Practical 5

CSE Spring 2023

1. **What does the root locus look like?**

   Using Python Control Systems Library's function `sisotool` (give it an open loop transfer function as an input - generate an appropriate transfer function yourself), find answers to the following questions.

   (a) How many loci are there on the root locus plot when the open loop transfer function has more poles than zeros? What if it has more zeros than poles? If the number of poles equals the number of zeros?

   (b) How are the loci related to the poles and zeros of the open loop transfer function?

   (c) What value of gain puts the closed loop transfer function to the locations of open loop poles? Which value positions them at open loop transfer function's zeros?

   (d) What determines how many loci tend towards infinity (in any direction)?

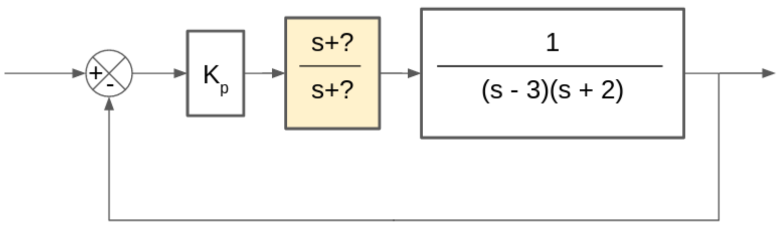
   (e) What are the angles at which loci tend towards infinity if there's one locus going to infinity? What if there are two? Three? Four?

2. **Lead compensators**

   We will now look at the following situation. Say we have a system with transfer function $\frac{1}{(s-3)(s+2)}$.

   (a) Plot the root locus. Can you make the system stable using just a proportional controller?

   Now we will add another block (highlighted in yellow).



   This block will have one real-valued pole and one real-valued zero. If the added pole is more negative than the added zero, then we call it a **phase lead compensator**.

   (b) Add this new block in series with the original transfer function. Try varying the distance between the new pole and zero as well as the location of the new pole and zero. How does this change the root locus?

   (c) Find values for the pole and zero such that you could find a value for K_p such that the system would be stable.

3. **Lag compensators**

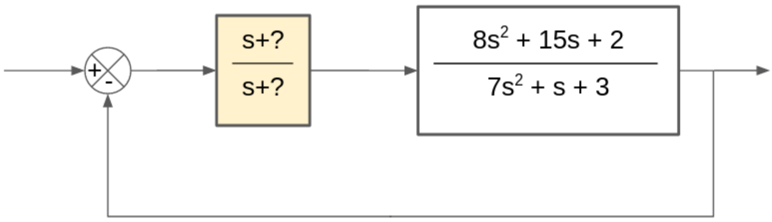
   If instead we add a block that has one pole and one zero, but the zero is more negative than the pole, then we call this a **phase lag compensator**.

   Let's now take a look at a system with the following transfer function:

$$\frac{8s^2 + 15s + 2}{7s^2 + s + 3}$$

(a) Plot the root locus. Also plot the step response of the system with unity feedback. What is the steady state offset?

Let's assume that we do not want the shape of the root locus (leave it unchanged as much as possible), but we do want to modify the steady state offset for this system with unity feedback. We can use a lag compensator to do this.



Let's say we want the steady state error of the system on the picture to be 5%. Through the magic of calculation, we can deduce from the constant terms of the numerator and denominator of our transfer function (they are 2 and 3, respectively) that we can do this if the added zero z and pole p satisfy the following ratio:

$$\frac{z}{p} = \frac{3 - 0.05 \cdot 3}{0.05 \cdot 2} = 28.5$$

(b) Find values for the new pole and zero such that they would not change the shape of the root locus, but that their ratio $\frac{z}{p}$ would be 28.5. Look at the step response of the system on the picture above, using the values of the zero and pole that you found. What's the steady state error now?