Lecture 6

CSE Spring 2023

SISO? MISO? MIMO?

SISO: **S**ingle-**I**nput **S**ingle-**O**utput

MISO: **M**ulti-**I**nput **S**ingle-**O**utput

MIMO: **M**ulti-**I**nput **M**ulti-**O**utput

Vectors and Matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

Input and Output

$\mathbf{z}(t) \in \mathbb{R}^n$ State vector

$\mathbf{u}(t) \in \mathbb{R}^p$ Input vector

$\mathbf{y}(t) \in \mathbb{R}^q$ Output (measured) vector

State-Space Model

$\mathbf{z}(t) \in \mathbb{R}^n$	State vector
$\mathbf{u}(t) \in \mathbb{R}^p$	Input vector
$\mathbf{y}(t) \in \mathbb{R}^q$	Output (measured) vector

A **state-space model** is a mathematical model of system's inputs, outputs and states represented as a set of 1st order ODEs.

State-Space Model for LTI Systems

$\mathbf{z}(t) \in \mathbb{R}^n$ State vector

$\mathbf{u}(t) \in \mathbb{R}^p$ Input vector

$\mathbf{y}(t) \in \mathbb{R}^q$ Output (measured) vector

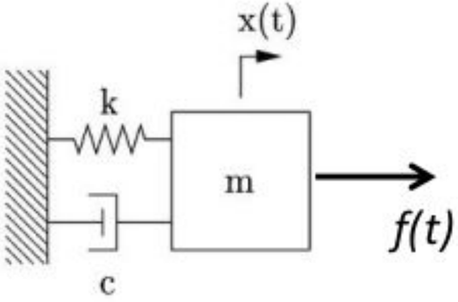
For **L**inear and **T**ime **I**nvariant systems:

$$\frac{d\mathbf{z}}{dt} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{u}$$

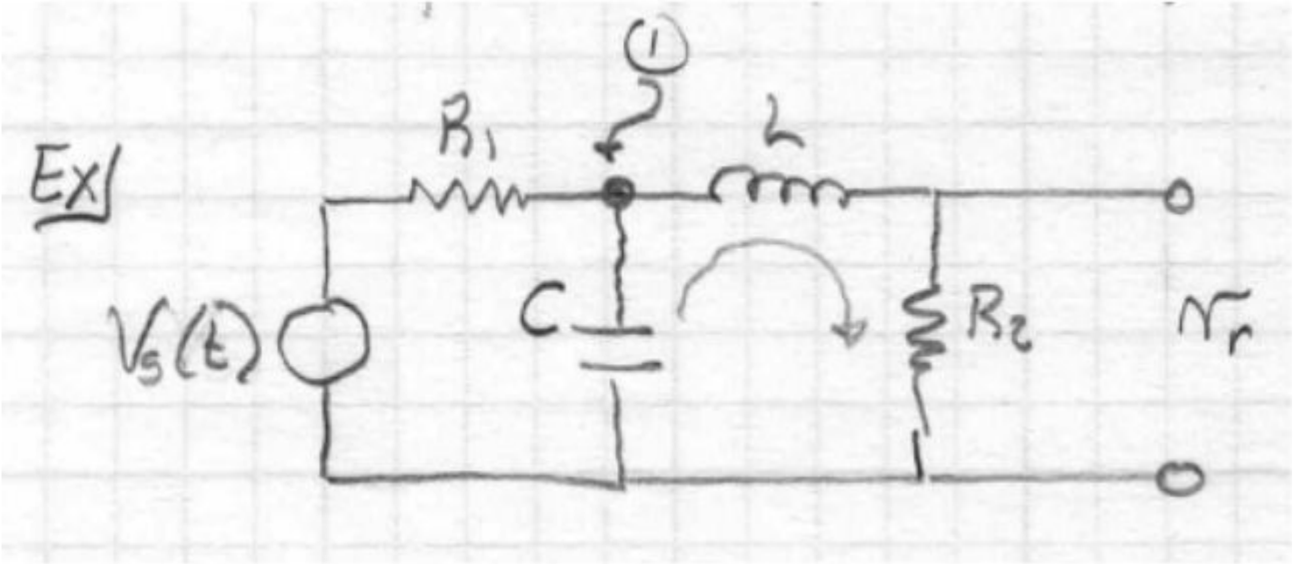
Mass Spring Damper

$$m\ddot{x} = -c\dot{x} - kx + f(t)$$

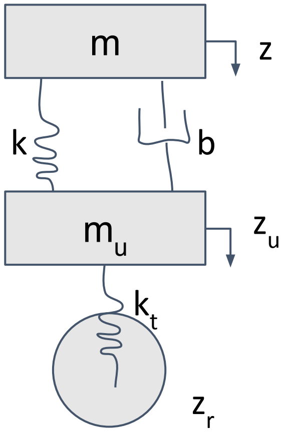


Let’s convert this to state-space form together on the whiteboard.

RLC Circuit Example



Landing Gear Example



$$\begin{cases} m\ddot{z} + b\left(\dot{z} - \dot{z}_u\right) + k\left(z - z_u\right) = 0 \\ m_u\ddot{z}_u + b\dot{z}_u + (k + k_t)z_u = k_t z_r + b\dot{z} + kz \end{cases}$$