

Problem Set 1

1. Show, by induction, the Bernoulli inequality: If $x > -1 \implies (1+x)^n \geq 1+nx \forall n \in \mathbb{N}$
2. Show, by contradiction, that the set of prime numbers is infinite.
3. Show the supremum of a set of real numbers is unique.
4. Let A and B be non-empty real-valued sets bounded above. Let $C = \{a+b, a \in A, b \in B\}$. Show $\sup C = \sup A + \sup B$
5. Given a real sequence (a_j) , define

$$b_n = \sum_{j=1}^n a_j \quad c_n = \sum_{j=1}^n |a_j|$$

Show (c_n) converges implies (b_n) converges. Give an example of (a_j) to show the converse may not hold.

6. Show if (x_m) is a bounded and monotonic real sequence then (x_m) converges.