Lab 3 – Inverted pendulum with a mobile base

Consider an inverted pendulum with a mobile base described by the equations (1.4) (1.5) given in the link:

$$(M+m)\ddot{x} - ml\ddot{\theta}cos(\theta) + ml\dot{\theta}^2 sin(\theta) = F$$

$$ml\ddot{\theta} - m\ddot{x}cos(\theta) = mgsin(\theta)$$

1. Adding a torque T to the inverted pendulum, show the system can be described by the following equations:

$$x_{dd} = \frac{-ml{\theta_d}^2 sin(\theta) + mg\, cos(\theta)sin(\theta) + Tcos(\theta)/l + F}{(M+m-mcos(\theta)^2)}$$

$$\theta_{dd} = \frac{(M+m)gl\, sin(\theta) - ml^2{\theta_d}^2 cos(\theta)sin(\theta) + (M+m)T/m + lcos(\theta)F}{(M+m-m\, cos(\theta)^2)\, l^2}$$

where $\theta_d = \dot{\theta}$, $\theta_{dd} = \ddot{\theta}$, $x_{dd} = \ddot{x}$

2. Choose a state vector X and give the state space representation of the nonlinear system.

3. Simulate the nonlinear system with Simulink.

4. Give all the equilibrium positions (solve $dX/dt = 0$) for input $T = 0$ and $F = 0$.

5. Give a linearized model around the vertical equilibrium points.

6. Study the stability of the system around each of these positions. Check the stability under Simulink for different initial conditions.

From now on, let $T = 0$.

7. Design pole placement controller to bring the system from any position ($\theta = 0$, $x \neq 0$) to the origin ($\theta = 0$, $x = 0$). Use **place** or **acker** to compute the control gain.

8. Simulate the controlled system (plant + pole placement control). Test the controller for different initial conditions (near and far from the equilibrium). Comment.

Bonus:

9. *Design a controller that bring the system from the origin ($\theta = 0$, $x = 0$) to any position ($\theta = 0$, $x \neq 0$).

10. *Simulate and test the controlled system for different initial conditions (near and far from the equilibrium). Comment.

11. *Design a controller that let the system move from the origin to a constant speed ($\dot{x} = 0$).

12. *Test the controller in simulation.