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Irregular Packing Problems: Industrial Applications and New Directions Using Computational Geometry ⋆

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Abstract: Cutting and Packing problems are hard Combinatorial Optimization problems that naturally arise in all industries and services where raw-materials or space must be divided into smaller non-overlapping items, so that waste is minimized. All the Cutting and Packing problems have in common the existence of a geometric sub-problem, originated by the natural item non-overlapping constraints. An important class of Cutting and Packing problems are the Irregular Packing problems that occur when raw materials have to be cut into items with irregular shapes. Irregular Packing problems, also known as Nesting problems, naturally arises in the garment, footwear, tools manufacturing and shipbuilding industries, among others. Each industrial application has its owns particular issues mainly related to the raw material's specific characteristics.

Several challenges remain open in the Irregular Packing problems field. Some are due to the combinatorial nature of these problems. Others are of geometric nature, due to the non-convex and non-regular geometry of the items involved. Moreover these geometric challenges do not allow the combinatorial ones being properly tackled. This paper is mainly focused on presenting and discussing efficient tools and representations to tackle the geometric layer of nesting algorithms that capture the needs of the real-world applications of Irregular Packing problems.

Keywords: Cutting and Packing, Irregular Packing, Nesting, Geometry.

1. INTRODUCTION

Cutting and Packing (C&P) problems are hard Combinatorial Optimization problems that naturally arise in all industries and services where a given resource, raw material or space, must be divided into smaller non-overlapping items, so that waste is minimized. All C&P problems have in common the existence of a geometric sub-problem, originated by the natural item non-overlapping constraints. In this geometric sub-problem the number of relevant dimensions and the geometry of the items play an important role. Another crucial issue is the type of assignment, maximizing output or minimizing input. In the first case, maximizing output, one wishes to obtain the maximum number (or value) of small items from a limited set of large objects. In the second case, minimizing input, one wishes to minimize the amount of resource needed to produce a given set of small items. These characteristics are synthesized in the C&P typology proposed by Wäscher et al. (2007).

Container Loading, Rectangular Strip and Bin Packing and Circle Packing are classical examples of C&P problems. In the Container Loading problem a set of given boxes need to be packed inside a container or a truck. In the Rectangular Strip and Bin Packing problems a set of rectangles need to be packed, respectively, on a minimum length strip or a minimum number of boards. The Circle Packing problem deals with the packing of a set of circles inside a circular or rectangular container. In the Irregular Packing problem the major difference comes from the more complex items' geometry, which can be irregular (i.e., non-convex and non-regular). These problems arise when a set of irregular items need to be packed, without overlapping, inside a minimum strip or an irregular board. From the previous examples one can derive the main characteristic which distinguish the Irregular Packing problem from the remaining C&P problems: the items' complex and non-regular geometry. This characteristic leads to the necessity of using efficient geometric tools and representations to achieve good quality feasible solutions.

This paper focus on presenting and discussing geometric tools and representations needed to develop efficient nesting algorithms to tackle real-world Irregular Packing problems. This paper is structured in five sections, being this the first one. Section two is devoted to presenting the Irregular Packing problem in detail. Geometric representations are presented in the third section, while the fourth section is devoted geometric tools. Section five introduces the use of graphical processing approaches to the Irregular

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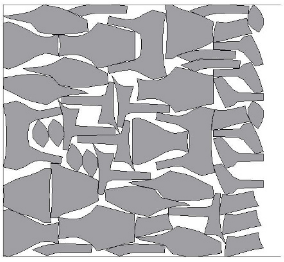

Figure 1. Irregular Packing problem layout example.

packing problem. Finally, the last section discusses industrial applications and presents some concluding remarks.

2. IRREGULAR PACKING PROBLEM

The Irregular Packing problem, also known as Nesting problem, is a hard combinatorial optimization problem where a set of two-dimensional irregular items need to be packed in minimum length strip or inside an irregular board. The irregular items cannot overlap each other and must be entirely placed inside the strip or board. Like in other C&P problems, the solution of a Irregular Packing problem is a cutting pattern or a layout (Fig. 1).

Irregular items are usually defined by just simple polygons (i.e., non-regular polygons), although they may also have internal holes (i.e., multi-connected regions) or even non-straight outlines. Both internal holes and non-straight outlines are removed on a pre-process stage. Internal holes are filled by smaller irregular items and afterwards eliminated. Non-straight outlines are approximated by a set of external edges. An alternative geometric representation is the raster representation, where irregular items are discretized and represented in matrices. These issues will be further discussed on section three.

The irregularity of the items presents a major challenge in developing nesting algorithms, increasing the difficulty of the underlying geometric problem. In fact, given two non-convex polygons and their relative positions, to know if they overlap is a non-trivial time-consuming task that enormously limits the computational effort that can be dedicated to optimizing nesting solutions. A detailed and comprehensive discussion on algorithms and approaches for the Irregular Packing problem is presented in (Bennell and Oliveira, 2009).

The simplest Nesting algorithms are based on constructive heuristic. These heuristics place items one by one, selecting, in each iteration, the next item to be placed (selection heuristics) and where to place it (placement rules). Selection heuristics can be static, where the items' order is previously set by a fixed rule, or dynamic, where in each iteration an item is chosen accordingly to the previous placed items (Oliveira et al., 2000). Placement rules actually place the items in the layout one by one, following a greedy rule, like the bottom-left placement (Dowsland et al., 2002) and the bottom-left-fill placement (Burke et al., 2006). The ability to fill holes among the previous placed items in the layout is one of the most important issues for placement rules. Constructive heuristics are quite fast, being able to deal with large instances.

Improvement approaches, like local search algorithms and metaheuristics frameworks, are another type of Nesting algorithms. In these approaches a better solution is searched in a neighborhood defined by a one or movements. In pure local search algorithms only better solutions are accepted (Gomes and Oliveira, 2002), which usually leads to local optima solutions. To overcome this limitation, metaheuristics approaches can accept poorer solutions. The balance between intensification, the search for a better solution in the current neighborhood (i.e, achieving local optima solutions), and diversification, moving to a different neighborhood in the solution space (i.e., escaping from local optima solutions), plays an important role in the design and implementation of any metaheuristic. Egeblad et al. (2007) propose a very effective approach to the Irregular Packing problem, where a fast neighborhood search algorithm based on a guided local search metaheuristic.

A recent trend in the combinatorial optimization field is the hybridization between metaheuristics and mathematical programming models (Maniezzo et al., 2010). These approaches, named Matheuristics, try to incorporate mathematical programming techniques into metaheuristic frameworks, metaheuristic concepts inside mathematical programming solvers or use both approaches cooperatively. Examples of nesting matheuristics are the hybridization between simulated annealing and linear programming models proposed by Gomes and Oliveira (2006) and the iterated local search based on nonlinear programming presented by Imamichi et al. (2009).

A final word about mathematical models for Irregular Packing problems. Until recently only very small toy instances (up to 7 items) where solved optimally using mixed integer programing models. However, due to the continuously increasing in the available computational power and the improvements in modern solvers a new generation of mathematical models for Irregular Packing problems is starting to appear. Martinez et al. (2012) uses a branch and bound algorithm that solves a mixed-integer programming model based on the horizontal slices formulation (H-Slice), being able to solve optimally instances up to 16 items. Toledo et al. (2013) propose a flexible mixed-integer programming model based on a discretization of the board. This approach takes advantage not only of the board discretization, but also of the existence of repeated items, to solve instances up to 56 items.

3. GEOMETRIC REPRESENTATIONS

An important issue that needs to be tackled by any nesting algorithm is the ability to deal with the geometric problem raised by the non-regularity and non-convexity of the irregular items. In the heart of this geometric problem is the necessity to achieve feasible placements for all irregular items, which means that items have to be checked against each other for overlaps and must be placed inside a strip or an irregular board. This necessity leads to the necessity of developing efficient and robust geometric tools and representations. Bennell and Oliveira (2008) present a tutorial on the geometric issues of irregular packing problems. This section and the next one are mainly based on that work, updating it when necessary.

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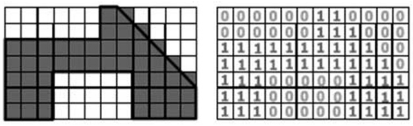
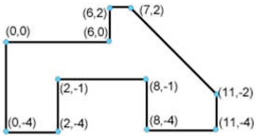


(a) Raster representation.



(b) Polygonal representation.

Figure 2. Geometric representations (Bennell and Oliveira, 2008).

There are two basic distinct ways of representing the geometry of nesting problems: a raster representation, where a grid of pixels is used to represent irregular items, or a polygonal representation, which saves only the polygons' vertices. Both approaches have advantages and disadvantages.

In a raster representation, the irregular items are represented on a matrix of pixels whose values differ according to what is present on that spot (figure 2a), it can be empty space or it can be an item or overlapped items; in some representations the contours of the items are also represented by distinct pixel values. Analyzing the matrix of pixels, one can easily detect the best feasible placements and overlapping situations, without requiring complex calculations. However, a grid representation always means a loss of precision than can be more or less significant depending on the matrix resolution. Another important issue of the raster representation is its flexibility and independence of the actual geometry of the irregular items

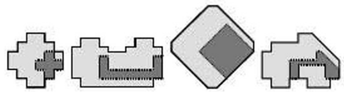
A polygonal representation only stores the polygon's vertices sequentially (figure 2b), which makes it much more economical in terms of memory. However, identifying feasible placements in a polygonal representation, due to the non-convex and non-regular items' geometry, requires complex and repetitive trigonometric calculations. The calculations time depends on the geometric complexity of the polygons. On the other hand, this representation does not limit the precision.

4. GEOMETRIC TOOLS

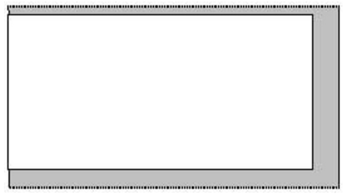
There are some geometric tools that simplify the placement of irregular items and allow the development of efficient nesting algorithms based on polygonal representations. One of those is the No-Fit Polygon (NFP), which prevents the item currently being placed from overlapping the already placed items. Another one is the Inner-Fit-Polygon (IFP) that prevents the item currently being placed from exceeding the boundaries.

The NFP of items A and B (NFP_{AB}) delimits the region of B's placements that will make the two items overlap. Therefore, B must be placed outside NFP_{AB}. NFP_{AB} is the polygon drawn by B's reference point when B slides around A in a way that the two items always touch but never overlap. Hence, if B's reference point is placed on the edge of NFP_{AB} the items will touch but the placement is feasible. However, if B's reference point is placed inside NFP_{AB}, the two items will overlap, which means that the placement is not feasible. Every placement outside of NFP_{AB} is feasible.

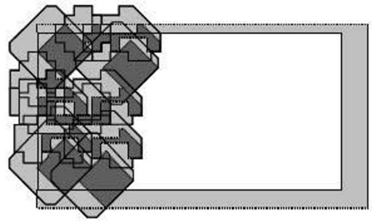
(a) Item to be placed (i).



(b) NFP's of item (i) with the other items.



(c) IFP of that item with the strip.



(d) Feasible placements for the item on the current layout.

Figure 3. Feasible placements of an item (Gomes and Oliveira, 2002).

The IFP of a surface A and an item B (IFP_{AB}) delimits the region of B's placements where B does not exceed the limits of A. Therefore, B must be placed inside IFP_{AB}. An IFP is obtained through an analogous process to the NFP's. One places B inside surface A and slides it around in a way that it is always in contact with the A's boundaries, but never crosses them. If B's reference point is placed inside IFP_{AB}, B will be inside A and if B's reference point is on IFP_{AB}'s edges B is touching A's limits. These are both feasible placements. If B's reference point is outside IFP_{AB}, the item is not completely inside the surface and, therefore, the placement is not feasible.

When searching for a feasible placement for an item in a partial layout, Nesting algorithms can use these tools to ensure the feasibility of the layout (i.e., all items inside the strip or board and no overlap between items). Additionally, to obtain compact layouts efficiently, Nesting algorithms should analyze all the IFP and NFP edges and intersections that lie inside the IFP and are not inside of any NFP. Figure 3 illustrates this situation when using a polygonal representation. This idea can also be used in a raster representation by discretizing all NFPs and IFPs in matrices. To obtain the feasible the feasible placements, the NFPs between the next item to place and items already placed should be added to the IFP of the next item to place. An example of using NFPs and IFPs with a raster representation is shown in section five.

Additionally, NFPs between all pairs of items and IFPs for all items can de calculated in a pre-processing stage. This should be done for all admissible orientations. However, obtaining a robust NFP generator for general non-convex items is non-trivial. An alternative is to decompose non-convex items in convex ones, since computing the NFP between two convex items is a trivial operation. It should be noted that obtaining the complete NFP from the partial NFPs between convex sub-items is a non-trivial operation due to degenerated cases. Bennell and Oliveira (2008) A detailed discussion on algorithms to compute NFP between two irregular items.

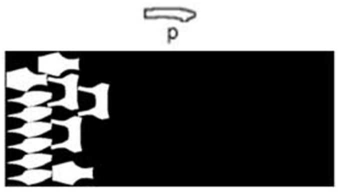
Another powerful geometrical tool, proposed by Stoyan et al. (2002), is the Φ-function, which in fact is a generalization of the NFP. Φ-functions are mathematical expressions that represent the relative positions of two objects. When its value is positive the two items are separated, when it is zero the two items overlap and when it is negative the two items overlap. Normalized Φ-function gives the actual Euclidean distance between two items, while non-nomalized Φ-function gives an estimate of the distance. Bennell et al. (2010) present Φ-functions for a set of primary objects (circles, rectangles, regular polygons, convex polygons and their complement). Items that are not primary objects are represented as a union or intersection of the primary objects.

NFPs/IFPs and Φ-functions can also be used to construct mathematical models for the Irregular Packing problem. The linear programming models used to compact and separate layouts in (Gomes and Oliveira, 2006) are based on NFPs and IFPs. The H-Slice model on (Martinez et al., 2012) is based on partitioning the feasible placement region for two items (i.e., the complement of the NFP) in horizontal slices. Chernov et al. (2010) presents a non-linear mathematical model based on Φ-functions that it is able to rotate items.

5. EXPLORING GRAPHICAL PROCESSING IN IRREGULAR PACKING PROBLEMS

Raster representations were popular in the eighties and early nineties mainly due to the limited computing power available and their simplicity. With the escalade in the computing power, they become less used in favor of polygonal representations. Raster representations are becoming once again attractive due to the development of graphical processing devices (often referred to as GPU – Graphical Processing Unit or General-Purpose Graphics Processing Units). These devices have evolved a lot lately, not only concerning its graphical processing capabilities but also regarding parallel processing.

A natural approach to solve Irregular Packing problems with GPU devices is to transfer the geometry processing to the GPU, using a raster representation of the cutting pattern. The layout is rendered in the graphical device and the image produced processed. An image processing step detects the feasible placements for an item by "drawing" the NFPs and IFP for the next item to be placed. This analysis efficiently identifies the possible placements of an item and is independent from the items' geometry since it is done over over the raster layout. Finally, a placement rule chooses the actual placement. Figure 4 illustrates the using of a GPU device to place one item on a partial layout.



(a) Current layout and next item to place (p).

(b) Contour of NFP/IFP's image of the current layout and the selected bottom-left placement.

(c) Layout with the new item placed.

Figure 4. Placement of an item on a partial layout by the graphical processing approach using a bottom-left placement rule (Sampaio, 2012).

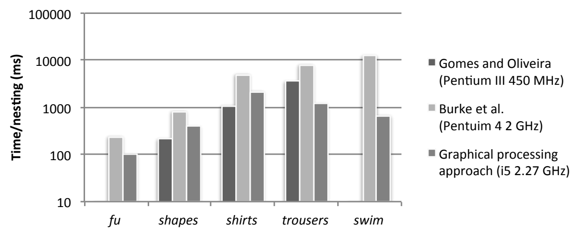


Figure 5. Nesting time comparison (times in ms and using a logarithmic scale).

Sampaio (2012) compares this graphical processing approach against the bottom-left placement heuristic presented in (Gomes and Oliveira, 2002) and the bottom-left-fill placement heuristic presented in (Burke et al., 2006). This comparison uses 5 data sets taken from ESICUP web site – EURO Special Interest Group on Cutting and Packing [1]: *fu*, *shapes*, *shirts*, *trousers*, and *swim* (problems are ordered by increasing items complexity). Results obtained by Sampaio (2012) are presented on figure 5.

These results allow concluding that the time needed to place an item in a partial layout by this graphical processing approach is smaller than traditional approaches and, more importantly, it is almost completely independent from the items' geometric complexity. Additionally, the computational times in the graphical approach does not tend to increase with the number of already placed items. Polygonal representations, even when using NFPs and IFPs, require a lot of repetitive trigonometric calculations to determine the region of feasible placements.

[1] http://www.fe.up.pt/esicup

These calculations take more time if the items are more complex and when the number of already placed items increases, as there are more edges to test. On the graphical processing implementation, these computations are replaced by rendering the partial layout, memory mapping and image processing, processes whose performance is not influenced by the problem's formulation. In fact, rendering complex irregular items and extracting the contours of a complicated region from an image is not significantly slower than in case of simpler items.

Nesting algorithms based on GPU devices can be further enhanced by exploiting the parallel processing capabilities of these devices.

6. INDUSTRIAL APPLICATIONS AND CONCLUDING REMARKS

Like all other C&P problems, the Irregular Packing problem is an applied problem with several industrial applications. Each industrial application has its own specific characteristics that can be used to overcome the complex geometric and combinatorial nature it of the Irregular packing problem, leading to the development of successful nesting algorithms. In the following paragraphs three different industrial applications are presented and discussed.

The variant of the Irregular Packing problem that appears on the garment and apparel industry is clearly the most well studied. This variant is modeled as an Irregular Strip Packing problem, since the raw material (fabric) is available in rolls with fixed width and a long length. Another important characteristic is the existence of drawing patterns and the physical properties of the fabric's weave, which limits the possible orientations just to two: 0^0 or 180^o. A few degrees tilting is usually allowed. This industrial application is well suited for using NFPs due to the discrete nature of the possible orientations. The tilting by a few degrees are usually tackled in a post-processing phase.

The team led by Milenkovic developed a successful application to a specific type of problem in the garment industry: the production of trousers. Li and Milenkovic (1995) present a compaction algorithm able to improve garment layouts made by human-experts with years of experience. (Daniels and Milenkovic, 1996) developed a successful constructive heuristic that builds columns of items.

In the footwear industry natural leather hides have irregular outlines, which leads to a Irregular Bin Packing problem. Additionally, they usually also have defective areas and different quality levels that need to be considered when items' placing. This variant is clearly more complex due not only to the irregular outline of the hides but also to defective areas. Pre-placed items can model defective areas, while different quality areas can be tackled with computing different IFPs. Heistermann and Lengauer (1995) present an greedy constructive heuristic able to compete with layouts generated human-experts.

Another variant of the Irregular Packing problem appear in a high precision tools factory. In this application a set of irregular items need to be placed on a circular plate made tungsten with a thin diamond dust layer. The cutting process is performed with the plate suspended by an "electrified copper string" in continuous way. This means that when an item is completed cut it falls, leading to necessity to cut the items in such a way that they fall one by one and completed cut. This problem was tackled in a two-stage approach. In the first stage, the Irregular Bin Packing problem is tackled by the bottom-left placement heuristic presented in (Gomes and Oliveira, 2002). The second stage deals with the non-trivial cutting path determination and is tackled by a constructive heuristic (Moreira et al., 2007).

The continuous increasing in the computing power available, together with the development of new and more powerful geometrical tools and mathematical models will allow the development of new nesting algorithms. Not only by improving previous results, but also by allowing tackling more complex Irregular Problems. Particularly interesting are the GPUs with the graphical processing and parallel processing capabilities.

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