Outline

- Project in brief — 2 bases for same V.S., trying to prove that the change of basis matrix is upper Δ.
- Young tableaux
- Band diagrams
- Rank
- Shadows
- shadow containment
- ex:

1	2	3	4	5	14
6	7	8	9	10	15
11	12	13	16	17	18

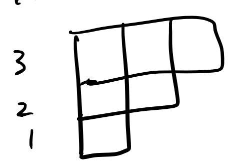
1	2	3	4	5	9
6	7	8	13	14	15
10	11	12	16	17	18

- $n \to$ partition into sum of natural numbers.

  $\lambda \vdash n$ ex: $(3,2,1) \vdash 6$ b/c $3+2+1=6$.

- given $\lambda \vdash n$, a Young diagram of shape λ is a top and left justified arr. of boxes where the i^{th} row has # of boxes equal to the i^{th} element of λ.

  ex: $\lambda = (3,2,1) \vdash 6$



- A Young tableau of shape λ is a filling of the Young diagram w/ #'s $\{1,\dots,n\}$.

ex:

1	3	5
4	2	
6		

not standard

ex:

1	2	4
3	5	
6		

standard

standard means inc $L \to R$ and $T \to B$.

$SYT(\lambda)$ = set of all stds of shape λ.

ex: $(2,2,2) \mapsto 6$.

1	2
3	4
5	6

1	3
2	4
5	6

1	4
2	5
3	6

1	3
2	5
4	6

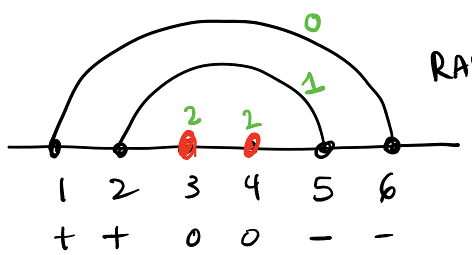
1	2
3	5
4	6

Interested in $SYT(n,n,n)$.

From a $T \in SYT(n,n,n)$, we can construct a band diagram.

+

1	2
3	4
5	6



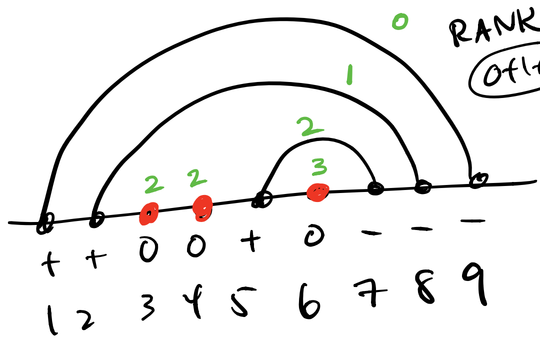
nesting dots

$\text{RANK} = (0+1) + (1+1)$
$= 3$

Begin w/ leftmost unpaired $-$ and connect to nearest unpaired $+$ to the left.

Repeat until all $+$ and $-$ are paired

1	2	5
3	4	6
7	8	9



$\text{RANK} = (0+1+2) + (1+1+2) = 7$

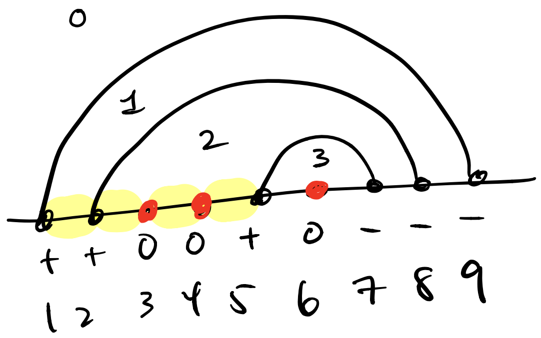
Statistics on band diagrams : rank + shadow.

Rank = sum of nesting number of each arc
+ sum of (depth − 1) for each dot
(or 0)

Shadow of a band diagram is a nested sequence of intervals

$S_d(B)$ = closure of the set of all points of depth d or greater along boundary.

$B =$



$$S(B) = S_1(B) \supseteq S_2(B) \supseteq S_3(B).$$

$$S_1(B) = [1,9]$$
$$S_2(B) = [2,8]$$
$$S_3(B) = [5,7]$$

We will say $S(B) \subseteq S(B')$ if

$S_d(B) \subseteq S_d(B')$ $\forall d$.

Looking for: Examples of band diagrams B, B' on same # points w/

- $S(B) \subseteq S(B')$ AND
- $r(B) > r(B')$

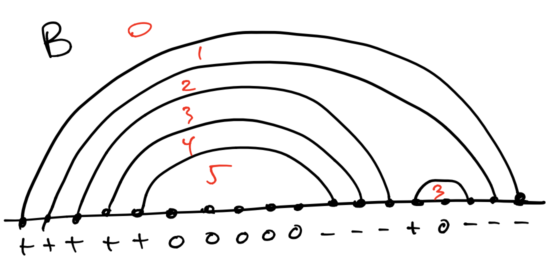
What is the smallest example?

• ex:

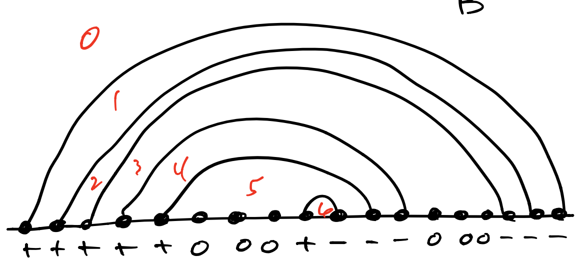
1	2	3	4	5	14
6	7	8	9	10	15
11	12	13	16	17	18

1	2	3	4	5	9
6	7	8	13	14	15
10	11	12	16	17	18

B



B'



	$r(B)=34$	$r(B')=33.$
S_1	$[1,18]$	$[1,18]$
S_2	$[2,17]$	$[2,17]$
S_3	$[3,13]\cup[14,16]$	$[3,16]$
S_4	$[4,12]$	$[4,12]$
S_5	$[5,11]$	$[5,11]$
S_6	$\emptyset$	$[9,10]$

$$S(B) \subseteq S(B')$$

$$\#SYT(\lambda \vdash n) = \frac{n!}{\text{prod of hook lengths}}$$

7	4	3	1
5	2	1	
2			
1			

$$SYT = \frac{9!}{7\cdot4\cdot3\cdot1\cdot5\cdot2\cdot1\cdot2\cdot1}$$

n+2	n+1	n	n-1	3	n
				2	n
				1	n

$$\frac{(3n)!}{}$$