Shock Wave Oscillation Driven by Turbulent Boundary-Layer Fluctuations

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Pressure fluctuations due to the interaction of a shock wave with a turbulent boundary layer are investigated. A simple model is proposed in which the shock wave is convected from its mean position by velocity fluctuations in the turbulent boundary layer. Displacement of the shock is assumed limited by a linear restoring mechanism. Predictions of peak rms pressure fluctuation and spectral density are in excellent agreement with available experimental data.

Nomenclature

a = speed of sound
D = diameter of protuberance; diameter of axisymmetric model
f = frequency
h = height of step or protuberance
M = Mach number
p = fluctuating pressure
P = mean static pressure
q = dynamic pressure
R = correlation function
t = time
u = velocity
x = displacement of shock wave from mean location
$\bar{x}$ = streamwise coordinate
β = see Eq. (2)
δ = boundary-layer thickness
ϵ = rms turbulent Mach number
$\mu(t)$ = random function representing turbulent velocity in boundary layer
ξ = variable of integration
ρ = density
τ = time separation
τ_μ, τ_x = integral scale; $\tau_i = \int_0^\infty R_i(\tau)\mathrm{d}\tau$
ϕ = spectral density
$\langle\ \rangle$ = ensemble average

Subscripts

0 = condition just ahead of shock wave
T = turbulent quantity
∞ = freestream condition

I. Introduction

An important source of surface pressure fluctuations on high-speed aerodynamic vehicles is the oscillation of shock waves. Figure 1 shows the static and fluctuating pressure levels associated with supersonic flow ahead of a flare or a three-dimensional proturberance. Other basic flow geometries containing oscillating shock waves exist, such as oscillation of a near normal terminal shock in transonic flow. Reference 1 provides detailed descriptions of fluctuating flowfields, and contains a comprehensive review of available experimental data. The flowfield of Fig. 1 will be briefly

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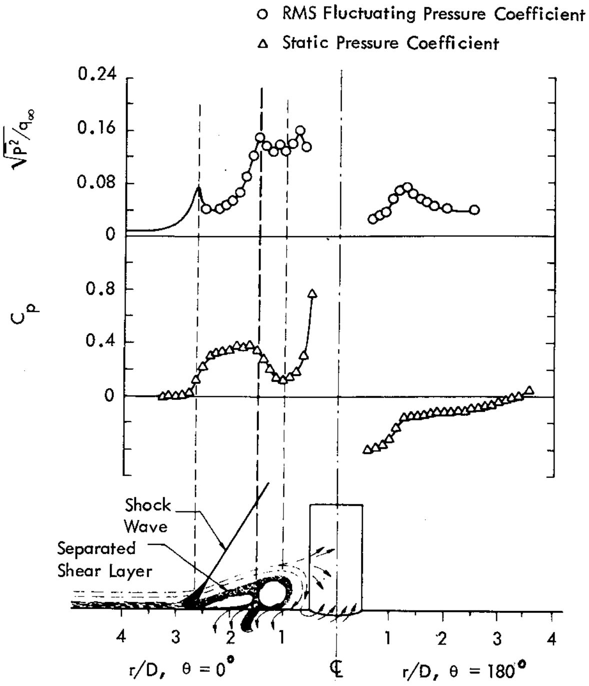

Fig. 1 Composite schematic of protuberance flowfield characteristics, $M_\infty = 1.60$, $h/D = 2.0$, from Ref. 1.

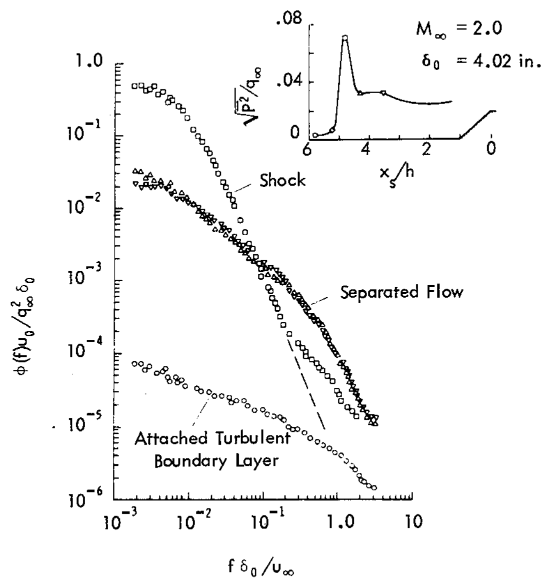

Fig. 2 Longitudinal distribution of pressure fluctuations and typical power spectra in vicinity of supersonic flow separation ahead of a 45° wedge, from Ref. 2.

described here to provide a framework for the shock oscillation model presented in Sec. II.

There are three basic sources of fluctuating pressure in the flowfield shown in Fig. 1: the attached turbulent boundary layer ahead of the shock, the separated region behind the shock, and the oscillation of the shock itself. Figure 2 shows spectra of pressure fluctuations, at the wall, in these three parts of the flowfield ahead of a 45° wedge.[2] Robertson[1] has pointed out that the spectrum at the shock location represents the spectrum associated with an oscillating shock wave alone, plus some fraction of the attached and detached spectra, as these two environments alternately appear at the mean shock location as the shock oscillates. Fluctuations from the attached boundary layer are much smaller than from separated flow and may be neglected, so the spectrum of fluctuations at the mean shock location is given by shock oscillation and separated flow environments. In Fig. 2, the spectral shape follows the separated flow spectrum for $f\delta_0/u_0>0.2$; the spectrum for shock oscillation alone would follow the dashed extrapolation.

Much of the analytical work on oscillating shock waves has followed the approach used by Trilling[3] for interaction of a shock wave with a laminar boundary layer. In that analysis, a harmonic perturbation to the mean flow was assumed, and it was shown that oscillation would be self sustaining for certain frequencies. This approach is felt to be unsatisfactory to the present problem for two reasons. First, the source of the initial disturbance is not identified. In the experimental investigation of Ref. 4, a spark discharge was used to artificially stimulate oscillation. Second, Trilling's analysis would lead us to expect dominant frequencies in the spectrum. The spectrum of Fig. 2 does not contain any such peaks.

Also related to the present problem is the work by Ribner[5,6] on shock-turbulence interaction. The basic gasdynamic problem of a single sinusoidal shear wave convected through a shock wave was solved, giving the strength of the acoustic signal generated and the modification to the shear wave.[5] This single frequency result was generalized, by Fourier synthesis, to broad-band shear waves representative of turbulence.[6] The major result of this work was the prediction of noise generated by the shock-turbulence interaction. The analysis also gave an expression for the motion of the shock wave convected by the incoming shear wave. This result for the motion was adopted by Lowson[7] into a model to predict the pressure fluctuations associated with an oscillating shock wave. While predicted pressure fluctuations agreed reasonably well with data, Lowson's model is felt to be less than satisfactory for two reasons. First, it does not provide for predicting the spectrum of the shock motion. Second, and more significant, it does not incorporate the random nature of the turbulent convection, nor the mean flow properties. Ribner's basic expression for the motion of the shock[5] is for a sinusoidal forcing function; the net amplitude of the motion is directly related to the turbulence intensity. Extension by Fourier synthesis to general turbulent spectra leads to a deterministic shock excursion dependent only on turbulence parameters. Further, since the very existence of a shock wave at a particular location is related to the mean flow conditions, it is not reasonable to expect perturbation of the shock from this position to be independent of the mean flow. For an analysis of the turbulent and acoustic properties into the separated region, Ribner's work is an excellent foundation; for consideration of the shock motion, the random nature of turbulence and the existence of mean flow stability must be accounted for.

The following model is therefore presented in the present paper. The mean location of the shock is a stable position, governed by mean flow conditions. (If this were not so, the shock would not be there.) As each turbulent eddy passes, the shock is convected upstream or downstream. The stability of the mean shock location causes a limit to the excursion distance. In Sec. II, an equation of motion for shock location is postulated, based on convective displacement and a linear restoring mechanism. Statistical properties of the shock motion are found from which mean-square pressure fluctuations and spectra are calculated.

II. Analysis

A. Equation of Motion of the Shock Wave

Velocity fluctuations in the x-direction (parallel to the wall) at a fixed point within the boundary layer may be represented as

$$u_{T_x}=a_\infty\,\epsilon\,\mu(t) \tag{1}$$

where $\mu(t)$ is a random function of time with $\langle\mu^2\rangle=1$, and ϵ is the turbulent Mach number $\langle u_{T_x}{}^2\rangle^{1/2}/a_\infty$. A one-dimensional model is adopted, neglecting velocity fluctuations in other directions. The shock wave within the boundary layer is then convected in the x-direction with speed u_{T_x}. If turbulent temperature fluctuations are significant, u_{T_x} may be considered to be the sum of turbulent velocity and sound speed fluctuations; the present analysis applies with no loss of generality. At the transonic and low supersonic speeds of interest here, however, temperature fluctuations are not important. This will be shown later.

After the shock wave is displaced a distance x (x small compared to scale length of the flow geometry), the flow geometry will be disturbed by an amount proportional to x. This will be either as a displacement of the entire shock, or as a rotation of the shock due to displacement of its foot. In either case, the pressure jump across the shock will be perturbed by an amount proportional to x. This is obvious for a rotation, where pressure jump is a function of shock angle so that linearization of the oblique shock relations gives a change in pressure proportional to change in shock angle. If the entire shock is displaced, then the flow behind it must be displaced relative to the fixed vehicle geometry, leading to a proportional change in flow angles. This again would result in a change to shock pressure jump proportional to x.

If the pressure jump across a shock changes by a small amount, the shock speed changes by a proportionate amount. Thus, the shock will then move with velocity proportional to x. Since the shock will move back toward its original position, this restoring velocity is given by:

$$u_{\text{restoring}}=-\beta x \tag{2}$$

where β is a constant depending on the flow geometry. This constant will be deduced as the analysis proceeds. Note that it is the velocity of the shock, not its acceleration, that is proportional to displacement. This is a departure from the familiar concepts of a spring-mass oscillation; of course, a shock wave does not have mass and a fluid flow is not a spring.

The net speed of the shock wave is the sum of Eqs. (1) and (2):

$$u=a_\infty\epsilon\,\mu(t)-\beta x \tag{3}$$

The equation for the shock location, x, is thus

$$\mathrm{d}x/\mathrm{d}t+\beta x=\epsilon\, a_\infty\mu(t) \tag{4}$$

Equation (4) may be integrated to give

$$x=e^{-\beta t}\epsilon\, a_\infty\int_0^t\mu(\xi)e^{\beta\xi}\mathrm{d}\xi \tag{5}$$

In Sec. IIA-D, various statistical properties of x are calculated, leading to predictions of the intensity and spectral distribution of pressure fluctuations.

B. Mean Square Shock Displacement

Squaring Eq. (5) and taking the ensemble average,

$$\langle x^2\rangle = \epsilon^2 a_\infty^2 e^{-2\beta}\int_0^t\int_0^t \langle \mu(\xi_1)\mu(\xi_2)\rangle e^{\beta(\xi_1+\xi_2)}\mathrm{d}\xi_1\mathrm{d}\xi_2 \quad (6)$$

It is straightforward to obtain the following equation for $t \gg \tau_\mu$:

$$\langle x^2\rangle = \epsilon^2 a_\infty^2(\tau_\mu/\beta)(1-e^{-\beta t}) \quad (7)$$

where:

$$\tau_\mu = \int_0^\infty R_\mu(\tau)\mathrm{d}\tau,\ R_\mu = \frac{\langle \mu_1\mu_2\rangle}{\langle \mu^2\rangle}$$

It is assumed that $x=0$ at $t=0$.

The calculation leading to Eq. (7), for the displacement of the shock wave, is identical to the one given in Ref. 8 for the velocity of a particle undergoing Brownian motion with linear damping. The mathematical analogy is useful in interpreting the present result. Details of the calculation may be found in Sec. III, where the analysis is extended to obtain the correlation function. For large time, $t \gg 1/\beta$, $\langle x^2\rangle$ reaches a finite limit:

$$\langle x^2\rangle = \epsilon^2 a_\infty^2(\tau_\mu/\beta) \quad (8)$$

Because of this constant limit, x for large time may be treated as a stationary random function.

It is interesting to compare Eq. (8) with the well-known result for $\beta=0$, corresponding to an unrestricted random walk:

$$\langle x^2\rangle_{\beta=0} = 2\,\epsilon^2 a_\infty^2\tau_\mu t \quad (9)$$

Without the restoring mechanism, displacement continues to grow with time. The asymptotic value given by Eq. (8) is also given by Eq. (9) when $t=1/(2\beta)$. If the restoring mechanism is thought of as limiting the time during which as unrestricted random walk takes place, then it is clear that β must be related to the integral time scale of the x motion. The integral time scale is found from the correlation function, which will now be calculated.

C. Correlation Function and Integral Scale

The correlation function $R_x(\tau)$ is defined as:

$$R_x(t_1,t_2) = \frac{\langle x(t_1)x(t_2)\rangle}{\langle x^2\rangle} \quad (10)$$

x is assumed to homogeneous and isotropic, so that

$$R_x(t_1,t_2) = R_x(t_1-t_2) = R_x(t_2-t_1) = R_x(\tau) \quad (11)$$

Working from Eq. (5), denoting $\langle x(t_1)x(t_2)\rangle$ as $\langle xx\rangle$, and taking $t_1=t$ and $t_2=t+\tau$,

$$\langle xx\rangle = e^{-2\beta t}e^{-\beta\tau}\epsilon^2 a_\infty^2\int_0^t \mathrm{d}\xi_1\int_0^{t+\tau}\mathrm{d}\xi_2 R_\mu(\xi_2-\xi_1)e^{\beta(\xi_1+\xi_2)} \quad (12)$$

The form of Eq. (12) suggests the obvious change of variable to center of mass and relative coordinates:

$$\xi_0 = (\xi_1+\xi_2)/2,\ \xi = \xi_2-\xi_1 \quad (13)$$

The Jacobian of this transformation equals one, so that $\mathrm{d}\xi_0\,\mathrm{d}\xi = \mathrm{d}\xi_1\,\mathrm{d}\xi_2$.

The range of ξ is from $-t$ to $t+\tau$. If the ξ_0 integration is performed first, this range gives the integration limits for ξ, and the limits for ξ_0 become a function of ξ as well as t and τ. The limits on ξ are:

$$\begin{aligned}&\xi\le\tau, \quad \tfrac{1}{2}|\xi| \le \xi_0 \le t+\tfrac{1}{2}\xi\\ &\xi\ge\tau, \quad \tfrac{1}{2}|\xi| \le \xi_0 \le t+\tau-\tfrac{1}{2}\xi\end{aligned} \quad (14)$$

Using these new variables in Eq. (12), the ξ_0 integration may be performed immediately, giving after some algebra:

$$\begin{aligned}\langle xx\rangle = (\epsilon^2 a_\infty^2/2\beta)e^{-\beta\tau}\Big[&\int_{-t}^{\tau} e^{\beta\xi}R_\mu(\xi)\,\mathrm{d}\xi\\ &+e^{2\beta\tau}\int_\tau^{t+\tau} e^{-\beta\xi}R_\mu(\xi)\,\mathrm{d}\xi\\ &- e^{-2\beta t}\int_{-t}^{t+\tau} e^{\beta|\xi|}R_\mu(\xi)\,\mathrm{d}\xi\Big]\end{aligned} \quad (15)$$

Now, $R_\mu(\xi)\to 0$ for $\xi \gg \tau_\mu = \int_0^\infty R(\xi)\,\mathrm{d}\xi$. (Note that ξ must simply be larger than several τ_μ for this condition to be satisfied.) If it is assumed that $R_\mu \to 0$ quickly enough so that $e^{\beta\xi}R_\mu(\xi)\to 0$, then for $t \gg \tau_\mu$ the limits $-t$ and $t+\tau$ may be replaced by $-\infty$ and $+\infty$. If $1/\beta \gg \tau_\mu$, then $e^{\beta\xi}$ and $e^{-\beta\xi}$ may be replaced by 1 in the integrals. Making these two approximations, and taking the limit $t\to\infty$ so that the last term in Eq. (5) vanishes,

$$\begin{aligned}\langle x\,x\rangle = (\epsilon^2 a_\infty^2/2\beta)\{&(e^{\beta\tau}+e^{-\beta\tau})\int_0^\infty R_\mu(\xi)\,\mathrm{d}\xi\\ &-(e^{\beta\tau}-e^{-\beta\tau})\int_0^\tau R_\mu(\xi)\,\mathrm{d}\xi\}\end{aligned} \quad (16)$$

For $\tau \gg \tau_\mu$, the upper limit on the second integral may be changed to ∞. It is expected that $1/\beta \gg \tau_\mu$ (and this approximation has been made above), so that τ will be larger than τ_μ over most of the range of the correlation function. This gives $\int_0^\infty R_\mu(\xi)\,\mathrm{d}\xi = \tau_\mu$ for both integrals, so that the result for $\tau \gg \tau_\mu$ is

$$\langle x\,x\rangle = \epsilon^2 a_\infty^2(\tau_\mu/\beta)e^{-\beta\tau} \quad (17)$$

The correlation function is therefore

$$R_x(\tau) = \langle x\,x\rangle/\langle x^2\rangle = e^{-\beta\tau} \quad (18)$$

The integral scale of x fluctuations is thus

$$\tau_x = \int_0^\infty e^{-\beta\tau}\mathrm{d}\tau = 1/\beta \quad (19)$$

This bears out the intuitive notion that β is closely related to the integral time scale of x fluctuations.

The mean square x excursion is given by using this in Eq. (8):

$$\langle x^2\rangle = \epsilon^2 a_\infty^2\tau_\mu\tau^X \quad (20)$$

For $\tau < \tau_\mu$, Eqs. (17-20) are not valid. For $\tau \ll \tau_\mu$, $\beta\tau \ll 1$, so that $e^{\pm\beta t}\approx 1\pm\beta\tau$.

Equation (16) may then be written:

$$\begin{aligned}\langle xx\rangle \approx (\epsilon^2 a_\infty^2/2\beta)\{&2\int_0^\infty R_\mu(\xi)\,\mathrm{d}\xi\\ &-2\beta\tau\int_0^\tau R_\mu(\xi)\,\mathrm{d}\xi\}\end{aligned} \quad (21)$$

For $\tau \ll \tau_\mu$, the second integral in Eq. (21) is approximately τ. Thus, for small τ,

$$\langle xx\rangle \approx (\epsilon^2 a_\infty^2/\beta)\tau_\mu[1-(\beta/\tau_\mu)\tau^2] \quad (22)$$

This will be used later in discussing the predicted spectrum.

D. Pressure Fluctuations

If the pressure due to the shock wave at its mean location is $P(\bar{x})$, then the pressure at time t is given by $P(\bar{x}-x(t))$. The fluctuating pressure from shock oscillation is then

$$p(\bar{x},t)=P(\bar{x}-x(t))-P(\bar{x}) \tag{23}$$

where $\bar{x}$ denotes the streamwise coordinate and $x(t)$ is the excursion. The mean square pressure fluctuation is

$$\langle p^2\rangle=\langle[P(\bar{x}-x(t))-P(\bar{x})]^2\rangle \tag{24}$$

If x is small, $P(\bar{x}-x)$ may be expanded in a Taylor series about $\bar{x}$. Retaining terms through the first order only,

$$\langle p^2\rangle=\langle[x(t)P'(\bar{x})]^2\rangle=[P'(\bar{x})]^2\langle x^2\rangle \tag{25}$$

This is accurate only when $\langle x^2\rangle^{1/2}$ is small compared to shock thickness. If $\langle x^2\rangle^{1/2}$ is not small, Eq. (24) would have to be used. This would require the probability distribution function of x, which has not been calculated.

E. Spectral Density

The spectral density of the mean square pressure fluctuations is defined as the Fourier transform of the correlation function:

$$\phi(f)=4\langle p^2\rangle\int_0^\infty R_p(\tau)\cos 2\pi f\,d\tau \tag{26}$$

where the factor 4 $\langle p^2\rangle$ appears so that $\int_0^\infty\phi(f)\,df=\langle p^2\rangle$. This definition of ϕ is consistent with the notation of Ref. 1. Assuming that $\langle x^2\rangle$ is small enough so that Eq. (25) is valid, $R_p(\tau)=R_x(\tau)$. Using Eq. (18) in Eq. (26), it is straightforward to obtain

$$\phi(f)=4\langle p^2\rangle/\{\beta[1+(2\pi f/\beta)^2]\} \tag{27}$$

Because Eq. (18) is not accurate for $\tau<\tau_\mu$, Eq. (26) is not accurate for $f>1/(2\pi\tau_\mu)$. For $f\gg 1/(2\pi\tau_\mu)$, $\phi(f)$ is given by the transform of Eq. (22). This behaves asymptotically as f^{-3}, compared to f^{-2} for Eq. (27). Equation (27), then, will tend to be too large at high frequencies $f>1/(2\pi\tau_\mu)$.

III. Prediction of Pressure Fluctuations ahead of a 45° Wedge

The fluctuating pressure intensity and spectra are given by Eqs. (25) and (27), with Eq. (8) giving $\langle x^2\rangle$ and Eq. (19) relating to τ_x to β. Given mean flow conditions, the parameters required are the boundary layer turbulent intensity ϵ and integral scale τ_μ, and the integral scale τ_x of the shock motion.

A. Flow Parameters

Figure 3 gives the mean pressure coefficient and rms fluctuating pressure for several test models in Ref. 2. The 8 in., 45° wedge is chosen for this sample calculation because of the simple two-dimensional geometry. Measuring the pressure gradient from Fig. 3,

$$P'(0)=0.42\ (q_\infty/h)=0.21\ (q_\infty/\delta_0) \tag{28}$$

The other flow parameters are given in the figure. From Eq. (25), the integral scale is given by:

$$\tau_i=\int_0^\infty R_i(\tau)\ d\tau=\tfrac{1}{4}[\phi_i(0)/\langle p^2\rangle] \tag{29}$$

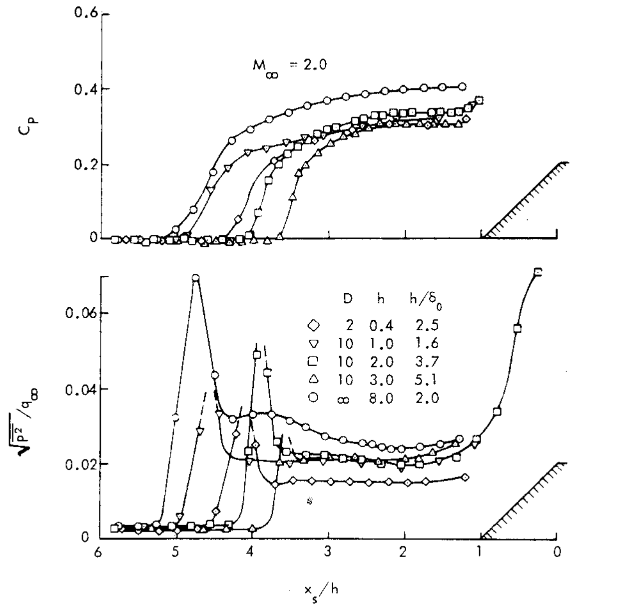

Fig. 3 Longitudinal distributions of steady and fluctuating wall pressures, from Ref. 2.

where i is μ or x. The integral scales are easily found from the zero frequency point in Fig. 2, and are

$$\tau_x=25\ (\delta_0/u_0),\quad \tau_\mu=1.9\ (\delta_0/u_0) \tag{30}$$

From Fig. 3, $\langle p^2\rangle_T^{1/2}/q_\infty=0.003$ for the boundary layer. Turbulent velocity fluctuations are related to pressure fluctuations by[9]

$$\langle p^2\rangle_T^{1/2}=0.7\,\rho\,\langle u^2\rangle=0.7\,\rho\,\epsilon^2a_\infty^2 \tag{31}$$

Taking $\rho=\rho_\infty$

$$\epsilon^2=(M_\infty^2/1.4)\ (\langle p^2\rangle_T^{1/2}/q_\infty) \tag{32}$$

Using $\langle p^2\rangle_T^{1/2}=0.003\,q_\infty$,

$$\epsilon^2=0.00214\ M_\infty^2 \tag{33}$$

This calculation is based on the assumption that density fluctuations may be neglected, so that $\rho=\rho_\infty$. It is not immediately obvious that this is valid at supersonic Mach numbers, and, indeed, at hypersonic speeds density and temperature fluctuations within a boundary layer become large.[10] Large temperature perturbations, however, are generally associated with hypersonic flow, and this is the case for turbulent boundary layers as well. Data taken by Kistler[11] over the range $M_\infty=1.7$ to 4.67 indicate that the maximum temperature fluctuations are approximately $T_T/T=0.14\ (T_o-T_\infty)/(T_0+T_\infty)$, where T_o and T_∞ are stagnation and freestream temperatures, respectively. The hypersonic data of Laderman and Demetriades[10] also fit this formula. At $M_\infty=2$, the expected temperature fluctuation is thus $T_T/T_\infty=0.04$. This leads to a sound speed fluctuation of $a_T/a_\infty=0.02$, which is small compared to $\epsilon=0.092$ from Eq. (33) at $M_\infty=2$. Equation (33) is in excellent agreement with Kistler's velocity data within the stronger part of the boundary layer.

B. Root Mean Square Pressure Fluctuations

Using the preceding values in Eq. (8),

$$\langle x^2\rangle=0.102\ \delta_0^2 \tag{34}$$

so that the rms displacement is

$$\langle x^2\rangle^{1/2}=0.32\ \delta_0 \tag{35}$$

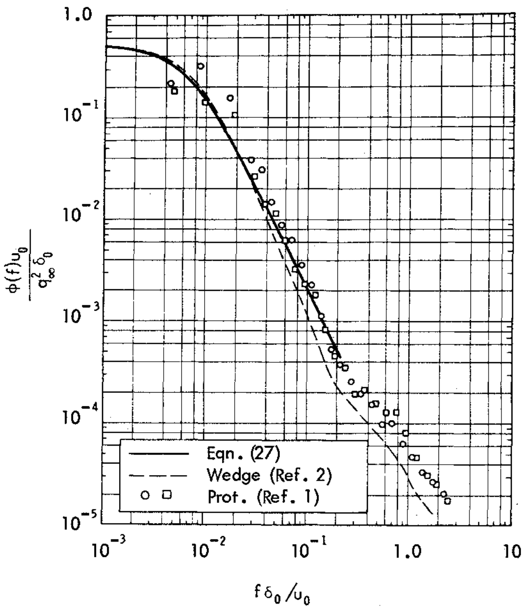

Fig. 4 Comparison of predicted spectrum with measured spectra.

This is smaller than the shock thickness [which is approximately 2 δ_0, based on a jump of $0.4q_\infty$ from Fig. 3 and the value of $P'(0)$ given in Eq. (28)] so that Eq. (25) may be used for the pressure. The fluctuation at the mean shock location, due to motion of the shock, is thus

$$\langle p^2 \rangle^{1/2}/q_\infty = 0.068 \tag{36}$$

This is in excellent agreement with the value of 0.07 in Fig. 3.

C. Spectrum

Figure 4 shows the predicted spectrum, Eq. (27), along with the shock spectrum from Fig. 2 and two spectra measured by Robertson[1] for three-dimensional protuberances. Except for being slightly high at higher frequencies, the agreement is excellent. $f\delta/u = 10^{-1}$ corresponds approximately to $f = 1$ $(2\pi\tau_\mu,)$ where Eq. (27) is not expected to be accurate. The slope of the measured spectra at this point corresponds to $f^{-2.6}$, which is between the f^{-2} behavior of Eq. (26) and the f^{-3} behavior of the high-frequency limit discussed previously. A spectrum calculated from Eq. (16) would show even better agreement with the data. Above $f\delta_0/u_\infty \approx 2 \cdot 10^{-1}$, the measured spectrum is dominated by separated flow fluctuations, as pointed out by Robertson,[1] and the present analysis does not apply.

IV. Conclusions

A simple mechanism has been proposed for the pressure fluctuations near a shock wave interacting with a turbulent boundary layer. The shock wave is convected from its mean location by velocity fluctuations in the turbulent boundary layer, while stability of the mean flow tends to restore the shock wave to its original position. The restoring mechanism is assumed to be linear. The motion of the shock wave, given by Eq. (5), has a mathematical analogy to the velocity of a particle undergoing Brownian motion. The mean-square displacement tends to a constant value at large time, given by Eq. (8), so that the motion may be treated as a stationary random function. The mean-square pressure fluctuation, Eq. (25), and spectral density, Eq. (27), calculated from this shock motion are in excellent agreement with experimental data. Because of the excellent agreement of the present theory with experimental data, and the straightforward physical model employed, it is concluded that turbulent boundary-layer fluctuations are the dominant cause of shock wave oscillation in the case of flare and protuberance-induced separated flow.

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