Applied Control Systems

Lecture 02

Sensors and Actuators

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2019

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Summary of AU47

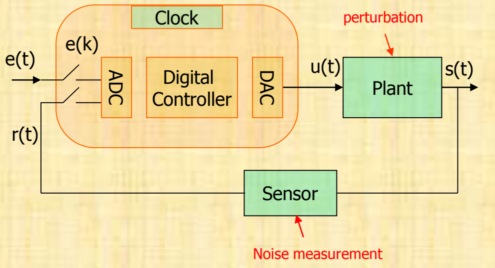
part 1
- Introduction, Internal/external model, Discrete/continuous transfer function, Discrete/continuous state space representation, SISO/MISO
  Conversion from continuous to discrete, Stability analysis…

part 2
- Sensors / Actuators :
  Measuring temperature, position, speed, acceleration, …
  Hydraulic, pneumatic, electrical actuators
  Power amplification, PWM drivers

part 3
- Practical aspects of control systems (PID, lead/lag; filtering, windup, …),
  Converting continuous controller into discrete controller,
  Control with state space representation …

A classical control scheme

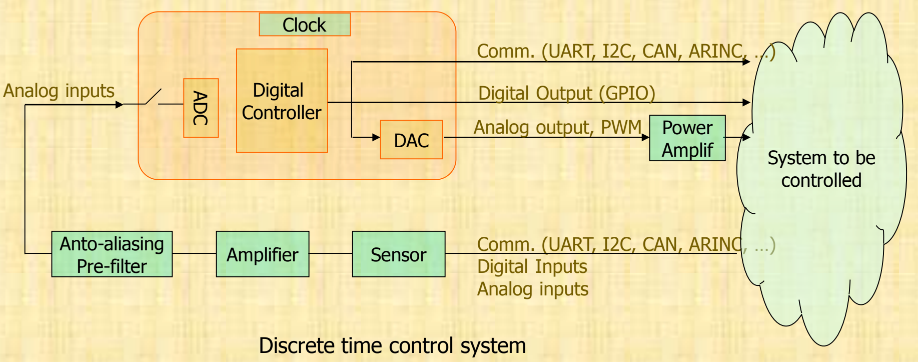
- Continuous control vs. Discrete control



Discrete time control system
(Digital control)

A classical control scheme

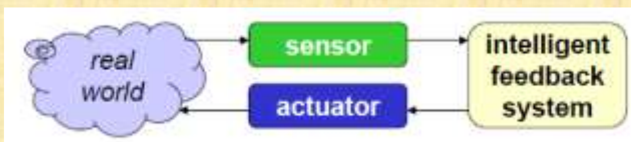
- A digital control scheme



Discrete time control system
(Digital control)

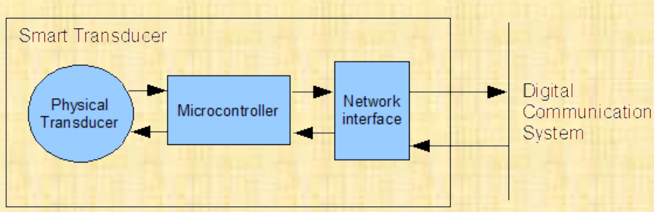
Sensors & Actuators = Transducer

- Transducer
  - A device that converts a primary form of energy (mechanical, thermal, optical, …) into a signal with a different energy form
- Sensor
  - Converts a physical quantity into Electrically measurable signal
  - Eg. Thermometer, voltmeter, tachometer
- Actuator
  - Converts an electrical input into a physical quantity
  - Eg. Heater, generator, DC motor



Sensors & Actuators

- Smart transducer
  - an analog or digital sensor/actuator combined with a **processing unit** and a **communication interface**
  - **Smart sensor/actuator = Sensor/actuator + integrated circuit**



  - Self calibration, multisensing, communication, computation, cost effective

Application: ESP

- ESP - Electronic Stability Program

Loss of steering
control detection

Braking automatically
applied to wheels
individually

Front wheel (avoid oversteer)
or
Inner wheel (avoid understeer)

ESP - Electronic Stability Program

- Sensors
  - Wheel speed sensor
  - Steering-angle sensor
  - Yaw-rate sensor
  - Lateral-accel. Sensor
- Actuators
  - Hydraulic brake
- Other
  - Control unit
  - Communication with engine management (to ajust engine torque)

Application: Aircraft flight control system

Digital technique was imported from military into civil aviation by Airbus, first with the A320, then by A319, A321, A330, A340, Boeing 777 and A380

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Measurement chain

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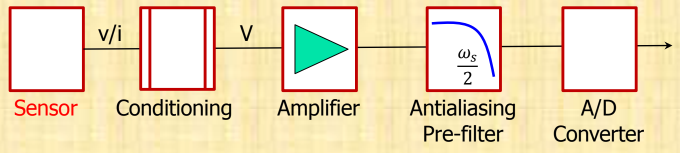
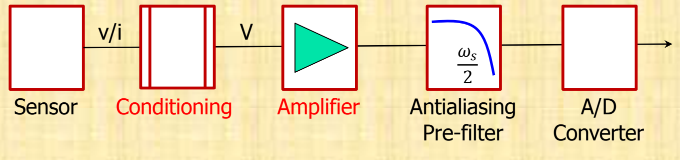
Measurement chain



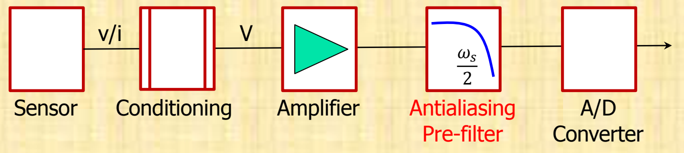
- Sensor
  - Converts a physical quantity into Electrically measurable signal
  - Two types: passive and active sensors
  - **Passive sensors:** measure energy that is naturally available
    can only be used to detect energy when the occurring energy is available
    eg. photogarphic, thermal, electric field, chemical, infrared, seismic
  - **Active sensors:** provide their own energy source for sensing
    eg. US sensor emits wave toward the target and compares it to the reflected wave.


Measurement chain

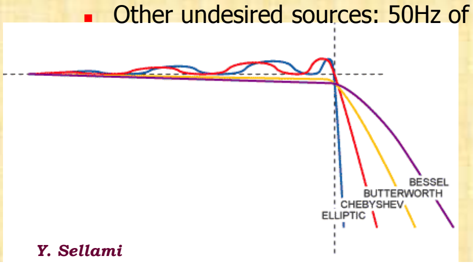


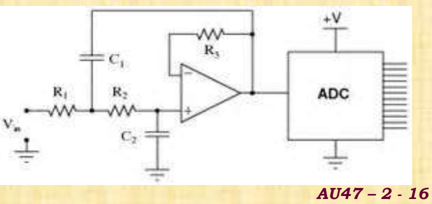
- First stage conditioning
  - Converts an electrical signal to an suitable signal for further processing
  - Output signal can be DC or AC voltage
  - External power supply is needed
  - Conditioning is necessary for passive sensors (R, C, …)
- Amplification
  - performs two functions: increases the resolution of the input signal, and increases its signal-to-noise ratio
  - ex: output range [0, 5v]

Measurement chain

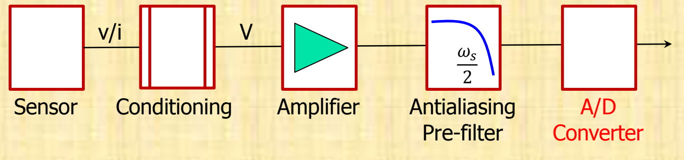


- Filtering
  - Not all the signal frequency spectrum contains valid data
  - Frequency higher than half sample rate $\omega_s/2$ must be filtered to avoid aliasing
  - Other undesired sources: 50Hz of power supply, current in neibour wires, …



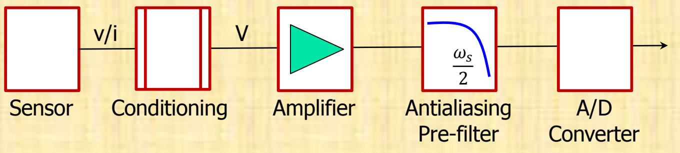


Measurement chain

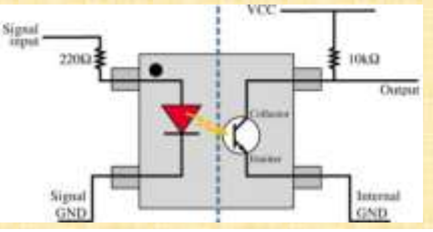


- Analog to Digital Converter
  - converts a continuous physical quantity (voltage) to a corresponding number
  - ADC converts periodically the input with a sample rate ω_s
  - ADC involves quantization of the input
  - Conversion from a continuous-time and continuous-amplitude analog signal ⇨ to discrete-time and discrete-amplitude digital signal.
  - Resolution: eg. ADC with $N = 8$ bits and a full scale range [0 , 5v]
    - ADC resolution : $2^8 = 256$ quantization levels
    - ADC voltage resolution: $(5 - 0)/256 = 18.5mV$
  - Quantization error: Signal to quantization noise ratio:
    - $SQNR = 20\log_{10}(2^N) \approx 6.02\ N\ dB$

Measurement chain



- Isolation
  - Useful to pass signal from source to the sensor without physical connection
  - Used to isolate signal perturbations or to isolate expensive equipment
  - ex. Magnetic isolation, Optic isolation, capacitive isolation

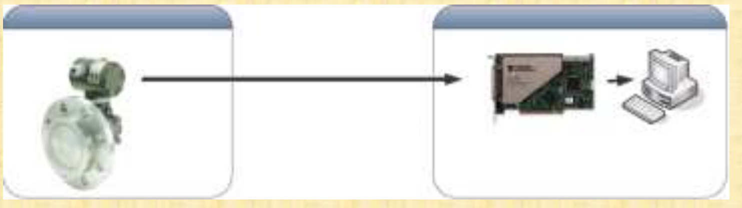


Connecting Smart Sensors to Microcontrollers

- Smart sensor
  - Smart sensor = sensor with built-in signal processing & communication
- Analog inputs
  - Many microcontrollers have ADC built-in peripherals (8-bit to 12 bit)
- Digital inputs
  - Use GPIO ports, synchronous (with clock), or asynchronous (without clock)
- Communication links
  - Asynchronous serial: **UART** (universal asynchronous receive and transmit)
    - TX, RX, nodes must match baud rate and protocol
    - RS232 on PCs uses UART format but at +/- 12 volts
  - Synchronous serial: **SPI** (serial peripheral interface)
    - 1 or 2 directional data MOSI, MISO, 1 clock SCLK, 1 chip select CS
  - **I2C** Inter Integrated Circuit bus
    - 2 wires

Connecting Smart Sensors to PC/Network

- Smart sensor = sensor with built-in signal processing & communication
- DAQ : Data acquisition cards
  - PC card with analog digital I/O



- Local area networks
  - CAN bus (controller area network) : in vehicles
  - ARINC 429 (Aeronautical Radio INC.) : two-wire data bus
  - AFDX (Avionics Full DupleX switched Ethernet) : on new Airbus aircrafts

Sensor calibration

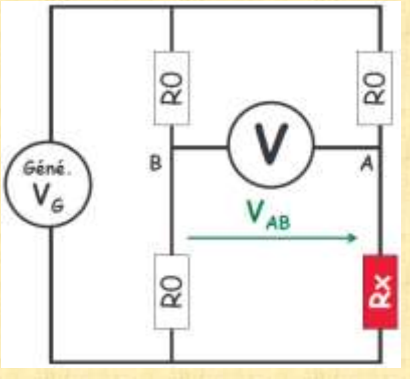
- Undesired effects
  - Offset : nominal output ≠ nominal physical parameter value
  - Nonlinearity : output is not linear, there are physical parameter changes
  - Cross parameter sensitivity : sensitive to other dependent parameters
- Calibration
  - Adjust output signal to match physical parameters
  - With signal conditioning, digital calibration, …
  - $T° = a + b\,V + c\,V^2$;

    $T°, V$: temperature (input) and sensor voltage

    a, b, c: calibration coefficients

Wheatstone Bridge

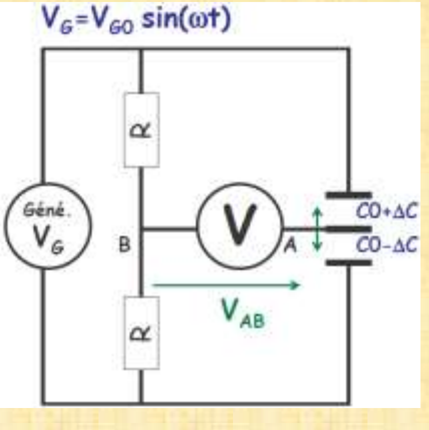
- Wheatstone Bridge
  - A balance measuring method
  - provides extremely accurate measurements
- Application 1



The current I_{AB} is neglected.
- Compute V_{AB}
- Compute V_{AB} if $\Delta R \ll R0$
- Give the equilibrium condition of the bridge

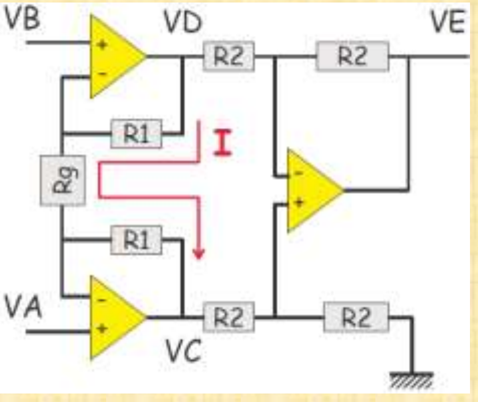
Wheatstone Bridge

- Application 2



The current I_{AB} is neglected.
- Compute V_{AB}
- Compute V_{AB} if $\Delta C \ll C0$

Op-Amp in instrumentation

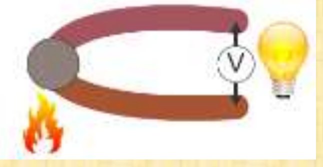


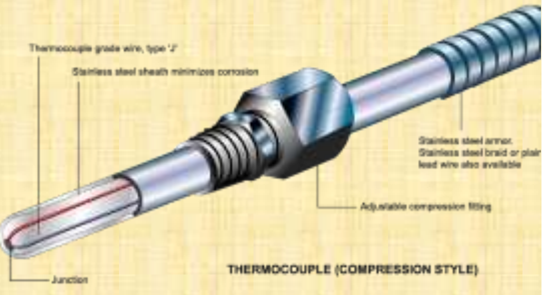
- Compute $V_E = f(V_A, V_B)$
- Role of this circuit?

Some Sensors technologies

Temperature sensor: Thermocouple

- Thermocouple
  - two different conductors forming electrical junctions at differing temperatures



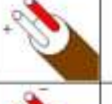
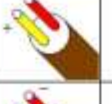
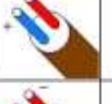


$$\Delta V = -S(T)\,\Delta T$$

$S(T)$: Seebeck coefficient is a temperature dependent material property

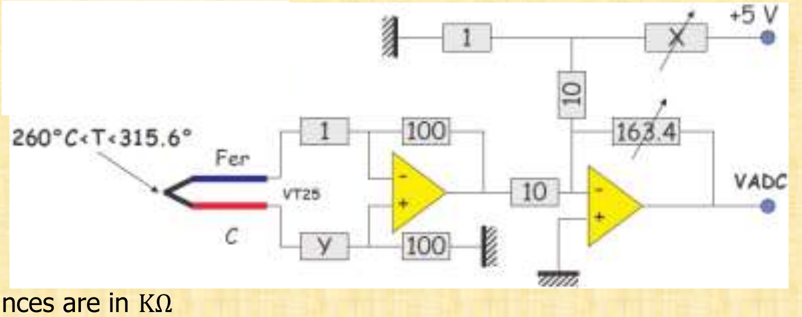
Temperature sensor: Thermocouple types

- Thermocouple Types
  - Common types of thermocouples

Type	Material		Color Code	Range (°C)	
Thermocouple Grade	Positive Wire	Negative Wire		Minimum	Maximum
J	Iron	Constantan		0	750
K	Chromel	Alumel		-200	1250
T	Copper	Constantan		-200	350
E	Chromel	Constantan		-200	900

Temperature sensor: Thermocouple example

- Application

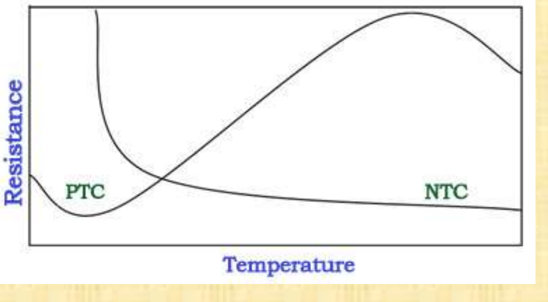


All resistances are in KΩ
Temperature range = [260° , 315.6°] ⇒ Our choice: thermocouple of type J
V_{T25} voltage range = [12.84mV , 15.90mV]
V_{ADC} voltage range = [0 , 5v]
$V_{ADC} = m\ V_{T25} + b$; a, b are obtained from the two above ranges
Resistors X, Y are adjusted so that the output VADC match the range [0 , 5v]

Temperature sensor: Thermistor

- Thermistor
  - A type of sensor whose resistance is dependent on temperature

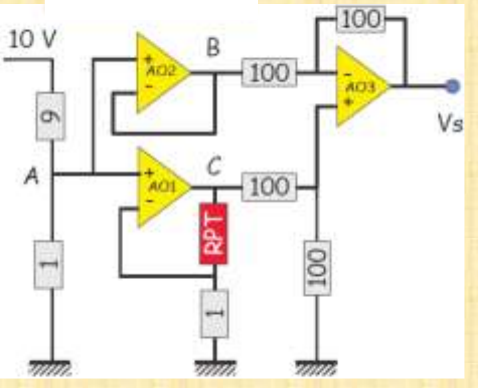
$$\Delta R = f(\Delta T)$$



  - PTC (positive temperature coef): whose resistance vary with temperature
  - NTC (negative temperature coef): resistance decreased as the temperature rises

Temperature sensor: Thermistor Example

- Thermistor : Application



All resistances are in KΩ

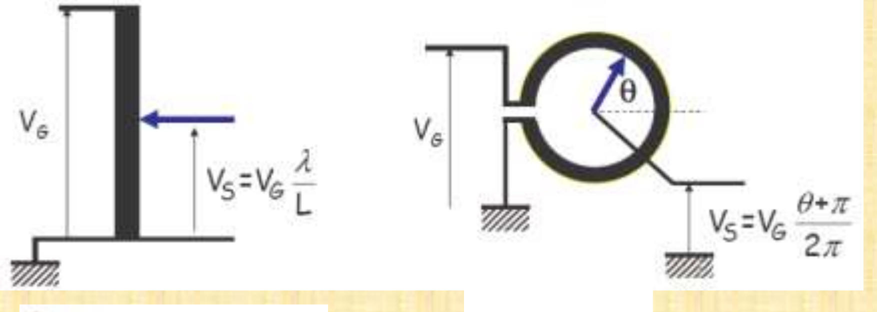
$RPT = 0.1\,(1 + 0.391\,T°)\ k\Omega$

- Compute VA, What is the role of AO2?

- Compute VC , VS and the sensitivity of the circuit $\frac{\Delta Vs}{\Delta T}$

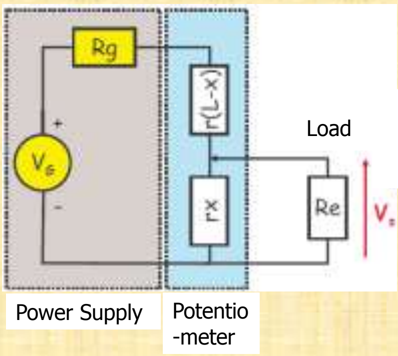
Position sensor: Potentiometer

- Potentiometer position sensor

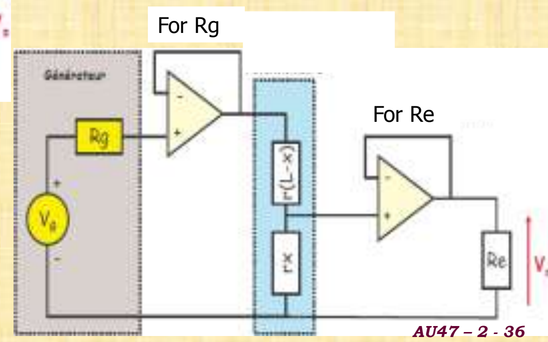


Position sensor: Potentiometer

- Conditioning : impact of Rg and Re



$$V_s = V_G \frac{x}{L}$$



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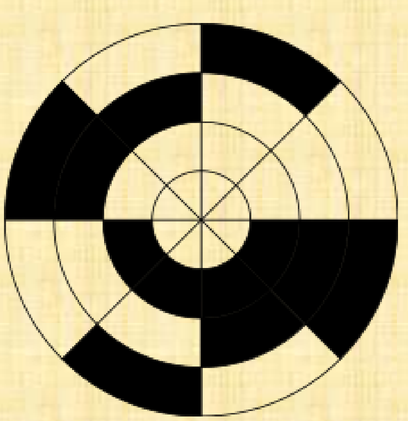
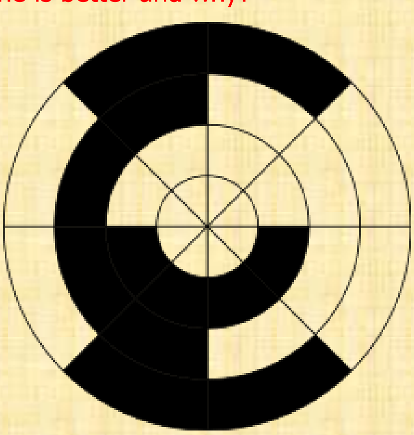
Position sensor: Absolute encoder

- Binary or Gray Code encoder? Which one is better and why?



3-bit binary encoder
3 channels



3-bit Gray encoder
3 channels

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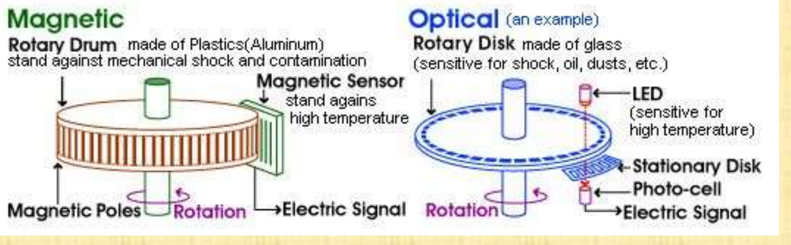
Position sensor: Absolute encoder

- Rotary (or linear) absolute encoder
  - provides a unique code for every single position of rotation (or translation).
- Construction
  - Mechanical absolute encoder
  - Optical absolute encoder
  - Magnetic absolute encoder

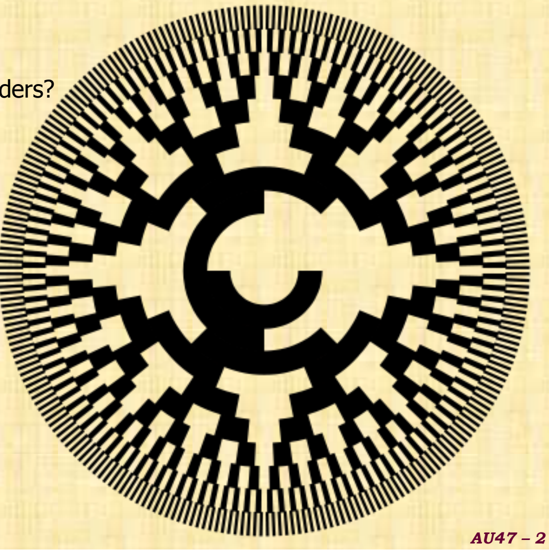


Position sensor: Absolute encoder

- A 10-bit rotary absolute encoder

  with 10 channels !

  What about 16 bits encoders?

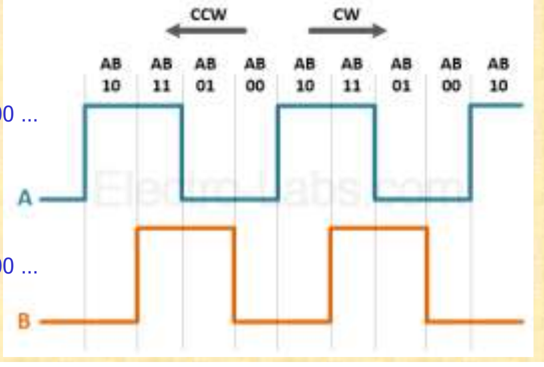


Position sensor: Incremental encoder

- Incremental rotary encoder (quadrature encoder, relative rotary encoder)
  - Produces a square wave pulses that, when counted, indicates the angular position and the rotating shaft (direction).
  - **Two channels : Two outputs in quadrature (90° of phase)**
  - Resolution: determined by the number of transparent/dark segments on the disk
  - A tachometer is a simple encoder with 1 single wave output (one directional application)

Clockwise rotation
AB = 00 10 11 01 00 …

Counter-clockwise
AB = 00 01 11 10 00 …



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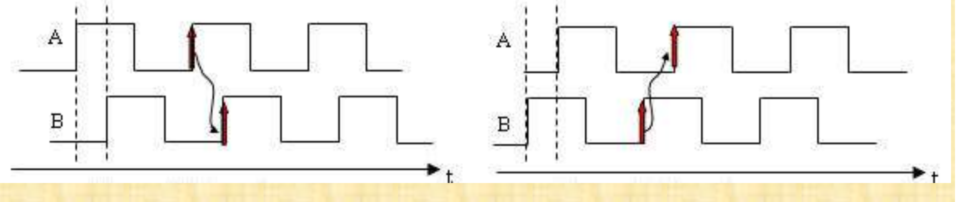
Position sensor: Incremental encoder

- Incremental rotary encoder (quadrature encoder, relative rotary encoder)
  - How the rotating shaft is obtained?



  - Method 1: When A rising edge, B is low (or high)
  - Method 2: When A rising and falling edges, A and B are equal (or different)
  - Method 3: More precise:
    when A and B rising and falling edges, A and B are equal (or different)

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Arduino code for encoder

- Method 1 : using interrupt 0 (method 2)

```
/* Read a rotary encoder with interrupts pinA to pin 2, pinB to pin 4 uses Arduino pullups on A & B turning
   on the pullups saves having to hook up resistors to A & B */
#define  pinA   2                               // encoder pin A
#define  pinB   4                               // encoder pin B
volatile unsigned int encoderPos = 0;      // position of the encoder

void setup() {
  pinMode(pinA, INPUT);      pinMode(pinB, INPUT);      // pinA and pinB as inputs
  digitalWrite(pinA, HIGH); digitalWrite(pinB, HIGH); // turn on pullup resistors
  attachInterrupt(0, doEncoder, CHANGE); // encoder pin on interrupt 0 - pin 2
                                          // CHANGE = FALLING + RISING edges
  Serial.begin (115200);  Serial.println("start");    // a personal quirk
}

void loop(){            // do something - the joy of interrupts is that they take care of themselves
}

void doEncoder() {  // If pinA and pinB are equal, it is spinning forward. If they're different, it's going backward.
  if (digitalRead(pinA) == digitalRead(pinB))
      {  encoderPos++;  }
  else {  encoderPos--; }

  Serial.println (encoderPos, DEC);
}
```


Arduino code for encoder

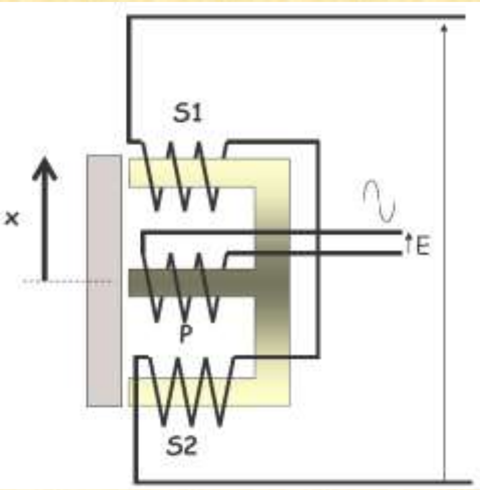
- Method 2 : using Encoder Library
  - See PJRC website: http://www.pjrc.com/teensy/td_libs_Encoder.html

```
Encoder myEnc(pinA, pinB);
// Create an Encoder object, using 2 pins.
// You may create mulitple Encoder objects, where each uses its own 2 pins.
// The first pin should be capable of interrupts.
// If both pins have interrupt capability, both will be used for best performance.
// Encoder will also work in low performance polling mode if neither pin has interrupts.

position = myEnc.read();// Returns the accumulated position. This number can be positive or negative.
myEnc.write(newPosition);  // Set the accumulated position to a new number (if necessary).
```

Position sensor : LVDT

- LVDT Linear Variable Differential Transformer
  - Robust, absolute linear displacement sensor



Output $U = f(x)$

Primary:
$E = E_0 \sin(\omega t)$

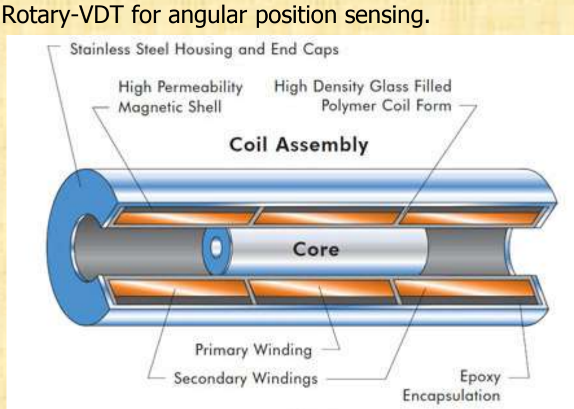
Secondary:
$S_1 = k_1 V_0 \sin(\omega t)$
$S_2 = k_2 V_0 \sin(\omega t)$

If $x \neq 0$ then
$k_1 = k_0 + a\,x$
$k_2 = k_0 - a\,x$

$U = S_1 - S_2 = 2aV_0\, x \sin(\omega t)$
$x \geq 0 \;\Rightarrow\; phase = 0, else\ 180°$

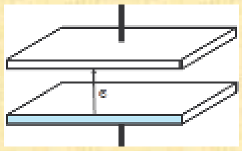
Position sensor : LVDT

- LVDT Linear Variable Differential Transformer
  - Robust, absolute linear displacement sensor
  - Has virtually infinite cycle life
  - Widely used in power turbines, hydraulics, automation, aircraft, satellites, nuclear reactors, and many others.
  - RVDT Rotary-VDT for angular position sensing.

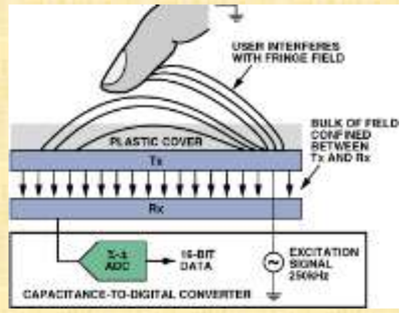


Capacitive sensors

- By changing distance between plates

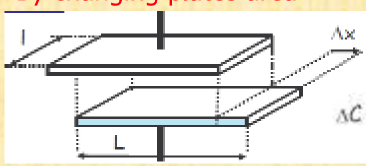


$C = \varepsilon_0 \varepsilon_r \frac{S}{e}$



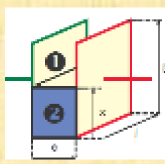
What about this sensor?

- By changing plates area



$\Delta C = -\varepsilon_0 \varepsilon_r \frac{l}{e} \Delta x$

- By changing dielectric type between plates



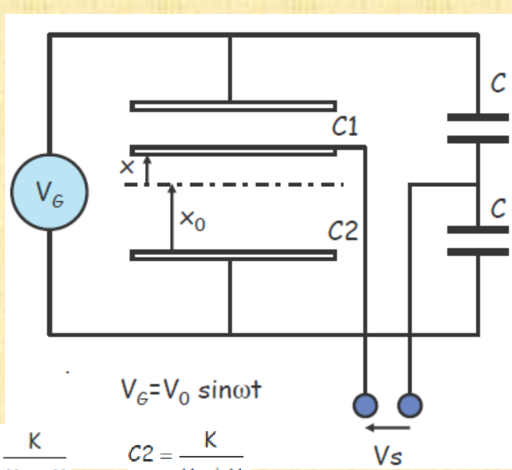
$C1 = \varepsilon_0 \frac{l(L-x)}{e}$

$C2 = \varepsilon_0 \varepsilon_r \frac{lx}{e}$

⇨ $\Delta C = \frac{\varepsilon_0 l(\varepsilon_r - 1)}{e} \Delta x$

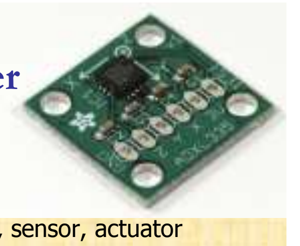
Capacitive sensors

- Example



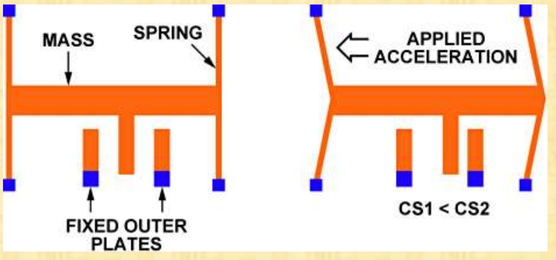
$C1 = \frac{K}{x_0 - x}$ $C2 = \frac{K}{x_0 + x}$

Capacitive sensor: accelerometer

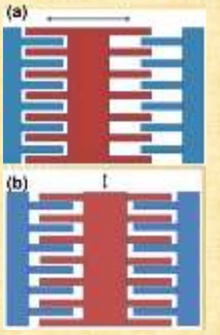


- MEMS capacitive accelerometer
  - MEMS: Micro-Electro-Mechanical System

    Integration of mechanical unit, electrical unit, sensor, actuator



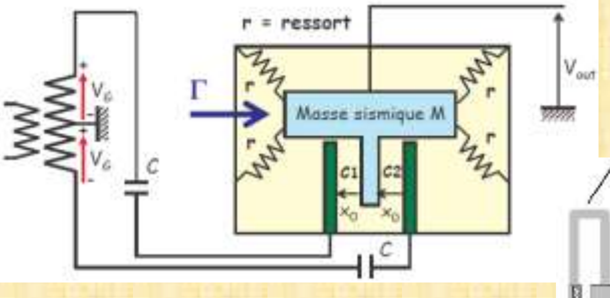
MEMS accelerometer principle
Sense mass motion and compute acceleration from capacitive variation

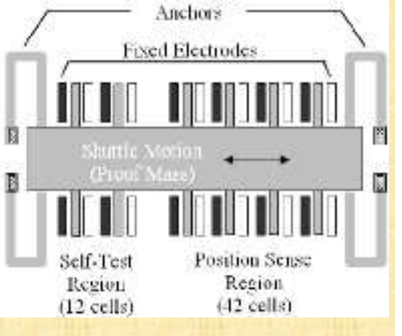


Two axis accelerometer
a) Lateral sensing
b) transverse sensing

Capacitive sensor: Example

- Application : Linear accelerometer. Analog Device ADXL 150



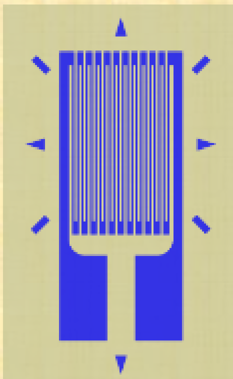


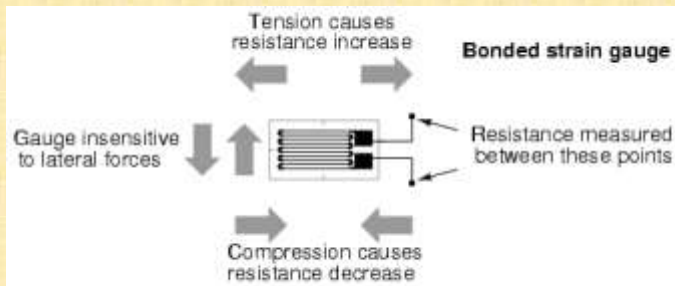
$$C1 = \frac{K}{x_0 - x} \quad ; \quad x = \frac{M}{k}\Gamma$$

Compute the acceleration $\Gamma = f(x)$.

Strain gauge (force sensor)

- Resistive sensors
  - Force => Gauge deformation => resistance variation => Wheatstone Bridge

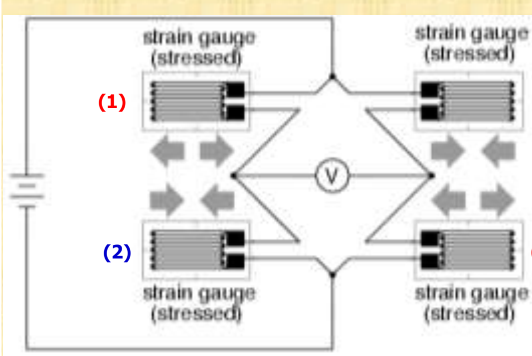


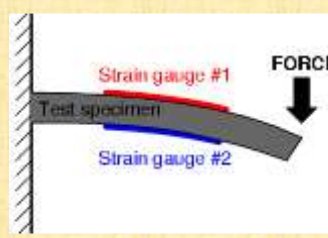


Metallic/semiconductor gauge

Strain gauge

- Full-bridge strain gauge circuit





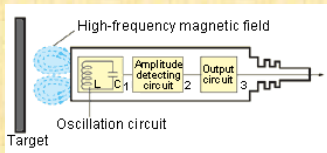
- Gauges 1,2,3,4 have the same nominal resistance R0
- Gauges 1,2,3,4 are sensitive to temperature (ΔR_T)
- Tension on gauges 1 and 4 : $(R0 + \Delta R_F + \Delta R_T)$
- Compression on gauges 2 and 3 : $(R0 - \Delta R_F + \Delta R_T)$

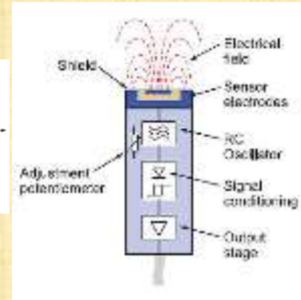
- Compute V_{AB} as a function of ΔR_F
- Does the bridge sensitive to temperature?
- At which condition we have linearity?

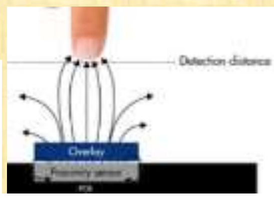
Proximity sensors

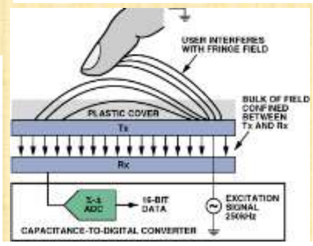
Proximity sensors are used to detect presence of object in proximity

- Touch sensors (electromechanical sensors)
- Inductive sensors
- Capacitive sensors
- Optical sensors (IR, camera, …)
- Hall Effect sensors
- Ultrasonic sensors









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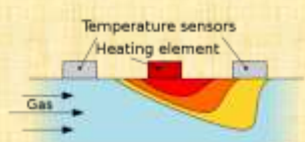
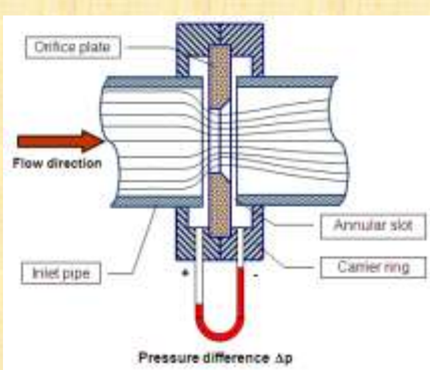
Flow sensor

Flow sensor measure the rate of fluid flow.

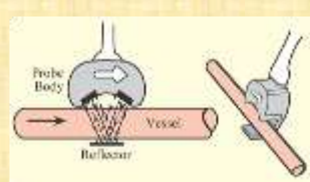
- Technologies:
  - Turbine, orifice plate, US, thermal, …



MEMS Thermal Mass Flow Sensor



Flow Sensor with orifice plate



Ultrasonic Flow Sensor
(Doppler principle)

Actuators

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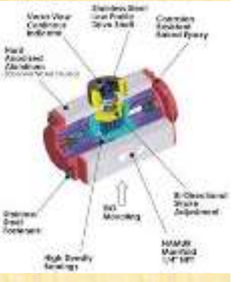
Actuators

- Objectives
  - Manufacturing cost
  - Electronic control cost (simple electronic control)
  - High torque / low inertia / small size / precision / …
  - Motor efficiency (low amount of lost energy, lower temperature)

- Actuators types
  - On/Off Controlled (open loop)
  - Uni- or bi-directional
  - Speed controlled (open loop)
  - Speed controlled (closed loop)
  - Servo motor (position controlled)
  - Stepper motor

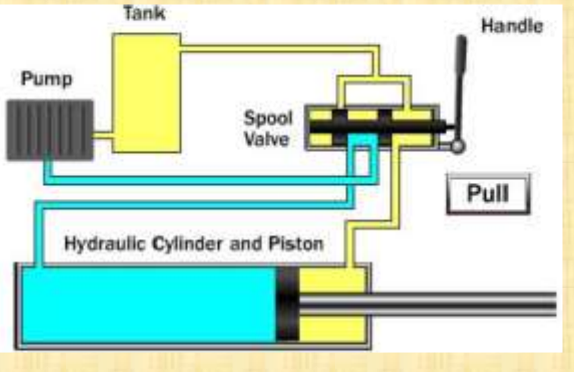
Actuators types

- Actuators types
  - Electrical drives
    - Convert energy from electrical to mechanical motion
    - 90% of drives are electrical
  - Pneumatic actuators
    - Convert energy from compressed gas (air) to mechanical motion
    - Time response is high due to air elasticity
  - Hydraulic actuators
    - Convert energy from hydraulic to mechanical motion
    - Almost 100% efficient (negligible loss due to fluid friction)
    - Good power/size ratio (high power with compact size)
    - Widely use in aircrafts



Hydraulic system

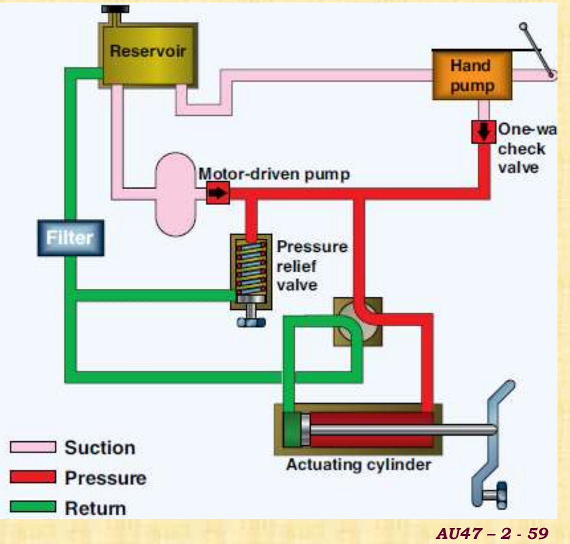
- Principle



Hydraulic system

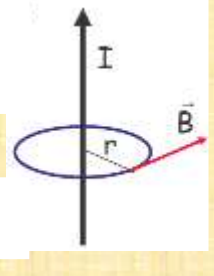
- Basic hydraulic system

Basic components:
- Reservoir (tank)
- Pumps
- Filter
- Pressure relief valve (accumulator)
- Hydraulic valve (distributor)
- Actuator (acting cylinder)

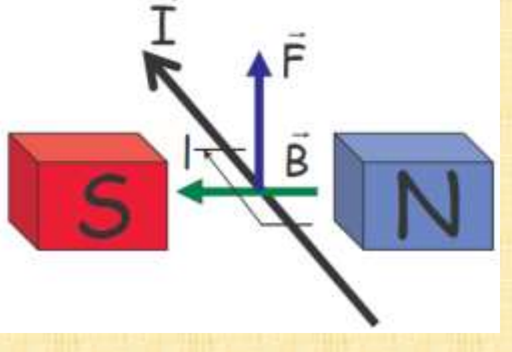


Electrical Drive

- Principle



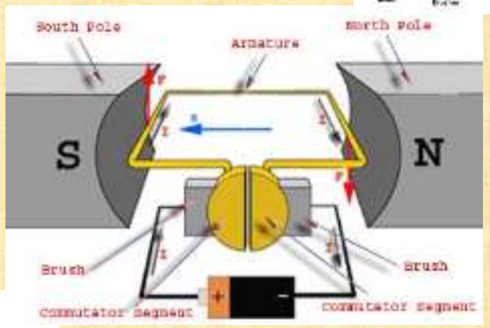
$$\vec{I} = \frac{2\pi}{\mu}\vec{r}\wedge\vec{B}$$



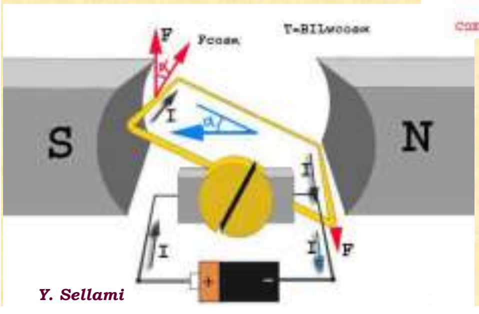
$$\vec{F} = \vec{I}.l\wedge\vec{B}$$

Brushed DC Motor

- DC Motor Principle



Try to understand how does it work?



What is the role of the commutator?

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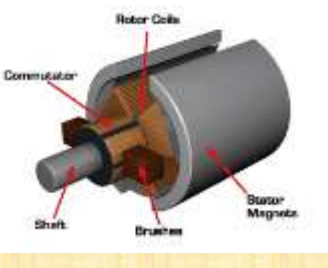
Brushed DC Motor

- Stator
  - Is the fixed part of DC Motor
  - Generate magnetic field from an external **DC current** or from a permanent **magnet** (Permanent Magnet DC Motor – PMDC Motor)
- Rotor
  - Rotates inside the stator magnetic field
  - Consists of core, windings and commutator



- Collector
  - The armature winding are connected to the commutator segments placed on the motor shaft.
  - Carbon or graphite brushes are placed on the commutator segments to supply current to the armature.

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Brushed DC Motor

- Advantages
  - Low initial cost, high reliability, simple control of motor speed
- Overcomes
  - High maintenance (replacing the carbon brushes, cleaning the commutator…)
  - Low life for high intensity uses

Y. Sellami *AU47 – 2 - 65*

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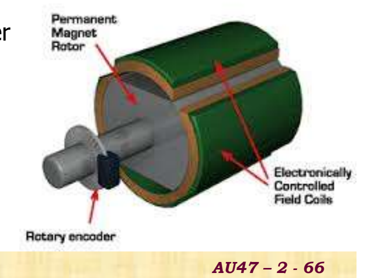
Brushless DC Motor

- Rotor
  - Includes one or more permanent magnets (no commutator needed)
- Stator
  - Powered by an external power supply. A motor controller converts DC to AC.
  - Sensing rotor position are needed (Hall effect sensors) to control the timing to optimize torque, regulate speed, brake, …
- Advantages
  - Long life, little or no maintenance, high efficiency
- Disadvantages
  - Higher cost, more complicated speed controller

=> Referred as Synchronous Motors



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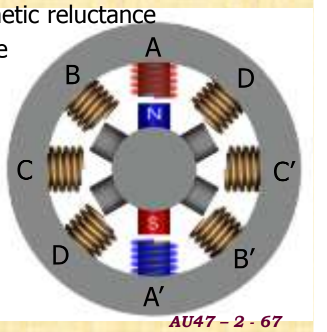
Stepper motors

- Permanent magnet stepper
  - Uses permanent magnet rotor to align to the stator magnetic field
  - Stator is commanded by electrical pulses
  - No teeth required
- Variable reluctance stepper
  - has toothed non-magnetic soft iron rotor
  - Rotor is ferromagnetic, torque is generated by magnetic reluctance
  - When the stator is powered, the rotor moves to have a minimum gap between the stator and its teeth.

Try to understand how does it work?

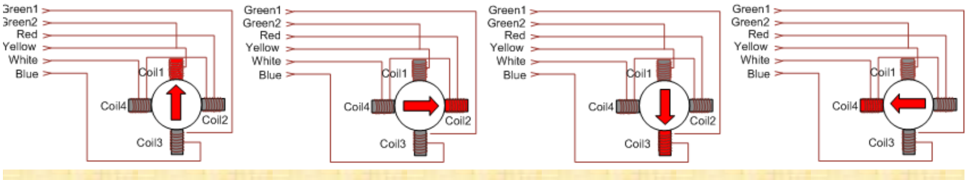
How the stator is powered?

What is the step angle? the rotation direction?

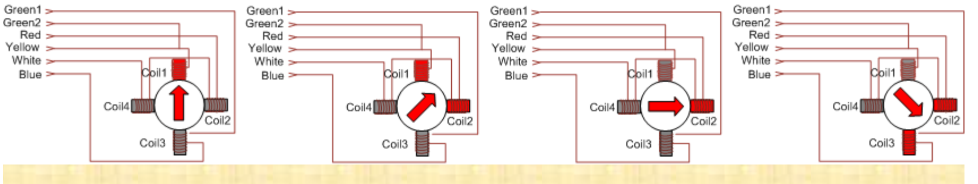


Stepper motors

- Full drive. One phase are powered at a time (see this link or this)

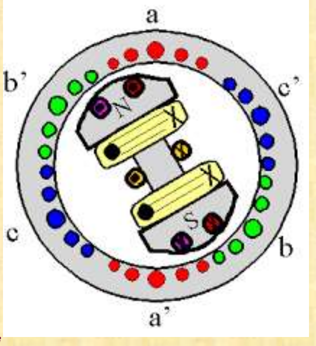


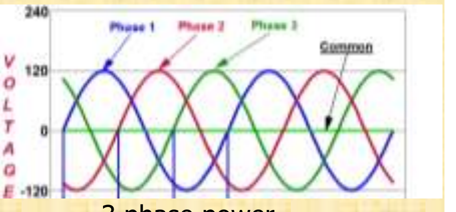
- Half drive. Alternatively one and two phases are powered.
  - **Inscresed resolution**



AC Synchronous Motor

- Rotor
  - Includes permanent magnets (no commutator needed)
- Stator
  - Powered by an external AC power supply.
- Advantages
  - Long life, little or no maintenance, high efficiency

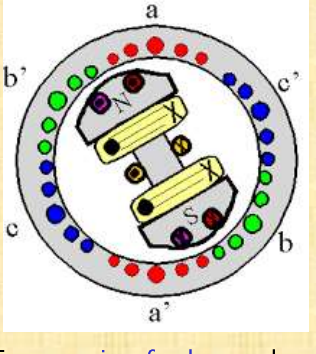


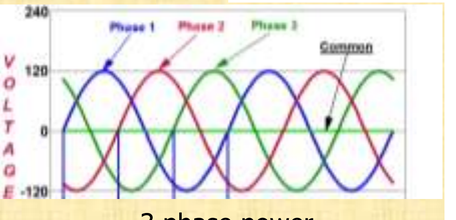


3 phase power

AC Synchronous Motor

- Synchronism
  - Mechanical speed is equal to frequency of the supply power





3 phase power

  - For a p pairs of poles synchrounous drive: $\omega_m = \frac{\omega_e}{p}$

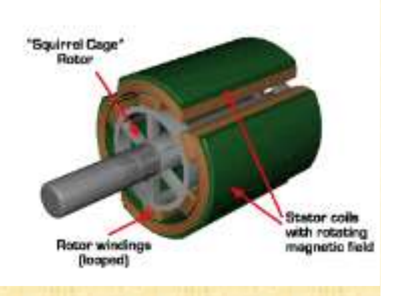
ω_m: mechanical speed; ω_e: electrical speed in Hz, rpm or rad/s (50Hz, 60Hz, …)

  - To vary the rotational speed we just vary the electrical speed

AC Asynchronous (Induction) Motor

- Rotor
  - Wound or squirrel-cage rotor (no commutator needed, no permanent magnet)
- Stator
  - Powered by an external AC power supply.
- Principle
  - Rotor electric current needed to produce torque is obtained by electromagnetic induction from the magnetic field of the stator winding.
- Advantages
  - Long life, little or no maintenance, cheaper
- Overcome
  - There is a slip between mechanical speed and electrical speed
  - The mechanical speed is slightly slower:

$$\omega_m = (1 - s)\frac{\omega_e}{p}$$

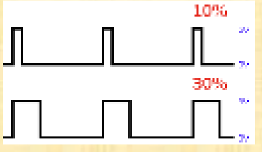


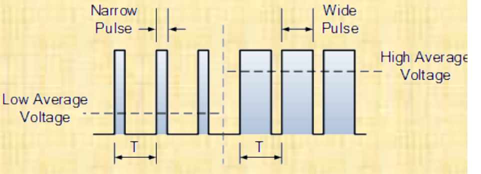
Powering electrical Motors

PWM (Pulse Wide Modulation)

Power Supply - PWM

- Voltage Control for DC drives
  - No necessary to use energy wasting rheostats to provide a variable voltage
  - Pulse Width Modulation (PWM) for DC voltages:
    - Switching regulators provide a variable DC voltage from a fixed DC supply.
    - The DC supply is switched on and off at high frequency using MOSFETs, IGBTs, … The average level of the output voltage can be controlled by varying the duty cycle





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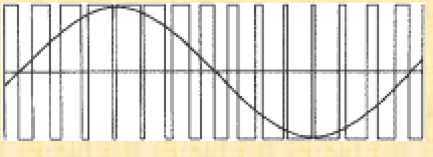
Power Supply - PWM

- Voltage Control for AC drives
  - Pulse Width Modulation (PWM) for AC voltages:
    - Similarly controlled using bi-directional pulses to have a sinusoidal waves.
    - By varying the pulse width, the amplitude of the sine wave can be changed.



Two levels: 0 , Vcc

Three levels: –Vcc , 0 , Vcc

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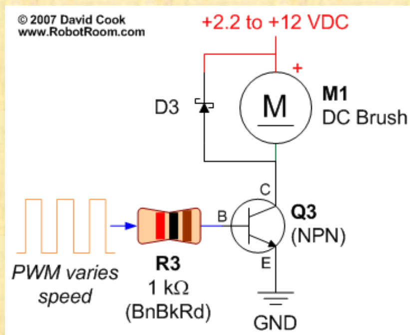
Power Supply - PWM

- PWM for controlling a DC motor speed
  - Using a transistor

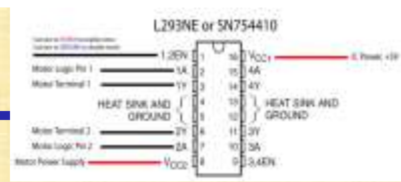


Why the diode D3?

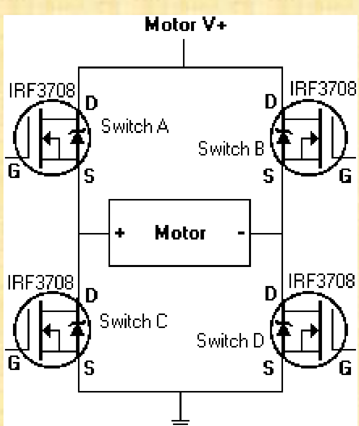
Is it possible to change the direction of the DC motor?

Source: http://www.robotroom.com/PWM5.html

Power Supply - PWM



- PWM for controlling a DC motor speed
  - H-Bridge

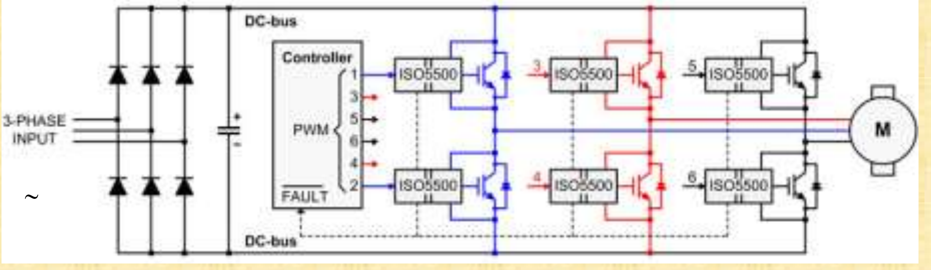


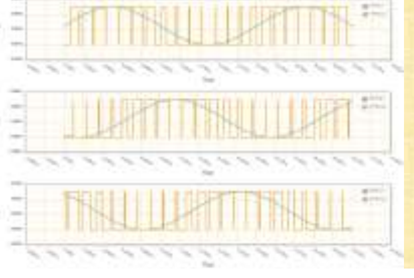
Commutation states

A	B	C	D	
1	0	0	1	Motor moves right (Sens positif)
0	1	1	0	Motor moves left (Sens opposé)
0	0	0	0	Motor free runs (Mouvement libre)
1	1	0	0	Motor brakes (Frein)
0	0	1	1	Motor brakes (Frein)
1	0	1	0	Shoot-through (Court-circuit)
0	1	0	1	Shoot-through (Court-circuit)
1	1	1	1	Shoot-through (Court-circuit)

Power Supply - PWM

- Controlling 3-phase AC drives





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Modeling

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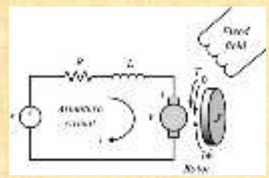
Modeling actuators / sensors

- Example of a DC motor
  - With **a simple gain**

$$\omega = K\, u$$

u in volts and ω in rpm. DC motor is supposed to act faster than the plant

  - With **a second order model** (classical linear model)



$$\begin{cases} L\frac{di}{dt} = u - Ri - K\omega \\ J\frac{d\omega}{dt} = Ki - \tau_L - b\omega \end{cases}$$

  - With **a first order model** (by neglecting inductance)

$$J\frac{d\omega}{dt} = -\frac{K^2}{R}\omega - \tau_L - b\omega + \frac{K}{R}u$$

  - With a complex model

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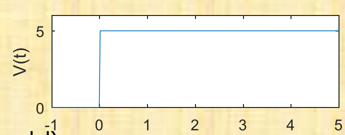
Modeling actuators / sensors

- Example of a DC motor:
  - With **a simple gain** (simplified model)

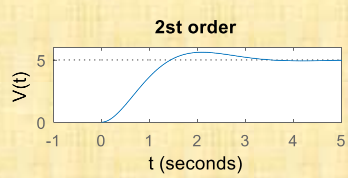
$$\omega = K\, u$$



  - With **a second order model** (classical linear model)

$$\begin{cases} L\frac{di}{dt} = u - Ri - K\omega \\ J\frac{d\omega}{dt} = Ki - \tau_L - b\omega \end{cases}$$

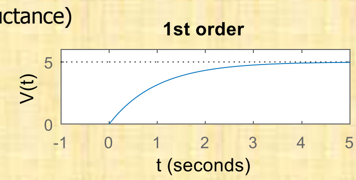
$$P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2}$$



  - With **a first order model** (by neglecting inductance)

$$J\frac{d\omega}{dt} = -\frac{K^2}{R}\omega - \tau_L - b\omega + \frac{K}{R}u$$

$$P(s) = \frac{\omega}{U} = \frac{cst}{1+\tau\, s}$$ with τ time constant



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- Questions?

Do not hesitate to let me know
if you find any errors!!!