Epidemic Disease Example

Given: Find the state-space model to simulate the spread of a disease throughout a population

Solution: In some cases, it is easier to define the states prior to determining the system model equations.

States:

- z_1 = number NOT infected but susceptible to disease
- z_2 = number of people infected
- z_3 = number of people cured or immunized
- z_4 = number of people who die

Note: Different assumptions lead to different answers. There may not be a “correct” answer when developing a model.

Inputs:

- u_1 = new uninfected (but susceptible) people (born, immigrated, etc.)
- u_2 = new infected people (born infected, immigrated infected, etc.)

Epidemic Disease Example

With the states defined, we can then determine the relationships between those states.

z_1 = # NOT infected

z_2 = # infected

z_3 = # immunized

z_4 = # removed

a = healthy who die*

b = healthy who are infected

c = healthy who are immunized

d = infected who die

e = infected who are cured

f = immunized who die

$$\dot{z}_4 = az_1 + dz_2 + fz_3$$
$$\dot{z}_3 = cz_1 + ez_2 - fz_3$$
$$\dot{z}_2 = bz_1 - dz_2 - ez_2 + u_2$$
$$\dot{z}_1 = -az_1 - bz_1 - cz_1 + u_1$$

$$\frac{d\mathbf{z}}{dt} = \mathbf{Az} + \mathbf{Bu}$$
$$= \begin{bmatrix} -a-b-c & 0 & 0 & 0 \\ b & -d-e & 0 & 0 \\ c & d & -f & 0 \\ a & d & f & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \mathbf{Cz} + \mathbf{Du}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

*rated in #/100/day.

Epidemic Disease Example

From the previous slide…

$$\frac{d\mathbf{z}}{dt} = \mathbf{Az} + \mathbf{Bu}$$

$$= \begin{bmatrix} -a-b-c & 0 & 0 & 0 \\ b & -d-e & 0 & 0 \\ c & d & -f & 0 \\ a & d & f & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \mathbf{Cz} + \mathbf{Du}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

Given the units on the coeeficients, it makes more sense to think of this as a discrete system.

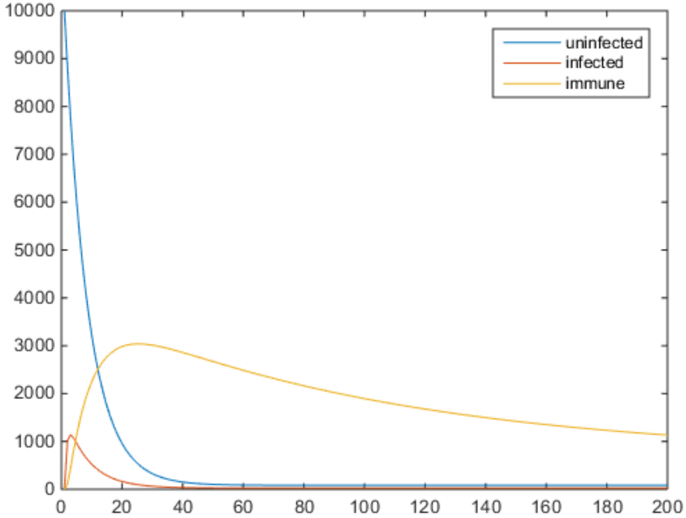
$$\mathbf{z}[i+1] - \mathbf{z}[i] = \begin{bmatrix} -a-b-c & 0 & 0 & 0 \\ b & -d-e & 0 & 0 \\ c & d & -f & 0 \\ a & d & f & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

*rated in #/100/day.

Epidemic Disease Example, MATLAB

Which makes it easy to utilize MATLAB to simulate our system

$$\mathbf{z}[i+1]=\mathbf{z}[i]+\begin{bmatrix}-a-b-c & 0 & 0 & 0\\ b & -d-e & 0 & 0\\ c & d & -f & 0\\ a & d & f & 0\end{bmatrix}\mathbf{z}+\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}\mathbf{u}$$

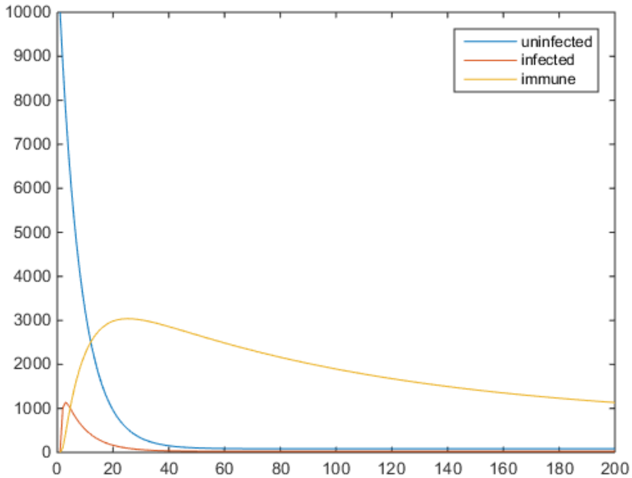



```
clear all;
z(1,1) = 10000; %initial uninfected pop
z(2,1) = 10; %initial infected pop
z(3,1) = 0; %initial immunized/cured
z(4,1) = 0; %dead

a=1; b=10; c=1; %#/100/day die, infected, immunized
d=50; e=25; %#/100/day of infected who die or are cured
f=1; %#/100/day of immune who die
u(1) = 10; u(2) = 10; %#/uninfected and infected added per day.

A = [ -a-b-c 0 0 0; b -d-e 0 0; c e -f 0; a d f 0; ]./100;
B = [ 1 0; 0 1; 0 0; 0 0 ];
C = [ eye(3) zeros(3,1) ];

day =1:200;
for count=1:length(day)-1
    z(:,count+1)= z(:,count) + A*z(:,count)+B*u';
    for(j=1:4)
        if z(j,count+1) < 0
            z(j,count+1) = 0;
        end
    end
end
plot(day,C*z)
legend('uninfected','infected','immune');
```

Easy to change parameters to see their impact.

Easy to change parameters

```
a=1; b=10; c=1 %#/100/day die, infected, immunized
d=50; e=25; %#/100/day of infected who die or are cured
f=1; %#/100/day of immune who die
```




```
a=1; b=10; c=50; %#/100/day die, infected, immunized
d=50; e=25; %#/100/day of infected who die or are cured
f=1; %#/100/day of immune who die
```