Problem Set 1

1. *Show, by induction, the Bernoulli inequality:* $x > -1 \implies (1+x)^n \geq 1 + nx \ \forall n \in \mathbb{N}$

2. *Show, by contradiction, that the set of prime numbers is infinite.*

3. *Show the supremum of a set of real numbers is unique.*

4. *Let A and B be non-empty real-valued sets bounded above. Let $C = \{a + b, a \in A, b \in B\}$. Show* $\sup C = \sup A + \sup B$

5. *Given a real sequence (a_j), define*

$$b_n = \sum_{j=1}^{n} a_j \qquad c_n = \sum_{j=1}^{n} |a_j|$$

Show (b_m) converges if (c_m) converges. Give an example of (a_j) to show the converse may not hold.

6. *Show if (x_m) is a bounded and monotonic real sequence then (x_m) converges.*