DM – AÉ411

Longitudinal Dynamic Stability of Airplane E (case – Subsonic)

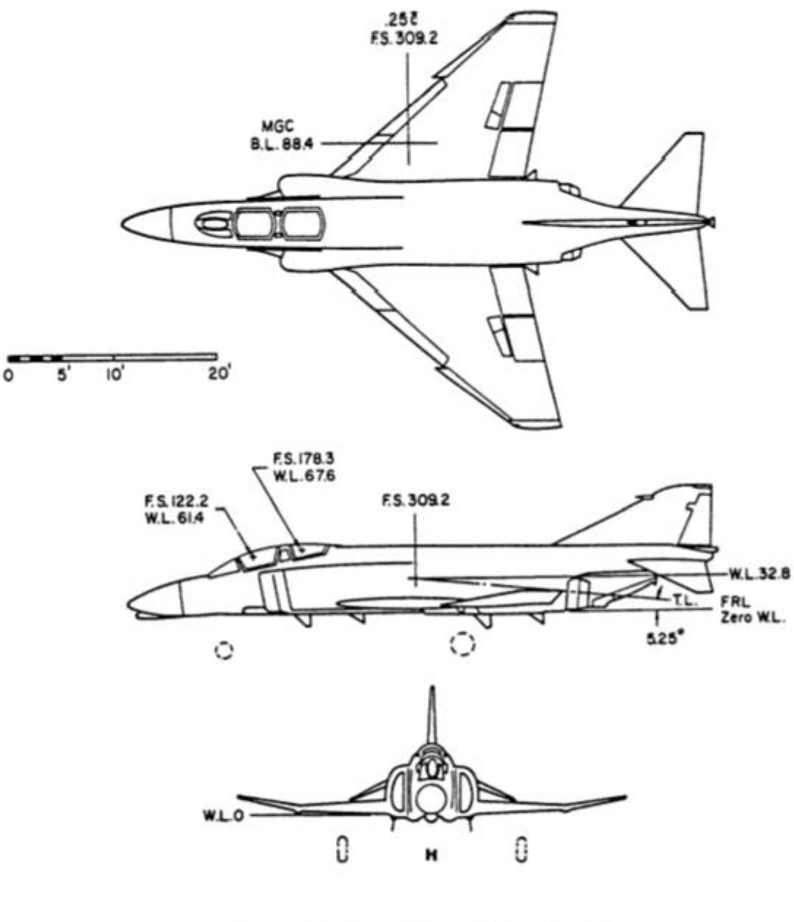

Figure C5 Three-View of Airplane E

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TABLE DES MATIERES

Given data

Following picture are the given date to do all the calculation.

Flight Condition	1	2	3
	Power Approach	Subsonic Cruise	Supersonic Cruise
Altitude (ft)	Sealevel	35,000	55,000
Air Density (slugs/ft^3)	.002378	.000739	.000287
Speed (fps)	230	876	1742
Center of Gravity ($\bar{x}_{cg}$)	.29	.29	.29
Initial Attitude (deg)	11.7	2.6	3.3
Geometry and Inertias			
Wing Area (ft^2)	530	530	530
Wing Span (ft)	38.7	38.7	38.7
Wing Mean Geometric Chord (ft)	16.0	16.0	16.0
Weight (lbs)	33,200	39,000	39,000
I_{xx_B} (slug ft^2)	23,700	25,000	25,000
I_{yy_B} (slug ft^2)	117,500	122,200	122,200
I_{zz_B} (slug ft^2)	133,700	139,800	139,800
I_{xz_B} (slug ft^2)	1,600	2,200	2,200
Steady State Coefficients			
C_L	1.0	.26	.17
C_D	.2	.03	.048
C_{T_X}	.2	.03	.048
C_m	0	0	0
C_{m_T}	0	0	0

Figure 1_Given table 1

Longitudinal Derivatives	1	2	3
C_{m_u}	0	-.117	+.054
C_{m_α}	-.098	-.40	-.78
$C_{m_{\dot\alpha}}$	-.95	-1.3	-.25
C_{m_q}	-2.0	-2.7	-2.0
$C_{m_{T_u}}$	0	0	0
$C_{m_{T_\alpha}}$	0	0	0
C_{L_u}	0	+.27	-.18
C_{L_α}	2.8	3.75	2.8
$C_{L_{\dot\alpha}}$	0	0	0
C_{L_q}	0	0	0
C_{D_α}	.555	.3	.4
C_{D_u}	0	+.027	-.054
$C_{T_{X_u}}$	0	0	0
$C_{L_{i_H}}$	.24	.40	.25
$C_{D_{i_H}}$	-.14	-.10	-.15
$C_{m_{i_H}}$	-.322	-.58	-.38

Figure 2_Given table 2

Unfortunately, few data are not given in the SI system, so some conversion must be done.

Parameter	*Symbol*	*Imperial unit*	*Equivalent SI unit*
Mass	m	1 slug	14.594 kg
Length	l	1 ft	0.3048 m
Velocity	V	1 ft/s	0.3048 m/s
Acceleration	a	1 ft/s^2	0.3048 m/s^2
Force	F	1 lb	4.448 N
Moment	M	1 lb ft	1.356 N m
Density	ρ	1 slug/ft^3	515.383 kg/m^3
Inertia	I	1 slug ft^2	1.3558 kg m^2

Figure 3_Given table 3

Question

Question 1: Find the Equation of longitudinal motion

From de lecture we have the initial equation:

$$m\,\dot{u} = X - mg\sin\theta + m\,(rv - qw)$$

$$m\,\dot{w} = Z + mg\cos\ \varphi\cos\theta + m\,(qu - pv)$$

$$I_{yy}\,\dot{q} = M + (I_{zz} - I_{xx})\,p\,r$$

Moreover, there are linearization equation that can be used to simplify the previous equation and also the steady flight conditions:

$$u = U_0 + u(t)$$

$$w = 0 + w(t)$$

$$q = 0 + q(t)$$

$$\theta = \theta_0 + \theta(t)$$

$$\varphi = 0 + \varphi(t)$$

$$0 = X_0 - mg\sin\theta_0$$

$$0 = Z_0 + mg\ \cos\theta_0$$

$$0 = M_0$$

So, with all the simplification, we can obtain the equation below:

$$m\,(U_0 + \dot{u}(t)) = X_0 + \Delta X - mg\sin\ (\theta_0 + \theta(t)) + m\,(rv - qw)$$

$$m\,w\dot{(t)} = Z_0 + \Delta Z + mg\cos\ \varphi(\mathrm{t})\cos\ (\theta_0 + \theta(t)) + m\,(q(U_0 + u(t)) - pv)$$

$$I_{yy}\,q\dot{(t)} = M_0 + \Delta M + (I_{zz} - I_{xx})\,p\,r$$

With the Taylor development we can re-arrange the equation

$$m\,\dot{u} = X_0 + \Delta X - mg\ \sin\theta_0 - mg\,\theta\cos\theta_0\ + m\,(rv - qw)$$

$$m\,\dot{w} = Z_0 + \Delta Z + mg\ \cos\theta_0 - mg\,\theta\sin\theta_0 + m\,(qU_0 + q\,u(t) - pv)$$

$$I_{yy}\,q(\dot{t}) = M_0 + \Delta M + (I_{zz} - I_{xx})\,p\,r$$

By subtracting of initial solution and by neglecting the second order, we have:

$$m\,\dot{u} = \Delta X - mg\,\theta\cos\theta_0$$

$$m\,\dot{w} = \Delta Z - mg\,\theta\sin\theta_0 + m\,(qU_0)$$

$$I_{yy}\,\dot{q} = \Delta M$$

By using the total differential definition and by substituting them in the equation we obtain:

$$\dot{u} = \frac{1}{m}\frac{\partial X}{\partial u}u + \frac{1}{m}\frac{\partial X}{\partial w}w + \frac{1}{m}\frac{\partial X}{\partial \delta_e}\delta_e + \frac{1}{m}\frac{\partial X}{\partial \delta_T}\delta_T - g\,\theta\cos\theta_0$$

$$\dot{w} = \frac{1}{m}\frac{\partial Z}{\partial u}u + \frac{1}{m}\frac{\partial Z}{\partial w}w + \frac{1}{m}\frac{\partial Z}{\partial \dot{w}}\dot{w} + \frac{1}{m}\frac{\partial Z}{\partial q}q + \frac{1}{m}\frac{\partial Z}{\partial \delta_e}\delta_e + \frac{1}{m}\frac{\partial Z}{\partial \delta_T}\delta_T - g\,\theta\sin\theta_0 + qU_0$$

$$\dot{q} = \frac{1}{I_{yy}}\frac{\partial M}{\partial u}u + \frac{1}{I_{yy}}\frac{\partial M}{\partial w}w + \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{w}}\dot{w} + \frac{1}{I_{yy}}\frac{\partial M}{\partial q}q + \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_e}\delta_e + \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_T}\delta_T$$

We take:

$X_u \equiv \frac{1}{m}\frac{\partial X}{\partial u}$; $M_u \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial u}$; $Z_u \equiv \frac{1}{m}\frac{\partial Z}{\partial u}$

$$X_w \equiv \frac{1}{m}\frac{\partial X}{\partial w} \quad ; \quad Z_w \equiv \frac{1}{m}\frac{\partial Z}{\partial w} \quad ; \quad Z_{\dot{w}} \equiv \frac{1}{m}\frac{\partial Z}{\partial \dot{w}} \quad ; \quad M_w \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial w} \quad M_{\dot{w}} \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{w}}$$

$$Z_q \equiv \frac{1}{m}\frac{\partial Z}{\partial q} \quad ; \quad MX_q \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial q} \quad ; \quad M_{\delta_e} \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_e}$$

$X_{\delta_e} \equiv \frac{1}{m}\frac{\partial X}{\partial \delta_e}$; $X_{\delta_T} \equiv \frac{1}{m}\frac{\partial X}{\partial \delta_T}$; $Z_{\delta_e} \equiv \frac{1}{m}\frac{\partial Z}{\partial \delta_e}$; $Z_{\delta_T} \equiv \frac{1}{m}\frac{\partial Z}{\partial \delta_T}$; $M_{\delta_T} \equiv \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_T}$;

So, we can now re-write the equation:

$$\dot{u} = X_u u + X_w w + X_{\delta_e}\delta_e + X_{\delta_T}\delta_T - g\,\theta\cos\theta_0$$

$$\dot{w} = Z_u u + Z_w w + Z_{\dot{w}}\dot{w} + Z_q q + Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T - g\,\theta\sin\theta_0 + qU_0$$

$$\dot{q} = M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_e}\delta_e + M_{\delta_T}\delta_T$$

To write the equation we must factorize by the simple value and not the derivative, so we can write:

$$(\frac{d}{dt} - X_u)u - X_w w + g\,\theta\cos\theta_0 = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T$$

$$-Z_u u + \left(\frac{d}{dt} - Z_w - Z_{\dot{w}}\frac{d}{dt}\right)w - (Z_q + U_0)q + g\,\theta\sin\theta_0 = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T$$

$$-M_u u - (M_w - M_{\dot{w}}\frac{d}{dt})\,w + (\frac{d}{dt} - M_q)q = M_{\delta_e}\delta_e + M_{\delta_T}\delta_T$$

The final equation obtain is: $I_n\dot{X} = A_n X + B_n * \eta$

Using the equation found, we can deduct two matrices to write the equation:

$$A_n = \begin{bmatrix} Xu & Xw & 0 & -g_0 * \cos(\theta_0) \\ Zu & Zw & U_0 + Zq & -g_0 * \sin(\theta_0) \\ Mu & Mw & Mq & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_n = \begin{bmatrix} X_{\delta e} & X_{\delta T} \\ Z_{\delta e} & Z_{\delta T} \\ M_{\delta e} & M_{\delta T} \\ 0 & 0 \end{bmatrix}$$

But we can rearrange, and we get $X = A\dot{X} + B\eta$

So, our new matrices are:

$$A_n = \begin{bmatrix} Xu & Xw & 0 & -g_0 * \cos(\theta_0) \\ Zu & Zw & U_0 + Zq & -g_0 * \sin(\theta_0) \\ Mu + M\dot{w} * Zu & Mw + M\dot{w} * Zu & Mq + U_0 * M\dot{w} & -M\dot{w} * g_0 * \sin(\theta_0) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_n = \begin{bmatrix} X_{\delta e} & X_{\delta T} \\ Z_{\delta e} & Z_{\delta T} \\ M_{\delta e} + M\dot{w} * Z_{\delta e} & M_{\delta T} + M\dot{w} * Z_{\delta T} \\ 0 & 0 \end{bmatrix}$$

Differential equation uses below

$$\Delta X = \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \delta_e} \delta_e + \frac{\partial X}{\partial \delta_T} \delta_T$$

$$\Delta Z = \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \dot{w}} \dot{w} + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial \delta_e} \delta_e + \frac{\partial Z}{\partial \delta_T} \delta_T$$

$$\Delta M = \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \delta_e} \delta_e + \frac{\partial M}{\partial \delta_T} \delta_T$$

Question 2: Find the matrix A of aircraft

To calculate the matrix found in the question1) we need to find the formula for all the coefficient, to not complicated this document, only the solution will be given. In fact, the solution has been found in the tutorial 3.

Coefficient for the A matrix:

Xu

$$X_u = -\frac{\bar{q}S}{mU_0}\left[(2C_{D_0} + MC_{D_M})\right]$$

Zu

$$Z_u = -\frac{\bar{q}S}{mU_0}\left[(2C_{L_0} + \frac{M^2}{1-M^2}C_{L_0})\right]$$

Mu

$$M_u = \frac{\bar{q}S\bar{C}}{I_{yy}U_0}\left[M\,C_{m_M}\right]$$

Xw

$$X_w = \frac{\bar{q}S}{mU_0}\left(C_{L_0} - \frac{2}{\pi\ e\ AR}(C_{L_0}C_{L_\alpha})\right)$$

Zw

$$Z_w = -\frac{\bar{q}S}{mU_0}\left[(C_{D_0} + C_{L_\alpha})\right]$$

Zẇ

$$Z_{\dot{w}} = \frac{\bar{q}S\bar{C}}{2m{U_0}^2}\left[(C_{D_0} + C_{L_{\dot{\alpha}}})\right]$$

Mw

$$M_w = \frac{\bar{q}S\bar{C}}{I_{yy}U_0}\left[\,C_{m_\alpha}\,\right]$$

$M\dot{w}$

$$M_{\dot{w}} = \frac{\bar{q}S\overline{C^2}}{2I_{yy}{U_0}^2}\left[\,C_{m_{\dot{\alpha}}}\,\right]$$

Zq

$$Z_q = \frac{\bar{q}S\bar{C}}{2mU_0}\left[C_{L_q}\right]$$

Mq

$$M_q = \frac{\bar{q}S\overline{C^2}}{2I_{yy}U_0}\left[\,C_{m_q}\,\right]$$

Coefficient for the B matrix:

X_{δ_e}

$$X_{\delta_e} = \frac{\bar{q}S}{mU_0} C_{D_{\delta_e}}$$

X_{δ_T}

$$X_{\delta_T} = \frac{\bar{q}S}{mU_0} C_{T_{\delta_T}}$$

Z_{δ_e}

$$Z_{\delta_e} = \frac{\bar{q}S}{mU_0} C_{L_{\delta_e}}$$

Z_{δ_T}

$$Z_{\delta_T} = \frac{\bar{q}S}{mU_0} C_{L_{\delta_T}}$$

M_{δ_e}

$$M_{\delta_e} = \frac{\bar{q}S\overline{C^2}}{2I_{yy}U_0} C_{M_{\delta_e}}$$

M_{δ_T}

$$M_{\delta_T} = \frac{\bar{q}S\bar{C}}{I_{yy}U_0} C_{M_{\delta_T}}$$

With the MATLAB code, we have obtained the coefficient use in the equation found below:

```
Question 1 : Equation of longitudinal motion
Below the different coefficient of the equation
Xu =
  -1.2315e-02

Zu =
  -2.3476e-01

Mu =
  -8.6220e-03

Xw =
   2.2538e-04

Zw =
  -5.3506e-01

Mw =
  -2.9477e-02

Mw_point =
  -8.7489e-04

Zq =
   1.0355e-01

Mq =
  -1.8171e-03
```

```
XdeltaE =
  -1.4155e-02

ZdeltaE =
   5.6620e-02

ZdeltaE =
   5.6620e-02

XdeltaT =
     0

ZdeltaT =
     0

MdeltaT =
     0
```

Then we obtain the result with MATLAB:

```
Question 2 : The matrix A of the aircraft
The matric of the arcraft is
  -1.2315e-02   2.2538e-04            0  -9.7999e+00
  -2.3476e-01  -5.3506e-01   2.6700e+02  -4.4501e-01
  -8.4166e-03  -2.9009e-02  -2.3542e-01   3.8933e-04
            0            0   1.0000e+00            0



The B matric of the arcraft is
  -1.4155e-02            0
   5.6620e-02            0
  -1.0427e-01            0
            0            0
```

Question 3: Find the characteristic equation

```
Question 3 : The characteristics equation
the characteristic coeffitient of the equation are :
   1.0000e+00   7.8279e-01   7.8806e+00   1.8511e-03   2.2442e-02

Finally we obtain the equation below
x^4 + 0.7828*x^3 + 7.881*x^2 + 0.001851*x + 0.02244
```

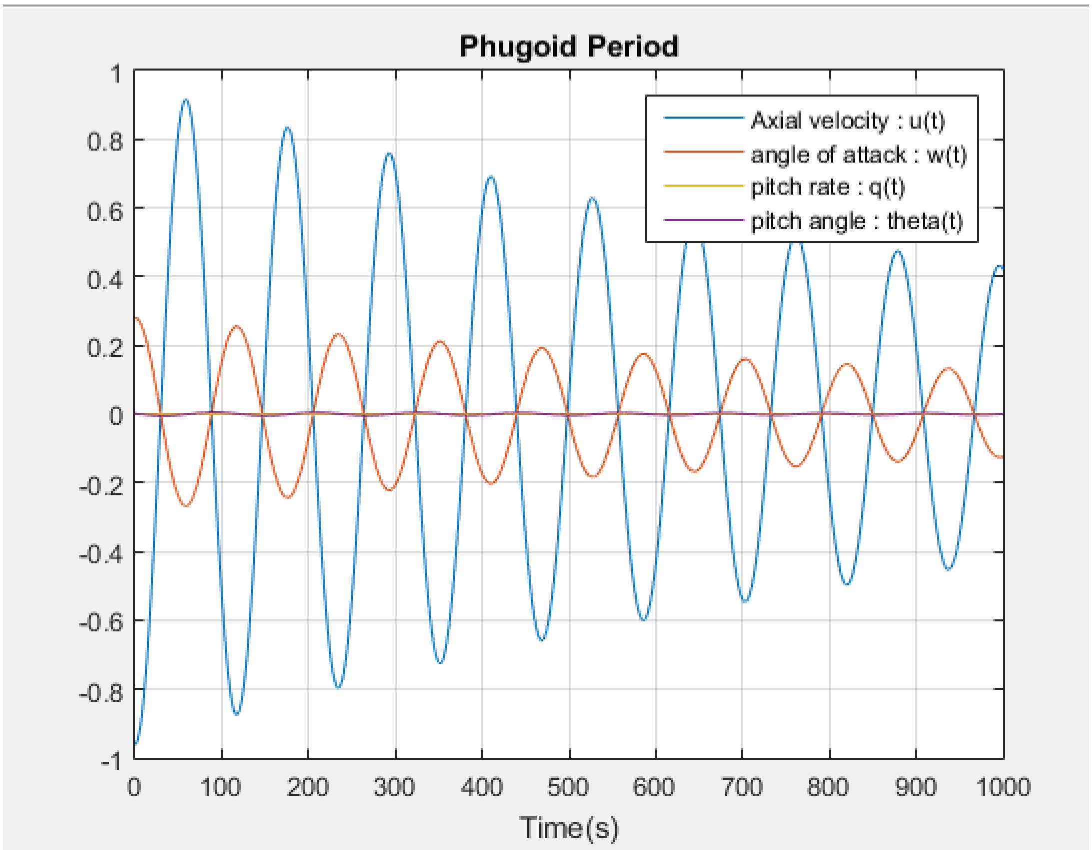
Question 4: Find the eigenvalues of the system

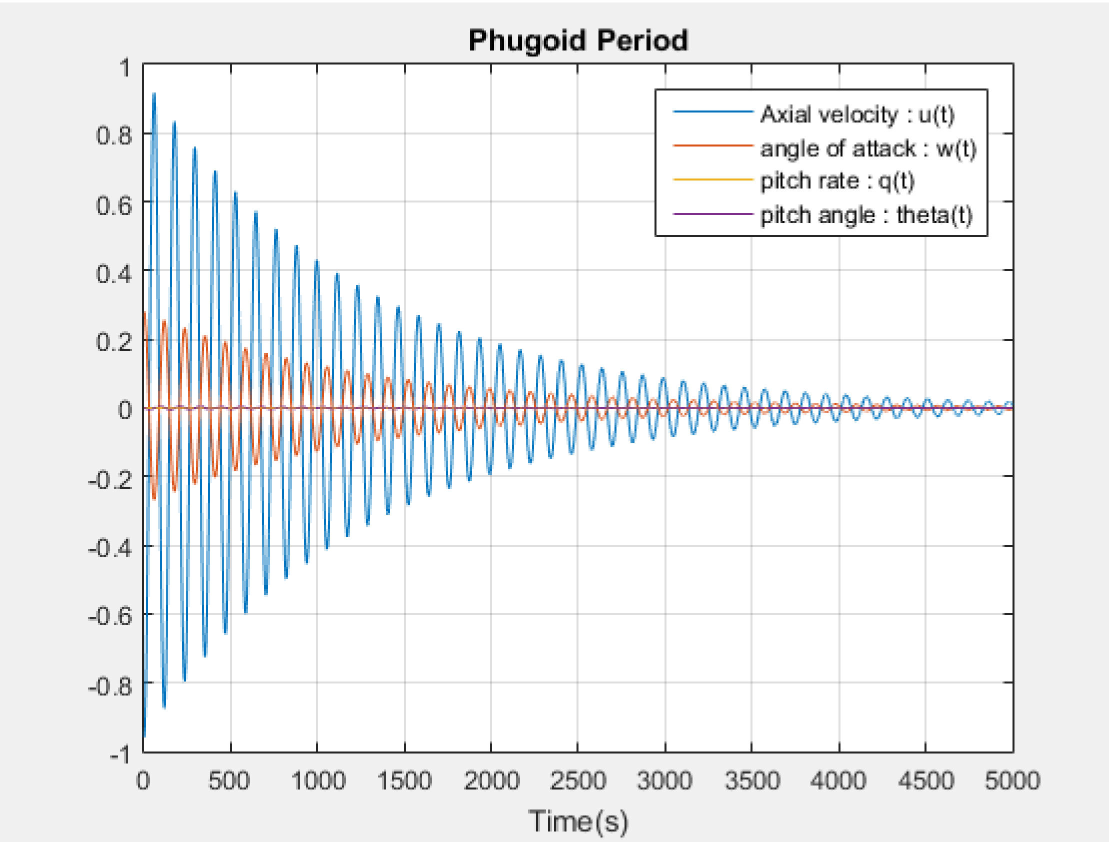
```
Question 4 : The eigenvalues of the system
The eigenvalue of the system are :
lambda 1
  -3.9142e-01 + 2.7793e+00i

Lambda 2
  -3.9142e-01 - 2.7793e+00i

Lambda 3
   2.4054e-05 + 5.3374e-02i

Lambda 4
   2.4054e-05 - 5.3374e-02i
```

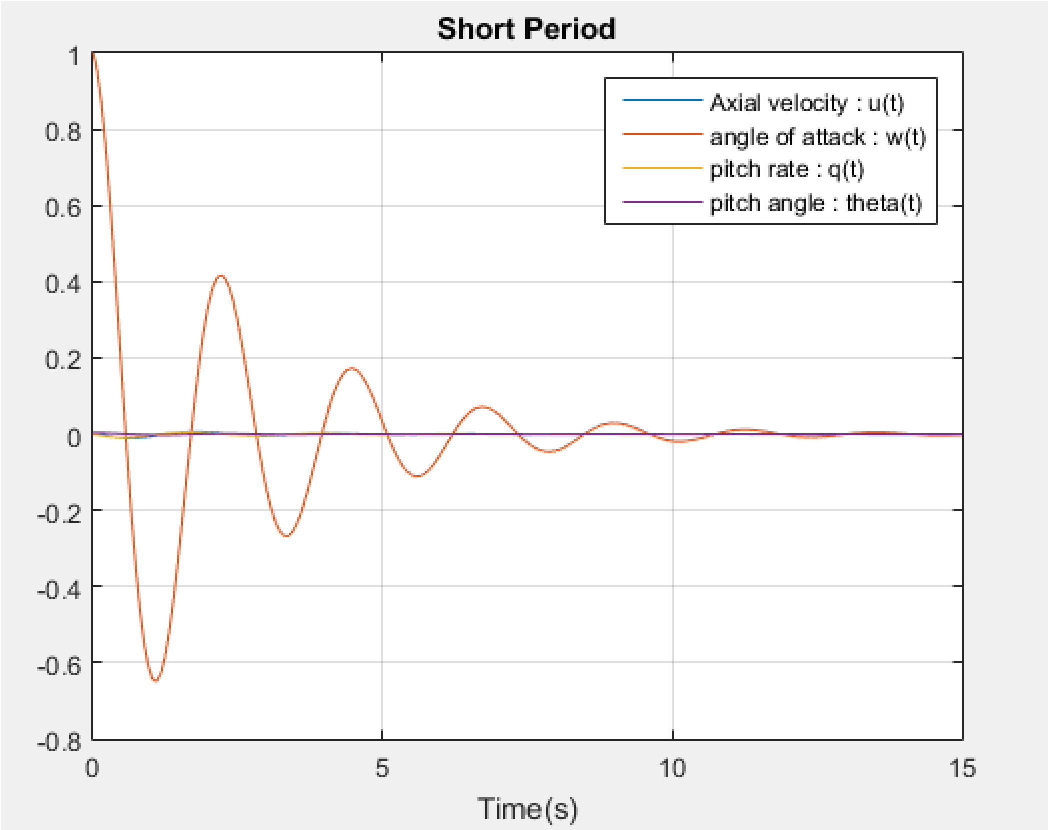
Question 5: Find the different modes of longitudinal stability

```
Question 5 : Different modes of longitudinal stability
Question a : Short period mode - Natural frequency/Damping Factor
The Damping ratio is
   7.1706e+00

The Undamped Natural Frequency coeffciient is
   5.4587e-02

-------------------------------------------------------------------------
Question b : Phugoid mode - Natural frequency/Damping Factor
The Damping ratio is
   4.5067e-04

The Undamped Natural Frequency coeffciient is
  -5.3374e-02
```

Question 6: plot des curves of longitudinal motion







We can observe that the short period is amortized, like it was supposed to do. The amortization is fast and take only 15 second. On the contrary the amortization of the phugoid mode take approximative 5000 second. I think it normal because the study is about a military aircraft. And the amortization is less important in this kind of aircraft. In fact, the most important think is to have a very high performance in maneuver.

Question 7: Find the transfer Function of each variable

We have:

$$\frac{X(s)}{\eta(s)} = (SI - A)^{-1} * B$$

Phugoid case

For the phugoid case, we note:

$$A_{ph} = SI - \begin{bmatrix} Xu & -g * \cos(\theta_0) \\ \frac{Zu}{U_0} & -g * \sin(\theta_0) \end{bmatrix} \quad \text{and} \quad B_{\delta e} = \begin{matrix} X_{\delta e} \\ Z_{\delta e} \end{matrix}$$

$$\Leftrightarrow \begin{bmatrix} \frac{U(s)}{\delta e} \\ \frac{\theta(s)}{\delta e} \end{bmatrix} = \frac{\begin{bmatrix} S - Xu & g * \cos(\theta_0) \\ -\frac{Zu}{U_0} & S + g * \sin(\theta_0) \end{bmatrix}}{\det(A_{ph})} * B_{\delta e}$$

Short case

And for the short case, we note:

$$A_{sh} = SI - \begin{bmatrix} Zw & U_0 \\ \frac{Mw + M\dot{w}}{U_0} & Mq + U_0 * M\dot{w} \end{bmatrix} \quad \text{and} \quad B_{\delta e} = \begin{matrix} Z_{\delta e} \\ M_{\delta e} + M\dot{w} * Z_{\delta e} \end{matrix}$$

$$\Leftrightarrow \begin{bmatrix} \frac{W(s)}{\delta e} \\ \frac{q(s)}{\delta e} \end{bmatrix} = \frac{\begin{bmatrix} S - Zw & -U_0 \\ -\frac{Mw + M\dot{w}}{U_0} & S - Mq - U_0 * M\dot{w} \end{bmatrix}}{\det(A_{ph})} * B_{\delta e}$$

The calculation is done with MATLAB and we obtained the result below:

```
Question 7 : transfer Functions of each variable
U(s) =
-(1.0*(367487697464885679045875985 28512.0*S + 163536035966492330211837739 00800.0))/(2.5961484292674138142652481 6461e33*S^2 + 1.18728581389388920931983880 8277e33*S - 20521364306832144768633009 1470422.0)

Teta(s) =
(0.056620445*S + 0.00057518768)/(S^2 + 0.45732586*S - 0.07904542)

W(s) =
0.056620444859323250386129444677863*S - 0.0030247784363184753754494327182556/(S^2 + 0.45732586030488242036240453813889*S - 0.079045420036414878809763814646693) + 0.013329381699887379287150446887909

q(s) =
- (1.0*(0.10427077689175302810387080398868*S + 0.055791455958991925824404049290847))/(S^2 + 0.45732586030488242036240453813889*S - 0.079045420036414878809763814646693) - 15.117930555574632833781834051479
```

Below the solution obtained with the MATLAB code. In case the screen shot is not enough clear.

$$U(s) = \frac{-367487697464885679045875985 28512.0*S - 16353603596649233021183773900800.0}{2.59614842926741381426524816461e33*S^2 + 1.187285813893889209319838808277e33*S - 205213643068321447686330091470422.0}$$

$$\theta(\mathrm{s}) = \frac{0.056620445*\mathrm{S} + 0.00057518768}{S^2 + 0.45732586*\mathrm{S} - 0.07904542}$$

$$\mathrm{W(s)} = \frac{0.056620444859323250386129444677863*\mathrm{S} - 0.0030247784363184753754494327182556}{S^2 + + 0.45732586030488242036240453813889*\mathrm{S} - 0.079045420036414878809763814646693} + 0.013329381699887379287150446887909$$

$$\mathrm{q(s)} = \frac{-0.10427077689175302810387080398868*\mathrm{S} - 0.055791455958991925824404049290847}{S^2 + + 0.45732586030488242036240453813889*\mathrm{S} - 0.079045420036414878809763814646693} - 15.117930555574632833781834051479$$

Matlab Code

Principal code

```
clear all

close all

clc


disp('DM YAZIGI')

disp('Joulin Lise MS3')

disp('----------------------------------------------------------------')

disp('')


%%Donnée%%

%Flight condition%

altitude = 35000*0.3048;

air_density = 0.000739*515.383;

speed = 876*0.3048;

Xcg = 0.29;

teta0 = (2.6*pi)/180;


%----------------------------------------------------------------------%


%Géométrie et Inertie%

wing_area = 530*0.0929;
```

```
wing_span = 38.7*0.3048;

mean_chord = 16*0.3048;

mass = 39000*0.4535  ;

Ixxb = 25000*1.3558;

Iyyb = 122200*1.3558;

Izzb = 139800*1.3558;

Ixzb = 2200*1.3558   ;


%--------------------------------------------------------------------%


%Coeffficient%

CL0 = 0.26;

CD0 = 0.03;

CTx = 0.03;

Cm = 0 ;

Cmt = 0;


%--------------------------------------------------------------------%


%Longitudinal Derivatives%


Cmu = -0.117;

Cm_alph = -0.40;

Cm_alpha_point = -1.3;
```

```
Cm_q = -2.7;

Cm_Tu = 0;

Cm_T_alpha = 0;

CL_u = 0.27;

CL_alpha = 3.75;

CL_alpha_point = 0;

CL_q = 0.3;

CD_alpha = 0.3;

CDu = 0.027;

CTxu = 0;

CL_deltaE = 0.40;

CD_deltaE = -0.10;

Cm_deltaE = -0.58;


%Calculated

P = 101325*(1-(0.0065*altitude)/288.15)^(9.81/(0.0065*287));

a = sqrt((1.4*P)/air_density);

M = speed/a;

g0 = 9.81;

q = (1/2)*air_density*(speed)^2;

AR = wing_span^2 / wing_area ;

e = 0.85;


%-----------------------------------------------------------------%
```

```
disp('Question 1 : Equation of longitudinal motion')

disp('')


Xu = -((q*wing_area)/(mass*speed))*(2*CD0 + CDu) ;  %CDu = M*CDm

Zu = -((q*wing_area)/(mass*speed))*(2*CL0 + M^2/(1-M^2)*CL0);

Mu = ((q*wing_area*mean_chord)/(Iyyb*speed))*(Cmu)   ;


Xw = ((q*wing_area)/(mass*speed))*(CL0 - (2*CL0*CL_alpha)/(pi*e*AR));

Zw = -((q*wing_area)/(mass*speed))*(CD0+CL_alpha);

Zw_point = ((q*wing_area*mean_chord)/(2*mass*speed^2))*(CD0+CL_alpha_point) ;

Mw = ((q*wing_area*mean_chord)/(Iyyb*speed))*Cm_alph;

Mw_point = ((q*wing_area*mean_chord^2)/(2*Iyyb*speed^2))*Cm_alpha_point;


Zq = ((q*wing_area*mean_chord)/(2*mass*speed))*CL_q ;

Mq = ((q*wing_area*mean_chord^2)/(2*Iyyb*speed^2))*Cm_q;


XdeltaE = ((q*wing_area)/(mass*speed)) * CD_deltaE ;

XdeltaT = 0 ;

ZdeltaE = ((q*wing_area)/(mass*speed)) * CL_deltaE;

ZdeltaT = 0;
```

```
MdeltaE = ((q*wing_area*mean_chord^2)/(2*Iyyb*speed))*
Cm_deltaE ;

MdeltaT = 0;


disp('Below the different coefficient of the equation ')

disp('Xu = ')

disp(Xu)

disp('')


disp('Zu = ')

disp(Zu)

disp('')


disp('Mu = ')

disp(Mu)

disp('')


disp('Xw = ')

disp(Xw)

disp('')


disp('Zw = ')

disp(Zw)

disp('')
```

```
disp('Mw = ')
disp(Mw)
disp('')

disp('Mw_point = ')
disp(Mw_point)
disp('')

disp('Zq = ')
disp(Zq)
disp('')

disp('Mq = ')
disp(Mq)
disp('')

disp('')
disp('---------------------------------------------------------------------
')
disp('---------------------------------------------------------------------
')
disp('')
disp('Question 2 : The matrix A of the aircraft')
```

```
A = [Xu,Xw,0,-g0*cos(teta0) ;

     Zu,Zw,speed,-g0*sin(teta0);

     (Mu + Mw_point*Zu) , (Mw + Mw_point*Zw) ,( Mq + speed*Mw_point) , -g0*Mw_point*sin(teta0);

     0,0,1,0 ];


 B = [XdeltaE , XdeltaT  ;

      ZdeltaE , ZdeltaT  ;

      MdeltaE + Mw_point*ZdeltaE , MdeltaT + Mw_point*ZdeltaT ;

      0 , 0  ];


 disp('The matric of the arcraft is ')

 disp(A)


disp('')

disp('-------------------------------------------------------------------')

disp('-------------------------------------------------------------------')

disp('')

disp('Question 3 : The characteristics equation')


%Première méthode

I = eye(4,4);

syms lambda ;
```

```
d = det(lambda*I-A);

eq = d ==0;


S1 = solve(eq,lambda);


%Deuxième méthode


S2 = poly(A);


disp('the characteristic coeffitient of the equation are : ')

disp(S2)


syms x ;

equation = vpa(simplify(det(A-x*eye(4))),4);

disp('Finally we obtain the equation below ')

disp(equation)


disp('')

disp('----------------------------------------------------------------------
')

disp('----------------------------------------------------------------------
')

disp('')

disp('Question 4 : The eigenvalues of the system')
```

```
[i,j] = eig(A) ;
lambda1 = j(1,1);
lambda2 = j(2,2);
lambda3 = j(3,3);
lambda4 = j(4,4);

disp('The eigenvalue of the system are :')
disp('lambda 1 ')
disp(lambda1)

disp('Lambda 2')
disp(lambda2)

disp('Lambda 3')
disp(lambda3)

disp('Lambda 4')
disp(lambda4)

disp('')
disp('----------------------------------------------------------------------
')
disp('----------------------------------------------------------------------
')
```

```
disp('')

disp('Question 5 : Different modes of longitudinal stability')

disp('Question a : Short period mode - Natural frequency/Damping Factor')

disp('The Damping ratio is ')

Zeta_s = sqrt(1/1+((imag(lambda1)/real(lambda1))^2));

disp(Zeta_s)


disp('The Undamped Natural Frequency coeffciient is ')

Omega_s = -(real(lambda1))/Zeta_s;

disp(Omega_s)


disp('----------------------------------------------------------------------')

disp('Question b : Phugoid mode - Natural frequency/Damping Factor')

disp('The Damping ratio is ')

Zeta_p = sqrt(1/(1+((imag(lambda3)/real(lambda3))^2)));

disp(Zeta_p)


disp('The Undamped Natural Frequency coeffciient is ')

Omega_p = -(real(lambda3))/Zeta_p;

disp(Omega_p)


disp('')
```

```
disp('-----------------------------------------------------------------------
')

disp('-----------------------------------------------------------------------
')

disp('')

disp('Question 6 : Curves of longitudinal motion')


%short period


grid on

figure (1)


X1 =i(:,2) ;

[t,x] = ode45('Courbe', [0 15], X1);

plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4))

legend('Axial velocity : u(t)','angle of attack : w(t)','pitch rate : q(t)','pitch angle : theta(t)')

xlabel('Time(s)')

title('Short Period')



%Phugoid period



grid on
```

```
figure (2)


X2 =i(:,3) ;

[t,x] = ode45('Courbe', [0 1000], X2);

plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4))

legend('Axial velocity : u(t)','angle of attack : w(t)','pitch rate : q(t)','pitch angle : theta(t)')

xlabel('Time(s)')

title('Phugoid Period')

grid on


disp('')

disp('----------------------------------------------------------------------')

disp('----------------------------------------------------------------------')

disp('')

disp('Question 7 : transfer Functions of each variable')



%Phugoid mode

syms S

format short e


Ap = [S-Xu , g0*cos(teta0) ;
```

```
     -(Zu*9.81)/speed , S+g0*sin(teta0)] ;

det1 = det(Ap) ;

TF_U = vpa(simplify(((S*XdeltaE)+(((g0*sin(teta0)*XdeltaE))))/det1,3)) ;

TF_teta = vpa(simplify(((-Zu*9.81* XdeltaE)/speed) - (Xu*ZdeltaE) + (S*ZdeltaE))/det1,3);

disp('U(s) = ')

disp(TF_U)

disp('Teta(s) = ')

disp(TF_teta)

%Short-period transfer function approximations

As = [S-A(2,2), A(2,3) ;

     -A(3,2) , S-A(3,3)] ;

det2 = det(As) ;

TF_W = vpa(simplify((S-A(3,3))*B(2,1) - A(3,2)*B(3,1)/det1,3)) ;

TF_q = vpa(simplify(-A(2,3)*B(2,1)+ (S-A(2,2))*B(3,1)/det1,3)) ;

disp('W(s) = ')
```

```
disp(TF_W)
disp('q(s) = ')
disp(TF_q)
```

Function code

```
function system = Courbe(t,u)


Xu=-0.0123;

Xw=  2.2538e-04;




Zu=-0.2348;

Zw=-0.5351;

Zq= 1.0355e-01 ;

Zw_point = 3.8781e-05;


Mu=-8.6220e-03

Mw=-0.0295;

Mq= -0.0018;

Mw_point= -8.7489e-04;


speed= 267.0048;


g0=9.81;

theta0= 0 ;


U=u(1);

W=u(2);
```

```
Q=u(3);

Theta=u(4);


du =Xu*U +Xw*W-g0*cos(theta0)*Theta;

dw = (Zu*U + Zw*W + (Zq+speed)*Q - g0*sin(theta0)*Theta)/(1-
Zw_point);

dq =(Mu+(Mw_point*Zu)/(1-Zw_point))*U+ (Mw+(Mw_point*Zw)/(1-
Zw_point))*W+(Mq+(Mw_point*(Zq+speed))/(1-Zw_point))*Q-
(g0*Mw_point*sin(theta0)/(1-Zw_point))*Theta;

dtheta = Q ;


system = [du;dw;dq;dtheta];

end
```