

R. Liégeois¹, M. Zorzi² and BT Thomas Yeo^{1,3}
¹ASTAR-NUS CIRC, Department of ECE, SINAPSE, National University of Singapore, Singapore, ²Department of Information Engineering, University of Padova, Italy, ³Martinos Center for Biomedical Imaging, MGH/HMS, USA

Abstract

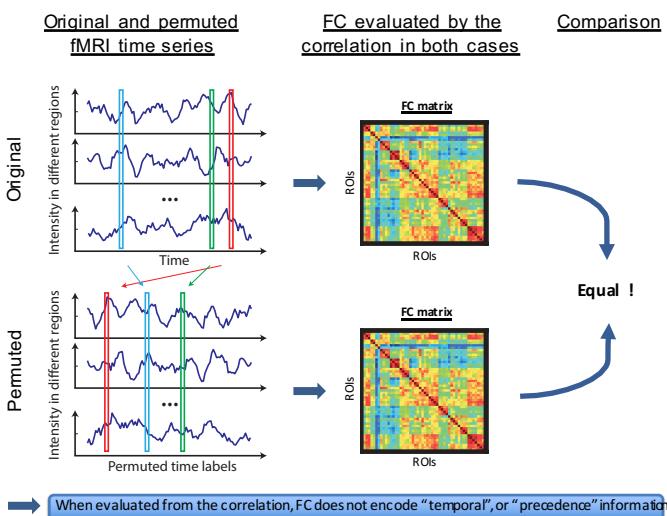
Motivation: Principal component analysis (PCA) is a popular multivariate framework that relies on a low-rank approximation of covariance matrices. It has been used in fMRI analyses to extract relevant information from functional connectivity (FC) matrices. Indeed, FC is classically evaluated from the correlation of the data, a normalized version of the covariance matrix (e.g. Gwady et al. 2001). However, a limitation of this approach is that it is *static*. Hence the fluctuations of FC, that have been shown to encode crucial information (Hutchison et al. 2013), are neglected when PCA or related methods are used.

→ How can we include this dynamical information in a component analysis framework?

Methods: We use our **dynamical generalization of PCA** (Liégeois et al. 2015) in order to extract **dynamical components** from synthetic and fMRI data. This framework is based on a Sparse plus Low-rank (S+L) decomposition of the inverse power spectral density of the time courses.

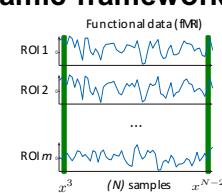
Results: We show that this approach provides a finer description of functional networks. In particular, using our framework the Default Mode Network (DMN) and the Executive Control Network (EXN) are merged into the same *dynamic* functional network whereas using classical static approaches they are considered to be two distinct *static* networks. This echoes recent findings about the complementarity between these two networks (Vanhaudenhuyse et al. 2011).

Motivating example: correlation is a static measure of FC



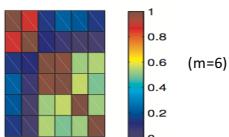
Methods 1: Static vs. dynamic framework

Starting from N observations $\{x^k\}_{k=1}^N, x^k \in \mathbb{R}^m$ unknown phenomenon \mathbb{F} ,



Static models consider that \mathbb{F} is a centered, gaussian random vector X taking values in \mathbb{R}^m .

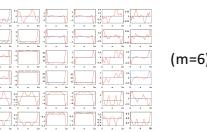
- X is completely described by its **correlation matrix** $\Sigma = \mathbb{E}[XX^T]$:



→ How to extract information from the PSD in the same way PCA does it from the correlation matrix?

Dynamical models consider that \mathbb{F} is a centered, gaussian, stationary stochastic process $\{X_t\}_{t \in \mathbb{Z}}$, where X_t takes values in $\mathbb{R}^m, \forall t \in \mathbb{Z}$.

- $\{X_t\}_{t \in \mathbb{Z}}$ is completely described by its **power spectral density**: $\Phi(e^{j\theta}) = \sum_{k=1}^{p+1} R_k e^{-jk\theta}$, estimated from the $p+1$ first lags $\{R_0, R_1, \dots, R_p\}$, where p is the order of the model.



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Methods 2: identifying latent components

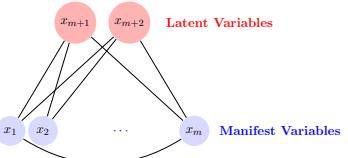
As in PCA we want (i) **manifest (observed)** variables to be modeled as the superposition of **few latent (hidden)** variables and (ii) few connections within the **manifest** variables. Considering:

- m **manifest** (i.e. **observed**) variables
- l **latent** variables ($l < m$)
- $x(t) = [(x^m(t))^T \quad (x^l(t))^T]^T$

$$\text{We have: } \Phi_x = \begin{bmatrix} \Phi_m & \Phi_{ml} \\ \Phi_{ml} & \Phi_l \end{bmatrix}, \quad \Phi_x^{-1} = \begin{bmatrix} \Upsilon_m & \Upsilon_{ml} \\ \Upsilon_{ml} & \Upsilon_l \end{bmatrix}.$$

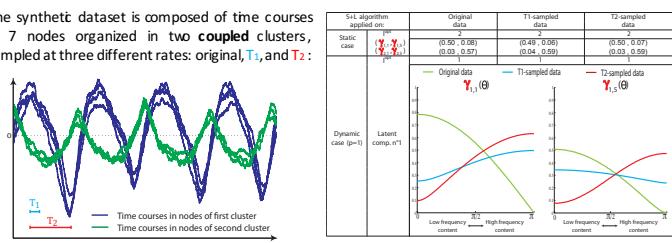
→ $\Phi_x^{-1} = \Upsilon_m - \Upsilon_m^* \Upsilon_l \Upsilon_l^* \Upsilon_m = S - L$ and the identified inverse power spectral density should be the sum of a **sparse** and a **low-rank** term (Zorzi and Sepulchre 2015).

Υ_m encodes the weight of each **latent** variable in each **manifest** variable and its k th column ($k < l$) is the k th **latent component** of the model. It is denoted $y_i(\theta)$, it is a function of θ when $p > 0$, and when m is too large, and for $p=1$, we define two contributions: $y_0 = y(\theta=0)$ and $y_n = y(\theta=\pi)$.



Toy example: spectral properties are encoded in latent components (for $p > 0$)

The synthetic dataset is composed of time courses in 7 nodes organized in two **coupled** clusters, sampled at three different rates: original, T_1 , and T_2 :



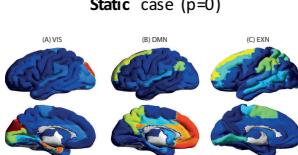
For each of the three datasets, and for $p=0$ (static case) and $p=1$ (dynamic case), we identify the optimal **S+L** model and corresponding **latent components**.

In the **static** case, we identify two latent components similar for each dataset and capturing the two clusters. In the **dynamic** case, the two clusters are merged into a single latent component due to their **similar spectral content**.

Real data: a new interpretation of classical components

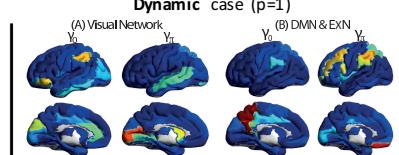
- Data was collected from 17 healthy volunteers during rest and functional (fMRI) time series were computed in 90 regions of interest (Tzourio-Mazoyer et al. 2002, Boveroux et al. 2010).
- For each subject, the optimal **S+L** model and corresponding **latent components** are computed for $p=0$ (static case) and $p=1$ (dynamic case).
- 10 to 12 latent components are identified, among which the following resting state networks.

Static case ($p=0$)



In the static case, classical resting state networks are identified (Moussa et al. 2012)

Dynamic case ($p=1$)



In the dynamic case, the visual network is identified in a single latent component. On the contrary, the DMN and EXN are found to be part of the same latent component, the DMN corresponding to y_0 and the EXN to y_n . This might be due to the anticorrelation of these networks (Vanhaudenhuyse et al. 2011).

Conclusion

- We used a new dynamical extension of PCA to compute **dynamical latent components** from synthetic and fMRI datasets.
- Our proposed dynamical latent components encode both **spatial** and **temporal** properties of the components.
- For example, we show that the DMN and EXN are intimately linked through a common spectral content, hereby refining recent findings about complementarity between these two networks (Vanhaudenhuyse et al. 2011).
- The code is available from the first author's personal webpage <http://www.morteforeldiacbe/~liegeois/>

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