

A: Gold rush

Let r_i , c_j and d_{j-i+n} be the estimated depths of gold deposits for row, column and diagonal of sector (i, j) . To solve the problem we would like to know the value

$$R = \sum_{i=1}^n \sum_{j=1}^m [\max(r_i, c_j, d_{j-i+n}) - \min(r_i, c_j, d_{j-i+n})].$$

To achieve this we will calculate two expressions

$$S = \sum_{i=1}^n \sum_{j=1}^m [r_i + c_j + d_{j-i+n}]$$

and

$$T = \sum_{i=1}^n \sum_{j=1}^m [2 \cdot \min(r_i, c_j, d_{j-i+n}) + \text{mid}(r_i, c_j, d_{j-i+n})],$$

where `mid` returns median of given values. Note that $R = S - T$.

To obtain S we can iterate over all estimations and multiply them by the number of sectors they apply to. So we would multiply row estimations by m , column estimations by n and diagonal estimations by an appropriate value between 1 and $\min(n, m)$.

Note that

$$T = \sum_{i=1}^n \sum_{j=1}^m [\min(r_i, c_j) + \min(r_i, d_{j-i+n}) + \min(c_j, d_{j-i+n})].$$

To obtain T we can solve the following problem. Given only two types of estimations calculate sum of minimums over every sector.

Let's resolve this subproblem given estimations for rows and diagonals. Other pairs are solved similarly. We use two segment trees. One for rows and one for diagonals. Each segment tree will store the information on how many sectors in a given row (diagonal) was not assigned a smaller diagonal (row) estimation, e.g. segment tree for rows will start with value m in every leaf. We iterate over all the estimations for rows and diagonals in an ascending order. Let r_i be an row estimation under consideration. We query segment tree for rows (i -th position) to find the number of sectors that were not assigned a value yet. Let x be the result of the query. We add $r_i \cdot x$ to the result of the subproblem. Before moving to the next estimation we have to update the segment tree for diagonals. We add -1 to all the diagonals that have a common sector with considered row. We follow a similar procedure when considering diagonal estimations.

Solving this subproblem for every pair of estimation types gives as T . So in the end we can return R as the result.

Time complexity: $O((n + m) \log(n + m))$.