 2019). That is, the agents each observe a random source with uniform probability during the first experimentation εT periods, where $\varepsilon \in (0,1)$. In the remaining periods, the agent chooses the source that minimizes expected loss, subject to the restriction that she chooses a source for which the posterior mean on α_j is weakly positive provided such a source is available.[7]

[7]The restriction rules out an agent choosing to observe sources she believes are negatively correlated with the truth over one she believes is positively correlated with the truth; such a strategy could improve the accuracy of beliefs in principle, but we see it as unrealistic from a behavioral point of view.

We are interested in agent i's beliefs about $\theta_i = (a_i, b_i, \alpha, \beta, \gamma, \Sigma)$ and ω_t during the exploitation phase in the limit where the number of periods is large, so $T \to \infty$. The true value of θ_i is denoted θ_{0i}, and where it adds clarity we use a_{0i}, b_{0i}, α_{0j}, β_{0j}, γ_0, and Σ_0 to refer to the true values of the individual components. The set of all possible θ_i is denoted as $\Theta_i = \Theta$. In our main case of interest, the value of a_{0i} is small and $\gamma_0 = 0$.

The correlation matrix for $(\omega_t, r_t, x_{it}, s_t)$, which we denote as $\tilde{\Omega}_i$, is fully parametrized by θ_i.[8] The covariance matrix is thus $\Omega_i = V_i^{\frac{1}{2}} \tilde{\Omega}_i V_i^{\frac{1}{2}}$, where V_i is a diagonal matrix containing the variances of $(\omega_t, r_t, x_{it}, s_t)$. We assume that the set of all possible variances V_i is a compact set $\mathscr{V}$. The Lebesgue space on $(\Theta, \mathscr{V})$ is denoted $((\Theta, \mathscr{V}), \mathscr{L}, \nu)$.

Example. **(Masks)** In period t, the state ω_t indexes the effectiveness of masks at preventing the transmission of disease during a pandemic. The ideological valence r_t is negative: conservatives believe masks are relatively ineffective, while liberals believe they are relatively effective. Note that it is natural to think of the magnitude $|r_t|$ as a finite number—neither group typically claims that masks are *completely* effective or ineffective. The signals s_{jt} could include reports on news or social media about scientific studies of mask effectiveness, anecdotes where people caught the disease despite wearing a mask, and so on. Right-biased sources might distort information in a negative direction by downplaying evidence that masks are effective; left-biased news reports might do the opposite. Reasoning x_{it} could capture logical inference, e.g., about the likelihood that masks fully block disease-carrying droplets, and direct observation, e.g., of people in the agents' own social network who either did or did not wear masks and may or may not have caught the disease.

Example. **(Election)** In period t, the state ω_t indexes the extent of fraud in a recent presidential election. The ideological valence r_t is positive: conservatives believe fraud was high while liberals believe fraud was low. It is again natural to think of the magnitude $|r_t|$ as a finite num-

[8]Specifically, given any θ_i, the correlation matrix for $(\omega_t, r_t, x_{it}, s_t)$ is given by

$$\tilde{\Omega}_i(\theta_i) = \begin{bmatrix} 1 & \gamma & a_i & \alpha' \\ \gamma & 1 & a_i\gamma + b_i\sqrt{1-\gamma^2} & \alpha'\gamma + \beta'\sqrt{1-\gamma^2} \\ a_i & a_i\gamma + b_i\sqrt{1-\gamma^2} & 1 & a_i\alpha' + b_i\beta' \\ \alpha & \alpha\gamma + \beta\sqrt{1-\gamma^2} & a_i\alpha + b_i\beta & \Sigma \end{bmatrix}.$$

Note then that Θ is the set of all $\theta \in (0,1] \times [-1,1]^{2J+2} \times [-1,1]^{J^2}$ such that $\tilde{\Omega}(\theta)$ is positive semi-definite and the diagonal entries of Σ are equal to one.

ber—neither group claims that *all* votes in the election were fraudulent nor that *no* votes in the election were fraudulent. The signals s_{jt} could include reports on news or social media about scientific studies of voter fraud, court cases challenging election outcomes, claims made by politicians, and so on. Right-biased sources might distort information in a positive direction by emphasizing anecdotes suggesting fraud; left-biased news reports might do the opposite by emphasizing evidence that fraud is rare. Reasoning x_{it} could capture logical inference, e.g., about the likelihood that widespread election fraud would be detected, and direct observation, e.g., of the way polling places, mail-in ballots, etc. were administered in the agents' own neighborhood.

2.2 Learning

At the beginning of the first period, agents have an absolutely continuous prior belief μ_0^i over $(\Theta, \mathbb{V})$ with a continuous density with respect to ν. For tractability, we assume that θ_i is independent of V_i under each agent's prior and we let $\mu_{0,\theta}^i$ and $\mu_{0,V}^i$ respectively denote agent i's prior distribution on θ_i and V_i. Our central assumption is that all θ_i in the support of $\mu_{0,\theta}^i$ have $b_i = 0$. In other words, all agents *a priori* believe that their own reasoning x_{it} is unbiased, even though their true bias may be nonzero. In Appendix B, we show that our main results extend to the case where agents believe their own bias to be $b_i = b^*$ for some small $b^* \neq 0$.

All θ_i in the support of $\mu_{0,\theta}^i$ also have $a_i \in \mathscr{A}_i$ for some set $\mathscr{A}_i$. In our baseline analysis, we assume that agents know the accuracy of their own reasoning, so $\mathscr{A}_i = \{a_0\}$. We then extend the model to allow for the possibility that agents could become overconfident. In this case, $\mathscr{A}_i = (0, a_i^{max}]$ for some $a_i^{max} \geq a_0$. We will assume that $\mu_{0,\theta}^i$ has full support on the subset Θ_i^{prior} of Θ consistent with the restrictions that $b_i = 0$ and $a_i \in \mathscr{A}_i$.

In our baseline case, agent i observes both x_{it} and s_{jt} for each j in many periods, and so, in the limit as $t \to \infty$, she learns the J-vector ρ_{is} of correlations between x_{it} and the elements of s_t . In the case where r_t is observed, agents also learn the J-vector ρ_{rs} of correlations between r_t and the elements of s_t and the correlation ρ_{ir} of x_{it} and r_t. Finally, in the case of multi-homing, agents observe all sources in each period, so they additionally learn the correlation matrix Σ_0 of the elements of s_t. We abuse notation and let R_{0i} denote the true value of the vector of correlations that can be learned by agent i—(ρ_{is}), $(\rho_{is}, \rho_{rs}, \rho_{ir})$, or $(\rho_{is}, \rho_{rs}, \rho_{ir}, \Sigma_0)$ depending on the case considered—and we let

$R_i(\theta_i)$ denote the observable subset of the correlations $\tilde{\Omega}_i$ that would occur under parameters θ_i.

Our first result defines agent i's limiting beliefs about θ_i. We define the *identified set* of parameters in the support of agent i's prior that is consistent with correlations R_{0i} as $I_i(R_{0i}) = \left\{\theta_i \in \Theta_i^{prior} : R_i(\theta_i) = R_{0i}\right\}$. The following proposition shows that agent i's beliefs converge asymptotically to this identified set. Because all θ_i in the identified set imply the same distribution of observed data, beliefs within the set remain proportional to the prior.

Proposition 1. *Suppose the true correlations of the observed data are R_{0i}. As $T \to \infty$, agent i's posterior distribution in any period $t > \varepsilon T$ converges to a limit $\mu^i_{\infty,\theta}$ such that for all measurable $\vartheta \subseteq \mathscr{L}_\Theta$,*

$$\mu^i_{\infty,\theta}(\vartheta) = \frac{\mu^i_{0,\theta}(\vartheta \cap I_i(R_{0i}))}{\mu^i_{0,\theta}(I_i(R_{0i}))}.$$

Proof. All proofs are in Appendix A. □

The second result derives the agents' posterior expectations of ω_t given their limiting beliefs about θ_i. For simplicity, we focus on an agent's belief before she observes her reasoning x_{it} or the ideological valence r_t, so we capture only the influence of the information source(s) she observes. This will be similar to her belief after seeing x_{it} in the main case of interest where a_{i0} is small and agents are aware of this. We denote the mean of the agent's posterior at this *ex interim* stage by $\overline{\omega}^i_t$. We define agent i's expectation of α_j under $\mu^i_{\infty,\theta}$, which we denoted as $\overline{\alpha}^i_j$, as agent i's *trust* in source j. The following characterization follows from standard conjugate prior results for the normal distribution.

Proposition 2. *Suppose agent i single homes and her posterior belief on θ_i is $\mu^i_{\infty,\theta}$. In any period t where she observes source j, the mean of her posterior on ω_t given s_{jt} (but not x_{it} or r_t) is*

$$\overline{\omega}^i_t(s_{jt}) = \overline{\alpha}^i_j \tilde{s}_{jt}, \tag{1}$$

where $\tilde{s}_{jt} = s_{jt}/\sqrt{Var(s_{jt})}$ is the standardized version of s_{jt}.

Our third result shows that given her limiting belief about θ_i, a single-homing agent observes whichever source with positive accuracy that she believes to be most accurate. To be precise, let $\mathscr{J}^i_+$ denote the set of sources j such that $\overline{\alpha}^i_j \geq 0$, which we assume here to be non-empty. The agent chooses to observe the source from this set that minimizes her expected mean squared error after observing both the source and her own reasoning:[9]

$$\min_{j \in \mathscr{J}_+^i} \mathrm{E}\left[\left(d_{ijt}^*\left(s_{jt}, x_{it}\right) - \omega_t\right)^2\right],$$

where the expectation is taken under the marginal distribution of $\left(s_{jt}, x_{it}, \omega_t\right)$ under μ_∞^i and $d_{ijt}^*\left(s_{jt}, x_{it}\right)$ is the optimal decision after seeing $\left(s_{jt}, x_{it}\right)$.

Proposition 3. *Suppose agent i single-homes, her posterior belief on θ_i is $\mu_{\infty,\theta}^i$, and the expected accuracy a_i of her own reasoning under $\mu_{\infty,\theta}^i$ is less than one. Then she chooses to observe in each period a source j for whom her trust $\overline{\alpha}_j^i$ is maximal (provided some source has $\overline{\alpha}_j^i \geq 0$).*

Proposition 3 is non-trivial. The agent's choice of j is equivalent to minimizing posterior variance integrating over both the uncertainty in $\left(s_{jt}, x_{it}, \omega_t\right)$ given Ω_i and the uncertainty in Ω_i. If we were only integrating over the former, this would be a standard case of choosing among signals with known distributions and it would be optimal to choose the one with the highest α_j. Once there is uncertainty over Ω_i, it is not obvious that the agent would choose the one with the highest expected α_j, since the whole distribution of Ω_i may matter for the expected loss.

2.3 Benchmark Cases

As shown above, information about source j's accuracy α_j allows the agents to learn about the underlying state ω_t from observing s_{jt}. As shown below in Section 3, agents rely on the observed correlation of reasoning x_{it} and signal s_{jt} to learn α_j. In this section, we discuss benchmark cases where either no signal or no reasoning is available. In these cases, agents do not endogenously learn α_j, and small ideological biases in the agents' reasoning are not amplified into large disagreements. For simplicity, we assume here that there is a single source ($J = 1$), agents know the accuracy of their own reasoning ($\mathscr{A}_i = \{a_0\}$), and r_t is never observed.

[9]Here it is unimportant whether the agent observes x_{it}. The same source j is chosen whether the agent minimizes her mean squared error after observing both x_{it} and s_{jt} or after observing s_{jt} but not x_{it}.

No Signals

First, suppose that agent i observes only her reasoning x_{it} in every period, but never observes any source j or the ideological valence r_t. In the main case of interest where the accuracy of her reasoning a_{0i} is small, her posterior beliefs will always be noisy. To see this, assume for the moment that x_{it} is known to have variance equal to one, and that agent i knows the accuracy of their own reasoning, so $\mathscr{A}_i = \{a_{0i}\}$. Then her posterior mean on ω_t after observing x_{it} is $\overline{\omega}_t^i = a_{0i}x_{it} = a_{0i}(a_{0i}\omega_t + \eta_{it}) = a_{0i}^2\omega_t + a_{0i}\eta_{it}$, where η_t is an error term that is normally distributed with mean zero and variance $1 - a_{0i}^2$. When a_{0i} is small, $\overline{\omega}_t^i$ is a very noisy estimate of ω_t.

No Reasoning; Exogenous Trust

Next, suppose agent i observes a single source j every period but never observes her reasoning x_{it} or the ideological valence r_t. In this case, her beliefs about the source's accuracy α_j never change. The mean of her posterior belief about ω_t after observing s_{jt} in any period t is $\overline{\omega}_t^i = \overline{\alpha}_j^0 s_{jt}$, where $\overline{\alpha}_j^0$ is the expectation of the source's accuracy under the agent's prior μ_0.

If $\overline{\alpha}_j^0$ is zero—roughly speaking, agents think the source is as likely to be a perverse "false news" source as an accurate "true news" source—then agent i's posterior belief about ω_t would be zero in all periods, no matter what value of s_{jt} is realized. She would therefore learn nothing from observing s_{jt}, even if s_{jt} is in fact highly accurate.

If instead $\overline{\alpha}_j^0 = \alpha_{0j}$, so the agents exogenously know the true accuracy of signals, then their posterior belief about ω_t will be the same as that of a Bayesian. Furthermore, if source j is highly accurate, $\overline{\omega}_t^i$ will be close to the true value ω_t. This will be true regardless of the agent's bias b_{0i} or the source's bias β_{0j}.

A closely related benchmark is one where agent i does observe x_{it} but trust is exogenous in the sense that the agent's prior on α_j is degenerate at $\overline{\alpha}_j^0$. Updating from s_{jt} in this case will be identical to that when x_{it} is not observed. The only difference is that agent i's beliefs about ω_t would also be affected by the realization of x_{it}. In the main case of interest where the accuracy a_{0i} of x_{it} is low and the agents know that it is low, the effect of x_{it} on beliefs will be small, and it will remain true that beliefs do not depend on bias. In this sense, endogenous trust is necessary in our model for bias to produce polarization.

Towards the Full Model

Now consider an agent i who observes both x_{it} and s_{jt} in every period and whose prior mean is $\overline{\alpha}_j^0 = 0$. In this case, her reasoning x_{it} functions as a reference point that allows her to learn the accuracy of source j. To see this, note that the agent believes that (1) x_{it} is positively correlated with ω_t (since $a_i > 0$ with probability one under μ_0^i) and (2) x_{it} is uncorrelated with $\tilde{r}_t$ (since $b_i = 0$ with probability one under μ_0^i). This means that she expects the correlation between x_{it} and s_{jt} to be increasing in the source's accuracy α_j. If she observed a positive correlation between x_{it} and s_{jt}, then she would endogenously revise her estimate of the source's accuracy upward. Having learned about source j's accuracy, the agent can then learn about ω_t from observing s_{jt}.

If agent i's reasoning is unbiased, her posterior belief on α_j converges to the true value α_{0j} as $t \to \infty$. If the source was highly accurate (i.e., α_{0j} was equal to one), then in the limit as $t \to \infty$, her posterior mean $\overline{\omega}_t^i$ approaches the true value ω_t. In other words, having access to noisy but unbiased reasoning allows an agent to accurately learn which sources to trust by aggregating over many periods. Unbiased agents can thereby learn the underlying states of the world as if they exogenously knew the accuracy of the information sources and therefore much more precisely than if they had access to only her reasoning. However, as we show in the following sections, endogenous trust can lead small biases in reasoning to become amplified into large disagreements.

2.4 Discussion: Reasoning and Bias

What does x_{it} represent in the real world? As a baseline case, we think of x_{it} as the result of the agent's reasoning and introspection about the likely value of ω_t. In the examples above, this could involve thinking about the likelihood that masks fully block disease-carrying droplets or that widespread election fraud is detected. If ω_t relates to economic stimulus policy, agents might reason from first principles about how large the plausible costs and benefits could be. (They might even write down and solve a model!)

There are a large number of well-studied psychological phenomena that could provide a micro-foundation for bias in agents' reasoning and introspection (i.e., $b_{0i} \neq 0$). Consider an agent who has grown up in a liberal family, benefitted from liberal policies, and taken actions (like voting) consistent with liberal ideology. Such an agent may engage in motivated reasoning (Kunda 1990), distorting her inferences to reach conclusions that support a liberal point of view. She may under-weight arguments or evidence pointing in the conservative direction in order to reduce cognitive dissonance (Festinger 1957). She may be more likely to remember evidence consistent with a liberal view (Eagly et al. 1999). She may tilt her assessment of the credibility of evidence due to confirmation bias (Lord, Ross, and Lepper 1979). She may also live in an environment in which information that supports her position is more "available" in the sense of Tversky and Kahneman (1973) – for example, if she had gone to high quality public schools, she may find it easier to think of the benefits of teachers' unions than their costs. Finally, it may be that evidence that supports the liberal position is more salient in the sense of Bordalo, Gennaioli, and Shleifer (2012). A large body of evidence suggests that individuals themselves are not aware of these biases or at a minimum significantly underestimate them (e.g., Pronin et al. 2002; Pronin 2007; Thaler 2022).

Another possibility is that x_{it} captures information about the true value of the state ω_t that the agent observes directly. This could be mask behavior or election procedures as in the examples above. It could capture weather events in the agents' locality (when ω_t relates to global warming), the agents' own experiences with public schools (when ω_t relates to education policy), or the agent's personal economic situation (when ω_t relates to economic policy). Such information could be observed either before or after seeing s_{jt}. Bias in these observations could arise through many of the same mechanisms as bias in reasoning. For example, an agent from a liberal background might be more likely to remember unusually hot days or unusually severe storms that suggest global warming is severe (Eagly et al. 1999). It could also be that certain kinds of direct observation tend to favor one side or the other inherently and agents fail to adjust for this selection. For example, observing the effort, wages, and impacts of public school teachers may naturally provide more evidence favorable to teachers' unions, and observing polling places where voting fraud is unlikely to be visible might naturally provide more evidence favorable to the view that elections are conducted fairly.

A final possibility is that x_{it} is the signal of a particular information source that agents believe *a priori* to be unbiased. This might be what their mothers say, or what the Bible says, or what scientists say, or even the report of a particular news source that they begin with extraordinary faith in.

What is crucial for our model is that x_{it} satisfies two conditions. First, agents believe that it is free from ideological bias ($b = 0$) with probability one. Second, contrary to the agents' beliefs, x_{it} may in fact be subject to ideological bias ($b_{0i} \neq 0$).

The ultimate source of the agents' biases is left unmodeled in our framework—b_{0i} is an exogenous parameter that we think of as determined by experiences and motivations prior to $t = 0$. We show below that non-zero bias leads to distortion in agents' trust in sources and their beliefs about states of the world, and also shapes what we label "ideology"—agent i's belief $\overline{\gamma}^i$ about the correlation between the truth ω_t and the ideological valence r_t. This point might lead to some confusion, as we show how bias can shape ideology, but psychological phenomena like motivated reasoning also provide reasons why ideology would shape bias. A richer model that included this two-way feedback between ideology and bias might well lead to even more dramatic divergence of trust and beliefs. We isolate one direction of causality for analytic tractability, taking bias to be a fixed characteristic that may depend on baseline ideology and other traits but does not change as ideology evolves.

It may seem strong to assume that the agent puts such dogmatic *ex ante* faith in her own reasoning or observation. However, if there is *no* source of truth in which she would place such faith, the agent would never learn what sources she can trust. More precisely, she would never be able to reject that any particular source is either perfectly positively correlated, uncorrelated, or perfectly negatively correlated with the state. We make this precise in Appendix D.

3 Trust and Polarization

In this section, we assume there are multiple available sources, so $J \geq 2$, but each agent single-homes and so observes only one source in each period. For simplicity, we focus on the case where the ideological valence r_t is never observed. This means that ideological valence is a latent source of bias in signals and reasoning but is not something agents make inferences from directly. Finally, we also focus on the case where agents know the accuracy a_{0i} of their own reasoning, so that $\mathscr{A}_i = \{a_0\}$. We consider the case where r_t is observed in Section 4, the case where agents are uncertain about the accuracy of their own reasoning in Section 5, and the case of multi-homing in Section 6.

For ease of exposition, we assume that there are three agents $i \in \{U, R, L\}$. All agents have accuracy $a_{0i} = a_0$, where $a_0 > 0$. Agent U's reasoning has no bias, so $b_{0U} = 0$. Agent R's reasoning has positive bias, while agent L's reasoning has negative bias. We set $b_{0R} = b_0$ and $b_{0L} = -b_0$, where $b_0 > 0$.

Assumption 1. *The support of $\mu^i_{0,\theta}$ is the set $\Theta^{prior}_i \subset \Theta$ for which $b_i = 0$ and $a_i = a_{0i}$.*

For agent U, this is a simple case of Bayesian learning with a correctly specified model. For agents R and L, the true parameters of the model lie outside the support of their priors (since $b_{0i} \neq 0$). Our model is thus an example of Bayesian learning under misspecification (Lian 2009).

Because r_t is not observed, agent i learns only the correlation ρ_{is} between s_t and x_{it}. This restricts agent i's limiting beliefs, since the model requires that $\rho_{is} = a_i\alpha + b_i\beta$. The identified set is then $I_i(R_{0i}) = \left\{\theta_i \in \Theta^{prior}_i : a_i\alpha + b_i\beta = \rho_{is}\right\}$.

Misspecified learning can lead to instability or lack of convergence (Berk 1966), but we focus our baseline analysis on parameter values such that the data agents observe do not violate their model of the world—i.e., the identified set is non-empty. A sufficient condition in the single-homing case is that $b_0 \leq a_0$ and $\left|\alpha_j\right| + \left|\beta_j\right| \leq 1$ for all j. We relax this assumption when we extend the model to allow overconfidence in Section 5 below.

Assumption 2. *$b_0 \leq a_0$ and $\left|\alpha_j\right| + \left|\beta_j\right| \leq 1$ for all j.*

We can then show that the identified set $I_i(R_{0i})$ is non-empty and contains a single value of α. It does not restrict the values of γ or β because when r_t is not observed, the observed correlations do not contain information about these parameters.

Proposition 4. *Suppose r_t is never observed and agents single home. Under Assumptions 1 and 2, agent i's identified set, $I_i(R_{0i})$, is non-empty and consists of all $\theta_i \in \Theta$ such that $a_i = a_{0i}$, $b_i = 0$, and $\alpha = \frac{\rho_{is}}{a_{0i}}$.*

Trust

A large body of evidence shows divergence between the sources trusted by conservatives and the sources trusted by liberals (e.g., Pew Research Center 2014a, 2020), and many have pointed to this as a key factor undermining the media's role in democracy (Gallup and Knight Foundation 2018, 2020). Consistent with this, agents in our model may come to trust biased sources more than unbiased sources. Divergence in trust may be large even when biases are small, provided that the accuracy of the agents' own reasoning is low.

To see this, notice that under Assumptions 1 and 2, Propositions 1 and 4 imply that agent i's trust in information source j is

$$\overline{\alpha}^i_j = \frac{\rho_{ij}}{a_{0i}} = \alpha_{0j} + \frac{b_{0i}\beta_{0j}}{a_{0i}}. \tag{2}$$

Therefore, when the accuracy a_{0i} of the agents' reasoning is low, small differences in biases b_{0i} and β_{0j} translate into large differences in trust. Agents come to trust source j more when their biases are more aligned (i.e., $b_{0i}\beta_{0j}$ is positive), and that divergent trust can be extreme even when b_{0i} is close to zero.

Corollary 1. *Suppose r_t is never observed, agents single home, and Assumptions 1 and 2 hold. Then agent R's (L's) trust in source j is increasing (decreasing) in the source's bias β_{0j} holding constant the source's accuracy α_{0j}. In the limit as $a_0 \downarrow b_0$, she will come to believe that a perfectly right-biased (left-biased) source is perfectly accurate, and trust it more than any unbiased source with $\alpha_{0j} < 1$.*

Polarization

Substantial literatures document large and growing disagreement between Democrats and Republicans on both policy issues (Pew Research Center 2014b; Boxell, Gentzkow, and Shapiro 2017) and questions of fact (Marietta and Barker 2019). Consistent with these findings, we show that disagreement in our model can be large even when the underlying bias is small and accurate information is widely available.

We focus on the exploitation periods $t > \varepsilon T$ in the limit as $T \to \infty$, where each agent observes the source j for which trust is maximal by Proposition 3. We define agents' *expected disagreement* when they observe these sources in such period to be $\pi = \mathrm{E}\left[\frac{1}{4}\left(\overline{\omega}^R_t - \overline{\omega}^L_t\right)^2\right]$. Scaling by one-fourth here ensures that $\pi \in [0,1]$.

The expected disagreement of single-homing agents is sensitive to the set of available sources. If all sources are unbiased, expected disagreement will be zero because all agents will come to trust the most accurate source and the trust of R and L agents for this source will be identical. However, if highly biased sources are available, disagreement can grow large.

To see this, suppose that all sources have accuracy $\alpha_{0j} < 1$ and there is one perfectly right-biased source and one perfectly left-biased source. By Corollary 1, agent R's trust for the perfectly right-biased source and agent L's trust for the perfectly left-biased source are both maximal in the limit as $a_0 \to b_0$. Proposition 3 then implies that each agent observes their like-minded perfectly biased source during the exploitation periods. Expected disagreement then reaches the maximum possible value $\pi = 1$.

Corollary 2. *Suppose r_t is never observed, agents single home, and Assumptions 1 and 2 hold. Further suppose that all sources have accuracy $\alpha_{0j} < 1$ and there is at least one perfectly right-biased source and at least one perfectly left-biased source. In the limit as $a_0 \downarrow b_0$, expected disagreement is one.*

4 Ideology and Perceived Bias

We now consider the case where the ideological valence r_t is observed in every period. Agents are able to learn the vector of correlations $(\rho_{is}, \rho_{ir}, \rho_{rs})$. In this case, the results from the previous section are broadly unchanged, but we can additionally derive each agent's beliefs about the correlation of the true states with the ideological valence and the bias of sources.

Remark 1. Given parameters $\theta_i = (a_i, b_i, \alpha, \beta, \gamma, \Sigma)$, the elements of $R(\theta_i)$ are given by

$$\begin{aligned} \rho_{is} &= a_i\alpha + b_i\beta \\ \rho_{ir} &= a_i\gamma + b_i\sqrt{1-\gamma^2} \\ \rho_{rs} &= \alpha\gamma + \beta\sqrt{1-\gamma^2}. \end{aligned}$$

For simplicity, we continue to focus on parameter values such that the data an agent observes do not violate her model of the world—i.e., the identified set is non-empty. In the current case, this will require $b_{i0}^2/a_{i0}^2 \le 1 - \alpha_{0j}^2/(1-\beta_{0j}^2)$ for all i and j. This assumption is more stringent than Assumption 2, because learning the values of ρ_{ir} and ρ_{rs} allows agents to reject their models of the world in more cases. We relax this assumption when we extend the model to allow overconfidence in Section 5 below.

Assumption 2'. $b_0^2/a_0^2 \le 1-\alpha_{0j}^2/(1-\beta_{0j}^2)$ for all j.

Under this assumption, the identified set $I_i(R_{0i})$ is non-empty.

Proposition 5. *Suppose r_t is observed and agents single home. Under Assumptions 1 and 2', agent i's identified set, $I_i(R_{0i})$, is non-empty and consists of all $\theta_i \in \Theta$ such that $a_i = a_{0i}$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, and $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$.*

Ideology

A striking feature of political polarization in the US is the significant correlation between citizens' liberal-conservative ideologies and their views across a range of diverse issues (Gentzkow 2016). Consistent with this fact, agents in our model when r_t is observed may form an *ex ante* conviction that either the conservative or the liberal point of view on *any* issue is likely to be closer to the truth on average. This conviction is captured in agents' limiting beliefs about the correlation γ between ω_t and r_t.

We define an agent's *ideology* $\overline{\gamma}^i$ to be her limiting posterior mean on γ. We say that agent i's ideology is *right-leaning* if $\overline{\gamma}^i > 0$. Under Assumptions 1 and 2' and focusing on the case of interest where $\gamma_0 = 0$, agent i's ideology is

$$\overline{\gamma}^i = \frac{b_{0i}}{a_0}. \tag{3}$$

Equation (3) implies that if agent R simply observes the ideological valence r_t, then her beliefs about ω_t become biased toward r_t, while agent L comes to believe the opposite. This is because agent R sees positive correlation between r_t and x_{it}, so she comes to believe that conservative views are on average closer to the truth than liberal views.

Corollary 3. *Suppose r_t is observed, agents single home, and Assumptions 1 and 2' hold. Then agent R's (L's) ideology is increasing (decreasing) in her bias b_0.*

Perceived Bias

Empirical evidence suggests that most Americans perceive media sources they distrust to be systematically biased (Gallup and Knight Foundation 2018, 2020). Consistent with this, agents in our model may perceive unbiased sources as biased, and also perceive like-minded sources as less biased than they actually are.

We define an agent's *perceived bias* $\overline{\beta}^i_j$ of information source j as the agent i's limiting posterior mean on β_j.[10] We say that agent i perceives a source j to be *oppositely biased* if $\text{sign}\left(\overline{\beta}^i_j\right) \neq \text{sign}\left(\overline{\gamma}^i\right)$. We say that agent i perceives a source j as *less right-biased than it actually is* if $\overline{\beta}^i_j < \beta_{0j}$.

Corollary 4. *Suppose r_t is observed, agents single home, and Assumptions 1 and 2' hold. Then agents R and L both perceive an unbiased source with $\alpha_{0j} > 0$ as oppositely biased. They also perceive a like-minded biased source with $\alpha_{0j} > 0$ as less biased than it actually is.*

Trust and Polarization when r_t is Observed

Our baseline results on trust (Corollary 1) and polarization (Corollary 2) continue to hold in the case where r_t is observed, as the identified set is still concentrated on the value $\alpha = \frac{\rho_{is}}{a_i}$, and observing r_t does not change either ρ_{is} or a_i. The one proviso is that the limit $a_0 \downarrow b_0$ can only be consistent with Assumption 2' if $\alpha_{0j} = 0$ for all j. Even when a_0 is bounded away from b_0, however, it will continue to be the case that trust in perfectly biased sources diverges as a_0 approaches b_0 , and polarization consequently grows large.

5 Overconfidence

We now allow for the possibility that agents are uncertain about the accuracy a_i of their own reasoning. Because this will enable a biased agent to rationalize any signal she might observe via a higher value of a_i, it allows us to dispense with restrictions on the parameter space (Assumptions 2 and 2'). We continue to assume the ideological valence r_t is observed in every period.

We now revise Assumption 1 to allow the agent to entertain any $a_i \in (0, a_i^{max}]$.

Assumption 1'. The support of $\mu^i_{0,\theta}$ is the set $\Theta^{prior}_i \subset \Theta$ for which $b_i = 0$ and $a_i \in (0, a_i^{max}]$, where $a_i^{max} \geq \sqrt{a_0^2 + b_0^2}$.

Agents' identified sets now may include multiple values of a_i, and this means they may include multiple values of α, β, and γ as well.

[10]Under Assumptions 1 and 2', the agent's perceived bias of source j is $\overline{\beta}^i_j = \beta_{0j}\sqrt{1-(b_{0i}/a_0)^2} - \frac{a_0 b_{0i}\alpha_{0j}}{a_0^2\sqrt{1-(b_{0i}/a_0)^2}}$.

Proposition 6. *Suppose r_t is observed and agents single home. Under Assumption 1', agent i's identified set, $I_i(R_{0i})$, is non-empty and consists of all $\theta_i \in \Theta$ such that $a_i \in [\underline{a}_i, a_i^{max}]$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, and $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$, where $\underline{a}_i = \max_j \sqrt{\zeta_{ij}}$ and ζ_{ij} is the population R^2 from a regression of x_{it} on r_t and s_{jt}.*

What will drive overconfidence is the fact that an agent's asymptotic belief in the accuracy of her own reasoning a_i is bounded from below by $\underline{a}_i$, which in turn depends on the extent to which x_{it} is correlated with the other data the agent observes. Intuitively, under the agent's maintained assumption that $b_i = 0$, the only source of correlation is the fact that x_{it} is related to ω_t, so correlation must be small if a_i is small.

An agent's own accuracy a_i and the accuracy α of the information sources are not separately identified. Since agents believe that $b_i = 0$, we have that $\rho_{is} = a_i\alpha$ for any $\theta_i \in I_i(R_{0i})$. A given value of ρ_{is} could result from a high value of a_i and low values of α_j, or a low value of a_i and high values of α_j; these cannot be distinguished by the observed data. Beliefs within the identified set will therefore remain proportional to the prior.

Overconfidence

A large literature in psychology, economics, and finance has documented overconfidence in many contexts, with early evidence including the pioneering study of Alpert and Raiffa (1982). Ortoleva and Snowberg (2015) explore in detail the implications of overconfidence for political behavior. While overconfidence is a primitive in their model, the results of this section show that it may arise endogenously as a consequence of other biases in reasoning.[11]

We refer to agent i's belief about the accuracy a_i of her own reasoning as her *confidence.* We say that she is *overconfident* if $a_i > a_0$ for all $\theta_i \in I_i(R_{0i})$ and *underconfident* if $a_i < a_0$ for all $\theta_i \in I(R_{0i})$.

Proposition 6 shows that agents are never underconfident and will be overconfident if and only if $\underline{a}_i > a_0$. The bound $\underline{a}_i$ is determined by the R^2 values ζ_{ij} from regressions of x_{it} on r_t and s_{jt}. Intuitively, agents are overconfident when they observe correlations between x_{it}, r_t and at least one

[11] In Ortoleva and Snowberg (2015), agents overestimate the precision of their information because they ignore correlation in the underlying signals they see. This leads overconfident citizens to have excess variance in their posterior beliefs. Overconfidence in our model has the same excess variance implication, but also has a further effect on polarization via endogenous trust.

s_{jt} that are infeasible at the true value a_0 (under the agent's maintained assumption that $b_i = 0$). The proof of Proposition 5 shows that this R^2 is given by

$$\zeta_{ij} = b_{0i}^2 + a_0^2 \left(\frac{\alpha_{0j}^2}{1 - \beta_{0j}^2} \right). \tag{4}$$

Therefore, agents R and L are overconfident if and only if

$$\frac{b_{0i}^2}{a_0^2} > 1 - \max_j \left\{ \frac{\alpha_{0j}^2}{1 - \beta_{0j}^2} \right\}. \tag{5}$$

Noting that the fraction in curly braces approaches one as $\alpha_{0j}^2 + \beta_{0j}^2$ approaches one yields the following result.

Corollary 5. *Suppose r_t is observed, agents single home, and Assumption 1' holds. Then agents are never underconfident. Agent U is never overconfident. Agents R and L are overconfident if (i) a_0 is sufficiently small or (ii) there is some source j with $\alpha_{0j}^2 + \beta_{0j}^2$ sufficiently close to one.*

Overconfidence emerges in our model in order to reconcile the agents' dogmatic belief that their reasoning x_{it} is unbiased with the observed correlations of the sources. No agent is ever underconfident, since it is always possible to rationalize the observed correlations with a sufficiently high value of a_i. Overconfidence can arise if an agent's bias is large. However, even when an agent's bias is small, she is overconfident if the accuracy of her reasoning a_0 is sufficiently low. She is also overconfident if at least one source in the market is sufficiently accurate. When there is at least one source with $\alpha_{0j} \neq 0$ and $\alpha_{0j}^2 + \beta_{0j}^2 = 1$, agents with any bias are overconfident regardless of the value of a_0, since $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$.

Trust, Polarization, Ideology, and Perceived Bias in the Presence of Overconfidence

Can divergence in trust, disagreement about ω_t, ideology, and perceived bias be large once we allow for overconfidence? The answer is yes if a_0 is small relative to b_0. To illustrate, we focus on the case where the upper bound of the agent's prior on a_i is $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$. This is the smallest value for which an agent's identified set is always non-empty, and it approximates a situation where each agent's prior on her own accuracy a is concentrated on values that are close to the true value

a_0.[12] We further focus on the case where the set of sources is sufficiently rich that it includes each agent's *trust-maximizing* source. These results are not knife-edge; they hold approximately if a_i^{max} is close to the assumed value and there is an available source that generates sufficiently high trust. More general results for trust and beliefs in the presence of overconfidence are derived in Appendix C.

Because we have relaxed Assumptions 1 and 2, agent i's trust-maximizing source is defined by the accuracy and bias (α_i, β_i) that maximizes the agent's trust over all pairs satisfying the feasibility condition $\alpha_i^2 + \beta_i^2 \leq 1$. We show in Appendix C that this is

$$(\alpha_i^{max}, \beta_i^{max}) \equiv \left(\frac{a_0}{\sqrt{a_0^2 + b_{0i}^2}}, \frac{b_{0i}}{\sqrt{a_0^2 + b_{0i}^2}} \right).$$

Note that the correlation between x_{it} and a trust-maximizing source is $\rho_{isj} = a_0 \alpha_i^{max} + b_{0i} \beta_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$.

Agent U's trust will be maximized by an unbiased source with accuracy $\alpha_j = 1$ and bias $\beta_j = 0$. For agents R and L, the trust-maximizing source will be biased. If b_0 is close to a_0, a biased agent will prefer a source with bias and accuracy close to $1/\sqrt{2}$. If the set of available sources includes agent i's trust-maximizing source, then in the limit as $T \to \infty$, the single-homing agent observes her trust-maximizing source in all periods $t > \varepsilon T$ by Proposition 3.

If the trust maximizing source is available, the agent will be maximally overconfident, with confidence degenerate at the maximal value $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$. This is because substituting $(\alpha_i^{max}, \beta_i^{max})$ for α_j and β_j in equation 4 yields $\zeta_{ij} = a_0^2 + b_{0i}^2$ which in turn implies $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$ by Proposition 6. It follows that the agent's trust in the trust-maximizing source will be one, and her posterior will be degenerate at

$$\overline{\omega}_t^i = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t. \tag{6}$$

We can now apply Proposition 6 to see that trust becomes highly divergent in the limit as $a_0 \to 0$. The fact that overconfidence is maximal means an agent R's trust $\overline{\alpha}_j^R$ for an arbitrary

[12] In the alternative case where each agent's prior on a is concentrated on values much larger than a_0, their initial beliefs about a are severely mistaken. Furthermore, if a_0 and b_0 are small, then the agent learns that all available sources are very noisy (i.e., $\overline{\alpha}_j$ is small for all j), and there is limited amplification of bias into divergent trust and disagreement.

source j (not necessarily her trust maximizing source) approaches the source's bias β_j in that limit. Her trust for a perfectly right-biased source approaches one. Her trust for a perfectly left-biased source approaches zero. The opposite is true of agent L. Agents may therefore underestimate the accuracy of unbiased sources and come to trust biased sources strictly more than even a perfectly accurate source.

The agents' ideologies and perceived biases also become highly polarized in the same limit. Agent R believes that the true state is almost perfectly correlated with r_t, i.e. $\overline{\gamma}^R \to 1$, and perceives a perfectly accurate source j to be almost perfectly left-biased. Agent L correspondingly believes that the true state is almost perfectly correlated with $-r_t$, i.e. $\overline{\gamma}^L \to -1$, and perceives a perfectly accurate source to be almost perfectly right-biased.

Expected disagreement increases with the agents' bias, since $\pi = b_0^2/\left(a_0^2+b_0^2\right)$. This will be at least $1/2$ if $a_0 \leq b_0$, and it will approach one in the limit as a_0 approaches zero. Thus, polarization is significant if a_0 is small relative to b_0.[13]

6 Multi-Homing

A common intuition is that divergent trust and polarization could be reduced or eliminated if agents were exposed to an ideologically diverse set of information sources. In this section, we show that it is possible for multi-homing to have beneficial effects consistent with this intuition, but also that this need not be the case. Multi-homing may leave trust and polarization unchanged, or even exacerbate them.

6.1 Trust under Multi-Homing

We first consider the agent's limiting beliefs about θ_i under multi-homing. In the *multi-homing* case, $R_{0i} = (\rho_{is}, \rho_{ir}, \rho_{rs}, \Sigma_0)$, where Σ is the matrix of correlations among elements of s_t.[14]

Proposition 7. *Under Assumption 1', agent i's identified set under multi-homing, $I_i(R_{0i})$, when r_t is observed, is non-empty and consists of all $\theta_i \in \Theta$ with $a_i \in \left[\underline{\underline{a}}_i, a_i^{max}\right]$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$,*

[13] Note that the requirement that the source be trust-maximizing is not knife-edge: π is continuous in α and β, so the result holds approximately when these are close to the trust-maximizing values.

[14] To avoid singular covariance matrices, we assume that none of x_t, s_t, and r_t are perfectly correlated with each other, so the vector of true correlations $R \in \text{int}(\mathscr{R})$.

$\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$*, and* $\Sigma = \Sigma_0$*, where* $\underline{\underline{a}}_i = \sqrt{\zeta_i}$ *and* ζ_i *is the population* R^2 *from a regression of* x_{it} *on* r_t *and the elements of* s_t*.*

In the multi-homing case, a cannot be smaller than $\underline{\underline{a}}_i = \sqrt{\zeta_i}$, where ζ_i is the population R^2 from a regression of x_{it} on r_t and all of the elements of s_t.[15] Since $\zeta_i \geq \zeta_{ij}$ for all j, the lower bound on a_i is tighter under multi-homing than under single-homing.

Since the multi-homing bound on confidence $\underline{\underline{a}}_i$ is weakly greater than the single-homing bound $\underline{a}_i$, a multi-homing agent's confidence will be weakly greater (in an FOSD sense) than a single-homing agent's confidence. This means that the difference in trust $\left|\overline{\alpha}^i_j - \overline{\alpha}^i_k\right|$ between any two sources can be weakly smaller under multi-homing, and that ideology $\overline{\gamma}^i$ will tend to be less extreme.

Suppose, for tractability, that the agent has an *a priori* belief that the accuracies or biases of observable external signals (i.e., s_t and r_t) are independent of her own accuracy.

Assumption 3. a_i *and* (α, β, γ) *are independently distributed under the agent's prior* μ^i_0*.*

When Assumption 3 holds, each agent's limiting (marginal) posterior distribution on a_i, which we denote by $\mu^i_{\infty,a}$, is the agent's prior marginal distribution on a except truncated at the lower bound of $\underline{\underline{a}}_i$. It follows that multi-homing may dampen divergent trust and ideology (while also increasing confidence).

Corollary 6. *Suppose Assumptions 1' and 3 hold. Then for any agent i, the difference in trust* $\left|\overline{\alpha}^i_j - \overline{\alpha}^i_k\right|$ *between any two sources and the ideology* $\left|\overline{\gamma}^i\right|$ *are both weakly smaller under multi-homing.*

While such an effect of multi-homing is possible, it need not be large, and it is possible for the limiting posterior μ^i_∞ of a multi-homing agent to be exactly the same as a single-homing agent's. For example, when at least one source with $\alpha_{0j} \neq 0$ is located on the frontier, $\underline{a}_i$ already achieves its maximal value under single-homing, so the limiting posterior under multi-homing is unchanged.

[15] That is, $\zeta_i = \tilde{\rho}_i{}' \tilde{\Sigma}^{-1} \tilde{\rho}_i$ where $\tilde{\rho}_i = \begin{pmatrix} \rho_{ir} \\ \rho_{is} \end{pmatrix}$ and $\tilde{\Sigma} = \begin{pmatrix} 1 & \rho_{rs}' \\ \rho_{rs} & \Sigma \end{pmatrix}$.

6.2 Polarization under Multi-Homing

How does expected disagreement about ω_t compare in the single- and multi-homing cases? As shown in this subsection, it is possible for multi-homing to reduce disagreement, but multi-homing may also increase disagreement in some situations.

To demonstrate this, we first characterize the agent's posterior expectation of ω_t when her beliefs about θ_i are given by the limiting posterior $\mu^i_{\infty,\theta}$. Under multi-homing, the average belief $\overline{\omega}^i_t$ is now a linear function of the observed signals, as shown below.

Lemma 1. *Suppose agent i multi homes and Assumption 1' holds. As $T \to \infty$, in any period $t > \varepsilon T$, the mean of her posterior on ω_t given s_t (but not x_{it} or r_t) is*

$$\overline{\omega}^i_t = A_i \rho'_{is} \Sigma^{-1} \tilde{s}_t,$$

where $\tilde{s}_t$ is the J-vector of standardized signals s_t and the amplification factor A_i is given by

$$A_i = \int_{\underline{a}_i}^{a_i^{max}} \frac{1}{a} d\mu^i_{\infty,a}(a).$$

We consider a special case wherein each agent may observe three sources with respective biases of β, 0,and $-\beta$, where $\beta > 0$. All of these sources are on the *frontier*, so $\alpha_j^2 + \beta_j^2 = 1$ for all j. As shown in Lemma 2 below, a multi-homing agent's posterior mean $\overline{\omega}^i_t$ is the same as that of a single-homing agent who observes her trust-maximing source. The reason is that a multi-homing agent observing two distinct frontier sources can construct a linear combination of the sources' signals whose value will be equal to the signal of the agent's trust-maximizing source. Appendix E show that we obtain the same result in the limit of a sequence of random markets with non-frontier sources.

Lemma 2. *Suppose Assumption 1' holds and there are at least two frontier sources with distinct biases. Then the mean of the multi-homing agent's posterior on ω_t given s_t is*

$$\overline{\omega}^i_t = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t.$$

It follows, as formalized in Proposition 8 below, that multi-homing does not in general reduce expected disagreement. In fact, multi-homing may make it worse. To see this, let $\phi_b = \tan^{-1}\left(\frac{b_0}{a_0}\right)$ denote the angle between vectors $(1,0)$ and (a_0, b_0) and let $\phi_\beta = \sin^{-1}(\beta)$ denote the angle between $(1,0)$ and $\left(\sqrt{1-\beta^2}, \beta\right)$. If $\phi_b \in \left(0, \frac{1}{2}\phi_\beta\right)$, all single-homers observe the unbiased source, while the multi-homers' beliefs are the same as would occur if they observed their trust-maximizing source, so multi-homing results in greater disagreement. If instead $\phi_b \in \left(\frac{1}{2}\phi_\beta, \phi_\beta\right)$, biased single-homers observe a source with more bias than the trust-maximizing source, so multi-homing results in less disagreement.

Proposition 8. *Suppose Assumption 1' holds and there are three frontier sources with respective biases* β, 0, *and* $-\beta$. *Then expected disagreement* π *is greater under multi-homing than under single-homing if* $\phi_b \in \left(0, \frac{1}{2}\phi_\beta\right)$, *but smaller if* $\phi_b \in \left(\frac{1}{2}\phi_\beta, \phi_\beta\right)$.

7 Extensions

What does our model imply about media competition and political behavior? Our first extension shows that media competition can intensify disagreements in a population with ideological biases. The second extension shows that ideological bias results in interpersonal mistrust and creates welfare losses in strategic games of collective decision-making.

7.1 Endogenous Media Bias

To explore how media competition affects ideological disagreement, this extension endogenizes the accuracies and biases of the information sources in a sequential positioning game.

We consider a unit mass of agents. The agents are divided into three types $i \in \{U, R, L\}$, with accuracies a_i and biases b_i defined in Section 2.2. We assume that agents' priors follow the setup of the full model in Section 5, specifically those in Assumption 1'. Each type i has mass $m_i > 0$ such that $m_U + m_R + m_L = 1$. For simplicity, we assume that $m_L = m_R$.

A set of E identical potential entrants sequentially choose whether or not to enter. If they enter, they may choose any accuracy α_{0j} and bias β_{0j} on the frontier (i.e., $\alpha_{0j}^2 + \beta_{0j}^2 = 1$). Prior to entry, each entering outlet observes all preceding entrants' choices of $\left(\alpha_{0j}, \beta_{0j}\right)$. We use subgame perfect equilibrium as our solution concept.

All agents are single-homers who choose a single outlet j to observe in a given period t. We focus on media viewership choices during their exploitation period, assuming that agents have beliefs about the accuracies of the outlets corresponding to the limiting posterior $\mu^i_{\infty,\theta}$.[16][17] If Proposition 3 identifies multiple potential sources to observe, agents randomize between them with equal probability.

We assume that the revenue of a media outlet is increasing in both the mass of viewers that choose it and the trust of its viewers. This is consistent with advertising-supported media where conditional on viewing an outlet a customer spends more time viewing when trust is high. It could also be consistent with paid media where the revenue an outlet can earn from a customer who chooses to view is greater when trust is high.

Let $\mathscr{J}_i$ be the set of outlets for which an i-type agent's trust $\overline{\alpha}^i_j$ is highest. Let $\xi\left(\overline{\alpha}^i_j\right)$ denote revenue per viewer of type i. We assume that $\xi(\cdot)$ is positive, strictly increasing, continuously differentiable, and concave, to capture the idea that firms make additional revenue from higher trust, but with declining marginal revenue. Firms also pay an entry cost $\lambda > 0$. Each firm j thus has expected profit:

$$\Pi_j = \sum_{i\in\{U,R,L\}} \mathbf{1}\{j \in \mathscr{J}_i\} \frac{m_i}{|\mathscr{J}_i|} \xi\left(\overline{\alpha}^i_j\right) - \lambda,$$

where $\mathbf{1}\{j \in \mathscr{J}_i\}$ is an indicator for whether outlet j is in the set of outlets that type-i agents observe, and $m_i/|\mathscr{J}_i|$ measures the probability of observing j within that set. Note that both $\mathscr{J}_i$ and $\overline{\alpha}^i_j$ are equilibrium outcomes that depend on the accuracy and bias choices of all media outlet entrants.

We can now solve for the media outlets' equilibrium choice of accuracies and bias via backward induction. We first consider outcomes in a monopoly market.

Proposition 9. *Suppose there is only one potential entrant ($E = 1$). Then for λ sufficiently low, this firm enters and becomes a monopolist with $\alpha_j = 1$ and $\beta_j = 0$. Biased agents are overconfident, but have expected disagreement $\pi = 0$.*

[16]This can be motivated by considering media viewership choices after a large number of exploration periods, in which beliefs about the sources' accuracies converged to the limit μ^i_∞.

[17]Strictly speaking, this is a behavioral assumption that the agents' inferences about media outlet accuracy do not condition on the equilibrium strategies chosen by the outlets, but only on the signals the outlets produce.

Proposition 9 shows that the monopolist becomes a completely accurate and unbiased source of information. Even though the monopolist has a captive audience, it still seeks to capture rising profits from trust. Since it faces a linear trade-off in trust between the L and R agents when it adds bias, the optimal choice under equal proportions of L and R agents and a concave revenue function ξ is to simply focus on accuracy instead and choose $\alpha_{0j} = 1$ and $\beta_{0j} = 0$. This results in no expected disagreement in the population, as agents observe a common outlet and have a common level of trust. Note that this trust is still suboptimal, however, and so beliefs are less than perfectly accurate. Note also that this result is not knife edge: If the proportions of L and R agents are slightly unequal, the resulting optimal position remains close to unbiased, and confidence, trust, and beliefs remain close to the characterization above.

Turning to the competitive case, we focus on the case where the set of potential entrants is sufficiently large that there are potential entrants who do not find it profitable to enter in equilibrium. We can see from Appendix C (Lemma 6) that sources gain maximum trust from biased agents by choosing those agents' trust-maximizing level of bias. It is then unsurprising that in the case of competition, some sources choose to be biased and successfully retain a large audience.

Proposition 10. *For λ sufficiently low and a set of potential entrants $E(\lambda)$ sufficiently large, all entrant outlets locate at positions on the frontier with $\beta_{0j} \in \{\beta^L, 0, \beta^R\}$, where β^L and β^R are the trust-maximizing biases for type L and type R agents respectively. At least one outlet chooses each of these positions. Biased agents are overconfident. Furthermore, expected disagreement will be at least $\pi = \frac{1}{2}$ if $a_0 \leq b_0$, and will approach $\pi = 1$ in the limit as $a_0 \to 0$. Thus, the entry of partisan media leads to greater divergence in beliefs.*

In contrast to the monopoly case, there is now significant disagreement in the population. This stems from their complete faith in the accuracy of like-minded outlets and their undivided attention to such outlets. Their beliefs about ω_t are simply degenerate at the signal s_{jt} of their trust-maximizing source. Since such outlets adopt the trust-maximizing bias and this bias approaches ± 1 as the ratio of b_0 to a_0 increases, competition can potentially give rise to maximal disagreement and perfectly negatively correlated beliefs. Note these results are not at all dependent on our assumption that $m_R = m_L$.

7.2 Mistrust of Motives and Partisan Conflict

This extension shows how ideological bias can lead to mistrust of motives across ideological divides and results in intensified conflict in political settings. This exploration is motivated by studies that show rising numbers of Americans hold negative views towards people on the other side of the partisan divide, for example, seeing them as unintelligent and selfish (Iyengar, Sood, and Lelkes 2012; Iyengar et al. 2019), with potentially important consequences such as reducing the efficacy of government (Hetherington and Rudolph 2015). [18]

We augment our model by adding an observable policy decision d_t to be made by one of two agents, R and L, and allow for ulterior motives B in decision making. We then characterize the agents' beliefs about the others' motive B when the agents assume both their and others' biases are $b = 0$ when in fact $b \neq 0$. This assumption is deliberately stark to illustrate that if people underestimate the extent to which others' reasoning is biased, they may attribute their behavior to biased motives instead. More precisely, we show that agents mistakenly learn that $B \neq 0$ even when in fact $B = 0$.

The setup is as follows. Suppose Assumption 1' holds and all agents are single-homers observing their trust-maximizing source $(\alpha_i^{max}, \beta_i^{max})$ in some period $t > \varepsilon T$ in the limit as $T \to \infty$. After observing the sources' signals in some period t, R makes an observable policy decision d_t to maximize the social welfare function, given by $-(\omega_t - d_t)^2$.

We assume that agents fail to appreciate both their own and others' ideological bias and believe that $b_i = 0$ for all agents i. Consequently, they believe that others have the same belief about the state ω_t as they do. At the same time, agents entertain the possibility that others may have ulterior motives. Specifically, we assume that L believes that R maximizes $-(\omega_t + B_R r_t - d_t)^2$, where B_R parameterizes R's ulterior motive and may not be equal to zero. Under these assumptions, it is immediate that people who underestimate the extent to which others' *reasoning* is biased attribute observed behavior to biased *motives* instead.

If L observes R's decision d_t in any period when $r_t \neq 0$, then L infers that $B_R = 2b_0/\sqrt{a_0^2 + b_0^2} > 0$. Similarly, if R were to observe L's decisions, R would also conclude that L had an ulterior motive $B_L = -2b_0/\sqrt{a_0^2 + b_0^2} < 0$. In other words, mistrust of motives arises when well-meaning agents

[18] Relatedly, Ortoleva and Snowberg (2015) and Levy and Razin (2015) explore how correlation neglect and resulting overconfidence impact polarization and political behavior.

fail to see how ideological bias colors inference about facts by both themselves and others. The magnitude of mistrust in other's motives is increasing in ideological bias b_0 of the agents.

It follows that the political behavior of well-meaning agents with ideological bias can mimic that of self-interested agents with actual conflicts of interest. For example, suppose that the above two agents engage in a contest for the power to decide d_τ for some $\tau > t$ after learning about the bias in each other's preferences. Tullock (1980) provides an elemental model of such a contest. R and L simultaneously invest in "arms" to obtain decision-making power, where the probability that R has power to decide d_τ depends on R's stock of arms relative to L's. In L's eyes, the payoff of obtaining decision-making power is zero when $B_R = 0$, since the two agents would choose the same decision. However, the gain from winning the contest becomes positive if L either perceives R to have a nonzero ulterior motive B_R or believes R's inference about ω_t to be biased. The symmetric Nash equilibrium therefore has positive expenditures on arms, even though in equilibrium the contestants have the same probabilities of winning as if neither had spent anything.

Other types of inefficient strategic behavior also arise from conflicts of interest in elemental game theoretic models of organizational behavior, including costly signaling, signal jamming, obfuscation and uninformative cheap talk (see Gibbons, Matouschek, and Roberts 2013). Ideological differences may therefore lead to welfare losses from uninformative communication across ideological divides, poor decision-making, as well as inefficient expenditures in the battle for power.

8 Conclusion

We present a model to explain why individuals persistently disagree about both objective facts and the trustworthiness of information sources. In contrast to recent theories, we assume that agents have Bayesian learning rules and can process information from an arbitrarily large set of high-quality sources. Agents in our model learn about policy-relevant states by observing signals from information sources whose accuracy is *ex ante* uncertain. Agents learn these accuracies by comparing their own reasoning about the states to the sources' reports.

We show that small biases in reasoning can result in large and persistent divergence in both trust and beliefs about facts, provided that the accuracy of their own reasoning is sufficiently low. Partisans end up trusting unreliable but ideologically aligned sources more than accurate neutral sources, and become overconfident in their own reasoning. They form a conviction that either the conservative or the liberal point of view is closer to the truth on average, and perceive unbiased sources to be oppositely biased. Divergent trust and beliefs can arise to a similar extent whether agents selectively view only ideologically aligned sources or are exposed to a diverse range of sources. Moving from a monopoly to a competitive market can deepen rather than mitigate ideological disagreement. Mistrust of motives results and leads to inefficient political outcomes.

Our theory highlights the outsized importance of trust in driving ideological differences in society. If individuals' reasoning has even a small amount of bias, then they may learn to trust biased sources, and hence form biased beliefs about facts. For this reason, ideological disagreement can persist even among otherwise Bayesian agents who can process information about an arbitrarily large set of high-quality sources. Reducing selective exposure may therefore fail to redress political polarization. Targeting the underlying drivers of divergent trust – for example, by reducing biases in the population's reasoning through scientific literacy, or increasing the prominence of commonly trusted sources – may yield larger gains.

References

Acemoglu, Daron, Victor Chernozhukov, and Muhamet Yildiz. 2016. “Fragility of asymptotic agreement under bayesian learning.” *Theoretical Economics* 11 (1): 187–225.

Alpert, Marc and Howard Raiffa. 1982. “A progress report on the training of probability assessors.” In *Judgment under Uncertainty: Heuristics and Biases*, edited by Daniel Kahneman, Paul Slovic, and Amos Tversky, 294–305. Cambridge: Cambridge University Press.

Andreoni, James and Tymofiy Mylovanov. 2012. “Diverging Opinions.” *American Economic Journal: Microeconomics* 4 (1): 209–32.

Baliga, Sandeep, Eran Hanany, and Peter Klibanoff. 2013. “Polarization and Ambiguity.” *American Economic Review* 103 (7): 3071-83.

Banerjee, Abhijit. 1992. “A simple model of herd behavior.” *Quarterly Journal of Economics* 107 (3): 797–817.

Barberá, Pablo. 2020. “Social media, echo chambers, and political polarization.” In *Social Media and Democracy: The State of the Field.* Nate Persily and Joshua Tucker, eds. Cambridge University Press: 34-55.

Benedictis-Kessner, Justin De, Matthew A. Baum, Adam J. Berinsky, and Teppei Yamamoto. 2019. “Persuading the enemy: Estimating the persuasive effects of partisan media with the preference-incorporating choice and assignment design,” *American Political Science Review* 113 (4): 902–916

Benoit, Jean-Pierre and Juan Dubra. 2019. “Apparent bias: What does attitude polarization show?” *International Economic Review* 60 (4): 1675–1703.

Berk, Robert. 1966. “Limiting behavior of posterior distributions when the model is incorrect.” *Annals of Mathematical Statistics* 37 (1): 51–58.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. 1992. “A theory of fads, fashion, custom, and cultural change as informational cascades.” *Journal of Political Economy* 100 (5): 992–1026.

Blackwell, David and Lester Dubins. 1962. “Merging of opinions with increasing information.” *Annals of Mathematical Statistics* 33 (3): 882–886.

Bohren, J. Aislinn and Daniel N. Hauser. 2021. “Learning with heterogeneous misspecified models: Characterization and robustness.” *Econometrica* 89 (6): 3025-3077.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer. 2012. “Salience theory of choice under risk.” *Quarterly Journal of Economics* 127 (3): 1243–1285.

Bowen, T. Renee, Danil Dmitriev, and Simone Galperti. 2021. “Learning from Shared News: When Abundant Information Leads to Belief Polarization.” NBER Working Paper.

Boxell, Levi, Matthew Gentzkow, and Jesse M. Shapiro. 2017. “Greater Internet Use is Not

Associated with Faster Growth in Political Polarization among US Demographic Groups." *Proceedings of the National Academy of Sciences* 114 (40): 10612–10617.

Boyd, Stephen and Lieven Vandenberghe. 2004. *Convex Optimization*. New York: Cambridge University Press.

Campante, Filipe and Daniel Hojman. 2013. "Media and polarization. Evidence from the introduction of broadcast TV in the United States." *Journal of Public Economics* 100: 79–92.

Che, Yeon-Koo and Konrad Mierendorff. 2019. "Optimal Dynamic Allocation of Attention." *American Economic Review* 109 (8): 2993–3029.

Cheng, Ing-Haw and Alice Hsiaw. 2022. "Distrust in Experts and the Origins of Disagreement." *Journal of Economic Theory* 200.

Cotton, John L. 1985. "Cognitive dissonance in selective exposure." In D. Zillmann and J. Bryant eds., *Selective Exposure to Communication*. Hillsdale, JM: Lawrence Erlbaum.

DeGroot, Morris. 1974. "Reaching a consensus." *Journal of the American Statistical Association* 69 (345): 118–121.

DeMarzo, Peter M., Dimitri Vayanos, and Jeffrey Zwiebel. 2003. "Persuasion bias, social influence, and unidimensional opinions." *Quarterly Journal of Economics* 118: 909-968.

Dixit, Avinash K. and Jörgen W. Weibull. 2007. "Political polarization." *Proceedings of the National Academy of Sciences* 104 (18): 7351–7356.

Eagly, Alice H., Serena Chen, Shelly Chaiken, and Kelly Shaw-Barnes. 1999. "The impact of attitudes on memory: An affair to remember." *Psychological Bulletin* 125 (1): 64-89.

Festinger, Leon. 1957. *A Theory of Cognitive Dissonance*. Stanford: Stanford University Press.

Flaxman, Seth, Sharad Goel, and Justin M. Rao. 2016. "Filter Bubbles, Echo Chambers, and Online News Consumption." *Public Opinion Quarterly* 80 (1): 298–320.

Frick, Mira, Ryota Iijima, and Yuhta Ishii. 2020. "Misinterpreting Others and the Fragility of Social Learning." *Econometrica* 88: 2281–2328.

Fryer, Roland G., Jr., Philipp Harms, and Matthew O. Jackson. 2019. "Updating beliefs when evidence is open to interpretation: Implications for bias and polarization." *Journal of the European Economic Association* 17 (5): 1470–1501.

Gallup and Knight Foundation. 2018. "American views: Trust, media and democracy." Accessed at <https://knightfoundation.org/reports/american-views-trust-media-and-democracy/> on May 2, 2020.

Gallup and Knight Foundation. 2020. "American Views 2020: Trust, Media and Democracy." Accessed at <https://knightfoundation.org/reports/american-views-2020-trust-media-and-democracy/> on May 2, 2020.

Gentzkow, Matthew. 2016. "Polarization in 2016." Toulouse Network for Information Technology white paper.

Gentzkow, Matthew and Jesse M. Shapiro. 2006. “Media bias and reputation.” *Journal of Political Economy* 114 (2): 280–316.

Gentzkow, Matthew and Jesse M. Shapiro. 2011. “Ideological segregation online and offline.” *Quarterly Journal of Economics* 126 (4): 1799–1839.

Gentzkow, Matthew, Jesse M. Shapiro and Daniel Stone. 2016. “Media Bias in the Marketplace: Theory.” In *Handbook of Media Economics*, edited by Simon Anderson, Joel Waldfogel, and David Strömberg, 623–645. Volume 2. Amsterdam and Oxford: Elsevier.

Gibbons, Robert, Niko Matouschek and John Roberts. 2013. “Decisions in Organizations.” In R. Gibbons and J. Roberts, eds., *Handbook of Organizational Economics*. Princeton and Oxford: Princeton University Press.

Glaeser, Edward L. and Cass R. Sunstein. 2014. “Does More Speech Correct Falsehoods?” *The Journal of Legal Studies* 43 (1): 65–93.

Glaeser, Edward L. and Bryce A. Ward. 2006. “Myths and realities of American political geography.” *Journal of Economic Perspectives* 20 (2): 119–144.

Golub, Benjamin and Matthew O. Jackson. 2010. “Naïve Learning in Social Networks and the Wisdom of Crowds.” *American Economic Journal: Microeconomics* 2 (1): 112–149.

Golub, Benjamin and Matthew O. Jackson. 2012. “How Homophily Affects the Speed of Learning and Best-Response Dynamics.” *Quarterly Journal of Economics* 127 (3): 1287–1338.

Heidhues, Paul, Botond Kőszegi, and Philipp Strack. 2018. “Unrealistic Expectations and Misguided Learning.” *Econometrica* 86: 1159–1214.

Heidhues, Paul, Botond Kőszegi, and Philipp Strack. 2019. “Overconfidence and Prejudice.” Working paper.

Hetherington, Marc J. and Thomas J. Rudolph. 2015. *Why Washington Won’t Work: Polarization, Trust, and the Governing Crisis.* Chicago: University of Chicago Press.

Iyengar, Shanto, Gaurav Sood, and Yphtach Lelkes. 2012. “Affect, not ideology: A social identity perspective on polarization.” *Public Opinion Quarterly* 76 (3): 405–431.

Iyengar, Shanto, Yphtach Lelkes, Matthew Levendusky, Neil Malhotra, and Sean J. Westwood. 2019. “The origins and consequences of affective polarization in the United States.” *Annual Review of Political Science* 22: 129–146.

Jerit, Jennifer and Yangzi Zhao. 2020. “Political misinformation.” *Annual Review of Political Science* 23: 77-94.

Jo, Donghee. 2020. “Better the Devil You Know: Selective Exposure Alleviates Polarization in an Online Field Experiment.” Working paper.

Keyes, Ralph. 2004. *The Post-Truth Era: Dishonesty and Deception in Contemporary Life.* New York: St Martin’s Press.

Kondor, Péter. 2012. “The More We Know about the Fundamental, the Less We Agree on the Price.” *The Review of Economic Studies* 79 (3): 1175–1207.

Kunda, Ziva. 1990. “The case for motivated reasoning.” *Psychological Bulletin* 108(3): 480–498.

Levendusky, Matthew S. 2013. “Why Do Partisan Media Polarize Viewers?” *American Journal of Political Science* 57 (3): 611–623.

Levy, Gilat and Ronny Razin. 2015. “Correlation Neglect, Voting Behavior, and Information Aggregation.” *American Economic Review* 105 (4): 1634–1645.

Lian, Heng. 2009. “On rates of convergence for posterior distributions under misspecification.” *Communications in Statistics - Theory and Methods* 38 (11): 1893–1900.

Liang, Annie and Xiaosheng Mu. 2020. “Complementary information and learning traps.” *The Quarterly Journal of Economics* 135 (1): 389-448.

Lord, Charles G., Lee Ross, and Mark R. Lepper. 1979. “Biased Assimilation and Attitude Polarization: The Effects of Prior Theories on Subsequently Considered Evidence.” *Journal of Personality and Social Psychology* 37 (11): 2098–2109.

Marietta, Morgan and David C. Barker. 2019. *One Nation, Two Realities: Dueling Facts in American Democracy.* New York: Oxford University Press.

McCarty, Nolan M., Keith T. Poole, and Howard Rosenthal. 2006. *Polarized America: The Dance of Ideology and Unequal Riches.* Cambridge, MA: MIT Press.

McIntyre, Lee. 2018. *Post-truth*. MIT Press.

Mullainathan, Sendhil and Andrei Shleifer. 2005. “The Market for News.” *American Economic Review* 95 (4): 1031–1053.

Nisbet, Erik C., Kathryn E. Cooper, and R. Kelly Garrett. 2015. “The Partisan Brain: How Dissonant Science Messages Lead Conservatives and Liberals to (Dis)Trust Science.” *The ANNALS of the American Academy of Political and Social Science* 658 (1): 36–66.

Ortoleva, Pietro and Erik Snowberg. 2015. “Overconfidence in Political Behavior.” *American Economic Review* 105 (2): 504–535.

Pew Research Center. 2014a. “Political Polarization and Media Habits.” Accessed at <www.journalism.org/2014/10/21/political-polarization-media-habits> on May 2, 2021.

Pew Research Center. 2014b. “Political Polarization in the American Public.” Accessed at <www.pewresearch.org/politics/2014/06/12/political-polarization-in-the-american-public/> on May 2, 2021.

Pew Research Center. 2020. “U.S. Media Polarization and the 2020 Election: A Nation Divided.” Accessed at <https://www.journalism.org/2020/01/24/u-s-media-polarization-and-the-2020-election-a-nation-divided> on June 17, 2020.

Prior, Markus. 2013. “Media and political polarization.” *Annual Review of Political Science* 16: 101–127.

Pronin, Emily. 2007. “Perception and misperception of bias in human judgment.” *Trends in Cognitive Sciences* 11 (1): 37-43.

Pronin, Emily, Daniel Y. Lin, and Lee Ross. 2002. “The bias blind spot: Perceptions of bias in self versus others.” *Personality and Social Psychology Bulletin* 28 (3): 369-381.

Rabin, Matthew and Joel L. Schrag. 1999. “First Impressions Matter: A Model of Confirmatory Bias.” *The Quarterly Journal of Economics* 114 (1): 37–82.

Rochet, Jean-Charles and Jean Tirole. 2003. “Platform Competition in Two-Sided Markets.” *Journal of the European Economic Association* 1 (4): 990–1029.

Sethi, Rajiv and Muhamet Yildiz. 2016. “Communication with Unknown Perspectives.” *Econometrica* 84 (6): 2029–2069.

Slivkins, Aleksandrs. 2019. *Introduction to Multi-Armed Bandits.* Now Publishers.

Thaler, Michael. 2020. “The ’Fake News’ Effect: Experimentally Identifying Motivated Reasoning Using Trust in News.” Working paper.

Tullock, Gordon. 1980. “Efficient Rent Seeking.” In *Toward a Theory of the Rent-seeking Society*, edited by James M. Buchanan, Robert D. Tollison, and Gordon Tullock, 97–112. College Station, TX: Texas A&M University Press.

Tversky, Amos and Daniel Kahneman. 1973. “Availability: a heuristic for judging frequency and probability.” *Cognitive Psychology* 5 (2): 207–232.

van der Vaart, Aad W. 1998. *Asymptotic Statistics*. New York: Cambridge University Press.

Appendices

A Proofs

A.1 Proof of Proposition 1

Let D_{it} denote the data observed by agent i in period t in three cases.

- Case 1. Agent i single-homes and does not observe r_t. We let $D_{it} = \left(s_{jt}, x_{it}\right)$.
- Case 2. Agent i single-homes and observes r_t. We let $D_{it} = \left(s_{jt}, x_{it}, r_t\right)$.
- Case 3. Agent i multi-homes and observes r_t. We let $D_{it} = (s_t, x_{it}, r_t)$.

Lemma 3. *$D_{i1}, \ldots, D_{it}$ is independent of θ_{0i} conditional on R_{0i} and V_{0i}.*

Proof. First consider Case 1. With slight abuse of notation, define $D_{i\tau}^{j} = \left(s_{j\tau}, x_{i\tau}\right)$ to be the τth time j's signal and reasoning are observed. The agent knows $D_{i\tau}^{j} \sim N\left(0, \Omega_{0ij}\right)$ for some positive definite Ω_{0ij}, where $\Omega_{0ij} = V_{0ij}^{\frac{1}{2}} R_{0ij} V_{0ij}^{\frac{1}{2}}$, where $V_{0ij} = \operatorname{diag}\left(\Omega_{0ij}\right)$ and R_{0ij} is the correlation matrix for $D_{i\tau}^{j}$, which in Case 1 is the correlation ρ_{ij}. Independence across periods then generalizes the result to the entire sequence of data observations. Cases 2 and 3 can be proved in the same way. □

Let $P_{Y|X}^{i}$ denote the posterior distributions of Y given X and the prior μ_0^i. Then, by Lemma 3, we can see that for all $\vartheta \in \mathscr{L}_{\Theta}$, $P_{\theta_i|R_i,V_i,D_1,\ldots,D_t}^{i}(\vartheta) = P_{\theta_i|R_i,V_i}^{i}(\vartheta)$ and hence that

$$
\begin{aligned}
P_{\theta_i|D_{i1},\ldots,D_{it}}^{i}(\vartheta) &= \int_{\mathscr{R},\mathscr{V}} P_{\theta_i|R_i,V_i,D_{i1},\ldots,D_{it}}^{i}(\vartheta)\, dP_{R_i,V_i|D_{i1},\ldots,D_{it}}^{i} \\
&= \int_{\mathscr{R},\mathscr{V}} P_{\theta_i|R_i,V_i}^{i}(\vartheta)\, dP_{R_i,V_i|D_{i1},\ldots,D_{it}}^{i}.
\end{aligned}
$$

We now characterize the limit of $P_{R_i,V_i|D_1,\ldots,D_t}^{i}$. First consider Case 3, where agent i multi-homes and observes r_t. Let $P_{D_i|R_i,V_i}$ denote the (true) distribution of D_{it} conditional on R_i and V_i. The experiment $\left(P_{D_i|R_i,V_i} : R_i \in \mathscr{R}; V_i \in \mathscr{V}\right)$ is Gaussian with known mean zero and known variance, and its parameter space $(\mathscr{R}, \mathscr{V})$ is compact. It is straightforward to verify the following regularity conditions: (i) $P_{D_i|R_i,V_i} \neq P_{D_i|(R_i,V_i)'}$ for any $(R_i, V_i) \neq (R_i, V_i)'$; (ii) the mapping

$(R_i, V_i) \mapsto P_{D_i|R_i,V_i}$ is continuous in total variation norm; (iii) $P_{D_i|R_i,V_i}$ has a nonsingular information matrix $I_{R_{0i},V_{0i}}$ at (R_{0i}, V_{0i}) (recalling from Remark 1 we focus on θ_0 such that $R_{0i} \in \operatorname{int}(\mathscr{R})$); (iv) $\left(P_{D_i|R_i,V_i} : R_i \in \mathscr{R}; V_i \in \mathscr{V}\right)$ is differentiable in quadratic mean at (R_{0i}, V_{0i}). Then by van der Vaart (1998) Lemma 10.6 and Theorem 10.1 (the Bernstein-von Mises Theorem), the limit of $P^i_{R_i,V_i|D_{i1},\ldots,D_{it}}$ as $t \to \infty$ is a distribution degenerate at the true correlations (R_{0i}, V_{0i}).

Next consider Case 1, where agent i single-homes and does not observe r_t. Reorder the experimentation periods so those where the agent observes (s_{1t}, x_{it}) occur first, those where the agent observes (s_{2t}, x_{it}) occur second, through those where the agent observes (s_{Jt}, x_{it}). Denote these respective subsequences of data by $D_i^1, \ldots, D_i^J$. Since posterior beliefs are invariant to the order of data, this reordering does not affect the limit of $P^i_{R_i,V_i|D_{i1},\ldots,D_{it}}$. The logic above implies that as $T \to \infty$, the agent's posterior belief $P^i_{R_i,V_i|D_{i1},\ldots,D_{it}}$ at the end of the first set of periods converges to a limit whose marginal distribution on ρ_{i1} is degenerate at the true values of these correlations. Note that for every finite t, $P^i_{R_i,V_i|D_i^1}$ has continuous density on $(\mathscr{R}, \mathscr{V})$ and so is a valid prior under our model. Applying the same logic again then implies that the agent's posterior belief $P^i_{R_i,V_i|D_i^1,D_i^2}$ at the end of the second set of periods converges to a limit whose marginal distribution on (ρ_{i1}, ρ_{i2}) is degenerate at the true value of these correlations. Iterating this logic repeatedly shows that $P^i_{R_i,V_i|D_i^1,\ldots,D_i^J} = P^i_{R_i,V_i|D_{i1},\ldots,D_{it}}$ converges to a limit whose marginal distribution $(\rho_{i1}, \ldots, \rho_{iJ})$ is degenerate at the full vector of true correlations R_{0i}. Case 2 can be proved in the same way.

Finally, note that for all $\vartheta \in \mathscr{L}_\theta$,

$$
\begin{aligned}
\mu^i_{\infty,\theta}(\vartheta) &= \lim_{T\to\infty} P_{\theta_i|D_{i1},\ldots,D_{it}}(\vartheta) \\
&= \lim_{T\to\infty} \int_{\mathscr{R},\mathscr{V}} P^i_{\theta_i|R_i,V_i}(\vartheta)\, dP^i_{R_i,V_i|D_{i1},\ldots,D_{it}} \\
&= P^i_{\theta_i|R_{0i},V_{0i}}(\vartheta) \\
&= \mu^i_{0,\theta}(\vartheta | R_i = R_{0i}) \\
&= \frac{\mu^i_{0,\theta}(\vartheta \cap I_i(R_{0i}))}{\mu^i_{0,\theta}(I_i(R_{0i}))}.
\end{aligned}
$$

where the third equality uses the convergence of $P^i_{R_i,V_i|D_{i1},\ldots,D_{it}}$.

A.2 Proof of Proposition 2

Given any value of $\theta_i \in I_i(R_{0i})$, the state ω_t and the (normalized) signals $\tilde{s}_{jt}$ are jointly distributed

$$\begin{pmatrix} \omega_t \\ \tilde{s}_{jt} \end{pmatrix} \sim N\left(0, \begin{bmatrix} 1 & \alpha_j \\ \alpha_j & 1 \end{bmatrix}\right).$$

The conditional expectation of ω_t given $\tilde{s}_{jt}$ (under single-homing) is then $\alpha_j \tilde{s}_{jt}$, by the properties of the multivariate normal distribution. The desired result follows from taking the expectation over the limiting posterior $\mu^i_{\infty,\theta}$.

A.3 Proof of Proposition 3

The expected loss for a given j is the same for all t in the exploitation period, so we can focus on minimizing the single-period loss for a single agent. Thus we can drop the i and t subscripts. For notational simplicity, we also focus on the simple case where all variances are known to be one. The agent solves

$$\min_{j \in \mathscr{J}_+} E_{s_j,x,\omega}\left[\left(d^*_j\left(s_j,x\right)-\omega\right)^2\right],$$

where the expectation is taken under the distribution of (s,x,ω) and $d^*_j\left(s_j,x\right)$ is the optimal decision after seeing $\left(s_j,x\right)$. Note that $d^*_j\left(s_j,x\right) = E_{\omega|s_j,x}[\omega]$. The law of iterated expectations implies that

$$E_{s_j,x,\omega}\left[\left(d^*_j\left(s_j,x\right)-\omega\right)^2\right] = 1 - E_{s_j,x}\left[d^*_j\left(s_j,x\right)^2\right].$$

Note that in all cases here expectations are taken under the joint distribution of (Ω,s,x,ω) given $\Omega \sim \mu_\infty$. Thus, an expression like $E_\omega[\cdot]$ refers to the expectation under the marginal distribution of ω in that distribution.

Define $d^*_j\left(s_j,x,\Omega\right)$ to be the optimal decision *conditional* on a particular Ω. Note that $d^*_j\left(s_j,x,\Omega\right) = E_{\omega|s_j,x,\Omega}[\omega]$, so we have

$$d^*_j\left(s_j,x,\Omega\right) = \begin{pmatrix} a \\ \alpha_j \end{pmatrix}' \begin{bmatrix} 1 & \alpha_j a \\ \alpha_j a & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix} = \begin{pmatrix} a \\ \alpha_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix},$$

where the last line follows from observing that for all $\left(a,\alpha_j\right)$ in the support of μ_∞, we must have $\alpha_j a = \rho_j$, where ρ_j is a constant equal to the empirical correlation observed in the data. Since $E_{\omega|s_j,x}[\omega] = E_\Omega E_{\omega|s_j,x,\Omega}[\omega] = E_\Omega d^*\left(s_j,x,\Omega\right)$, we have

$$d_j^*\left(s_j,x\right) = \begin{pmatrix} \overline{a} \\ \overline{\alpha}_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix}.$$

It follows from some algebraic manipulation that

$$E_{s_j,x}\left[d_j^*\left(s_j,x\right)^2\right] = \begin{pmatrix} \overline{a} \\ \overline{\alpha}_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} \overline{a} \\ \overline{\alpha}_j \end{pmatrix} = \frac{\overline{a}^2 - 2\rho_j \overline{a}\overline{\alpha}_j + \overline{\alpha}_j^2}{1-\rho_j^2}.$$

By Jensen's inequality

$$\overline{\alpha}_j = E_\Omega\left[\frac{\rho_j}{a}\right] \geq \frac{\rho_j}{\overline{a}},$$

so we have $\rho_j = c\overline{a}\overline{\alpha}_j$ for some $c \leq 1$, holding μ_∞ fixed.Therefore, if we compare the expected variance in decisions when the agent observes sources with different trust $\overline{\alpha}_j$, holding μ_∞ fixed, we have that

$$\frac{\partial E_{s_j,x}\left[d_j^*\left(s_j,x\right)^2\right]}{\partial \overline{\alpha}_j} = \frac{\partial}{\partial \overline{\alpha}_j}\left[\frac{\overline{a}^2 - 2c\overline{a}^2\overline{\alpha}_j^2 + \overline{\alpha}_j^2}{1-c^2\overline{a}^2\overline{\alpha}_j^2}\right] \geq \frac{\partial}{\partial \overline{\alpha}_j}\left[\frac{\overline{a}^2 - 2\overline{a}^2\overline{\alpha}_j^2 + \overline{\alpha}_j^2}{1-\overline{a}^2\overline{\alpha}_j^2}\right] > 0,$$

where the first inequality follows from $c \leq 1$ and the second inequality is strict under the maintained assumption that $\overline{a} < 1$. This then implies that for any j and k such that $\overline{\alpha}_j^2 > \overline{\alpha}_k^2$, we have

$$E_{s_j,x,\omega}\left[\left(d_j^*\left(s_j,x\right) - \omega\right)^2\right] < E_{s_k,x,\omega}\left[\left(d_k^*\left(s_k,x\right) - \omega\right)^2\right]$$

and so the problem is solved by choosing a j with the lowest value of $\overline{\alpha}_j^2$.

A.4 Proof of Proposition 4

Recall that $I_i(R_{0i}) = \left\{\theta_i \in \Theta_i^{prior} : a_i\alpha + b_i\beta = \rho_{is}\right\}$, where any $\theta_i \in \Theta_i^{prior}$ must give rise to a positive semi-definite correlation matrix $\tilde{\Omega}(\theta_i)$ for $(\omega_t, r_t, x_{it}, s_t)$. Assumption 1 implies that $\theta_i \in$

$I_i(R_{0i})$ if and only if $a_i = a_0$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite. Assumption 2 implies that $|\alpha_{0j}| = |\rho_{ij}/a_{0i}| = |\alpha_{0j} + (b_{0i}/a_{0i})\beta_{0j}| \leq ||\alpha_{0j}| + |\beta_{0j}|| \leq 1$. We can then pick $\beta = 0$, $\gamma = 0$, and assume that s_{jt} are mutually independent and independent of r_t and x_{it} conditional on ω_t, so $\Sigma = \text{Corr}(s_t) = \alpha\alpha' + K$, where K is a diagonal matrix with entries equal to $1 - \alpha_j^2$. It follows that $\tilde{\Omega}(\theta_i)$ is positive semi-definite, so $I_i(R_{0i})$ is non-empty.

A.5 Proof of Proposition 5

The proof is similar to the proof of Proposition 4, except r_t is now observed. By Assumption 1, Remark 1, and the definition of $I_i(R_{0i})$, $\theta_i \in I_i(R_{0i})$ if and only if $a_i = a_0$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite. We first show that the correlation matrix for (ω_t, r_t, s_{jt}), i.e. $\begin{bmatrix} 1 & \tilde{\alpha}_{ij} \\ \tilde{\alpha}_{ij} & \tilde{\Sigma}_j \end{bmatrix}$, where $\tilde{\alpha}_{ij} = \begin{pmatrix} \frac{\rho_{ir}}{a_i} \\ \frac{\rho_{ij}}{a_i} \end{pmatrix}$ and $\tilde{\Sigma}_j = \begin{bmatrix} 1 & \rho_{rj} \\ \rho_{rj} & 1 \end{bmatrix}$, is positive semi-definite for all j. This is equivalent to $1 - \tilde{\alpha}_{ij}'\tilde{\Sigma}_j^{-1}\tilde{\alpha}_{ij} \geq 0$ by standard matrix results (see Boyd and Vandenberghe 2004 Appendix A.5.5). This in turn requires that for all j,

$$a_i^2 \geq \zeta_{ij} = \frac{\rho_{ir}^2 + \rho_{xj}^2 - 2\rho_{rj}\rho_{ij}\rho_{ir}}{1 - \rho_{rj}^2}. \tag{7}$$

Algebraic substitution shows that $\zeta_{ij} = b_0^2 + a_0^2 \left(\frac{\alpha_{0j}^2}{1-\beta_{0j}^2} \right)$. Since $a_i = a_0$, this condition is guaranteed by Assumption 2'. We can then assume that s_{jt} are mutually independent and independent of x_{it} conditional on ω_t and r_t, so $\Sigma = \alpha\alpha' + \beta\beta' + K$, where K is a diagonal matrix with entries equal to $1 - \alpha_j^2 - \beta_j^2$. It follows that $\tilde{\Omega}(\theta_i)$ is positive semi-definite, so $I_i(R_{0i})$ is non-empty.

A.6 Proof of Proposition 6

The proof is similar to the proof of Proposition 5, except the agent may become overconfident. By Assumption 1', Remark 1, and the definition of $I_i(R_{0i})$, $\theta_i \in I_i(R_{0i})$ if and only if$a_i \leq a_i^{max}$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite. By the same logic as the proof of Proposition 5, the last condition is true if and only if $a_i \geq \underline{a}_i = \max_j \sqrt{\zeta_{ij}}$. Therefore, $\theta_i \in I_i(R_{0i})$ if and only if $a_i \in [\underline{a}_i, a_i^{max}]$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, and $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$.

A.7 Proof of Proposition 7

This proof is the same as the proof of Proposition 6, except that agent i multi-homes. By Assumption 1', Remark 1, and the definition of $I_i(R_{0i})$, $\theta_i \in I_i(R_{0i})$ if and only if $a_i \le a_i^{max}$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite. Since the agent believes that $x_{it} = a_i\omega_t + \eta_{it}$ for some $a_i \in (0, a_i^{max}]$, $\tilde{\Omega}(\theta_i)$ is positive semi-definite if and only if the covariance matrix for (ω_t, r_t, s_t), given by $\begin{bmatrix} 1 & \tilde{\alpha}' \\ \tilde{\alpha} & \tilde{\Sigma} \end{bmatrix}$, where $\tilde{\alpha} = \begin{pmatrix} \frac{\rho_{ir}}{a_0} \\ \frac{\rho_{is}}{a_0} \end{pmatrix}$ and $\tilde{\Sigma} = \begin{bmatrix} 1 & \rho_{rs}' \\ \rho_{rs} & \Sigma \end{bmatrix}$, is postive semi-definite. This holds if and only if $a_i^2 \ge \zeta_i = \tilde{\rho}_i'\tilde{\Sigma}^{-1}\tilde{\rho}_i$, where $\tilde{\rho}_i = \begin{pmatrix} \rho_{ir} & \rho_{is}' \end{pmatrix}'$, by Boyd and Vandenberghe 2004 Appendix A.5.5. Therefore, $\theta_i \in I_i(R_{0i})$ if and only if $a_i \in \left[\underline{\underline{a}}_i, a_i^{max}\right]$, $b_i = 0$, $\alpha = \frac{\rho_{is}}{a_i}$, $\gamma = \frac{\rho_{ir}}{a_i}$, and $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$, where $\underline{\underline{a}}_i = \sqrt{\zeta_i}$.

A.8 Proof of Lemma 1

Given any value of $\theta_i \in I_i(R_{0i})$, the state ω_t and the (normalized) signals $\tilde{s}_{jt}$ are jointly distributed

$$\begin{pmatrix} \omega_t \\ \tilde{s}_t \end{pmatrix} \sim N\left(0, \begin{bmatrix} 1 & \alpha' \\ \alpha & \Sigma \end{bmatrix}\right).$$

Under multi-homing, the conditional expectation of ω_t given s_t is then $\alpha'\Sigma^{-1}\tilde{s}_t$. By Proposition 7, $\alpha = \frac{\rho_{is}}{a_i}$. Furthermore, $I_i(R_{0i})$ includes all $a_i \in \left[\underline{\underline{a}}_i, a_i^{max}\right]$. The desired result then follows from taking the expectation over the limiting posterior $\mu^i_{\infty,a}$.

A.9 Proof of Lemma 2

We first prove the following Lemma.

Lemma 4. *Suppose that the signals s_{jt} are mutually independent conditional on ω_t and r_t. Then in the multi-homing case, we have*

$$\rho_{is}'\Sigma^{-1}\tilde{s}_t = y'Z\left(Z'Z + K\right)^{-1}\left(Z'\varphi_t + \varepsilon_t\right)$$

and

$$\rho_{is}'\Sigma^{-1}\rho_{is} = y'Z\left(Z'Z + K\right)^{-1}Z'y,$$

where $y = \left[\begin{array}{cc} a_0 & b_{0i} \end{array}\right]'$, *$Z$ is the* $2 \times J$ *matrix where the jth column is* $\left[\begin{array}{cc} \alpha_{0j} & \beta_{0j} \end{array}\right]'$, *$K$ is a diagonal matrix such that the jth diagonal is* $\kappa_{0j}^2 = 1 - \alpha_{0j}^2 - \beta_{0j}^2$, $\varphi_t = \left[\begin{array}{cc} \omega_t & \tilde{r}_t \end{array}\right]'$, and ε_t is the J-vector of $\varepsilon_{jt} = s_{jt} - \alpha_{0j}\omega_t - \beta_{0j}\tilde{r}_t$.

Proof. The lemma follows from noting that $\rho_{is} = Z'y$, $\Sigma = Z'Z + K$, and $s_t = Z'\varphi_t + \varepsilon_t$. $\square$

When all sources are on the frontier and have distinct biases, $K = 0$ and Z spans $\mathbb{R}^2$, so $Z\left(Z'Z+K\right)^{-1}Z' = I$. Lemma 4 then implies that $\rho_{is}'\Sigma^{-1}s_t = a_0\omega_t + b_{0i}r_t$. Note that the R^2 of the population regression of x_{it} on s_t is $\rho_{is}'\Sigma^{-1}\rho_{is}$, which is equal to $a_0^2 + b_{0i}^2$ by Lemma 4. Therefore, the R^2 of the population regression of x_t on s_t and r_t must be weakly greater than $a_0^2 + b_{0i}^2$. However, the R^2 from a regression of x_{it} on s_t and r_t cannot exceed the R^2 from a regression of x_{it} on r_t and ω_t, which is $a_0^2 + b_{0i}^2$. Therefore, $\underline{\underline{a}}_i = \sqrt{\zeta_i} = \sqrt{a_0^2 + b_{0i}^2}$. Since $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$ by assumption, we have that $A_i = 1/\sqrt{a_0^2 + b_{0i}^2}$.

Next consider the case with additional non-frontier sources. Let the vector of signals of the frontier sources be s_t^F. Note that the elements of $\rho_{is}'\Sigma^{-1}$ are the coefficients from a population regression of x_t on the elements of s_t. Further note that $x_{it} = a_0\omega_t + b_{0i}\tilde{r}_t + \eta_t$ where η_t is orthogonal to ε_{jt} for all j, while each element of s_t^F is a linearly independent linear combination of ω_t and r_t. Thus x_t is orthogonal to s_{jt} conditional on s_t^F for all non-frontier sources. By the Frisch–Waugh–Lovell theorem, the elements of $\rho_{is}'\Sigma^{-1}$ corresponding to non-frontier sources must be equal to zero, and the elements of $\rho_{is}'\Sigma^{-1}$ corresponding to frontier sources are the same as in the case with all frontier sources. We conclude that $\rho_{is}'\Sigma^{-1}\tilde{s}_t = a_0\omega_t + b_{0i}r_t$. Furthermore, we can conclude that $\underline{\underline{a}}_i = \sqrt{a_0^2 + b_{0i}^2}$ using the same argument as in the previous paragraph.

A.10 Proof of Proposition 8

In the multi-homing case, there are two or more frontier sources, so $\underline{\underline{a}}_i = \sqrt{a_0^2 + b_{0i}^2}$, $\overline{\alpha}_j^i = 1$, and $\overline{\omega}_t^i = \alpha_i^{max}\omega_t + \beta_i^{max}\tilde{r}_t$. Expected disagreement is $b_0^2/\left(a_0^2 + b_0^2\right) = \sin^2\left(\phi_b\right)$. In the single-homing case, r_t is observed along with a frontier source, so $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$ and $\overline{\alpha}_j^i = \frac{a_0\alpha_{0j} + b_{0i}\beta_{0j}}{\sqrt{a_0^2 + b_{0i}^2}}$. Note that a biased agent observing a bias source results in trust equal to $\cos\left(\phi_b - \phi_\beta\right)$, while a biased agent observing an unbiased source results in trust equal to $\cos\left(\phi_b\right)$. If $\phi_b < \frac{1}{2}\phi_\beta$, then the agent's trust for the unbiased source is higher, so all single-homers observe the unbiased source. This implies that $\pi = 0$ under single-homing, while disagreement is positive under multi-homing. If $\phi_b > \frac{1}{2}\phi_\beta$, then biased single-homers observe the similarly biased source. In this case, disagreement under multi-homing is weakly greater than disagreement under single-homing if and only if $\sin^2(\phi_b) \geq \cos^2(\phi_b - \phi_\beta)\sin^2(\phi_\beta)$, which can be shown to be true if and only if $\phi_b \geq \phi_\beta$ using trigonometric identities and by noting that $0 < \phi_b, \phi_\beta \leq \pi$.

A.11 Proof of Proposition 9

Because all agents will observe the monopolist's signal in every period, the monopolist's profit maximization problem simplifies to choosing accuracy α_{0j} and bias β_{0j} to maximize

$$\Pi_j = \sum_{i \in \{L,U,R\}} m_i \xi\left(\overline{\alpha}_j^i\right) - \lambda,$$

where $\overline{\alpha}_j^i$ is type-i consumers' trust in the monopolist.

The derivative of trust $\overline{\alpha}_j^i$ with respect to β_{0j} along the frontier is:

$$\delta^i\left(\beta_{0j}\right) = \left.\frac{\partial \overline{\alpha}_j^i}{\partial \beta_{0j}}\right|_{\alpha_{0j}^2+\beta_{0j}^2=1} = \frac{1}{\sqrt{a_{0i}^2+b_{0i}^2}}\left(b_{0i} - a_{0i}\frac{\beta_{0j}}{\sqrt{1-\beta_{0j}^2}}\right).$$

Letting $m = m_R = m_L$, the optimal frontier location must satisfy the first order condition that

$$\left.\frac{\partial \Pi}{\partial \beta_{0j}}\right|_{\alpha_{0j}^2+\beta_{0j}^2=1} = (1-2\mu)\,\xi'\left(\overline{\alpha}^U\right)\delta^U\left(\beta_{0j}\right) + m\left[\xi'\left(\overline{\alpha}^R\right)\delta^R\left(\beta_{0j}\right) + \xi'\left(\overline{\alpha}^L\right)\delta^L\left(\beta_{0j}\right)\right] = 0. \quad (8)$$

This condition is satisfied at $\beta_{0j} = 0$ because $\delta^U(0) = 0$, $\overline{\alpha}_j^R = \overline{\alpha}_j^L$, and $\delta^R\left(\beta_{0j}\right) = -\delta^L\left(\beta_{0j}\right)$. When $\beta_{0j} > 0$, $\xi'\left(\overline{\alpha}_j^R\right) \leq \xi'\left(\overline{\alpha}_j^L\right)$ because $\xi(\cdot)$ is assumed to be concave and $\overline{\alpha}_j^R \geq \overline{\alpha}_j^L$. Moreover it is straightforward to show that $\delta^R\left(\beta_{0j}\right) + \delta^L\left(\beta_{0j}\right) < 0$ and $\delta^L\left(\beta_{0j}\right), \delta^U\left(\beta_{0j}\right) < 0$. Thus the derivative in equation (8) is strictly negative. Symmetric reasoning shows that this derivative is strictly positive when $\beta_{0j} < 0$. Thus, the unique solution is for the monopolist to choose $\beta_{0j} = 0$ and $\alpha_{0j} = 1$.

Since revenues at this position are strictly positive, the monopolist will enter when λ is sufficiently low. Overconfidence for biased agents follows from noting that $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$, and the expected disagreement result follows from noting that $\overline{\alpha}_j^R = \overline{\alpha}_j^L$.

A.12 Proof of Proposition 10

Suppose one of the positions $\{\beta^L, 0, \beta^R\}$ is not occupied by any outlet. Then a potential entrant j can enter into this position and become the unique outlet with maximum trust from the associated type of agent. This outlet will have a trust of one and hence positive revenue $\xi(1)$ from the associated agent type, so entry will be profitable for sufficiently low λ. Let λ be any λ small enough to support positive profit for at least two trust-maximizing outlets in every position earning revenue exclusively from their corresponding agent type, i.e.,[19]

$$\left\{\lambda \,\middle|\, \min_{i\in\{U,R,L\}} \frac{1}{2} m_i \xi(1) - \lambda > 0\right\}.$$

Furthermore, when these positions are occupied, an outlet at any other position earns zero revenue and strictly negative profit since $\lambda > 0$. Thus, in any equilibrium all entrants must locate at one of these positions.

It remains to show that an equilibrium exists with at least one outlet in each position. The above result reduces the problem to a standard sequential entry game with three possible locations. Let $\Pi_L(J_L, J_U, J_R)$ denote the profit earned by an outlet in position β^L in a market with J_L, J_U, J_R firms in the three positions. We show two properties about Π_L that will be useful later. First, once the other positions (0 and β^R) have each been occupied by at least one outlet, the specific number of such outlets no longer affect Π_L: for any $J_L, J_U, J'_U, J_R, J'_R$ such that $\min(J_U, J'_U, J_R, J'_R) \geq 1$,

$$\Pi_L(J_L, J_U, J_R) = \Pi_L\left(J_L, J'_U, J'_R\right).$$

Moreover, by definition Π_L is strictly decreasing in J_L and $\Pi_L(2, J_U, J_R) > 0$ for any J_U, J_R due to our earlier choice of λ, so there exists a number of potential entrants $E_L(\lambda) < \infty$ such that $\Pi_L(E_L, J_U, J_R) < 0$ for any $J_U, J_R \geq 1$. Combining this with our preceding result, we obtain the

[19] Requiring enough profit for two trust-maximizing outlets in every position eliminates edge cases where the market only supports a limited number of outlets. Such outlets could instead choose to pool in one or two of the locations rather than spread across all three.

second property: there exists a unique $J_L^* \in [2, E_L)$ such that for any $J_U, J_R \geq 1$,

$$\begin{aligned} \Pi_L(J_L^*, J_U, J_R) &\geq 0 \\ \Pi_L(J_L^* + 1, J_U, J_R) &< 0. \end{aligned}$$

This unique J_L^* is the threshold beyond which entry into position β^L is no longer profitable. Let Π_U and Π_R denote similar objects for positions 0 and β^R, where similar arguments show that these two properties hold as well. Let J_U^* and J_R^* denote their counterparts to J_L^*.

The tuple (J_L^*, J_U^*, J_R^*) of outlets is an equilibrium if the following conditions hold for Π_L:

$$\begin{aligned} \Pi_L(J_L^*, J_U^*, J_R^*) &\geq 0 \\ \Pi_L(J_L^* + 1, J_U^*, J_R^*) &< 0 \\ \Pi_L(J_L^*, J_U^*, J_R^*) &\geq \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*) \\ \Pi_L(J_L^*, J_U^*, J_R^*) &\geq \Pi_R(J_L^* - 1, J_U^*, J_R^* + 1), \end{aligned}$$

and similar conditions hold for Π_U and Π_R. The first two conditions follow from the definition of J_L^*. Next, at (J_L^*, J_U^*, J_R^*) we have $\Pi_L(J_L^*, J_U^*, J_R^*) \geq 0$ by definition of J_L^* and $0 > \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*)$ by definition of J_U^* (noting that $J_L^* \geq 2$ and so $J_L^* - 1 \geq 1$). Hence $\Pi_L(J_L^*, J_U^*, J_R^*) > \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*)$, giving us the third condition. The fourth condition follows similarly. The same arguments prove that the corresponding conditions hold for Π_U and Π_R. The tuple (J_L^*, J_U^*, J_R^*) is also the unique equilibrium: by uniqueness of (J_L^*, J_U^*, J_R^*), no other candidate tuple can satisfy the first two conditions simultaneously. Thus, at least one outlet chooses each of the positions $\{\beta^L, 0, \beta^R\}$ in equilibrium.

Applying Corollary 5 shows that agents are overconfident. Expected disagreement follows from equation (10), noting that all outlets have a trust of one and the positions $\{\beta^L, \beta^R\}$ approach ± 1 as $a_0 \to 0$.

B Trust and Polarization when Agents Believe their own Bias is Nonzero

We consider a case where agents place a dogmatic prior on $b_i = b^*$ for some $b^* \neq 0$. We follow the same setup as those in Section 3, namely that there exist multiple available sources, but agents are single-homing and ideological valence r_t is never observed. As in Section 3, we make the following assumptions, with the first having been modified to incorporate the dogmatic prior on b_i:

Assumption 4. *The support of $\mu^i_{0,\theta}$ is the set $\Theta^{prior}_i \subset \Theta$ for which $b_i = b^*$ and $a_i = a_{0i}$.*

Assumption 5. *$b_0 \leq a_0$ and $|\alpha_j| + |\beta_j| \leq 1$ for all j.*

The following proposition (originally Proposition 4) only requires minor modification.[20][21]

Proposition 11. *Suppose r_t is never observed and agents single home. Under Assumptions 4 and 5, agent i's identified set, $I_i(R_{0i})$, is non-empty and consists of all $\theta_i \in \Theta$ such that $a_i = a_{0i}$, $b_i = b^*$, $\alpha = \frac{\rho_{is} - \beta b^*}{a_i}$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite, where $\tilde{\Omega}(\theta_i)$ is the correlation matrix for $(\omega_t, r_t, x_{it}, s_t)$.*

Proof. The proof for Proposition 4 (reproduced with only minor modification below) applies the expressions from Assumption 4 and then shows that the identified set is non-empty by picking (among other parameters) $\beta = 0$. The same argument can be used to prove the new setting, as the degenerate prior on $b_i = b^*$ has no effect in the case with $\beta = 0$.

Recall that $I_i(R_{0i}) = \left\{\theta_i \in \Theta^{prior}_i : a_i\alpha + b_i\beta = \rho_{is}\right\}$, where any $\theta_i \in \Theta^{prior}_i$ must give rise to a positive semi-definite correlation matrix $\tilde{\Omega}(\theta_i)$ for $(\omega_t, r_t, x_{it}, s_t)$. Assumption 4 implies that $\theta_i \in I_i(R_{0i})$ if and only if $a_i = a_0$, $b_i = b^*$, $\alpha = \frac{\rho_{is} - \beta b^*}{a_i}$, and $\tilde{\Omega}(\theta_i)$ is positive semi-definite. Assumption 5 implies that $|\alpha_{0j}| = |\rho_{ij}/a_{0i}| = |\alpha_{0j} + (b_{0i}/a_{0i})\beta_{0j}| \leq ||\alpha_{0j}| + |\beta_{0j}|| \leq 1$. We can then pick $\beta = 0$, $\gamma = 0$, and assume that s_{jt} are mutually independent and independent of r_t and x_{it} conditional on ω_t, so $\Sigma = \mathrm{Corr}(s_t) = \alpha\alpha' + K$, where K is a diagonal matrix with entries equal to $1 - \alpha_j^2$. It follows that $\tilde{\Omega}(\theta_i)$ is positive semi-definite, so $I_i(R_{0i})$ is non-empty. $\square$

[20]The results from Section 2 do not rely on the agent's prior on b_i.

[21]Unlike in the original proposition, the presence of a nonzero b^* here means that the identified set may sometimes exclude extreme values of β. The specific bounds are not necessary for the results that follow, and we do not derive explicit expressions for them.

Under Assumptions 4 and 5, Propositions 1 and 11 imply the agent's trust in information source j is[22]

$$\overline{\alpha}^i_j = \frac{\rho_{ij} - \overline{\beta}^i_j b^*}{a_{0i}} = \frac{a_{0i}\alpha_{0j} + b_{0i}\beta_{0j} - \overline{\beta}^i_j b^*}{a_{0i}} = \alpha_{0j} + \frac{b_{0i}\beta_{0j} - \overline{\beta}^i_j b^*}{a_{0i}},$$

where $\overline{\beta}^i_j$ denotes agent i's expectation of β_j under $\mu^i_{\infty,\theta}$.

Proposition 12. *Suppose r_t is never observed, agents single home, and Assumptions 4 and 5 hold. Then given any prior $\mu^i_{0,\theta}$, there exists $b^*_{max} > 0$ such that for all $b^* \in [-b^*_{max}, b^*_{max}]$, agent R's (L's) trust in source j is increasing (decreasing) in the source's bias β_{0j} holding constant the source's accuracy α_{0j}. Additionally, in the limit as $b^* \to 0$ and then as $a_0 \downarrow b_0$, she will come to believe that a perfectly right-biased (left-biased) source is perfectly accurate, and trust it more than any unbiased source with $\alpha_{0j} < 1$.*

Proof. Without loss of generality, let i be an R-agent. For the first result, note the derivative of her trust $\overline{\alpha}^i_j$ with respect to in a source j's bias β_{0j} holding constant its accuracy α_{0j} is:

$$\frac{\partial}{\partial \beta_{0j}} \overline{\alpha}^i_j \quad \propto \quad b_{0i} - \left(\frac{\partial}{\partial \beta_{0j}} \overline{\beta}^i_j\right) b^*.$$

For any continuous prior $\mu^i_{0,\theta}$, $\frac{\partial}{\partial \beta_{0j}} \overline{\beta}^i_j$ is bounded above by some finite $\overline{M}_j$ and below by some finite $\underline{M}_j$ for the closed interval $\beta_{0j} \in [-1,1]$. First, consider any $b^* > 0$. If $\overline{M}_j \leq 0$ then the second term above is always strictly negative. If $\overline{M}_j > 0$, then for any $b^* < b_{0i}/\overline{M}_j$ we have $b_{0i} > \overline{M}_j b^* \geq \left(\frac{\partial}{\partial \beta_{0j}} \overline{\beta}^i_j\right) b^*$. In both cases, we have the desired result of $\frac{\partial}{\partial \beta_{0j}} \overline{\alpha}^i_j > 0$ for all β_{0j}. A similar argument with $\underline{M}_j$ proves the result for $b^* < 0$. Hence we can set $b^*_{max} = \min\left\{\min_j \left|b_{0i}/\overline{M}_j\right|, \min_j \left|b_{0i}/\underline{M}_j\right|\right\}$ to guarantee the result for all $b^* \in [-b^*_{max}, b^*_{max}]$ and across all sources j.

For the second result, the R-agent's trust with respect to a perfectly right-biased source is $\overline{\alpha}^i_j = \frac{b_{0i} - \overline{\beta}^i_j b^*}{a_{0i}}$. Since $\overline{\beta}^i_j$ is bounded within $[-1,1]$ for any prior, taking the limit as $b^* \to 0$ and then $a_0 \downarrow b_0$ gives $\overline{\alpha}^i_j = 1$. For any unbiased source with $\alpha_{0j} < 1$, the agent's trust is $\overline{\alpha}^i_j = \alpha_{0j} - \frac{\overline{\beta}^i_j b^*}{a_{0i}}$, which approaches $\alpha_{0j} < 1$ in the limit as $b^* \to 0$. $\square$

[22] For any $\theta_i \in I_i(R_{0i})$, by Proposition 11 $\alpha^i_j = \frac{\rho_{ij} - \beta^i_j b^*}{a_{0i}}$. The expression for trust $\overline{\alpha}^i_j$ is then obtained by taking the expectation over the limiting posterior $\mu^i_{\infty,\theta}$.

Corollary 7. *Suppose r_t is never observed, agents single home, and Assumptions 4 and 5 hold. Further suppose that all sources have accuracy $\alpha_{0j} < 1$ and there is at least one perfectly right-biased source and at least one perfectly left-biased source. In the limit as $b^* \to 0$ and then $a_0 \downarrow b_0$, expected disagreement is one.*

Proof. Follows from Proposition 12. □

C Trust and Disagreement with Overconfidence

In this section, we derive general results for trust and beliefs in the presence of overconfidence, as discussed in Section 5. Here we drop the simplifying assumption that $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$, and instead assume that $a_i^{max} \geq \sqrt{a_0^2 + b_{0i}^2}$, as in Assumption 1'.

Lemma 5. *Under Assumption 1', agent i's trust in information source j is*

$$\overline{\alpha}_j^i = A_i \rho_{ij} = A_i \left(a_0 \alpha_{0j} + b_{0i} \beta_{0j} \right),$$

where the amplification factor A_i is given by

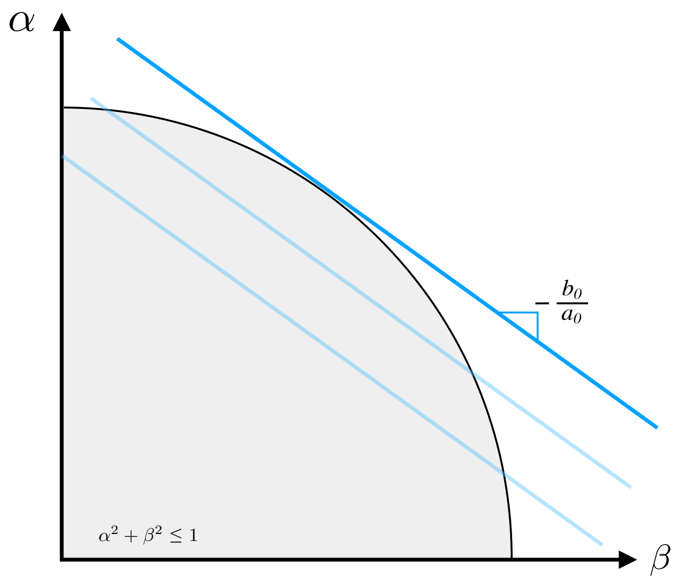
$$A_i = \int_{\underline{a}_i}^{a_i^{max}} \frac{1}{a} d\mu_{\infty,a}^i (a). \tag{9}$$

Proof. This follows from combining Propositions 1 and 6. □

Lemma 5 shows that, as before, small differences in biases b_0 and β_{0j} translate into large differences in trust, provided the amplification factor A_i is large. The amplification factor A_i is typically large whenever a_0 and b_{0i} are small. To see this, suppose that the posterior marginal probability density function on a_i is nonincreasing. It follows then that $A_i \to \infty$ as $\underline{a}_i \to 0$. In other words, A_i is large if $\underline{a}_i \leq \sqrt{a_0^2 + b_{0i}^2}$ is small and the agent's posterior places sufficient weight on values of a_i close to $\underline{a}_i$.

Lemma 6. *The accuracy and bias (α_i, β_i) that maximizes the agent i's trust over all pairs satisfying*

Figure 1: Iso-trust curves



the feasibility condition $\alpha_i^2+\beta_i^2\le 1$ *is given by*

$$(\alpha_i^{max},\beta_i^{max})\equiv\left(\frac{a_0}{\sqrt{a_0^2+b_{0i}^2}},\frac{b_{0i}}{\sqrt{a_0^2+b_{0i}^2}}\right).$$

Proof. By Lemma 5, we choose (α_i,β_i) to maximize $A_i(a_0\alpha_i+b_{0i}\beta_i)$ subject to $\alpha_i^2+\beta_i^2\le 1$. Standard constrained optimization techniques yield the result. □

For intuition, Figure 1 provides a graphical illustration of the forces that determine trust in our model. The gray shaded area shows the set of all feasible signals—i.e., the (α,β) satisfying the constraint that $\alpha^2+\beta^2\le 1$. The curved boundary of this area is defined as the set of *frontier sources* which have maximum possible accuracy given their bias. The blue lines in the figure plot the set of *iso-trust curves*: combinations of α and β that yield the same trust. The slope of these lines is $-\frac{b_0}{a_0}$. Sources that fall on higher iso-trust curves are trusted more. From this graphical analysis, it is immediately apparent that the trust-maximizing source $(\alpha^{max},\beta^{max})$ will be the point on the frontier tangent to the iso-trust curves.

Lemma 7. *In the limit as* $T\to\infty$*, whenever a single-homing agent* i *observes source* j *in any* *period $t > \varepsilon T$, the mean of the agent's posterior on ω_t given s_{jt} is*

$$\overline{\omega}_t^i(j) = \overline{\alpha}_j^i \tilde{s}_{jt} = A_i \rho_{ij} \tilde{s}_{jt}, \tag{10}$$

where $\tilde{s}_{jt} = s_{jt} / \sqrt{Var(s_{jt})}$ is the standardized version of s_{jt}.

Proof. This follows from combining Lemma 5 with Proposition 2. □

D Asymptotic Learning without Reasoning

Can agent i learn about ω_t if the agent does not observe any information source that she *ex ante* believed to be unbiased? As discussed in Section 2.4 and shown in this section, learning about ω_t from s_t is not possible if reasoning x_{it} were not available. Since ρ_{is} and ρ_{ir} are not observed in this case, the distribution of observable data is given by $R_{0i} = \rho_{rs}$ alone. The identified set $I_i(R_{0i})$ consistent with observed data ρ_{rs} contains a wide range of parameter values, including $\alpha_j = 1$ for some source j, or $\alpha_j = -1$ for the same source j. The agent thus cannot rule out the extreme possibilities that any of the sources is perfectly positively correlated, uncorrelated, or perfectly negatively correlated with the true state.

Without reasoning x_{it}, the agent's posterior mean on ω_t may always be zero regardless of what signals are available. This occurs whenever the agent's prior are (α, γ)-symmetric, meaning that $\mu_{0,\theta}^i(\vartheta) = \mu_{0,\theta}^i(\vartheta')$ for all measurable $\vartheta \subseteq \mathscr{L}_\Theta$, where $\vartheta' = \{(a, -\alpha, b, \beta, -\gamma, \Sigma) \mid (a, \alpha, b, \beta, \gamma, \Sigma) \in \vartheta\}$. Intuitively, the agent's average belief about ω_t does not change after observing s_t if she believes *a priori* that the correlation of ω_t with any observable source (i.e. any element of s_t or r_t) is zero in expectation.

Proposition 13. *Suppose agent i does not observe x_{it} in any period, but still observes r_t. Under Assumption 1', agent i's identified set, $I_i(R_{0i})$, includes $\theta_i \in \Theta$ such that $\alpha_j = z$, for any source j and any $z \in [-1, 1]$. Furthermore, the mean of the agent's posterior on ω_t given s_t in any period $t > \varepsilon T$ is zero in the limit as $T \to \infty$ if the agent's prior is (α, γ)-symmetric.*

Proof. Consider the multi-homing case first. Take any $z \in [-1, 1]$ and $j \in \{1, \dots, J\}$. Define θ_i as follows. Set $a = a_0$, $b = 0$, $\alpha_j = z$, $\alpha_k = z\Sigma_{jk}$ for all $k \neq j$, $\gamma = z\rho_{rj}$ and $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$.

It is immediate that $\rho_{rs} = \alpha\gamma + \beta\sqrt{1-\gamma^2}$ (as required by Remark 1). Furthermore, note that θ_i corresponds to a well-defined correlation matrix for the joint distribution of (ω_t, r_t, s_t).[23] Therefore, we have that $\theta_i \in I_i(R_{0i})$. The same θ_i works in the single-homing case, which only requires a well-defined correlation matrix for the unit-normal joint distribution of (ω_t, r_t, s_{kt}) for each k.

By Propositions 1 and the properties of the multivariate normal distribution, the mean of the multi-homing agent's posterior on ω_t given s_t is

$$\frac{\int_{I_i(R_{0i})} \alpha' \Sigma^{-1} \tilde{s}_t d\mu^i_{0,\theta}(\theta_i)}{\mu^i_{0,\theta}(I_i(R_{0i}))},$$

where $I_i(R_{0i}) = \left\{\theta_i \in \Theta : \tilde{\alpha}'\tilde{\Sigma}^{-1}\tilde{\alpha} \leq 1; b = 0\right\}$. Under single-homing, the analogous mean is

$$\frac{\int_{I_i(R_{0i})} \alpha_j \tilde{s}_{jt} d\mu^i_{0,\theta}(\theta_i)}{\mu^i_{0,\theta}(I_i(R_{0i}))},$$

where $I_i(R_{0i}) = \left\{\theta \in \Theta : \tilde{\alpha}_j'\tilde{\Sigma}_j^{-1}\tilde{\alpha}_j \leq 1 \forall j; b = 0\right\}$.[24] Since $\mu^i_{0,\theta}$ is (α,γ)-symmetric, both integrals above are zero. $\square$

E Large Market of Non-frontier Sources

This section considers a situation wherein there are many non-frontier sources that together provide a relatively "large" quantity of information about both ω_t and r_t, as discussed in Section 6. In particular, suppose that the number of sources is large, the signals s_{jt} are mutually independent conditional on ω_t and r_t, and there is at least a minimal amount of diversity in their biases. We formalize this notion of a "large and diverse" set of sources as follows.

Definition 1. A sequence of *random markets* is indexed by $J = 1, 2, ..., \infty$. Random market J has J sources, indexed by $j = 1, ..., J$, each with accuracy and bias $(\alpha_{0j}, \beta_{0j})$ drawn i.i.d. from

[23] This correlation matrix is given by

$$\overline{\Omega} = \begin{bmatrix} 1 & \gamma & \alpha' \\ \gamma & 1 & \rho_{rs}' \\ \alpha & \rho_{rs} & \Sigma \end{bmatrix}.$$

Setting $\gamma = z\rho_{rsj}$, $\alpha_j = z$ and $\alpha_k = z\Sigma_{jk}$ for all $k \neq j$, where $z \in [-1, 1]$, corresponds to supposing that $\omega_t = z\tilde{s}_{jt} + (1-z)e_t$, where $e_t \sim N(0, 1)$ and is independent of (r_t, s_t). Since it follows that $\omega_t \sim N(0, 1)$, $\overline{\Omega}$ is well-defined.

[24] See the proof of Propositions 6 and 7 for the definitions of $\tilde{\alpha}_j$, $\tilde{\Sigma}_j$, $\tilde{\alpha}$ and $\tilde{\Sigma}$.

some distribution F. The sources' signals s_{jt} are mutually independent conditional on ω_t and r_t. Furthermore, under F,

1. Both $\alpha_{0j} \neq 0$ and $\beta_{0j} \neq 0$ have nonzero probability;

2. α_{0j} and β_{0j} are not perfectly correlated; and

3. $\alpha_{0j}^2 + \beta_{0j}^2 < 1$ with probability one.

As shown in Proposition 14 below, a multi-homing agent's posterior mean $\overline{\omega}_t^i$ in a large random market is the same as a single-homing agent when she observes her trust-maximing source. The reason is that a multi-homing agent in a random market can construct a linear combination of the sources' signals whose value will approach the signal of the agent's trust-maximizing source, as in Lemma 2.

Proposition 14. *Suppose Assumption 1' holds. Then the mean of the multi-homing agent's posterior on ω_t given s_t under $\mu_{\infty,\theta}^i$ in the probability limit of a sequence of random markets is*

$$\overline{\omega}_t^i = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t.$$

Proof. By Lemma 1, it suffices to show that $\lim_{J\to\infty} \rho_{is}' \Sigma^{-1} \tilde{s}_t = a_0 \omega_t + b_{0i} \tilde{r}_t$ and $\lim_{J\to\infty} A_i = 1/\sqrt{a_0^2 + b_{0i}^2}$. We use notation developed in Lemma 4, and let $Q = ZK^{-1}Z'$, $d_{\alpha\alpha} = \frac{1}{J}\sum_{j=1}^J \alpha_{0j}^2/\kappa_{0j}^2$, $d_{\beta\beta} = \frac{1}{J}\sum_{j=1}^J \beta_{0j}^2/\kappa_{0j}^2$, and $d_{\alpha\beta} = \frac{1}{J}\sum_{j=1}^J \alpha_{0j}\beta_{0j}/\kappa_{0j}^2$. It follows that $Q = J\begin{bmatrix} d_{\alpha\alpha} & d_{\alpha\beta} \\ d_{\alpha\beta} & d_{\beta\beta} \end{bmatrix}$. Next let $W = Q(I+Q)^{-1}$. By Woodbury's matrix identity, we can write $Z(Z'Z+K)^{-1} = (I-W)ZK^{-1}$. It is easy to check that

$$W = \begin{bmatrix} \frac{\frac{1}{J}d_{\alpha\alpha} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2}{\frac{1}{J^2} + \frac{1}{J}d_{\alpha\alpha} + \frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2} & \frac{-\frac{1}{J}d_{\alpha\beta}}{\frac{1}{J^2} + \frac{1}{J}d_{\alpha\alpha} + \frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2} \\ \frac{-\frac{1}{J}d_{\alpha\beta}}{\frac{1}{J^2} + \frac{1}{J}d_{\alpha\alpha} + \frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2} & \frac{\frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2}{\frac{1}{J^2} + \frac{1}{J}d_{\alpha\alpha} + \frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2} \end{bmatrix}.$$

It is also easy to check that $Z(Z'Z+K)^{-1}Z' = (I-W)Q = W$. By Definition 1, $\mathrm{E}\left[\alpha_{0j}^2/\kappa_{0j}^2\right]$, $\mathrm{E}\left[\beta_{0j}^2/\kappa_{0j}^2\right]$, and $\mathrm{E}\left[\alpha_{0j}\beta_{0j}/\kappa_{0j}^2\right]$ exist and are finite. Therefore, $d_{\alpha\alpha}$, $d_{\beta\beta}$, and $d_{\alpha\beta}$ converge in probability in the limit as $J \to \infty$ by the weak law of large numbers. Furthermore, Definition

1 implies that α_{0j} and β_{0j} are not linearly dependent, so neither are $\alpha_{0j}/\sqrt{\kappa_{0j}^2}$ and $\beta_{0j}/\sqrt{\kappa_{0j}^2}$. By Cauchy-Schwarz, we have that $\mathrm{E}\left[\alpha_{0j}\beta_{0j}/\kappa_{0j}^2\right]^2 < \mathrm{E}\left[\alpha_{0j}^2/\kappa_{0j}^2\right]\mathrm{E}\left[\beta_{0j}^2/\kappa_{0j}^2\right]$. This implies that plim $\left(d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2\right) > 0$. Therefore, $Z\left(Z'Z+K\right)^{-1}Z' = W \to_p I$.

Further algebraic manipulation shows that

$$y'Z\left(Z'Z+K\right)^{-1}\varepsilon_t = \frac{\left(a_0 d_{\beta\beta} + b_0 d_{\alpha\beta}\right)\left(\frac{1}{J}\sum_{j=1}^{J}\frac{\alpha_{0j}}{\sqrt{\kappa_j^2}}\tilde{\varepsilon}_{jt}\right) + \left(a_0 d_{\alpha\beta} + b_0 d_{\alpha\alpha}\right)\left(\frac{1}{J}\sum_{j=1}^{J}\frac{\beta_{0j}}{\sqrt{\kappa_j^2}}\tilde{\varepsilon}_{jt}\right)}{\frac{1}{J^2} + \frac{1}{J}d_{\alpha\alpha} + \frac{1}{J}d_{\beta\beta} + d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2} + o_p(1)$$

where $\tilde{\varepsilon}_{jt} = \varepsilon_{jt}/\sqrt{\kappa_j^2} \sim N(0,1)$ are mututally independent across j as well as independent of α_{0j} and β_{0j}. Therefore, by the weak law of large numbers, $y'Z\left(Z'Z+K\right)^{-1}\varepsilon_t \to_p 0$. It immediately follows from Lemma 4 that $\rho_{is}'\Sigma^{-1}\tilde{s}_t = y'Z\left(Z'Z+K\right)^{-1}\left(Z'\varphi_t + \varepsilon_t\right) \to_p y'\varphi_t = a_0\omega_t + b_{0i}r_t$.

Next note that the R^2 of the population regression of x_{it} on s_t is $\rho_{is}'\Sigma^{-1}\rho_{is}$. By Lemma 4 and the first paragraph in this proof, $\rho_{is}'\Sigma^{-1}\rho_{is} = y'Z\left(Z'Z+K\right)^{-1}Z'y \to_p y'y = a_0^2 + b_{0i}^2$. Therefore, the R^2 of the population regression of x_{it} on s_t and r_t also converges in probability to $a_0^2 + b_{0i}^2$. This implies that $\underline{\underline{a}}_i \to_p \sqrt{a_0^2 + b_{0i}^2}$. By Assumption 3, $\mu_{\infty,a}^i = \mu_{0,a}^i$, so $A_i \to_p 1/\sqrt{a_0^2 + b_{0i}^2}$. $\square$