

# DOCUMENTOS DE TRABAJO

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## **A Macro Financial Model for the Chilean Economy\***

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### **Abstract**

This paper presents a dynamic stochastic general equilibrium (DSGE) model built with a focus on frictional financial intermediation. The model, estimated for the Chilean economy, expands the quantitative analysis toolkit of the Central Bank of Chile, allowing for the study of how financial frictions shape the transmission mechanisms of several macroeconomic and financial shocks. The model builds on a simplified version of the Central Bank of Chile's main DSGE model, described in Garcia et al. (2019), augmented to include a rich financial sector and financial frictions. The extensions include optimizing financial intermediaries, corporate and mortgage lending, long-term government bonds within a segmented bonds market, and the possibility for households, firms, and banks to default. The result is the Central Bank of Chile's Macro Financial Model. The model captures many features of the Chilean economy and allows for a quantitative analysis of the financial system's role in explaining the business cycle and of the interaction between the real and financial sides of the economy.

### **Resumen**

Este artículo presenta un modelo dinámico-estocástico de equilibrio general (DSGE) que incorpora de modo central la intermediación financiera bancaria. Estimado para la economía Chilena, extiende la batería de herramientas para el análisis cuantitativo del Banco Central de Chile, permitiendo el estudio de cómo las fricciones financieras pueden afectar los mecanismos de transmisión de varios shocks macroeconómicos y financieros. Este modelo se basa en una versión simplificada del principal modelo DSGE del Banco Central de Chile, descrito en García et al. (2019), extendido para incluir un sector y fricciones financieras. En particular, se incluyen la presencia de intermediarios financieros optimizadores, créditos corporativos e hipotecarios, bonos soberanos de largo plazo dentro de un mercado segmentado, y la posibilidad de impago para hogares, empresas y bancos, entre otras. El resultado es el modelo macro financiero del Banco Central de Chile, que permite un análisis cuantitativo del rol del sistema financiero en ciclos económicos, y en la interacción entre variables reales y financieras.

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# 1 Introduction

How can financial frictions shape the dynamics of macroeconomic variables over the business cycle? Can the inclusion of a financial sector in macro models change our understanding of the transmission channels of monetary policy?. Up until the financial crisis of 2008-2009, it was not a common practice for central banks to incorporate a rich financial sector or detailed financial frictions into the models used for forecasting and monetary policy analysis. Therefore, many such questions remained untouched by most economic research and played only a peripheral role in macroeconomic policy analysis. However, the importance for central banks of having robust analytical tools to study the interaction between the financial and real sectors of the economy only became evident after the crisis. During this period, these institutions had to rely on unconventional monetary policy. Although there is consensus that these policies had expansionary effects, quantitative impact estimates incorporated high degrees of uncertainty. Moreover, the crisis also made it clear that the financial sector not only has a prominent role in propagating non-financial economic shocks but can also be the source of economic volatility.

The following influx of new questions regarding the economic role of the financial system in explaining the business cycle led to a surge in the development of DSGE models featuring a prominent role for financial frictions and the financial system. As an example, Christiano et al. (2010), building on the canonical framework of Smets and Wouters (2003), models an economy populated with financial intermediaries and financial frictions *à la* Bernanke et al. (1999). Near the same time, Gertler and Karadi (2011) developed a quantitative monetary model with constrained financial intermediaries, used to evaluate the effects of unconventional monetary policy during the financial crisis. In the same avenue, Christiano et al. (2015) using an NK model, argued that most of the real economy movements during the great recession were due to financial frictions interacting with the zero lower bound.

Motivated by the need to better understand the role of the financial system and financial frictions explaining the business cycle, we introduce the Central Bank of Chile's Macro Financial Model. The model is based on a simplified version of the one described in Garcia et al. (2019)<sup>1</sup>, augmented with a frictional financial system following Clerc et al. (2014), long term bonds as in Woodford (2001), a preferred habitat framework for the modeling of the term structure as in Vayanos and Vila (2009) and imperfect asset substitution as in Andres et al. (2004). The model's features are chosen to allow for the inclusion of the most relevant characteristics of the Chilean economy. In particular, Chile is a small open economy with a sizable commodity-exporting sector that plays a prominent role in government revenues. In addition, the Chilean financial system is mainly formed by a highly regulated classic banking sector which is the primary source of financing for firms. Finally, the Chilean economy features a relevant

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<sup>1</sup>Garcia et al. (2019)'s Extended Model for Analysis and Simulations (XMAS) is currently the main DSGE model used at the Central Bank of Chile for forecast and analysis.

role for short-term and long-term financing, both in nominal and real terms.

The Central Bank of Chile is not alone in its quest to introduce a rich financial sector and financial frictions in DSGE models. Other central banks have also included these advances into the set of models used for policy analysis, for example, to understand the effects of shocks that originate in the financial sector and the role of financial frictions in the propagation of shocks. Central banks also use these models to understand the role of the financial market shaping the transmission of monetary policy and to assess the effect of non-conventional policies from a structural perspective. In addition, these models are used to analyze the financial system's stability and for macro-prudential decision-making, calibration of instruments, and stress testing.

Among the group of central banks' models that share financial elements with the model presented in this paper we can find the New Area-Wide Model II (NAWM II), the Norwegian Economy Model (NEMO) and the RAMSES II model.<sup>2</sup> The NAWM II, developed by the ECB for the Eurozone, extends the original NAWM with a rich financial sector, financial frictions, and long term loans. For Norway, the Norges Bank uses the NEMO, a DSGE model featuring a banking sector, a role for housing services and house prices, and long-term debt. For Sweden, the Riksbank developed the RAMSES II model as an extension of the original RAMSES model, augmented to include financial friction in the style of Bernanke et al. (1999). In addition to those efforts, the Banque de France, as part of a suite of models used to calibrate their macroprudential policy, developed two DSGE models with a banking sector and a central role for capital banking in the transmission of economic shocks, described in Clerc et al. (2014) and Gerali et al. (2008). For the Central Bank of Chile's Macro Financial model here described, we incorporate the features from these models that are more relevant to the Chilean economy.

The document is structured as follows. In Section 2 we present a detailed description of the theoretical structure of the model. Section 3 describes the Bayesian estimation of the model, the calibration, the choice of priors and presents the results. Section 4 discusses the role of default and financial frictions in the model. Section 5 concludes.

## 2 A Small Open Economy Model with Financial Frictions

In the following section, we augment a standard New Keynesian small open economy model with financial frictions in the economy's entrepreneurial, banking, and housing sectors. To do this, we introduce new agents taking Clerc et al. (2014) as starting point: entrepreneurs and bankers. The former are the sole owners of capital, who finance their capital investment through banking loans, while the latter are the owners of the banks who lend resources for capital investment and housing investment.

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<sup>2</sup>For a comprehensive description of the NAWM II, NEMO, and RAMSES II models, refer to, respectively, Coenen et al. (2018), Motzfeldt Kravik and Mimir (2019), and Adolfson et al. (2013)

Households are divided between patients, who save using the financial market, and impatient, who borrow using the financial market. We also introduce the segmented financial markets concept in the spirit of Vayanos and Vila (2009). Following Andres et al. (2004) and Chen et al. (2012), saving households can be unrestricted, who can save in short or long term financial assets, or unrestricted, who can save only in short term assets. All households derive utility from a consumption good, leisure, and housing stock.

From the production side, we use a simplified version of Garcia et al. (2019) in which a final good is produced using capital and labor and facing prices *à la* Calvo and a labor market facing quadratic adjustment cost in the style of Lechthaler and Snower (2011). In addition, we introduce three kinds of firms (capital producers, housing producers, and banks). Concerning debt, we include not only short-term deposits but also long-term government and bank bonds as perpetuities that pay exponentially decaying coupons introduced by Woodford (2001)

## 2.1 Households

There are two continuums of households of measure one, risk-averse and infinitely lived. These agents differ in their discount factor:  $\beta_I$  for impatient households ( $I$ ), and  $\beta_P$  for patient households ( $P$ ), with  $\beta_P > \beta_I$ . In equilibrium, impatient households borrow from banks and are ex-ante identical in asset endowments and preferences to others of their same patience.

In terms of patient households, following Andres et al. (2004) and Chen et al. (2012), we allow for a distinction between two types of patient households: Restricted (R) and Unrestricted (U) depending on which assets they can access for saving purposes. While Unrestricted households can buy both long and short-term assets with a transaction cost, Restricted households can only buy long-term bonds but do not face any transaction cost. Their combined measure is of size one.

Restricted and Unrestricted households' preferences depend on consumption of a final good  $C_t$  relative to external habits  $\tilde{C}_{t-1}$ , their stock of housing from last period  $H_{t-1}$  relative to external habits  $\tilde{H}_{t-2}$ , and labor supplied (hours worked)  $n_t$  in each period. The consumption of the aggregate good  $\hat{C}_t^i \equiv \hat{C}(C_t^i, \tilde{C}_{t-1}^i, H_{t-1}^i, \tilde{H}_{t-2}^i)$  for households of type  $i = \{U, R, I\}$  is assumed to be a constant elasticity of substitution (CES) as shown in (1):

$$\hat{C}_t^i = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( C_t^i - \phi_c \tilde{C}_{t-1}^i \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \xi_t^h \left( H_{t-1}^i - \phi_{hh} \tilde{H}_{t-2}^i \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \quad (1)$$

where  $o_{\hat{C}} \in (0, 1)$  is the weight on housing in the aggregate consumption basket,  $\eta_{\hat{C}}$  is the elasticity of substitution between the final good and the housing good,  $\xi_t^h$  is an exogenous preference shifter shock and  $\phi_c, \phi_{hh} \geq 0$  are parameters guiding the strength of habits in consumption and housing respectively. Households of type  $i = \{U, R, I\}$

maximize the following expected utility

$$\max_{\{\hat{C}_t^i, H_t^i\}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[ \frac{1}{1-\sigma} \left( \hat{C}_t^i \right)^{1-\sigma} - \Theta_t^i A_t^{1-\sigma} \xi_t^n \frac{(n_t^i)^{1+\varphi}}{1+\varphi} \right] \quad (2)$$

where  $\beta_i \in (0, 1)$  is the respective discount factor,  $\varrho_t$  is an exogenous shock to intertemporal preferences,  $\xi_t^n$  is a preference shock that affects the (dis)utility from labor,  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution,  $\varphi \geq 0$  is the inverse elasticity of labor supply.

As in Galí et al. (2012), we introduce an endogenous preference shifter  $\Theta_t$ , that satisfies the following conditions

$$\Theta_t^i = \tilde{\chi}_t^i A_t^\sigma \left( \hat{C} \left( \tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{-\sigma} \quad (3)$$

and

$$\tilde{\chi}_t^i = (\tilde{\chi}_{t-1}^i)^{1-v} A_t^{-\sigma v} \left( \hat{C} \left( \tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{\sigma v} \quad (4)$$

where the parameter  $v \in [0, 1]$  regulates the strength of the wealth effect, and  $\tilde{C}_t^i$  and  $\tilde{H}_{t-1}^i$  are taken as given by the households. In equilibrium  $C_t^i = \tilde{C}_t^i$  and  $H_t^i = \tilde{H}_t^i$ .

### 2.1.1 Patient Households

**Unrestricted Households.** This group is formed by fraction  $\varphi_U$  of the patient households. In equilibrium, they save in one-period government bond,  $BS_t^U$ , long-term government bonds,  $BL_t^U$ , short-term bank deposits  $D_t^U$ , long-term bank-issued bonds,  $BB_t^U$ , and one-period foreign bonds quoted in foreign currency  $B_t^{*U}$ . All these assets being non-state-contingent.

The structure of long term financial assets follows Woodford (2001), in this framework, long-term instruments are perpetuities, each paying a coupon of unitary value (in units of final goods) in the period after issuance, and a geometrically declining series of coupons (with a decaying factor  $\kappa < 1$ ) thereafter. That is, a bond issued in period- $t$  implies a series of coupon payments starting in  $t+1$ :  $\{1, \kappa, \kappa^2, \dots\}$ . Also, let  $B_{t-1}$ , where  $B_{t-1} = \{BL_{t-1}^U, BB_{t-1}^U\}$  represent the total liabilities due in period  $t$  from all past bond issues up to period  $t-1$ . That is

$$B_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \dots,$$

thus,  $CI_{t-1} = B_{t-1} - \kappa B_{t-2}$ . Let  $Q_t^B$  denote the period- $t$  price of a new issue, then  $Q_t^B$  summarizes the prices at all maturities. For instance,  $Q_{t|t-1}^B = \kappa Q_t^B$  is the price in  $t$  of a perpetuity issued in period  $t-1$ . Importantly, note that  $B_{t-1}$  denotes both, total liabilities in period- $t$  from previous debt, and –because of the particular coupon

structure— the total number of outstanding bonds. Then, the total value of financial asset debt in period  $t$  is given by  $Q_t B_t$ . Finally, the yield to maturity of holding long term assets at period  $t$ ,  $R_t^B$ , as,

$$R_t^B = \frac{P_t}{Q_t^B} + \kappa$$

Unrestricted households must pay a transaction cost  $\zeta_t^L$  per unit of long-term bond purchased. These costs are paid to a financial intermediary as a fee. This financial intermediary distributes its nominal value profits  $\Pi^{FI}$ , as dividends to its shareholders. Then, unrestricted patient households' period budget constraint is

$$\begin{aligned} BS_t^U + (1 + \zeta_t^L) Q_t^{BL} BL_t^U + D_t^U + (1 + \zeta_t^L) Q_t^{BB} BB_t^U + S_t B_t^{\star U} + P_t C_t^U + Q_t^H H_t^U = \\ R_{t-1} BS_{t-1}^U + Q_t^{BL} R_t^{BL} BL_{t-1}^U + \tilde{R}_t^D D_{t-1}^U + \tilde{R}_t^{BB} Q_t^{BB} BB_{t-1}^U + S_t B_{t-1}^{\star U} R_{t-1}^{\star} + W_t n_t^U \\ + Q_t^H (1 - \delta_H) H_{t-1}^U + \Psi_t \end{aligned} \quad (5)$$

where  $R_t^{BL}$  and  $R_t^{BB}$  are the gross yield to maturity for long-term government and bank-issued bonds at time  $t$ ,  $P_t$  denotes the price of the consumption good,  $Q_t^H$  denotes the price of housing good,  $\delta_H$  is the depreciation rate of housing,  $S_t$  denotes the nominal exchange rate (units of domestic currency per unit of foreign currency), and  $R_t^{\star}$  denotes the the foreign one-period bond and  $R_t$  denotes de short term nominal government bond.

Further,  $\tilde{R}_t^D = R_{t-1}^D (1 - \gamma_D P D_t^B)$ ,  $\tilde{R}_t^{BB} = R_t^{BB} (1 - \gamma_{BB} P D_t^B)$  denote the net return on resources loaned to banks in the form of deposits and bank-issued bonds,  $R_t^D$  is the gross interest rate received at  $t$  on the bank deposits at  $t-1$ , and  $R_t^{BB}$  is the gross return of saving on long term bank bonds,  $P D_t^B$  denotes the fraction of resources in banks that fail in period  $t$  and  $\gamma_D (\gamma_{BB})$  is a linear transaction cost that households must pay in order to recover their funds. Finally,  $W_t$  denotes the nominal wage and,  $\Psi_t$  denotes lump-sum payments that include taxes  $T_t$ , dividend income from entrepreneurs  $C_t^e$ , bankers  $C_t^b$ , rents from ownership of foreign firms  $REN_t^*$ , profits from ownership of domestic firms, and profits from the financial intermediary in the long-term bond transactions,  $\Pi^F = \zeta_t^L (Q_t^{BL} BL_t^U + Q_t^{BB} BB_t^U)$ .

Chen et al. (2012) show that the discounted value of future transaction costs implies a term premium. We assume that the period transaction cost is a function of the ratio of the aggregate market value of long-term to short-term assets and a disturbance term. Further, households do not internalize the effect of their choices on this transaction cost, yet in equilibrium  $\widetilde{BL}_t^U = BL_t^U$  and  $\widetilde{BS}_t^U = BS_t^U$ . This ratio captures the idea that holding long-term debt implies a loss of liquidity that households hedge by increasing the amount of short-term debt. Specifically,

the functional form is given by

$$\zeta_t^L = \left( \frac{Q_t^{BL} \widetilde{BL}_t^U + Q_t^{BB} \widetilde{BB}_t^U}{\widetilde{BS}_t^U + S_t \widetilde{B}_t^{*U} + \widetilde{D}_t^U} \right)^{\eta_{\zeta_L}} \epsilon_t^L \quad (6)$$

Households supply differentiated labor services to a continuum of unions which act as wage setters on behalf of the households in monopolistically competitive markets. The unions pool the wage income of all households and then distribute the aggregate wage income in equal proportions among households, hence, they are insured against variations in household-specific wage income. Defining for convenience the multiplier on the budget constraint as  $\lambda_t^U A_t^{-\sigma} / P_t$ , then, Unrestricted Households solve (2) subject to (1), (3), (4), and (5). From this problem we obtain the following first-order conditions:

$$[C_t^U] : \quad \lambda_t^U A_t^{-\sigma} = \left( \hat{C}_t^U \right)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{C}_t^U}{\left( C_t^U - \phi_c \tilde{C}_{t-1}^U \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (7)$$

$$[H_t^P] : \quad \varrho_t \frac{\lambda_t^U A_t^{-\sigma} Q_t^H}{P_t} = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left( \hat{C}_{t+1}^U \right)^{-\sigma} \xi_{t+1}^h \left( \frac{o_{\hat{C}} \hat{C}_{t+1}^U}{\xi_{t+1}^h \left( H_t^U - \phi_{hh} \tilde{H}_{t-1}^U \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1 - \delta_H) \frac{\lambda_{t+1}^U A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\} \quad (8)$$

$$[BS_t^U] : \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \quad (9)$$

$$[BL_t^U] : \quad \varrho_t \lambda_t^U A_t^{-\sigma} \left( \frac{1 + \zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} \left( \frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \quad (10)$$

$$[B_t^U] : \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t^* \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \quad (11)$$

$$[D_t^U] : \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^D A_{t+1}^{-\sigma} \right\} \quad (12)$$

$$[BB_t^U] : \quad \varrho_t \lambda_t^U A_t^{-\sigma} (1 + \zeta_t^L) Q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^{BB} A_{t+1}^{-\sigma} Q_{t+1}^{BB} \right\} \quad (13)$$

In equilibrium, we have that  $\tilde{C}_t^P = C_t^P$  and  $\tilde{H}_t^P = H_t^P$ , which applies for impatient households as well. The implied discount factor for nominal claims is, by iterating upon (9):

$$r_{t,t+s} = \frac{1}{\prod_{i=0}^{s-1} R_{t+i}} = \beta_U^s \frac{\varrho_{t+s} \lambda_{t+s}^U A_{t+s}^{-\sigma} P_t}{\varrho_t \lambda_t^U A_t^{-\sigma} P_{t+s}} \quad (14)$$

**Restricted households.** This group of households have a mass  $\varphi_R$  which complements the mass of unrestricted households  $\varphi_U$ , then  $\varphi_R = 1 - \varphi_U$ . The main difference with Unrestricted Household is that can only access long-term government bonds. In addition, Restricted Patient Household do not face transaction costs. They are subject

to the period-by-period budget constraint

$$P_t C_t^R + Q_t^H H_t^R + Q_t^{BL} BL_t^R = W_t n_t^R + Q_t^H (1 - \delta_H) H_{t-1}^R + Q_t^{BL} R_t^{BL} BL_{t-1}^R \quad (15)$$

Let us define, for convenience, the multiplier on the budget constraint as  $\lambda_t^R A_t^{-\sigma}/P_t$ . Then, restricted households solve (2) subject to (1), (3), (4), from which we obtain the following first-order conditions:

$$[C_t^R] : \quad \lambda_t^R A_t^{-\sigma} = \left( \hat{C}_t^R \right)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{C}_t^R}{\left( C_t^R - \phi_c \tilde{C}_{t-1}^R \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (16)$$

$$[H_t^P] : \quad \varrho_t \frac{\lambda_t^R A_t^{-\sigma} Q_t^H}{P_t} = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left( \hat{C}_{t+1}^R \right)^{-\sigma} \left( \frac{o_{\hat{C}} \hat{C}_{t+1}^R}{\xi_{t+1}^h (H_t^R - \phi_{hh} \tilde{H}_{t-1}^R)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \right. \\ \left. + (1 - \delta_H) \frac{\lambda_{t+1}^R A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\} \quad (17)$$

$$[BL_t^R] : \quad \varrho_t \lambda_t^R A_t^{-\sigma} Q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^R}{\pi_{t+1}} R_{t+1}^{BL} Q_{t+1}^{BL} A_{t+1}^{-\sigma} \right\} \quad (18)$$

### 2.1.2 Impatient Households

Impatient households work, consume, and purchase housing goods. In addition, they take long-term loans in equilibrium from banks to finance their purchases of housing goods, which we model using the same structure presented in the previous section.

We follow the Clerc et al. (2014) by assuming that these mortgage loans are non-recourse and limited liability contracts, which enables the possibility of default for households. For the household, the only consequence of default is losing the housing good on which the mortgage is secured, therefore default is optimal when the value of the total outstanding debt is higher than the value of the assets,  $R_t^I Q_t^L L_{t-1}^H > \omega_t^I Q_t^H (1 - \delta_H) H_{t-1}^I$ . Then the impatient household budget constraint is given by:

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L L_t^H = W_t n_t^I + \int_0^\infty \max \{ \omega_t^I Q_t^H (1 - \delta_H) H_{t-1}^I - R_t^I Q_t^L L_{t-1}^H, 0 \} dF_I(\omega_t^I) \quad (19)$$

Let's define  $\omega_t^I$  as an idiosyncratic shock to the efficiency units of housing of impatient households, which can be interpreted as a reduced-form representation of any shock to the value of houses. The shock  $\omega_t^I$  is i.i.d. across households and follows a log-normal distribution with pdf  $f_I(\omega_t^I)$  and cdf  $F_I(\omega_t^I)$ .

After the realization of aggregate and idiosyncratic shocks individual households decide whether to default, then the resulting net worth is distributed evenly across members of this type, who optimally decide to choose the

same debt, consumption, housing and hours worked. Let

$$R_t^H = \frac{Q_t^H(1 - \delta_H)}{Q_{t-1}^H}.$$

Then, in order for the impatient household to pay for its loan, the idiosyncratic shock  $\omega_t^I$  must exceed the threshold  $\bar{\omega}_t^I$ . Then, if  $\omega_t^I \geq \bar{\omega}_t^I$  the household pays liabilities due in the period  $t$  in the amount  $R_t^I Q_t^L L_{t-1}^H$ , and rolls over remaining outstanding value of debt,  $\kappa Q_t^L L_{t-1}^H$ , to obtain positive net worth,  $(\omega_t^I - \bar{\omega}_t^I) Q_t^H (1 - \delta_H) H_{t-1}^I$ . Otherwise, the household debt becomes non-performing, defaults and receives nothing. On the other hand, the bank receives  $R_t^I Q_t^L L_{t-1}^H$  from performing loans, but it only recovers  $(1 - \mu_I) \omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I$  from non performing loans. With the definition of the  $\bar{\omega}_t^I$  threshold, we can define  $PD_t^I = F_I(\bar{\omega}_t^I)$  as the default rate of impatient households on their housing loans. The functional form for the default threshold is given by

$$\bar{\omega}_t^I = \frac{R_t^I Q_t^L L_{t-1}^H}{R_t^H Q_{t-1}^H H_{t-1}^I}$$

We model the default decision threshold based on a smoothed valuation of the outstanding debt,  $\log Q_t^{\hat{L}} \equiv \alpha_{Q^L}^1 (\alpha_{Q^L}^2 \log Q_{t-1}^{\hat{L}} + (1 - \alpha_{Q^L}^2) \log Q^L) + (1 - \alpha_{Q^L}^1) \log Q_t^L$ . We use this functional form to avoid excessive volatility of the default threshold due to the influence of the revaluation of long-term debt, leading to a counterfactual behavior of the households' consumption decisions. With the definition of the  $\bar{\omega}_t^I$  threshold, we can define  $PD_t^I = F_I(\bar{\omega}_t^I)$  as the default rate of impatient households on their housing loans.

Out of all the loans, the share of the gross return that goes to the bank is denoted as  $\Gamma_I(\bar{\omega}_t^I)$  whereas the share of gross return that goes to the impatient household is  $(1 - \Gamma_I(\bar{\omega}_t^I))$  where:

$$\Gamma_I(\bar{\omega}_t^I) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I(\omega_t^I) d\omega_t^I + \bar{\omega}_t^I \int_{\bar{\omega}_t^I}^{\infty} f_I(\omega_t^I) d\omega_t^I$$

The first integral on the right denotes the share of the return that is defaulted while the second integral denotes the share of return that is paid in full. This allows us to rewrite the budget condition from (19) as

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L L_t^H = W_t n_t^I + [1 - \Gamma_I(\bar{\omega}_t^I)] R_t^H Q_{t-1}^H H_{t-1}^I \quad (20)$$

Also, let

$$G_I(\bar{\omega}_t^I) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I(\omega_t^I) d\omega_t^I$$

denote the part of those returns that comes from the defaulted loans. Taking into consideration the share of the

return that is lost due to verification cost as  $\mu_I G_I(\bar{\omega}_t^I)$ , then the net share of return that goes to the bank is

$$\Gamma_I(\bar{\omega}_t^I) - \mu_I G_I(\bar{\omega}_t^I).$$

The terms of the loan must imply the net expected profits of the bank must equal its alternative use of funds, therefore it must satisfy a participation constraint:

$$\mathbb{E}_t \{ [1 - \Gamma^H(\bar{\omega}_{t+1}^H)] [\Gamma_I(\bar{\omega}_{t+1}^I) - \mu_I G_I(\bar{\omega}_{t+1}^I)] R_{t+1}^H Q_t^H H_t^I \} \geq \rho_{t+1}^H \phi_H Q_t^L L_t^H \quad (21)$$

Where  $\Gamma^H(\bar{\omega}_{t+1}^H)$  is the fraction of bank gross returns that is used to pay depositors or is lost due to bank defaults when their own idiosyncratic shock  $\omega_{t+1}^H$  is too low. The rest of the left hand side expression is the total amount of returns on the housing project that goes to the lender bank. The right hand side indicates the opportunity cost, which is investing an amount of equity  $\phi_H Q_t^L L_t^H$  at a market-determined rate of return of  $\tilde{\rho}_{t+1}^H$ , where  $\phi_H$  is a regulatory capital constraint. We elaborate on the bank's problem on subsection 2.3, for now note that we can write (21) with equality without loss of generality.

Thus, following the timing described above, the impatient household's optimization problem can be written as maximizing (2) for  $i = I$  subject to their budget constraint (20) and the bank participation constraint (21). For this, define for convenience  $\lambda_t^I A_t^{-\sigma}/P_t$  and  $\lambda_t^H A_t^{-\sigma}/P_t$  as the multipliers for each constraint respectively. Define also  $x_t^I \equiv R_t^I L_t^H / Q_t^H H_t^I$ , a measure of household leverage. This yields the following FOC's:

$$[C_t^I] : \quad \lambda_t^I A_t^{-\sigma} = \left\{ \left( \hat{C}_t^I \right)^{-\sigma} \right\} \left( \frac{(1 - o_{\hat{C}}) \hat{C}_t^I}{(C_t^I - \phi_c \tilde{C}_{t-1}^I)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (22)$$

$$[H_t^I] : \quad \varrho_t \frac{\lambda_t^I A_t^{-\sigma} Q_t^H}{P_t} = \mathbb{E}_t \left\{ \begin{array}{l} \beta_I \varrho_{t+1} \left( \left( \hat{C}_{t+1}^I \right)^{-\sigma} \left( \frac{o_{\hat{C}} \hat{C}_{t+1}^I}{\varepsilon_{t+1}^h (H_t^I - \phi_{hh} \tilde{H}_{t-1}^I)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \right. \\ \left. + \frac{\lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} [1 - \Gamma_I(\bar{\omega}_{t+1}^I)] R_{t+1}^H Q_t^H \right) \\ + \frac{\varrho_t \lambda_t^H A_t^{-\sigma}}{P_t} [1 - \Gamma^H(\bar{\omega}_{t+1}^H)] [\Gamma_I(\bar{\omega}_{t+1}^I) - \mu_I G_I(\bar{\omega}_{t+1}^I)] R_{t+1}^H Q_t^H \end{array} \right\} \quad (23)$$

$$[L_t^H] : \quad \lambda_t^I = \lambda_t^H \tilde{\rho}_{t+1}^H \phi_H \quad (24)$$

$$[x_t^I] : \quad \frac{\varrho_t \lambda_t^H A_t^{-\sigma}}{P_t} \mathbb{E}_t \{ [1 - \Gamma^H(\bar{\omega}_{t+1}^H)] [\Gamma'_I(\bar{\omega}_{t+1}^I) - \mu_I G'_I(\bar{\omega}_{t+1}^I)] \} = \beta_I \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \Gamma'_I(\bar{\omega}_{t+1}^I) \right\} \quad (25)$$

Regarding the idiosyncratic shock, we assume that  $\ln(\omega_t^I) \sim N(-\frac{1}{2}(\sigma_t^I)^2, (\sigma_t^I)^2)$ , therefore its unconditional expectation is  $\mathbb{E}\{\omega_t^I\} = 1$ , and its average conditional on truncation is

$$\mathbb{E}_t \{ \omega_t^I | \omega_t^I \geq \bar{\omega}_t^I \} = \frac{1 - \Phi(z_t^I - \sigma_t^I)}{1 - \Phi(z_t^I)},$$

where  $\Phi$  is the c.d.f. of the standard normal and  $z_t^I$  is an auxiliary variable defined as  $z_t^I \equiv (\ln(\bar{\omega}_t^I) + 0.5(\sigma_t^I)^2)/\sigma_t^I$ .

Then, we can obtain the following functional forms:

$$\Gamma_I(\bar{\omega}_t^I) = \Phi(z_t^I - \sigma_t^I) + \bar{\omega}_t^I(1 - \Phi(z_t^I))$$

and

$$\Gamma_I(\bar{\omega}_t^I) - \mu_I G_I(\bar{\omega}_t^I) = (1 - \mu_I)\Phi(z_t^I - \sigma_t^I) + \bar{\omega}_t^I(1 - \Phi(z_t^I))$$

Finally, we allow for fluctuations in the variance of the idiosyncratic shock, as  $\sigma_t^I$  is modeled as an exogenous process.

## 2.2 Entrepreneurs

As in Clerc et al. (2014), we introduce risk-neutral entrepreneurs that follow an overlapping generations structure, where each generation lives across two consecutive periods. The entrepreneurs are the sole owners of productive capital, which is bought from capital producers to be, in turn, rented to the firms that produce different varieties of the home good.

Entrepreneurs born in period  $t$  draw utility in  $t+1$  from transferring part of final wealth as dividends,  $C_{t+1}^e$ , to unrestricted patient households and from leaving the rest as bequests,  $N_{t+1}^e$ , to the next generation of entrepreneurs in the form:

$$\max_{C_{t+1}^e, N_{t+1}^e} (C_{t+1}^e)^{\xi_{\chi_e} \chi_e} (N_{t+1}^e)^{1-\xi_{\chi_e} \chi_e} \text{ subject to}$$

$$C_{t+1}^e + N_{t+1}^e = \Psi_{t+1}^e$$

where  $\Psi_{t+1}^e$  is entrepreneurial wealth at  $t+1$ , explained below, and  $\xi_{\chi_e}$  is a stochastic shock to their preferences.

The first order conditions to this problem may be written as:

$$[C_{t+1}^e] : \xi_{\chi_e} \chi_e (C_{t+1}^e)^{(\xi_{\chi_e} \chi_e - 1)} (N_{t+1}^e)^{1-\xi_{\chi_e} \chi_e} - \lambda_t^{\chi_e} = 0$$

$$[N_{t+1}^e] : (1 - \xi_{\chi_e} \chi_e) (C_{t+1}^e)^{\xi_{\chi_e} \chi_e} (N_{t+1}^e)^{-\xi_{\chi_e} \chi_e} - \lambda_t^{\chi_e} = 0$$

$$[\lambda_t^{\chi_e}] : C_{t+1}^e + N_{t+1}^e - \Psi_{t+1}^e = 0$$

From first order conditions we get the following optimal rules

$$C_{t+1}^e = \chi_e \Psi_{t+1}^e$$

$$N_{t+1}^e = (1 - \chi_e) \Psi_{t+1}^e$$

In their first period, entrepreneurs will try to maximize expected second period wealth,  $\Psi_{t+1}^e$ , by purchasing capital at nominal price  $Q_t^K$ , which will be productive (and rented) in the next period. These purchases are financed using the resources left as bequests by the previous generation of entrepreneurs and borrowing an amount  $L_t^F$  at nominal rate  $R_t^L$  from F banks. In borrowing from banks, entrepreneurs also face an agency problem of the type faced by impatient households i.e. in  $t + 1$  entrepreneurs receive an idiosyncratic shock to the efficiency units of capital that will ultimately determine their ability to pay their liabilities to banks. Banks cannot observe these shock, but entrepreneurs can. Depreciated capital is sold in the next period to capital producers at  $Q_{t+1}^K$ . Entrepreneurial leverage, as measured by assets over equity, is  $lev_t^e = Q_t^K K_t / N_t^e$ .

In this setting, entrepreneurs solve, in their first period,

$$\max_{K_t, L_t^F} \mathbb{E}_t (\Psi_{t+1}^e) \text{ subject to}$$

$$Q_t^K K_t - L_t^F = N_t^e$$

$$\Psi_{t+1}^e = \max [ \omega_{t+1}^e (R_{t+1}^k + (1 - \delta_K) Q_{t+1}^K) K_t - R_t^L L_t^F, 0 ]$$

and a bank participation condition, which will be explained later. The factor  $\omega_{t+1}^e$  represents the idiosyncratic shock to the entrepreneurs efficiency units of capital. This shock takes place after the loan with the bank has taken place but before renting capital to consumption goods producers. It is assumed that this shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one. Let

$$R_{t+1}^e = \left[ \frac{R_{t+1}^k + (1 - \delta_K) Q_{t+1}^K}{Q_t^K} \right] \quad (26)$$

be the gross nominal return per efficiency unit of capital obtained in period  $t + 1$  from capital obtained in period  $t$ . Then in order for the entrepreneur to pay for its loan the efficiency shock,  $\omega_{t+1}^e$ , must exceed the threshold

$$\bar{\omega}_{t+1}^e = \frac{R_t^L L_t^F}{R_{t+1}^e Q_t^K K_t}$$

If  $\omega_{t+1}^e \geq \bar{\omega}_{t+1}^e$  the entrepreneurs pays  $R_t^L L_t^F$  to the bank and gets  $(\omega_{t+1}^e - \bar{\omega}_{t+1}^e) R_{t+1}^e Q_t^K K_t$ . Otherwise,

the entrepreneurs defaults and receives nothing. While F-banks only recover  $(1 - \mu_e)\omega_{t+1}^e R_{t+1}^e Q_t^K K_t$  from non performing loans, and  $R_t^L L_t^F$  from performing loans. With the threshold, we can define  $PD_t^e = F_e(\bar{\omega}_t^e)$  as the default rate of entrepreneurs on their loans.

The share of the gross return that goes to the bank is denoted as  $\Gamma_e(\bar{\omega}_{t+1}^e)$  whereas the share of gross return that goes to the entrepreneur is  $(1 - \Gamma_e(\bar{\omega}_{t+1}^e))$  where:

$$\Gamma_e(\bar{\omega}_{t+1}^e) = \int_0^{\bar{\omega}_{t+1}^e} \omega_{t+1}^e f_e(\omega_{t+1}^e) d\omega_{t+1}^e + \bar{\omega}_{t+1}^e \int_{\bar{\omega}_{t+1}^e}^{\infty} f_e(\omega_{t+1}^e) d\omega_{t+1}^e$$

also let

$$G_e(\bar{\omega}_{t+1}^e) = \int_0^{\bar{\omega}_{t+1}^e} \omega_{t+1}^e f_e(\omega_{t+1}^e) d\omega_{t+1}^e$$

denote the part of those returns that come from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as  $\mu_e G_e(\bar{\omega}_{t+1}^e)$ , then the net share of return that goes to the bank is

$$\Gamma_e(\bar{\omega}_{t+1}^e) - \mu_e G_e(\bar{\omega}_{t+1}^e).$$

Taking this into account then the maximization problem of the entrepreneur can be written as

$$\max_{\bar{\omega}_{t+1}^e, K_t} \mathbb{E}_t \{ \Psi_{t+1}^e \} = \mathbb{E}_t \{ [1 - \Gamma_e(\bar{\omega}_{t+1}^e)] R_{t+1}^e Q_t^K K_t \}, \text{ subject to}$$

$$\mathbb{E}_t \{ [1 - \Gamma_F(\bar{\omega}_{t+1}^F)] [\Gamma_e(\bar{\omega}_{t+1}^e) - \mu_e G_e(\bar{\omega}_{t+1}^e)] R_{t+1}^e Q_t^K K_t \} \geq \rho_{t+1}^F \phi_F L_t^F, \quad (27)$$

that says that banks will participate in the contract only if its net expected profits are at least equal to their alternative use of funds. This yields the following optimality conditions

$$(1 - \Gamma_{t+1}^e) = \lambda_t^e \left( \frac{\rho_{t+1}^F \phi_t^F}{R_{t+1}^e} - (1 - \Gamma_{t+1}^F) [\Gamma_{t+1}^e - \mu^e G_{t+1}^e] \right) \quad (28)$$

$$\Gamma_{t+1}^{e'} = \lambda_t^e (1 - \Gamma_{t+1}^F) \left[ \Gamma_{t+1}^{e'} - \mu^e G_{t+1}^{e'} \right] \quad (29)$$

Further, it is assumed that  $\ln(\omega_t^e) \sim N(-0.5(\sigma_t^e)^2, (\sigma_t^e)^2)$ , leading to analogous properties as with impatient households for  $\bar{\omega}_t^e$ ,  $\Gamma_e$  and  $G_e$ .

## 2.3 Bankers and Banks

### 2.3.1 Bankers

Bankers are modeled as in Clerc et al. (2014) and in a similar way to entrepreneurs: They belong to a sequence of overlapping generations of risk-neutral agents who live 2 periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital.

In the first period, the banker receives a bequest  $N_t^b$  from the previous generation of bankers and must distribute it across the two types of existing banks: banks specializing in corporate loans (F banks) and banks specializing in housing loans (H banks). That is, a banker who chooses to invest an amount  $E_t^F$  of inside equity in F banks will invest the rest of her bequest in H banks,  $E_t^H = N_t^b - E_t^F$ . Then, in the second period bankers receive their returns from both investments, and must choose how to distribute their net worth  $\Psi_{t+1}^b$  between transferring dividends  $C_{t+1}^b$  to households and leaving bequests  $N_{t+1}^b$  to the next generation. Additionally, disturbances to the exogenous variable  $\xi_t^{X^b}$  capture transitory fluctuations in the banker's dividend policy

Given  $\Psi_{t+1}^b$ , the banker will distribute it by solving the following maximization problem:

$$\max_{C_{t+1}^b, N_{t+1}^b} (C_{t+1}^b)^{\xi_{t+1}^{X^b} \chi^b} (N_{t+1}^b)^{1-\xi_{t+1}^{X^b} \chi^b}, \text{ subject to}$$

$$C_{t+1}^b + N_{t+1}^b = \Psi_{t+1}^b$$

which leads to the following optimal rules

$$C_{t+1}^b = \xi_{t+1}^{X^b} \chi^b \Psi_{t+1}^b \quad (30)$$

$$N_{t+1}^b = (1 - \xi_{t+1}^{X^b} \chi^b) \Psi_{t+1}^b \quad (31)$$

In turn, net worth in the second period is determined by the returns on bankers' investments in period- $t$ :

$$\Psi_{t+1}^b = \rho_{t+1}^F E_t^F + \xi_t^{b, roe} \rho_{t+1}^H (N_t^b - E_t^F)$$

where  $\xi_t^{b, roe}$  is a shock to the bankers' required return to equity invested in the housing branches,  $\rho_{t+1}^j$  is the period  $t+1$  ex-post gross return on inside equity  $E_t^j$  invested in period  $t$  in bank of class  $j$ . In order to capture the fact that most of mortgage debt takes the form of non endorsable debt —meaning the issuer bank retains it in its balance sheet to maturity— we assume that the banker  $j = H$  invests in the banking project  $H$  through a mutual fund which pays the expected average return to housing equity  $\rho_{t+1}^H$  every period. Thus, letting  $\tilde{\rho}_t^H$  represent the period

return on housing portfolio, then  $\rho_t^H = \kappa\tilde{\rho}_t^H + (1 - \kappa)\rho_{t+1}^H$ . The banker then chooses

$$\max_{E_t^F} \mathbb{E}_t \left\{ \Psi_{t+1}^b \right\} = \mathbb{E}_t \left\{ \rho_{t+1}^F E_t^F + \xi_t^{b,roe} \rho_{t+1}^H (N_t^b - E_t^F) \right\}$$

An interior equilibrium in which both classes of banks receive strictly positive inside equity from bankers will require the following equality to hold:

$$\mathbb{E}_t \left\{ \rho_{t+1}^F \right\} = \mathbb{E}_t \left\{ \xi_t^{b,roe} \rho_{t+1}^H \right\} = \bar{\rho}_t$$

where  $\bar{\rho}_t$  denotes banks' required expected gross rate of return on equity investment undertaken at time  $t$ .

### 2.3.2 Banks

Banks are institutions specialized in extending either corporate or housing loans drawing funds through deposits, and bonds from unconstrained household, and equity from bankers. We assume a continuum of identical banking institutions of  $j$  class banks  $j = \{F, H\}$ . In particular, banks of class  $j$  are investment projects created in period- $t$  that in  $t+1$  generate profits  $\Pi_{t+1}^j$  before being liquidated with:

$$\Pi_{t+1}^F = \max \left[ \omega_{t+1}^F \tilde{R}_{t+1}^F L_t^F - R_t^D D_t^F, 0 \right], \quad \Pi_{t+1}^H = \max \left[ \omega_{t+1}^H \tilde{R}_{t+1}^H Q_t^L L_t^H - R_{t+1}^{BB} Q_{t+1}^{BB} BB_t, 0 \right]$$

where  $\tilde{R}_{t+1}^j$  is the realized return on a well-diversified portfolio of loans to entrepreneurs or households and  $\omega_{t+1}^j$  is an idiosyncratic portfolio return shock, which is i.i.d across banks of class  $j$  with a cdf of  $F_j(\omega_{t+1}^j)$  and pdf  $f_j(\omega_{t+1}^j)$ . Due to limited liability, the equity payoff may not be negative, which defines thresholds  $\bar{\omega}_{t+1}^j$ :

$$\bar{\omega}_{t+1}^F \equiv \frac{R_t^D D_t^F}{\tilde{R}_{t+1}^F L_t^F}, \quad \bar{\omega}_{t+1}^H \equiv \frac{R_{t+1}^{BB} Q_{t+1}^{BB} BB_t}{\tilde{R}_{t+1}^H Q_t^L L_t^H}$$

Similar to households and entrepreneurs,  $\Gamma_j(\bar{\omega}_{t+1}^j)$  denotes the share of gross returns to bank  $j$  investments which are either paid back to depositors or bond holders, implying that  $[1 - \Gamma_j(\bar{\omega}_{t+1}^j)]$  is the share that the banks will keep as profits. We also define  $G_j(\bar{\omega}_{t+1}^j)$  as the share of bank  $j$  assets which belong to defaulting  $j$  banks, and thus  $\mu_j G_j(\bar{\omega}_{t+1}^j)$  is the total cost of bank  $j$  defaults expressed as a fraction of total bank  $j$  assets.

The balance sheet of banks of class F is given by  $L_t^F = E_t^F + D_t^F$ , and they face a regulatory capital constraint given by  $E_t^F \geq \phi_F L_t^F$ , where  $\phi_F$  is the capital-to-asset ratio, and is binding at all times in equilibrium so that the loans can be written as  $L_t^F = E_t^F / \phi_F$  and the deposits as  $D_t^F = (1 - \phi_F / \phi_F) E_t^F$ . Likewise, balance sheet of banks of class H is given by  $Q_t^L L_t^H = E_t^H + Q_t^{BB} BB_t$ , with binding capital regulation determining  $E_t^H = \phi_H Q_t^L L_t^H$ , and

$Q_t^{BB} BB_t = (1 - \phi_H)/\phi_H E_t^H$ . Further, using the threshold definitions and the binding capital constraints, we obtain<sup>3</sup>

$$\begin{aligned}\bar{\omega}_{t+1}^F &= (1 - \phi_F) \frac{R_t^D}{\tilde{R}_{t+1}^F} \\ \bar{\omega}_{t+1}^H &= (1 - \phi_H) \frac{R_{t+1}^{BB}}{\tilde{R}_{t+1}^H} \left( \frac{Q_{t+1}^{BB}}{Q_t^{\bar{B}\bar{B}}} \right)\end{aligned}$$

Finally, we define the realized rate of return of equity invested in a bank of class  $j$ :

$$\rho_{t+1}^j = \left[ 1 - \Gamma_j (\bar{\omega}_{t+1}^j) \right] \frac{\tilde{R}_{t+1}^j}{\phi_j} \quad (32)$$

For completeness, notice that derivations in prior sections imply that following expressions for  $\tilde{R}_{t+1}^j$ ,  $j = \{F, H\}$ :

$$\begin{aligned}\tilde{R}_{t+1}^F &= (\Gamma_e (\bar{\omega}_{t+1}^e) - \mu_e G_e (\bar{\omega}_{t+1}^e)) \frac{R_{t+1}^e Q_t^K K_t}{L_t^F} \\ \tilde{R}_{t+1}^H &= (\Gamma_I (\bar{\omega}_{t+1}^I) - \mu_I G_I (\bar{\omega}_{t+1}^I)) \frac{R_{t+1}^H Q_t^H H_t^I}{Q_t^L L_t^H}\end{aligned}$$

As with households and entrepreneurs, it is assumed that the bank idiosyncratic shock follows a log-normal distribution:  $\log(\omega_t^j) \sim N(-\frac{1}{2}(\sigma_t^j)^2, (\sigma_t^j)^2)$ , leading to analogous properties for  $\bar{\omega}_t^j$ ,  $\Gamma_j$  and  $G_j$ .

## 2.4 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. Monopolistically competitive unions act as wage setters by selling household's differentiated varieties of labor supply  $n_{it}$  to a perfectly competitive firm, which packs these varieties into a composite labor service  $\tilde{n}_t$ . There is a set of monopolistically competitive firms producing different varieties of a home good,  $Y_{jt}^H$ , using wholesale good  $X_t^Z$  as input; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties,  $X_{jt}^F$ ; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good,  $X_t^H$ , one packing the imported varieties into a composite foreign good,  $X_t^F$ , and, finally, another one that bundles the composite home and foreign goods to create a final good,  $Y_t^C$ . This final good is purchased by households ( $C_t^P, C_t^I$ ), capital and housing producers ( $I_t^K, I_t^H$ ), and the government ( $G_t$ ).

Similarly to Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms.

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<sup>3</sup>As with impatient households, to avoid excessive volatility of the default threshold due to the influence of the revaluation of long term debt, we model the default decision based on a smoothed valuation of the outstanding debt,  $Q_t^{\bar{B}\bar{B}}$ , where  $\log Q_t^{\bar{B}\bar{B}} \equiv \alpha_{Q_{BB}}^1 (\alpha_{Q_{BB}}^2 \log Q_{t-1}^{\bar{B}\bar{B}} + (1 - \alpha_{Q_{BB}}^2) \log Q_t^{BB}) + (1 - \alpha_{Q_{BB}}^1) \log Q_t^{BB}$ .

Both types of firms are owned by patient households and their technology is subject to an adjustment cost. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. However, we depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment. Finally, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and which follows an exogenous process). The total mass of firms in each sector is normalized to one.

#### 2.4.1 Capital goods

There is a continuum of competitive capital firm producers who buy an amount  $I_t$  of final goods at price  $P_t$  and use their technology to satisfy the demand for new capital goods not covered by depreciated capital, i.e.  $K_t - (1 - \delta_K)K_{t-1}$ , where new units of capital are sold at price  $Q_t^K$ . As is usual in the literature, we assume that the aggregate stock of new capital considers investment adjustment costs and evolves according to following law of motion:

$$K_t = (1 - \delta_K) K_{t-1} + \left[ 1 - \frac{\gamma_K}{2} \left( \frac{I_t}{I_{t-1}} - a \right)^2 \right] \xi_t^i I_t$$

Where  $\xi_t^i$  is a shock to investment efficiency. Therefore a representative capital producer chooses how much to invest in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^K \left[ 1 - \frac{\gamma_K}{2} \left( \frac{I_{t+i}}{I_{t+i-1}} - a \right)^2 \right] \xi_{t+i}^i I_{t+i} - P_{t+i} I_{t+i} \right\}$$

Discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of capital to the level of investment

$$\begin{aligned} P_t &= Q_t^K \left\{ \left( 1 - \frac{\gamma_K}{2} \left( \frac{I_t}{I_{t-1}} - a \right)^2 \right) - \gamma_K \left( \frac{I_t}{I_{t-1}} - a \right) \frac{I_t}{I_{t-1}} \right\} \xi_t^i \\ &\quad + E_t \left\{ r_{t,t+1} Q_{t+1}^K \gamma_K \left( \frac{I_{t+1}}{I_t} - a \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \xi_{t+1}^i \right\} \end{aligned} \tag{33}$$

#### 2.4.2 Housing goods

The structure of housing producers is similar to that of capital good producers with the difference that housing goods also face investment adjustment costs in the form of time to build Kydland and Prescott (1982) and Uribe and Yue (2006). As such, there is a continuum of competitive housing firm producers who authorize housing investment projects  $I_t^{AH}$  in period  $t$ , which will increase housing stock  $N_H$  periods later, the time it takes to build.<sup>4</sup> Thus, the law of motion for the aggregate stock of housing in  $H_t$  will consider projects authorized  $N_H$  periods before, and

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<sup>4</sup>Notice that if  $N_H = 0$ , the structure is symmetric to the capital producers.

includes investment adjustment costs,

$$H_t = (1 - \delta_H) H_{t-1} + \left[ 1 - \frac{\gamma_H}{2} \left( \frac{I_{t-N_H}^{AH}}{I_{t-N_H-1}^{AH}} - a \right)^2 \right] \xi_t^{ih} I_{t-N_H}^{AH}$$

where  $\xi_t^{ih}$  is a shock to housing investment efficiency, and the sector covers all demand for new housing,  $H_t - (1 - \delta_H) H_{t-1}$ , by selling units at price  $Q_t^H$ .

The firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price  $P_t$ ) by the firm in  $t$  to produce housing is given by

$$I_t^H = \sum_{j=0}^{N_H} \varphi_j^H I_{t-j}^{AH}$$

Where  $\varphi_j^H$  (the fraction of projects authorized in period  $t-j$  that is outlaid in period  $t$ ) satisfy  $\sum_{j=0}^{N_H} \varphi_j^H = 1$  and  $\varphi_j^H = \rho^{\varphi H} \varphi_{j-1}^H$ .<sup>5</sup>

Therefore a representative housing producer chooses how much to authorize in new projects  $I_t^{AH}$  in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^H \left[ 1 - \frac{\gamma_H}{2} \left( \frac{I_{t-N_H+i}^{AH}}{I_{t-N_H+i-1}^{AH}} - a \right)^2 \right] \xi_{t-N_H+i}^{ih} I_{t-N_H+i}^{AH} - P_{t+i} I_{t+i}^H \right\}$$

Where discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of housing to the level of housing investment

$$\begin{aligned} E_t \sum_{j=0}^{N_H} r_{t,t+j} \varphi_j^H P_{t+j} &= E_t r_{t,t+N_H} Q_{t+N_H}^H \left\{ \left[ 1 - \frac{\gamma_H}{2} \left( \frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right)^2 \right] - \gamma_H \left( \frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right) \frac{I_t^{AH}}{I_{t-1}^{AH}} \right\} \xi_t^{ih} \\ &\quad + E_t r_{t,t+N_H+1} Q_{t+N_H+1}^H \left\{ \gamma_H \left( \frac{I_{t+1}^{AH}}{I_t^{AH}} - a \right) \left( \frac{I_{t+1}^{AH}}{I_t^{AH}} \right)^2 \xi_{t+1}^{ih} \right\} \end{aligned} \quad (34)$$

#### 2.4.3 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts  $X_t^H$  and  $X_t^F$ , respectively, and combines them according to the following technology:

$$Y_t^C = \left[ \omega^{1/\eta} (X_t^H)^{1-1/\eta} + (1-\omega)^{1/\eta} (X_t^F)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \quad (35)$$

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<sup>5</sup>Notice that  $\rho^{\varphi H} > 1$  implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for  $\rho^{\varphi H} < 1$ .

where  $\omega \in (0, 1)$  is inversely related to the degree of home bias and  $\eta > 0$  measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by  $P_t$ , while the prices of the domestic and foreign inputs are  $P_t^H$  and  $P_t^F$ , respectively. Subject to the technology constraint (35), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X_t^H, X_t^F} P_t \left[ \omega^{1/\eta} (X_t^H)^{1-1/\eta} + (1-\omega)^{1/\eta} (X_t^F)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} - P_t^H X_t^H - P_t^F X_t^F$$

The first-order conditions of this problem determine the optimal input demands:

$$X_t^H = \omega \left( \frac{P_t^H}{P_t} \right)^{-\eta} Y_t^C \quad (36)$$

$$X_t^F = (1-\omega) \left( \frac{P_t^F}{P_t} \right)^{-\eta} Y_t^C \quad (37)$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t = \left[ \omega (P_t^H)^{1-\eta} + (1-\omega) (P_t^F)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (38)$$

#### 2.4.4 Home composite goods

A representative home composite goods firm demands home goods of all varieties  $j \in [0, 1]$  in amounts  $X_{jt}^H$  and combines them according to the technology

$$Y_t^H = \left[ \int_0^1 (X_{jt}^H)^{\frac{\epsilon_H-1}{\epsilon_H}} dj \right]^{\frac{\epsilon_H}{\epsilon_H-1}} \quad (39)$$

with  $\epsilon_H > 0$ . Let  $P_{jt}^H$  denote the price of the home good of variety  $j$ . Subject to the technology constraint (39), the firm maximizes its profits  $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$  over the input demands  $X_{jt}^H$  taking prices as given:

$$\max_{X_{jt}^H} P_t^H \left[ \int_0^1 (X_{jt}^H)^{\frac{\epsilon_H-1}{\epsilon_H}} dj \right]^{\frac{\epsilon_H}{\epsilon_H-1}} - \int_0^1 P_{jt}^H X_{jt}^H dj$$

This implies the following first-order conditions for all  $j$ :

$$\partial X_{jt}^H : P_t^H (Y_t^H)^{1/\epsilon_H} (X_{jt}^H)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left( \frac{P_{jt}^H}{P_t^H} \right)^{-\epsilon_H} Y_t^H \quad (40)$$

Substituting (40) into (39) yields the price of home composite goods:

$$P_t^H = \left[ \int_0^1 (P_{jt}^H)^{1-\epsilon_H} dj \right]^{\frac{1}{1-\epsilon_H}} \quad (41)$$

#### 2.4.5 Home goods of variety $j$

There is a continuum of  $j$ 's firms, with measure one, that demand a domestic wholesale good  $X_t^Z$  and differentiate into home goods varieties  $Y_{jt}^H$ . To produce one unit of variety  $j$ , firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \quad (42)$$

The firm producing variety  $j$  satisfies the demand given by (40) but it has monopoly power for its variety. For varieties, the nominal marginal cost in terms of the composite good price is given by  $P_t^H mc_{jt}^H$ . Given that, every firm buys their input from the same wholesale market. It implies that all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \quad (43)$$

Given nominal marginal costs  $P_t^H mc_{jt}^H$ , firm  $j$  chooses its price  $P_{jt}^H$  to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability  $1 - \theta_H$ , and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights  $\kappa_H \in [0, 1]$  and  $1 - \kappa_H$  respectively. A firm reoptimizing in period  $t$  will choose the price  $\tilde{P}_{jt}^H$  that maximizes the current market value of the profits generated until it can reoptimize again.<sup>6</sup> As the firms are owned by the households, profits are discounted using the households' stochastic discount factor for nominal payoffs,  $r_{t,t+s}$ . A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} (P_{jt+s}^H - P_{t+s}^H mc_{jt+s}^H) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left( \frac{\tilde{P}_{jt}^H \prod_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H} \right)^{-\epsilon_H} Y_{t+s}^H$$

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<sup>6</sup>Therefore, the following relation holds:  $P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \dots \pi_{t+s}^{I,H}$ , where  $\pi_t^{I,H} = (\pi_{t-1}^H)^{\kappa_H} (\pi_t^T)^{1-\kappa_H}$ ,  $\pi_t^H = P_t^H / P_{t-1}^H$ , and  $\pi_t^T$  denotes the inflation target in period  $t$ .

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[ \left( \tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} (P_{t+s}^H)^{\epsilon_H} - mc_{jt+s}^H \left( \tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} (P_{t+s}^H)^{1+\epsilon_H} \right] Y_{t+s}^H$$

The first-order conditions determining the optimal price  $\tilde{P}_t^H$  can be written as follows:<sup>7</sup>

$$\begin{aligned} 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[ (1 - \epsilon_H) \left( \tilde{P}_t^H \right)^{-\epsilon_H} \left( \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} (P_{t+s}^H)^{\epsilon_H} \right. \\ &\quad \left. + \epsilon_H mc_{t+s}^H \left( \tilde{P}_t^H \right)^{-\epsilon_H-1} \left( \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} (P_{t+s}^H)^{1+\epsilon_H} \right] Y_{t+s}^H \\ \Leftrightarrow 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[ \frac{\epsilon_H - 1}{\epsilon_H} \left( \tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \frac{(P_{t+s}^H)^{\epsilon_H}}{P_t^H} \right. \\ &\quad \left. - mc_{t+s}^H \left( \tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \frac{(P_{t+s}^H)^{1+\epsilon_H}}{P_t^H} \right] Y_{t+s}^H \\ \Leftrightarrow 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[ \frac{\epsilon_H - 1}{\epsilon_H} \left( \tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left( \frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} \right. \\ &\quad \left. - mc_{t+s}^H \left( \tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left( \frac{P_{t+s}^H}{P_t^H} \right)^{1+\epsilon_H} \right] Y_{t+s}^H \end{aligned}$$

where the second step follows from multiplying both sides by  $-\tilde{P}_t^H / (P_t^H \epsilon_H)$ , and the third by defining  $\tilde{p}_t^H = \tilde{P}_t^H / P_t^H$ .

The first-order condition can be rewritten in recursive form as follows, defining  $F_t^{H_1}$  as

$$\begin{aligned} F_t^{H_1} &= \frac{\epsilon_H - 1}{\epsilon_H} (\tilde{p}_t^H)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} \frac{\epsilon_H - 1}{\epsilon_H} \left( \tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left( \frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} Y_{t+s}^H \\ &= \frac{\epsilon_H - 1}{\epsilon_H} (\tilde{p}_t^H)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \left( \tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left( \frac{P_{t+s+1}^H}{P_t^H} \right)^{\epsilon_H} Y_{t+s+1}^H \\ &= \frac{\epsilon_H - 1}{\epsilon_H} (\tilde{p}_t^H)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} (\pi_{t+1}^H)^{\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \right. \\ &\quad \times \left. \left( \tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H} \right)^{1-\epsilon_H} \left( \frac{P_{t+s+1}^H}{P_{t+1}^H} \right)^{\epsilon_H} Y_{t+s+1}^H \right\} \\ &= \frac{\epsilon_H - 1}{\epsilon_H} (\tilde{p}_t^H)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} (\pi_{t+1}^H)^{\epsilon_H} F_{t+1}^{H_1} \right\} \end{aligned} \tag{44}$$

<sup>7</sup>Notice that the subscript  $j$  has been removed from  $\tilde{P}_t^H$ ; this simplifies notation and underlines that the prices chosen by all firms  $j$  that reset prices optimally in a given period are equal as they face the same problem by (43).

and, analogously,  $F_t^{H_2}$  as

$$\begin{aligned}
F_t^{H_2} &= (\tilde{p}_t^H)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} mc_{t+s}^H \left( \tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left( \frac{P_{t+s}^H}{P_t^H} \right)^{1+\epsilon_H} Y_{t+s}^H \\
&= (\tilde{p}_t^H)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} mc_{t+s+1}^H \left( \tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left( \frac{P_{t+s+1}^H}{P_t^H} \right)^{1+\epsilon_H} Y_{t+s+1}^H \\
&= (\tilde{p}_t^H)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} (\pi_{t+1}^H)^{1+\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} mc_{t+s+1}^H \right. \\
&\quad \times \left. \left( \tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H} \right)^{-\epsilon_H} \left( \frac{P_{t+s+1}^H}{P_{t+1}^H} \right)^{1+\epsilon_H} Y_{t+s+1}^H \right\} \\
&= (\tilde{p}_t^H)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} (\pi_{t+1}^H)^{1+\epsilon_H} F_{t+1}^{H_2} \right\}
\end{aligned} \tag{45}$$

such that

$$F_t^{H_1} = F_t^{H_2} = F_t^H \tag{46}$$

Using (41), we have

$$\begin{aligned}
1 &= \int_0^1 \left( \frac{P_{jt}^H}{P_t^H} \right)^{1-\epsilon_H} dj \\
&= (1 - \theta_H) (\tilde{p}_t^H)^{1-\epsilon_H} + \theta_H \left( \frac{P_{t-1}^H \pi_t^{I,H}}{P_t^H} \right)^{1-\epsilon_H} \\
&= (1 - \theta_H) (\tilde{p}_t^H)^{1-\epsilon_H} + \theta_H \left( \frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H}
\end{aligned} \tag{47}$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period  $t$  corresponds to the distribution of aggregate prices in period  $t-1$ , though with total mass reduced to  $\theta_H$ .

#### 2.4.6 Wholesale Domestic Goods

There is a representative firm producing a homogeneous wholesale home good, combining capital and labor according to the following technology:

$$Y_t^Z = z_t K_{t-1}^\alpha (A_t \tilde{n}_t)^{1-\alpha} \tag{48}$$

with capital share  $\alpha \in (0, 1)$ , an exogenous stationary technology shock  $z_t$  and a non-stationary technology  $A_t$ . Production of the wholesale good composite labor services  $\tilde{n}_t$  and capital  $K_{t-1}$ . Additionally, following Lechthaler and Snower (2010), the firm faces a quadratic adjustment costs of labor which is a function of parameter  $\gamma_n$ , and of

aggregate wholesale domestic goods  $\tilde{Y}_t^Z$ , which in equilibrium are equal to  $Y_t^Z$  and which the representative firm takes as given. In a first stage, the firm hires composite labor and rents capital to solve the following problem:

$$\begin{aligned} \min_{\tilde{n}_{t+s}, K_{t+s-1}} & \sum_{s=0}^{\infty} r_{t,t+s} \left\{ W_{t+s} \tilde{n}_{t+s} + \frac{\gamma_n}{2} \left( \frac{\tilde{n}_{t+s}}{\tilde{n}_{t+s-1}} - 1 \right)^2 \widetilde{Y_{t+s}^Z} P_t^Z + R_t K_{t+s-1} \right\} \\ \text{s.t. } & Y_{t+s}^Z = X_{t+s}^Z = z_{t+s} K_{t+s-1}^\alpha (A_{t+s} \tilde{n}_{t+s})^{1-\alpha} \end{aligned}$$

Then, the optimal capital and labor demands are given by:

$$\tilde{n}_t = (1 - \alpha) \left\{ \frac{mc_t^Z Y_t^Z}{W_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) \widetilde{Y_t^Z} P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) \widetilde{Y_{t+1}^Z} P_{t+1}^Z} \right\} \quad (49)$$

$$K_{t-1} = \alpha \left( \frac{mc_t^Z}{R_t^k} \right) Y_t^Z \quad (50)$$

Where  $mc_t^Z$  is the lagrangian multiplier on the production function and  $r_{t,t+1}$  the households' stochastic discount factor between periods  $t$  and  $t + 1$ . The, combining both optimality conditions:

$$\frac{K_{t-1}}{\tilde{n}_t} = \frac{\alpha}{(1 - \alpha) R_t^k} \left\{ W_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) \widetilde{Y_t^Z} P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) \widetilde{Y_{t+1}^Z} P_{t+1}^Z \right\}$$

Substituting (49) and (50) into (48) we obtain an expression for the real marginal cost in units of the wholesale domestic good:

$$\begin{aligned} mc_t^Z = & \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{(R_t^k)^\alpha}{z_t A_t^{1-\alpha}} \left\{ W_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) \widetilde{Y_t^Z} P_t^Z \right. \\ & \left. - r_{t,t+1} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) \widetilde{Y_{t+1}^Z} P_{t+1}^Z \right\}^{1-\alpha} \end{aligned}$$

In a second stage, the wholesale firm maximize its profits from the production of  $Y_t^Z$ , which is sold as  $X_t^Z$  at  $P_t^Z$ . The problem is:

$$\max_{Y_t^Z} (P_t^Z - mc_t^Z) Y_t^Z$$

The first-order condition implies that

$$P_t^Z = mc_t^Z.$$

#### 2.4.7 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties  $j \in [0, 1]$  in amounts  $X_{jt}^F$  and combines them according to the technology

$$Y_t^F = \left[ \int_0^1 (X_{jt}^F)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj \right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \quad (51)$$

with  $\epsilon_F > 0$ . Let  $P_{jt}^F$  denote the price of the foreign good of variety  $j$ . Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left( \frac{P_{jt}^F}{P_t^F} \right)^{-\epsilon_F} Y_t^F \quad (52)$$

for all  $j$ , and substituting (52) into (51) yields the price of foreign composite goods:

$$P_t^F = \left[ \int_0^1 (P_{jt}^F)^{1-\epsilon_F} dj \right]^{\frac{1}{1-\epsilon_F}} \quad (53)$$

#### 2.4.8 Foreign goods of variety $j$

Importing firms buy an amount  $M_t$  of a homogeneous foreign good at the price  $P_t^{M*}$  abroad and convert this good into varieties  $Y_{jt}^F$  that are sold domestically, and where total imports are  $\int_0^1 Y_{jt}^F dj$ . We assume that the import price level  $P_t^{M*}$  cointegrates with the foreign producer price level  $P_t^*$ , i.e.,  $P_t^{M*} = P_t^* \xi_t^m$ , where  $\xi_t^m$  is a stationary exogenous process. The firm producing variety  $j$  satisfies the demand given by (52) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety  $j$ , nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M*} = S_t P_t^* \xi_t^m \quad (54)$$

Given marginal costs, the firm producing variety  $j$  chooses its price  $P_{jt}^F$  to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability  $1 - \theta_F$ , and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights  $\kappa_F \in [0, 1]$  and  $1 - \kappa_F$  respectively. A firm reoptimizing in period  $t$  will choose the price  $\tilde{P}_{jt}^F$  that maximizes the current market value of the profits generated until it can reoptimize.<sup>8</sup> The solution to this problem is analogous to the case of domestic varieties, implying the

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<sup>8</sup>As in the home varieties case, the following relation holds:  $P_{jt+s}^F = \tilde{P}_{jt}^F \pi_t^{I,F} \dots \pi_{t+s}^{I,F}$ , where  $\pi_t^{I,F} = (\pi_{t-1}^F)^{\kappa_F} (\pi_t^T)^{1-\kappa_F}$ , and, in turn,  $\pi_t^F = P_t^F / P_{t-1}^F$ .

first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \quad (55)$$

where, defining  $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$ ,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F} (\tilde{p}_t^F)^{1-\epsilon_F} Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} (\pi_{t+1}^F)^{\epsilon_F} F_{t+1}^{F_1} \right\}$$

and

$$F_t^{F_2} = (\tilde{p}_t^F)^{-\epsilon_F} m c_t^F Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left( \frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} (\pi_{t+1}^F)^{1+\epsilon_F} F_{t+1}^{F_2} \right\}$$

Using (53), we further have

$$1 = (1 - \theta_F) (\tilde{p}_t^F)^{1-\epsilon_F} + \theta_F \left( \frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \quad (56)$$

#### 2.4.9 Wages

Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties  $i \in [0, 1]$  of labor services in amounts  $n_t(i)$  and combine them in order to produce composite labor services  $\tilde{n}_t$ . The production function, variety  $i$  demand, and aggregate nominal wage are respectively given by:

$$\tilde{n}_t = \left[ \int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \quad \epsilon_W > 0. \quad (57)$$

$$n_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\epsilon_W} \tilde{n}_t \quad (58)$$

$$W_t = \left[ \int_0^1 W_t(i)^{1-\epsilon_W} di \right]^{\frac{1}{1-\epsilon_W}}. \quad (59)$$

Regarding the supply of differentiated labor, as in Erceg et al. (2010), there is a continuum of monopolistically competitive unions indexed by  $i \in [0, 1]$ , which act as wage setters for the differentiated labor services supplied by households. These unions allocate labor demand uniformly across patient and impatient households, so  $n_t^P(i) = n_t^I(i)$  and  $n_t^P(i) + n_t^I(i) = n_t(i) \forall i, t$ , with  $n_t^P(i) = \varphi_U n_t^U(i) + (1 - \varphi_U) n_t^R(i)$ , which also holds for the aggregate  $n_t^P, n_t^I$  and  $n_t$ .

The union supplying variety  $i$  satisfies the demand given by (58) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability  $1 - \theta_W$ . The wages of unions that cannot optimally adjust, are indexed to a weighted average of past and steady state productivity and inflation, with a gross growth rate of

$$\pi_t^{I,W} \equiv a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W}$$

Where  $\Gamma_{t,s}^W = \Pi_{i=1}^s \pi_{t+i}^{I,W}$  is the growth of indexed wages  $s$  periods ahead of  $t$ . A union reoptimizing in period  $t$  chooses the wage  $\tilde{W}_t$  (equal for patient and impatient households) that maximizes the households' discounted lifetime utility. This union weights the benefits of wage income by considering the agents' marginal utility of consumption –which will usually differ between patient and impatient households– and weighs each household equally by considering a lagrangian multiplier of  $\lambda_t^W = (\lambda_t^P + \lambda_t^I)/2$ , with  $\lambda_t^P = \varphi_U \lambda_t^U + (1 - \varphi_U) \lambda_t^R$ . We assume, for the sake of simplicity, that  $\beta_W = (\beta_P + \beta_I)/2$  with  $\beta_P = \varphi_U \beta_U + (1 - \varphi_U) \beta_R$ , and  $\Theta_t = (\Theta_t^P + \Theta_t^I)/2$  with  $\Theta_t^P = \varphi_U \Theta_t^U + (1 - \varphi_U) \Theta_t^R$ .

All things considered, taking the aggregate nominal wage as given, the union  $i$ 's maximization problem can be expressed as

$$\begin{aligned} & \max_{\tilde{W}_t(i)} E_t \sum_{s=0}^{\infty} (\beta_U \theta_W)^s \varrho_{t+s} \left( \frac{\lambda_{t+s}^U A_{t+s}^{-\sigma}}{P_{t+s}} \tilde{W}_t \Gamma_{t,s}^W n_{t+s}(i) - \Theta_{t+s} (A_{t+s})^{1-\sigma} \xi_{t+s}^n \frac{n_{t+s}(i)^{1+\varphi}}{1+\varphi} \right), \\ & \text{s.t. } n_{t+s}(i) = \left( \frac{\tilde{W}_t \Gamma_{t,s}^W}{W_{t+s}} \right)^{-\epsilon_W} \tilde{n}_{t+s}, \end{aligned}$$

Which, after some derivation, results in the FOCs in a recursive formulation:

$$\begin{aligned} f_t^{W1} &= \tilde{w}_t^{1-\epsilon_W} \left( \frac{\epsilon_W - 1}{\epsilon_W} \right) \tilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left( \frac{\pi_{t+1}^{\tilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^{W1} \right\} \\ f_t^{W2} &= \tilde{w}_t^{-\epsilon_W(1+\varphi)} m c_t^W \tilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left( \frac{\pi_{t+1}^{\tilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^{W2} \right\} \end{aligned}$$

Where  $f_t^{W1} = f_t^{W2} = f_t^W$  are the LHS and RHS of the FOC respectively,  $m c_t^W = -(U_n/U_C)/(W_t/A_t P_t) = \xi_t^n(\tilde{n}_t)^\varphi / \lambda_t^U (\frac{A_t P_t}{W_t}) \Theta_t$ , is the gap with the efficient allocation when wages are flexible<sup>9</sup>,  $\pi_{t+1}^W = W_{t+1}/W_t$ ,  $\pi_{t+1}^{\tilde{W}} = \tilde{W}_{t+1}/\tilde{W}_t$  and  $\tilde{w}_t = \tilde{W}_t/W_t$ .

Further, let  $\Psi^W(t)$  denote the set of labor markets in which wages are not reoptimized in period  $t$ . By (59), the aggregate wage index  $W_t$  evolves as follows:

$$\begin{aligned} (W_t)^{1-\epsilon_W} &= \int_0^1 W_t(i)^{1-\epsilon_W} di = (1 - \theta_W) \left( \tilde{W}_t \right)^{1-\epsilon_W} + \int_{\Psi^W(t)} \left[ W_{t-1}(i) \pi_t^{I,W} \right]^{1-\epsilon_W} di, \\ &= (1 - \theta_W) \left( \tilde{W}_t \right)^{1-\epsilon_W} + \theta_W \left[ W_{t-1} \pi_t^{I,W} \right]^{1-\epsilon_W}, \end{aligned}$$

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<sup>9</sup> $U_n$  and  $U_C$  are the first derivatives of the utility function with respect to labor and consumption respectively.

or, dividing both sides by  $(W_t)^{1-\epsilon_W}$ :

$$1 = (1 - \theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left( \frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W}.$$

The third equality above follows from the fact that the distribution of wages that are not reoptimized in period  $t$  corresponds to the distribution of effective wages in period  $t - 1$ , though with total mass reduced to  $\theta_W$ .

Finally, the clearing condition for the labor market is

$$n_t = \int_0^1 n_t(i) di = \tilde{n}_t \int_0^1 \left( \frac{W_t(i)}{W_t} \right)^{-\epsilon_W} di = \tilde{n}_t \Xi_t^W,$$

Where  $\Xi_t^W$  is a wage dispersion term that satisfies

$$\Xi_t^W = (1 - \theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_W \left( \frac{\pi_t^{I,W}}{\pi_t^W} \right)^{-\epsilon_W} \Xi_{t-1}^W.$$

#### 2.4.10 Commodities

We assume the country receives an exogenous and stochastic endowment of commodities  $Y_t^{Co}$ . Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price  $P_t^{Co*}$ , which is assumed to evolve exogenously. We further assume that the government receives a share  $\chi \in [0, 1]$  of this income and the remaining share goes to foreign agents.

### 2.5 Fiscal and monetary policy

The government consumes an exogenous stream of final goods  $G_t$ , pays through an insurance agency  $IA_t$  for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households  $T_t^P$ , and issues one-period bonds  $BS_t^G$  and long-term bonds  $BL_t^G$ . Hence, the government satisfies the following period-by-period constraint:

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co*} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \quad (60)$$

where

$$T_t = \alpha^T GDPN_t + \epsilon_t (BS_{SS}^G - BS_t^G + Q_{SS}^{BL} BL_{SS}^G - Q_t^{BL} BL_t^G) \quad (61)$$

and

$$IA_t = \gamma_D PD_t^D R_{t-1}^D D_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} Q_t^{BB} BB_{t-1}^{Pr} \quad (62)$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on  $BL_t^G$ . In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\alpha_R} \left[ \left( \frac{(1 - \alpha_E) \pi_t + \alpha_E \mathbb{E}_t \{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left( \frac{GDP_t/GDP_{t-1}}{a} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \quad (63)$$

where  $\alpha_R \in [0, 1]$ ,  $\alpha_\pi > 1$ ,  $\alpha_y \geq 0$ ,  $\alpha_E \in [0, 1]$  and where  $\pi_t^T$  is an exogenous inflation target and  $e_t^m$  an i.i.d. shock that captures deviations from the rule.<sup>10</sup>

## 2.6 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level  $P_t^*$  is identical to the foreign consumption-based price index. Further, let  $P_t^{H*}$  denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e.  $P_t^H = S_t P_t^{H*}$  and  $P_t^{Co} = S_t P_t^{Co*}$ . That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e.,  $P_t^F mc_t^F = S_t P_t^* \xi_t^m$  from (54). The real exchange rate  $rer_t$  therefore satisfies

$$rer_t = \frac{S_t P_t^*}{P_t} = \frac{P_t^F}{P_t} \frac{mc_t^F}{\xi_t^m} \quad (64)$$

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^*}{\pi_t} \quad (65)$$

where  $\pi_t^s = S_t / S_{t-1}$ . Foreign demand for the home composite good  $X_t^{H*}$  is given by

$$X_t^{H*} = \left( \frac{P_t^H}{S_t P_t^*} \right)^{-\eta^*} Y_t^* \quad (66)$$

with  $\eta^* > 0$  and where  $Y_t^*$  denotes foreign aggregate demand or GDP. Both  $Y_t^*$  and  $\pi_t^*$  evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate  $R_t^W$  plus a country

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<sup>10</sup>We do not need a time-varying target, so we will set it to a constant.

premium that decreases with the economy's net foreign asset position (expressed as a ratio of nominal GDP):

$$R_t^* = R_t^W \exp \left\{ -\frac{\phi^*}{100} \left( \frac{S_t B_t^*}{GDPN_t} - \bar{b} \right) \right\} \xi_t^R z_t^R \quad (67)$$

with  $\phi^* > 0$  and where  $\xi_t^R$  is an exogenous shock to the country premium.

## 2.7 Aggregation and Market Clearing

### 2.7.1 Aggregation across patient households

Aggregate variables add up the per-capita amounts from unrestricted and restricted patient households, according to their respective mass  $\varphi_U$  and  $1 - \varphi_U$ :

$$C_t^P = \varphi_U C_t^U + (1 - \varphi_U) C_t^R$$

$$H_t^P = \varphi_U H_t^U + (1 - \varphi_U) H_t^R$$

$$n_t^P = \varphi_U n_t^U + (1 - \varphi_U) n_t^R$$

$$n_t^U = n_t^R$$

$$D_t^{Tot} = \varphi_U D_t^U$$

$$B_t^{*,Tot} = \varphi_U B_t^{*,U}$$

$$BS_t^{Pr} = \varphi_U BS_t^U$$

$$BL_t^{Pr} = \varphi_U BL_t^U + (1 - \varphi_U) BL_t^R$$

$$BB_t^{Pr} = \varphi_U BB_t^U$$

### 2.7.2 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \tau_t/P_t \quad (68)$$

where  $\Upsilon_t$  includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and the cost of adjusting labor.

$$\begin{aligned}\Upsilon_t = & \gamma_D P D_t^B R_{t-1}^D D_{t-1}^{Tot} + \gamma_D P D_t^B Q_t^{BB} R_t^{BB} B B_{t-1}^{Pr} + \mu_e G_e (\bar{\omega}_t^e) R_t^e Q_{t-1}^K K_{t-1} + \mu_I G_I (\bar{\omega}_t^I) R_t^H Q_{t-1}^H H_{t-1}^I \\ & + \mu_H G_H (\bar{\omega}_t^H) \tilde{R}_t^H Q_{t-1}^L L_{t-1}^H + \mu_F G_F (\bar{\omega}_t^F) \tilde{R}_t^F L_{t-1}^F + \frac{\gamma_n}{2} \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right)^2 Y_t^Z\end{aligned}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H*} \quad (69)$$

and

$$Y_t^F = X_t^F \quad (70)$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^H = X_{jt}^H \quad (71)$$

and

$$Y_{jt}^F = X_{jt}^F \quad (72)$$

for all  $j$ .

In the market for the wholesale domestic good, we have

$$Y_t^Z = X_t^Z \quad (73)$$

Finally, in the market for housing, demand from both households must equal supply from housing producers:

$$H_t = H_t^P + H_t^I \quad (74)$$

### 2.7.3 Factor market clearing

In the market for labor, the clearing conditions are:

$$n_t^P + n_t^I = n_t = \tilde{n}_t \Xi_t^W \quad (75)$$

$$n_t^P = n_t^I = \frac{n_t}{2} \quad (76)$$

Combining (50) and (49), the capital-labor ratio satisfies:

$$\frac{K_{t-1}}{\tilde{n}_t} = \frac{\alpha}{(1-\alpha)R_t^k} \left\{ W_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) Y_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) Y_{t+1}^Z P_{t+1}^Z \right\} \quad (77)$$

#### 2.7.4 Deposits clearing

Bank F takes deposits, and its demand must equal the supply from unrestricted households:

$$D_t^F = D_t^{Tot} \quad (78)$$

#### 2.7.5 Domestic bonds clearing

The aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \quad (79)$$

$$BS_t^{Pr} + BS_t^G = 0 \quad (80)$$

Where  $BL_t^{CB}$  is an exogenous process that represents the long-term government bond purchases done by the Central Bank.

#### 2.7.6 No-arbitrage condition in bond markets

The no-arbitrage condition implies the following relation between short and long-term interest rates:

$$R_t \left( \frac{1 + \zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} \left( \frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \left( \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \right)^{-1}$$

which can be further rearranged (up to a first order) by using the definition of  $R_t^{BL}$

$$R_t (1 + \zeta_t^L) \approx \mathbb{E}_t \left\{ \left( \frac{Q_{t+1}^{BL}}{Q_t^{BL}} R_{t+1}^{BL} \right) \right\} \quad (81)$$

### 2.7.7 Inflation and relative prices

The following holds for  $j = H, F$ :

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

### 2.7.8 Aggregate supply

Using the productions of different varieties of home goods (42)

$$\int_0^1 Y_{jt}^H dj = X_t^Z$$

Integrating (71) over  $j$  and using (40) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 (p_{jt}^H)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = X_t^Z$$

where  $\Xi_t^H$  is a price dispersion term satisfying

$$\begin{aligned} \Xi_t^H &= \int_0^1 \left( \frac{P_{jt}^H}{P_t^H} \right)^{-\epsilon_H} dj \\ &= (1 - \theta_H) (\tilde{p}_t^H)^{-\epsilon_H} + \theta_H \left( \frac{\pi_t^{I,H}}{\pi_t^H} \right)^{-\epsilon_H} \Xi_{t-1}^H \end{aligned}$$

### 2.7.9 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to  $Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t$ . The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H*} + S_t P_t^{Co*} Y_t^{Co} - S_t P_t^{M*} M_t \quad (82)$$

Integrating (72) over  $j$  and using (52) allows us to write imports as

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left( \frac{P_{jt}^F}{P_t^F} \right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where  $\Xi_t^F$  is a price dispersion term satisfying

$$\Xi_t^F = (1 - \theta_F) (\tilde{p}_t^F)^{-\epsilon_F} + \theta_F \left( \frac{\pi_t^{I,F}}{\pi_t^F} \right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real GDP as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP,  $Y_t^{NoCo}$ , is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t (C_t^P + C_t^I + I_t + I_t^H + G_t) + TB_t \quad (83)$$

Note that by combining (83) with the zero profit condition in the final goods sector, i.e.,  $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$ , and using the market clearing conditions for final and composite goods, (68)-(69), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

### 2.7.10 Balance of payments

Aggregate nominal profits, dividends, rents and taxes are given by

$$\begin{aligned}
\Psi_t &= \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\
&\quad + \underbrace{\int_0^1 Y_{jt}^H (P_{jt}^H - P_t^Z) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 (P_{jt}^F Y_{jt}^F - S_t P_t^{M*} Y_{jt}^F) dj}_{\int_0^1 \Pi_{jt}^F dj} \\
&\quad + \underbrace{Q_t^K (K_t - (1 - \delta_K) K_{t-1}) - P_t I_t}_{\Pi_t^I} + \underbrace{Q_t^H (H_t - (1 - \delta_H) H_{t-1}) - P_t I_t^H}_{\Pi_t^{I^H}} + \underbrace{(P_t^Z - mc_t^Z) Y_t^Z}_{\Pi_t^Z} + \underbrace{\zeta_t^L \left( \frac{1}{R_t^{BL} - \kappa_B} \right) BL_t^U}_{\Pi_t^F} \\
&\quad + C_t^e + C_t^b + S_t REN_t^* - T_t \\
&= P_t (C_t + G_t) + \Upsilon_t + P_t^H X_t^{H*} - S_t P_t^{M*} M_t - W_t n_t - R_t^k K_{t-1} \\
&\quad + Q_t^K (K_t - (1 - \delta_K) K_{t-1}) + Q_t^H (H_t - (1 - \delta_H) H_{t-1}) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left( \frac{1}{R_t^{BL} - \kappa_B} \right) BL_t^U \\
&= P_t (C_t + G_t) + \Upsilon_t + TB_t - S_t P_t^{Co*} Y_t^{Co} - W_t n_t - R_t^k K_{t-1} \\
&\quad + Q_t^K (K_t - (1 - \delta_K) K_{t-1}) + Q_t^H (H_t - (1 - \delta_H) H_{t-1}) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left( \frac{1}{R_t^{BL} - \kappa_B} \right) BL_t^U
\end{aligned}$$

Where the second equality uses the market clearing conditions (68)-(80), and the third equality uses the definition of the trade balance, (82). Substituting out  $\Psi_t$  in the households' budget constraint (5) and using the government's budget constraint (60) to substitute out taxes  $T_t$  shows that the net foreign asset position evolves according to

$$S_t B_t^* = S_t B_{t-1}^* R_{t-1}^* + TB_t + S_t REN_t^* - (1 - \chi) S_t P_t^{Co*} Y_t^{Co}$$

## 3 Parameterization strategy and estimation results

The model parameters are calibrated and estimated. The calibrated parameters include those characterizing model dynamics for which we have a data counterpart, those drawn from related studies, and those chosen to match the Chilean economy's sample averages or long-run ratios. In particular, we follow closely the calibration strategy from Garcia et al. (2019) and Clerc et al. (2014), as the models described there form the basis of this paper's framework. We estimate the non-calibrated parameters using Bayesian techniques as discussed below.

### 3.1 Calibration

Table 1 presents the values of the parameters related to the real sector of the economy that are either chosen from previous studies in the relevant literature or chosen in order to match exogenous steady state moments. The value of the parameters  $\alpha$ ,  $\alpha_E$ ,  $\beta_U$ ,  $\beta_R$ ,  $\chi$ ,  $\epsilon_F$ ,  $\epsilon_H$ ,  $\epsilon_W$ ,  $\omega$  and  $\pi^T$  are taken from Garcia et al. (2019). We assume that the housing capital depreciation rate,  $\delta_H$  is equal to the productive capital depreciation rate,  $\delta_K$ , whose value is taken from Adolfson et al. (2013). The value for  $\beta_I$  is taken from Clerc et al. (2014).

Table 1: Calibration, Real Sector

Parameter	Description	Value	Source
$\alpha$	Capital share in production function	0.34	Garcia et al. (2019)
$\alpha_E$	Expected Inflation weight in Taylor Rule	0.50	Garcia et al. (2019)
$\alpha^{BSG}$	Short-term govt. bonds as percentage of GDP	-0.40	Data: 2009-2019
$\alpha^{BLG}$	Long-term govt. bonds as percentage of GDP	-4.50	Data: 2009-2019
$\beta_U, \beta_U$	Patient HH Utility Discount Factors	0.9997	Garcia et al. (2019)
$\beta_I$	Impatient Utility HH Discount Factor	0.98	Clerc et al. (2014)
$\delta_K$	Capital Annual depreciation rate	0.01	Adolfson et al. (2013)
$\delta_H$	Housing Annual Depreciation rate	0.01	Same as capital depreciation
$\epsilon_F$	Elasticity of substitution among foreign varieties	11	Garcia et al. (2019)
$\epsilon_H$	Elasticity of substitution among home varieties	11	Garcia et al. (2019)
$\epsilon_W$	Elasticity of substitution among types of workers	11	Garcia et al. (2019)
$\epsilon_\tau$	Convergence speed towards SS Gov debt	0.10	Normalization
$N_H$	Time-to-build periods in housing goods	6	CBC's 2018S2 Financial report
$\kappa$	Coupon discount in housing loans	0.98	10 years duration of loan contract
$\kappa_{BL}$	Coupon discount in long term government bonds	0.98	10 years bond duration
$\kappa_{BB}$	Coupon discount in long term banking bonds	0.95	5 years bond duration
$\pi_t^T$	Annual inflation target of 3%	$1.03^{1/4}$	Garcia et al. (2019)
$\rho_{\varphi h}$	Spending profile for long term housing investment	1	Even investment distribution
$\sigma$	Log Utility	1	Garcia et al. (2019)
$v$	Strength of households wealth effect	0	No wealth effect
$\chi$	Government share in commodity sector	0.33	Garcia et al. (2019)
$\omega$	Home bias in domestic demand	0.79	Garcia et al. (2019)
$\omega_U$	Fraction of unrestricted patient households	0.70	Chen et al. (2012)
$\omega_{BL}$	Ratio of long term assets to short assets	0.82	Chen et al. (2012)

The parameters that set the steady state value of short term and long term government bonds as a percentage of GDP,  $\alpha^{BSG}$  and  $\alpha^{BLG}$ , respectively, were calculated from data obtained from DCV<sup>11</sup>. The value used for the time that takes a house to be built,  $N_H$  is taken from the second semester of 2018 IEF.<sup>12</sup> The parameters that determine the coupons' geometric decline of the long term housing debt,  $\kappa$ , and government bonds,  $\kappa_{BL}$ , are set so their duration is 10 years. The duration of the bank bonds,  $\kappa_{BB}$ , is set to 5 years.

For the housing investment sector, we set the time to built duration, defined by the parameter  $N_H$ , to 6 quarters in order to match the average length of construction projects, and assume an even investment spending profile for housing capital, consistent with a value of 1 for  $\rho_{\varphi h}$ . Following Garcia et al. (2019), we set the value of the parameter that determines the strength of the wealth effect,  $v$ , to 0, to avoid undesired dynamics in the labor

<sup>11</sup>DCV is an entity that processes and registers transfer operations that take place in several exchange markets.

<sup>12</sup>IEF stands for Financial Stability Report published twice a year by the Central Bank of Chile.

market.

For the calibration of the parameters related to the financial sector, shown in Table 2, the values of  $\chi_b$ ,  $\chi_e$ ,  $\gamma_{bh}$ ,  $\gamma_d$ ,  $\mu_e$ ,  $\mu_F$ ,  $\mu_H$  and  $\mu_I$  come from Clerc et al. (2014). The values for the parameters related to bank capital requirements,  $\phi_F$  and  $\phi_H$ , are set as the ratio between the average level of TIER I capital of over the risk weighted assets of the banking system from the year 2000 to the year 2020. In particular, we calculate 4.3% excess of TIER I capital in addition to legal 8%. For corporate banks we assume 100% weight in corporate loans, while for housing bank we assume 60% weight in housing loans.

Table 2: Calibration, Financial Sector

Parameter	Description	Value	Source
$\chi_b$	Banks dividend policy	0.05	Clerc et al. (2015)
$\chi_e$	Entrepreneurs dividend policy	0.05	Clerc et al. (2015)
$\gamma_{bh}$	Household cost bank bonds default	0.10	Clerc et al. (2015)
$\gamma_d$	Cost of recovering defaulted bank deposits	0.10	Clerc et al. (2015)
$\mu_e$	Entrepreneurs bankruptcy cost	0.30	Clerc et al. (2015)
$\mu_F$	Corporate bank bankruptcy cost	0.30	Clerc et al. (2015)
$\mu_H$	Housing bank bankruptcy cost	0.30	Clerc et al. (2015)
$\mu_I$	Impatient Household bankruptcy cost	0.30	Clerc et al. (2015)
$\phi_F$	Bank Capital Requirement (RWA)	0.12	Data (2000-2020)
$\phi_H$	Bank Capital Requirement (RWA)	0.09	Data (2000-2020)

### 3.2 Estimation and Results

We compute the model solution by a linear approximation around the deterministic steady state. The parameters whose values are not calibrated are estimated using Bayesian methods. The data for the estimation, described in Table 3, includes 25 macroeconomic and financial variables from between 2001Q3 and 2019Q3. Data for the real Chilean sector is obtained from the Central Bank of Chile's National Accounts database, while prices and labor statistics are obtained from the National Statistics Institute (INE). Finally, local financial data is obtained from the Financial Markets Committee (CMF), and foreign data is obtained from Bloomberg. Variables regarding the real sector are log-differentiated with respect to the previous quarter. All variables are demeaned. Our estimation strategy also includes i.i.d. measurement errors for all local observables with the exception of the interest rate. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable.

The posterior estimates are obtained from a random walk Metropolis–Hasting chain with 1,000,000 draws after discarding the first 500,000 draws. To facilitate optimization, following Christiano et al. (2011), we scale some of the parameters for the shocks' standard deviations to have a similar posterior order of magnitude. For the prior selection, we follow the endogenous prior strategy used in Christiano et al. (2011) and Coenen et al. (2013), where the joint prior distribution of the estimated parameters is computed as the product of the initial prior distribution

Table 3: Observable Data

Non Financial	Financial
$\Delta \log Y_t^{NoCo}$	Non mining real GDP
$\Delta \log Y_t^{Co}$	Copper real GDP
$\Delta \log C_t$	Total Consumption
$\Delta \log G_t$	Goverment Consumption
$\Delta \log I_t^K$	Real Capital Investment
$\Delta \log I_t^H$	Real Housing Investment
$TB_t/GDPN_t$	Trade Balance-GDP Ratio
$\Delta \log N_t$	Total Employment
$\Delta \log WN_t$	Nominal Cost of labor
$\pi_t$	Core CPI
$\Delta \log y_t^*$	Real External GDP
$\pi_t^*$	Foreign Price Index
$\pi_t^M$	Imports Deflactor
$\pi_t^{Co*}$	Nominal Copper Price
$\pi_t^H$	Housing Price Index

Sources: INE, BCCh, CMF and Bloomberg.

and the likelihood that the model generated standard deviations match the volatility of the observed variables. We choose the type of priors according to the related literature from distributions that have supported distributions consistent with the theoretical values expected for the parameters. In columns three, four and five of Table (4) we show the chosen prior distributions and prior distribution moments of the estimated values of the deep parameters. The sixth and seventh columns of the same table show the posterior mean and the 90% interval of the estimation. On the other hand, on Table 5 we show the estimation priors and results of the parameters related to shock variables. For all autocorrelation coefficient we use a beta distribution while for the standard deviation we use a inverse gamma distribution.

Table (6) reports the standard deviations, the correlation with the non-commodity GDP growth, and the first-order auto-correlation coefficients for a set of selected domestic variables implied by the posterior mean of the parameters, and compares these statistics with their corresponding empirical moments. The model does a good job of matching the unconditional volatility of most variables. In terms of the business cycle correlations, the table shows that the model captures a significant share of the cyclical correlations observed in the data. The variables auto-correlations are also fairly well matched. Overall, the model performs reasonably well fitting the data's second moments.

## 4 The role of financial frictions

In the following section, we highlight the role that financial frictions play in explaining the dynamics of the economy. First, in Section 4.1 we show the impact of non-financial shocks on the economy and analyze how the introduction

Table 4: Estimation, Deep Parameters

Parameter	Description	Prior		Posterior		
		Dist	Mean	St Dev	Mean	90% Inter
$\alpha_\pi$	Inflation weight in Taylor Rule	N	1.70	0.10	2.22	[2.10 2.33]
$\alpha_{Q^{BB}}^1$	Smoothed valuation of bank bonds, first parameter	$\beta$	0.85	0.03	0.84	[0.80 0.88]
$\alpha_{Q^{BB}}^2$	Smoothed valuation of bank bonds, second parameter	$\beta$	0.25	0.08	0.30	[0.15 0.43]
$\alpha_{Q^L}^1$	Smoothed valuation of housing debt, first parameter	$\beta$	0.85	0.03	0.50	[0.45 0.55]
$\alpha_{Q^L}^2$	Smoothed valuation of housing debt, second parameter	$\beta$	0.25	0.08	0.86	[0.80 0.92]
$\alpha_R$	Lagged interest rate weight in Taylor Rule	$\beta$	0.85	0.03	0.74	[0.71 0.76]
$\alpha_W$	Weight on past productivity on wage indexation	$\beta$	0.25	0.08	0.23	[0.10 0.35]
$\alpha_y$	Output weight in Taylor Rule	N	0.13	0.08	0.19	[0.10 0.29]
$\eta$	Elasticity of subst. home and foreign goods	$\gamma$	1.00	0.25	2.11	[1.55 2.66]
$\eta_{\hat{C}}$	Elasticity of subst. consumption and housing goods	$\gamma$	1.00	0.25	0.86	[0.79 0.92]
$\eta^*$	Foreign demand elasticity of substitution	$\gamma$	0.25	0.08	0.17	[0.08 0.25]
$\gamma_H$	Housing investment adjustment cost parameter	$\gamma$	3.00	0.25	2.56	[2.20 2.92]
$\gamma_K$	Capital investment adjustment cost parameter	$\gamma$	3.00	0.25	2.84	[2.46 3.20]
$\gamma_n$	Labor adjustment cost parameter	$\gamma$	3.00	0.25	1.61	[1.35 1.86]
$\kappa_F$	Weight on past inflation on foreign good indexation	$\beta$	0.50	0.08	0.56	[0.45 0.67]
$\kappa_H$	Weight on past inflation on home good indexation	$\beta$	0.50	0.08	0.69	[0.59 0.80]
$\kappa_W$	Weight on past inflation on wages indexation	$\beta$	0.85	0.03	0.82	[0.77 0.87]
$\phi^*$	Country premium elasticity to NFA position	$\gamma^{-1}$	1.00	Inf	0.24	[0.18 0.30]
$\phi_c$	Habit formation in good consumption	$\beta$	0.85	0.03	0.75	[0.70 0.79]
$\phi_{hh}$	Habit formation in housing consumption	$\beta$	0.85	0.03	0.86	[0.84 0.89]
$\theta_F$	Calvo param. foreign goods producers	$\beta$	0.50	0.08	0.77	[0.74 0.81]
$\theta_H$	Calvo param. domestic goods producers	$\beta$	0.50	0.03	0.78	[0.76 0.81]
$\theta_W$	Calvo param. wage setters	$\beta$	0.50	0.08	0.76	[0.72 0.80]
$\varphi$	Inverse Frisch elasticity	$\gamma$	7.50	1.50	7.10	[5.59 8.96]
$\eta_{\zeta_L}$	Term premium elasticity to relative bond liquidity	$\gamma$	0.85	0.03	0.15	[0.10 0.20]

Notes.— Reported posterior means and standard deviations are based on a 1.000.000 draws Metropolis-Hastings chain where the first 500.000 draws were discarded.

of financial frictions alters those effects. For this purpose, we calibrate an alternative version of the model that is absent of financial frictions. For the model without financial frictions, we assume agents do not default on their debt, there is no costly state verification, and there are no long-term bonds holding liquidity costs. Except for the changes needed to eliminate the frictions, the alternative specification uses the same parameterization as the baseline to allow for a cleaner analysis. Then, in Section 4.2, we analyze the economic impact of purely financial shocks, defined as shocks that have no effects in the absence of financial frictions. In particular, we study the implications of risk, liquidity, term premium, and credit supply shocks.

#### 4.1 The financial multiplier and the response to non-financial shocks

Modeling financial frictions not only allows for the inclusion of purely financial shocks. It also has material effects on the transmission mechanism of non-financial shocks.

Frictional financial intermediation affects the propagation of shocks through different channels. First, shocks directly affect households' consumption decisions, firms' investment decisions, and banks' lending decisions. Then, these effects are either reinforced or subdued by indirect effects through the ability of agents to obtain external

Table 5: Estimation, exogenous variables AR1 processes

Shock process	A.R.	Prior				Posterior				S.D.	Prior				Posterior			
		Mean	S.D.	Mean	90% HPD	Mean	S.D.	Mean	90% HPD		Mean	S.D.	Mean	90% HPD	Mean	S.D.	90% HPD	
Non stat. productivity	$\rho_a$	0.25	0.08	0.31	[0.18 0.44]	$100 \times \sigma_a$	0.50	Inf	0.27	[0.19 0.35]								
Monetary Policy	$\rho_{e^m}$	0.15	0.08	0.09	[0.02 0.15]	$100 \times \sigma_{e^m}$	0.50	Inf	0.13	[0.11 0.15]								
Government spending	$\rho_g$	0.75	0.08	0.73	[0.62 0.85]	$100 \times \sigma_g$	0.50	Inf	1.29	[1.15 1.43]								
Copper price	$\rho_{p^{co}}$	0.75	0.08	0.87	[0.84 0.91]	$10 \times \sigma_{p^{co}}$	0.50	Inf	11.6	[10.2 12.8]								
Foreign inflation	$\rho_{\pi^*}$	0.75	0.08	0.37	[0.32 0.41]	$100 \times \sigma_{\pi^*}$	0.50	Inf	2.59	[2.38 2.82]								
Foreign interest rate	$\rho_{RW}$	0.75	0.08	0.82	[0.78 0.86]	$100 \times \sigma_{RW}$	0.50	Inf	0.12	[0.10 0.13]								
Entrepreneurs risk	$\rho_{\sigma^e}$	0.75	0.08	0.89	[0.81 0.97]	$100 \times \sigma_{\sigma^e}$	0.50	Inf	1.24	[0.84 1.72]								
Corporate bank risk	$\rho_{\sigma^F}$	0.75	0.08	0.56	[0.45 0.68]	$100 \times \sigma_{\sigma^F}$	0.50	Inf	7.18	[5.58 8.84]								
Housing bank risk	$\rho_{\sigma^H}$	0.75	0.08	0.75	[0.63 0.88]	$100 \times \sigma_{\sigma^H}$	0.50	Inf	0.45	[0.12 0.85]								
Housing valuation risk	$\rho_{\sigma^I}$	0.75	0.08	0.76	[0.64 0.90]	$100 \times \sigma_{\sigma^I}$	0.50	Inf	1.82	[0.10 8.26]								
Current consumption prefs.	$\rho_\varrho$	0.75	0.08	0.48	[0.37 0.58]	$10 \times \sigma_\varrho$	0.50	Inf	0.42	[0.33 0.51]								
Housing consumption prefs	$\rho_{\xi^h}$	0.75	0.08	0.93	[0.90 0.96]	$1 \times \sigma_{\xi^h}$	0.50	Inf	0.66	[0.27 1.03]								
Investment mg. eff.(K)	$\rho_{\xi^i}$	0.75	0.08	0.47	[0.39 0.56]	$10 \times \sigma_{\xi^I}$	0.50	Inf	0.55	[0.42 0.68]								
Investment mg. eff.(H)	$\rho_{\xi^{ih}}$	0.75	0.08	0.52	[0.43 0.62]	$10 \times \sigma_{\xi^{ih}}$	0.50	Inf	1.47	[0.99 1.91]								
Import prices	$\rho_{\xi^m}$	0.75	0.08	0.65	[0.56 0.73]	$100 \times \sigma_{\xi^m}$	0.50	Inf	2.06	[1.67 2.46]								
Labor disutility	$\rho_{\xi^n}$	0.75	0.08	0.42	[0.32 0.52]	$1 \times \sigma_{\xi^n}$	0.50	Inf	1.89	[0.95 2.82]								
Country premium	$\rho_{\xi^R}$	0.75	0.08	0.69	[0.63 0.76]	$100 \times \sigma_{\xi^R}$	0.50	Inf	0.07	[0.06 0.08]								
Banker dividend	$\rho_{\xi^{xb}}$	0.75	0.08	0.46	[0.32 0.59]	$10 \times \sigma_{\xi^{xb}}$	0.50	Inf	0.64	[0.37 0.91]								
Entrepreneur dividend	$\rho_{\xi^{xe}}$	0.75	0.08	0.66	[0.55 0.78]	$10 \times \sigma_{\xi^{xe}}$	0.50	Inf	0.56	[0.28 0.84]								
Banker required return	$\rho_{\xi^{roe}}$	0.75	0.08	0.89	[0.80 0.98]	$100 \times \sigma_{\xi^{roe}}$	0.50	Inf	0.65	[0.32 1.01]								
Foreign demand	$\rho_{\xi^{y*}}$	0.85	0.08	0.88	[0.80 0.96]	$100 \times \sigma_{\xi^{y*}}$	0.50	Inf	0.36	[0.28 0.44]								
Mining productivity	$\rho_{\xi^{yco}}$	0.85	0.08	0.77	[0.65 0.90]	$100 \times \sigma_{\xi^{yco}}$	0.50	Inf	3.01	[2.62 3.40]								
Stat. productivity	$\rho_z$	0.85	0.08	0.94	[0.90 0.98]	$100 \times \sigma_z$	0.50	Inf	0.36	[0.26 0.47]								
UIP shock	$\rho_{\zeta^u}$	0.75	0.08	0.63	[0.55 0.72]	$100 \times \sigma_{z_\tau}$	0.50	Inf	0.68	[0.46 0.89]								
Liquidity costs	$\rho_{\epsilon^L}$	0.75	0.05	0.91	[0.88 0.95]	$1 \times \sigma_{\epsilon^L}$	0.50	Inf	0.87	[0.51 1.23]								

Notes.— Reported posterior means and standard deviations are based on a 1.000.000 draws Metropolis-Hastings chain where the first 500.000 draws were discarded. All of the autocorrelation parameters were estimated assuming a beta distribution while the standard deviation parameters were estimated using an inverse gamma distribution.

Table 6: Second Moments, selected domestic variables

Variable	Description	$\sigma(x_t)$		$\rho(x_t, \Delta \log Y_t^{NoCo})$		$\rho(x_t, x_{t-1})$	
		Model	Data	Model	Data	Model	Data
$\Delta \log Y_t^{NoCo}$	Non mining real GDP growth	0.89	0.92	1.00	1.00	0.23	0.45
$\Delta \log Y_t^{Co}$	Mining GDP growth	3.21	3.27	0.01	0.09	-0.11	-0.16
$\Delta \log C_t$	Total Consumption growth	1.16	1.10	0.84	0.76	0.30	0.37
$\Delta \log G_t$	Government Consumption growth	1.42	1.58	0.22	-0.22	-0.12	-0.21
$\Delta \log I_t^K$	Real Capital Investment growth	3.35	4.35	0.44	0.51	0.48	0.32
$\Delta \log I_t^H$	Real Housing Investment growth	5.22	4.26	0.12	0.30	0.92	0.65
$\Delta \log N_t$	Employment growth	1.39	1.37	0.47	0.33	0.08	-0.53
$\Delta \log WN_t$	Nominal wage growth	0.47	0.43	-0.07	-0.15	0.65	0.07
$TB_t/GDPN_t$	Trade Balance-GDP Ratio	2.69	5.28	-0.01	0.41	0.85	0.81
$\pi^{core}$	Core Inflation	0.39	0.49	0.12	-0.36	0.79	0.80
$\pi_t^H$	Housing Price Index growth	1.78	1.41	0.26	0.14	0.01	0.22
$R_t$	Monetary Policy Rate	0.40	0.40	-0.06	-0.19	0.91	0.88
$R_t^L$	Commercial Loans interest rate	0.40	0.43	-0.13	-0.20	0.86	0.89
$R_t^I$	Housing Loans Interest Rate	0.12	0.15	-0.19	0.02	0.84	1.00
$R_t^D$	Nominal Interest Rate on Deposits	0.41	0.40	-0.05	-0.16	0.91	0.89
$R_t^{LG}$	10 Year CBC bond nominal Rate	0.22	0.22	-0.04	0.31	0.90	0.97
$\Delta \log(L_t)$	Housing and Corporate Loan	1.01	1.38	0.34	0.34	-0.01	0.67
$ROE_t$	Banks ROE	0.45	0.50	0.02	0.51	0.83	0.96
$r_{et}$	Real Exchange Rate	3.58	4.11	0.11	-0.28	0.68	0.73

financing, which is, in turn, determined by the endogenous response of the agents' equity valuation.

In the model, the propagation of shocks through financial channels in this model is defined, in general, by two of the model's features. First, as in Bernanke et al. (1999), as the model's frictions impose a wedge between the savings and borrowing rates that, crucially, depend on the cycle, a shock that increases the value of assets leads to an increase in borrowers' net worth, allowing them to increase their borrowing. The mechanism further amplifies the shocks' first-round output effects in what is usually called a financial multiplier. A second financial propagation mechanism featured in the model relates to a Fisherian debt-deflation type effect, as discussed by Iacoviello (2005) and Christiano et al. (2010), and related to what was first proposed by Fisher (1933). The mechanism appears when unexpected changes in inflation alter the ex-post real financial burden of existing loans taken in the form of non-contingent nominal debt contracts. As the nominal payments are pre-agreed, higher than expected inflation reduces the real value of the outstanding debt obligations, thus improving the borrower's financial standing. When a shock leads to higher output and higher inflation, both mechanisms will reinforce each other to further amplify the initial output expansion.<sup>13</sup>

For example, a positive demand shock increases the demand for capital, increasing the value of equity and decreasing entrepreneurs' leverage, which allows them to obtain more debt, increase investment, and further stimulate output. Additionally, the increase in inflation that comes with the positive demand shock lowers the real value of entrepreneurs' debt, decreasing their financial burden and allowing them to increase their borrowing and further stimulate the economy. However, facing shocks that initially lead to higher output and lower inflation, the amplifying effect on output from the financial multiplier is counteracted by a dampening effect from a debt-deflation mechanism. This will be the case, for example, after a positive productivity shock. On the one hand, the shock reduces the entrepreneurs' leverage by increasing the demand for capital and thus raising the value of equity. However, on the other hand, the decrease in inflation that comes with higher productivity and lower marginal costs increases the real financial burden of entrepreneurs, constraining their ability to borrow and partially offsetting the initial boost in output coming from the productivity shock's first-round effect.

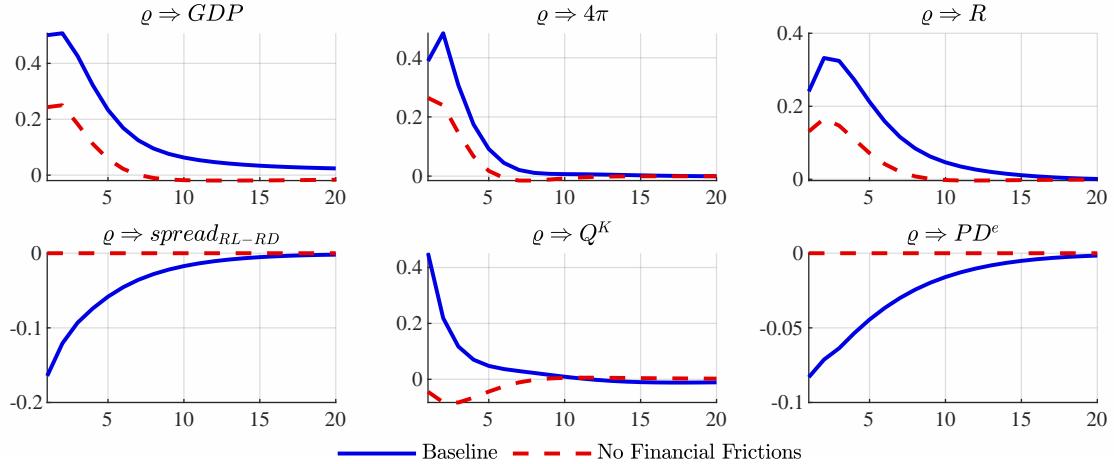
We start the analysis of how financial frictions affect the transmission mechanism of the different shocks present in the model with a disturbance to households' preference for current consumption. In addition to the standard new-Keynesian transmission channels, and due to the model's financial frictions, this demand shock also increases the equity value of borrowers and decreases the default likelihood, relaxing the borrowing constraints. All of

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<sup>13</sup>The strength of both channels depends crucially on how much the interest rate paid on loans changes due to variations in the borrowers' leverage. While for the first mechanism, the fundamental driver of the change in leverage is the revaluation of the borrowers' assets, the second mechanism works mainly through changes in the real valuation of the liabilities. Additionally, while the Fisherian debt-deflation type of mechanism requires non-contingent nominal debt contracts, the financial multiplier, as shown in Bernanke et al. (1999), still appears with real contingent debt contracts. Christiano et al. (2010) show that the debt deflation channel disappears if debt contracts are defined in real terms and interest payments are contingent on shocks that happen during the payment period. They use this alternative modeling of debt contracts to show the relevance of the channel for the US and the Euro Area.

which lead to lower lending interest rates and a credit expansion. The additional financial easing allows for higher investment, employment, and output. By lowering the interest rate spreads, the financial multiplier also helps reduce the financial burden on firms, lowering marginal costs and thus allowing for higher output with only limited inflation pressures. Then, compared with a similar economy with no financial frictions, this shock, which in general equilibrium leads to higher output and higher inflation, generates higher output volatility but with proportionally less effect in inflation. Figure 1 shows that, for this shock, the increase in GDP in the baseline model doubles the one from the version without financial frictions. The difference in the shock's inflationary effect is, however, proportionally milder. The discrepancy can be attributed to an attenuation of marginal cost increases due to lower financial costs.

Figure 1: Response to a Shock to Households' Demand

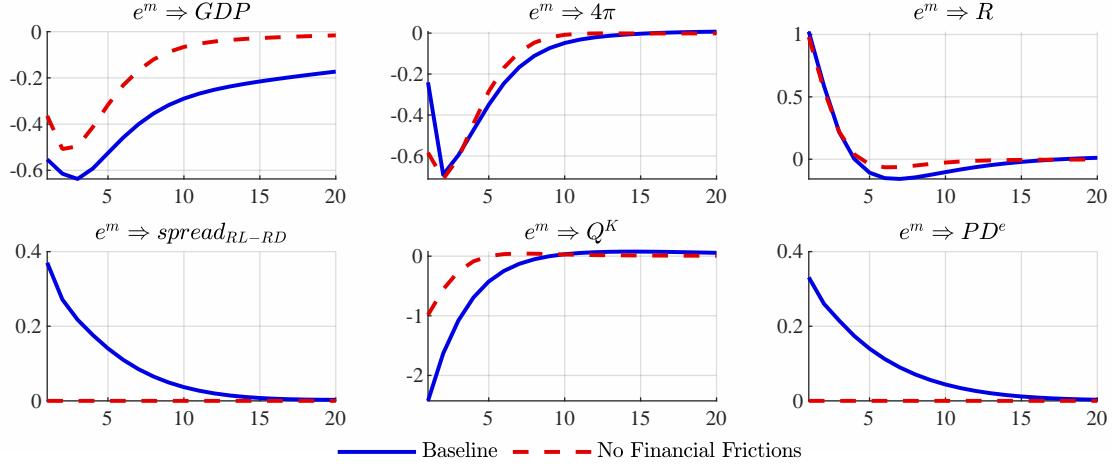


Notes.— The figure shows the the impulse response to a shock to households' preference for current consumption ( $\varrho_t$ ). The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

The same happens in reaction to monetary shocks. Figure 2 illustrates how, from the point of view of a central bank, the sacrifice ratio, —i.e., the level of employment that the economy needs to forgo to attain lower inflation— significantly increases due to the financial multiplier. In the exercise, we compare an economy with and without financial frictions where the central bank tries to lower the annualized four periods ahead expected rate of quarterly inflation in 100 bp. In the example, while the required increase in interest rates is similar in both scenarios, the financially constrained economy has to endure a GDP drop 80 bp larger, on average, during the first eight quarters. These results are consistent and comparable with those found, for the Euro Area, by Coenen et al. (2018). They show that a short-term nominal interest rate shock has more substantial real effects but marginally weaker impacts on inflation under frictional financial intermediation.

Figure 3 describes the case of a one standard deviation shock to the marginal efficiency of investment (MEI). These types of shocks are interesting to analyze because, although they lead to higher aggregate demand and

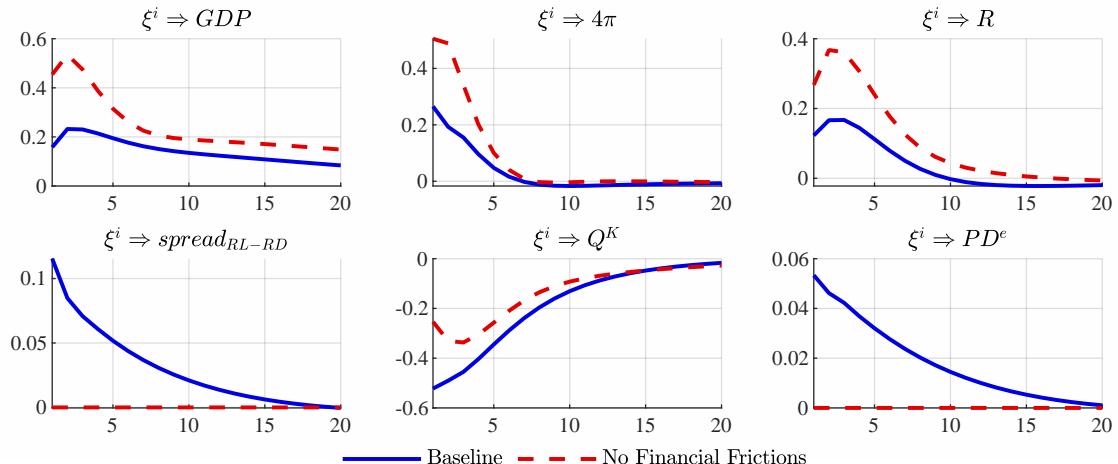
Figure 2: Response to a Monetary Policy Shock



Notes.— The figure shows the impulse response to a monetary policy shock ( $e_t^m$ ). The shock's size is calibrated to generate a 100 bps increase, on impact, on  $R_t$ , the monetary policy rate. The simulation considers the estimated persistence of the shock, as described in Table 5. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

higher inflation, they are in essence, as highlighted by Christiano et al. (2014), supply shocks where the underlying mechanism is characterized by a technological improvement in the production of capital goods, which increases the equilibrium quantities while decreasing sectoral prices. Following a positive shock to the MEI, the counter-cyclical dynamics of the capital goods valuation lead to a fall in the entrepreneur's net worth, followed by an increase in their default probability and, subsequently, to higher interest rates on their loans. Thus, for the same increase in the MEI, the impact on output and inflation in the economy with financial frictions is actually smaller due to financial frictions acting as an offsetting mechanism against the shock's underlying transmission mechanisms.

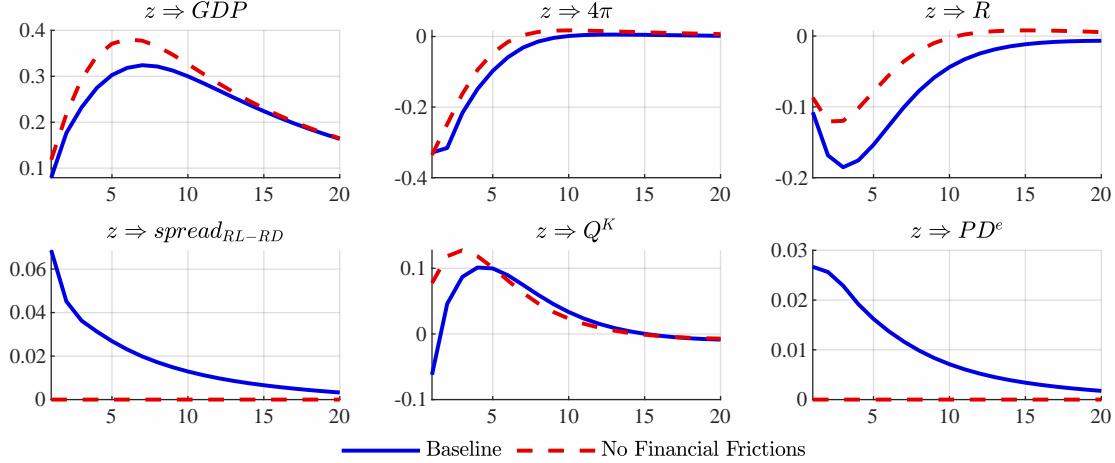
Figure 3: Response to a Shock to the Marginal Efficiency of Investment



Notes.— The figure shows the impulse response to a shock to the marginal efficiency of investment on productive capital ( $\xi_t^i$ ). The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. The monetary policy rate,  $R_t$ , is expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

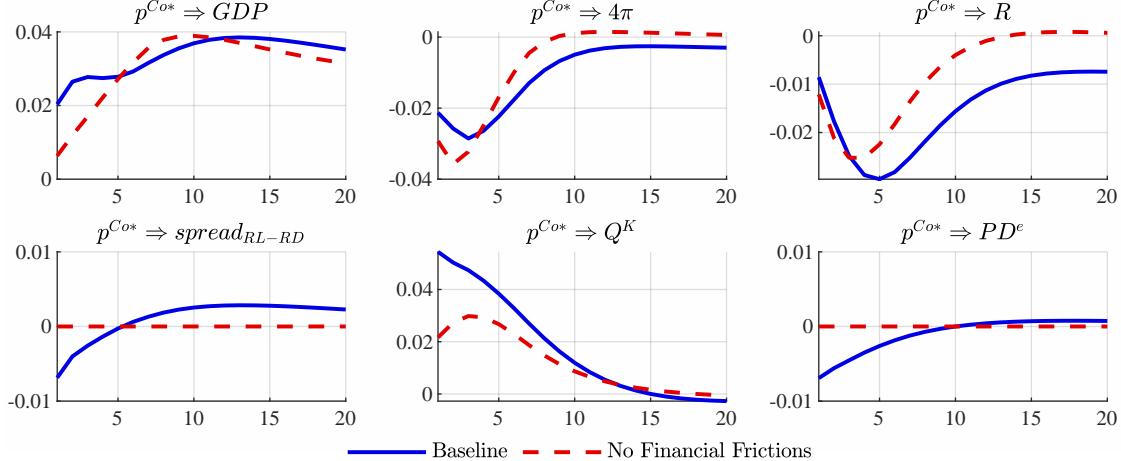
For shocks that move GDP and inflation in opposing directions, the financial multiplier channel still leads to an amplification of output volatility. However, in these cases, the debt-deflation mechanism will partially offset the overall results. Figures 4 and 5 illustrate the underlying mechanisms for two of such shocks: a transitory productivity shock and a commodity export price shock, both calibrated at one standard deviation.

Figure 4: Response to a Transitory Productivity Shock



Notes.— The figure shows the impulse response to a transitory productivity shock ( $z_t$ ). The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. The monetary policy rate,  $R_t$ , and the spread between the interest rates for corporate loans ( $R_t^L$ ) and deposits ( $R_t^D$ ) are all expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

Figure 5: Response to a Commodity Export's Price Shock



Notes.— The figure shows the impulse response to a shock to the international price of the exported commodity ( $p_t^{Co*}$ ). The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. The monetary policy rate,  $R_t$ , and the spread between the interest rates for corporate loans ( $R_t^L$ ) and deposits ( $R_t^D$ ) are all expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

The increase in output followed by a productivity shock leads, on the one hand, to higher demand for loans. However, as higher productivity also leads to lower marginal costs and thus lower inflation, real debt valuation increases, leading to higher leverage and financial costs, thus dampening the financial multiplier amplification

effect. A shock to the price of the exported commodity is also characterized by higher output and lower inflation<sup>14</sup>, the propagation of shocks to commodity export prices are likewise associated with a debt-deflation channel that tends to counteract and dampen the effects of the financial multiplier mechanism.

## 4.2 Risk, liquidity and credit shocks

In addition to affecting the transmission mechanism of non-financial shocks, introducing financial friction also allows for analyzing the impact of surprises that would otherwise not have real effects. One type of those surprises is a shock to the idiosyncratic risk faced by the borrowers. Due to the heterogeneity in the projects' returns, some borrowers will find it optimal to default on their financial obligations. Higher risk on the returns increases the share of the projects expected to default.

Default rates act as adverse economy-wide supply shocks, decreasing GDP and increasing inflation. The transmission flows primarily through two channels. First, as more projects default, more resources will be spent on verification costs. In addition, expectations of higher default rates lead to higher lending spreads, raising all borrowers' financial burden.

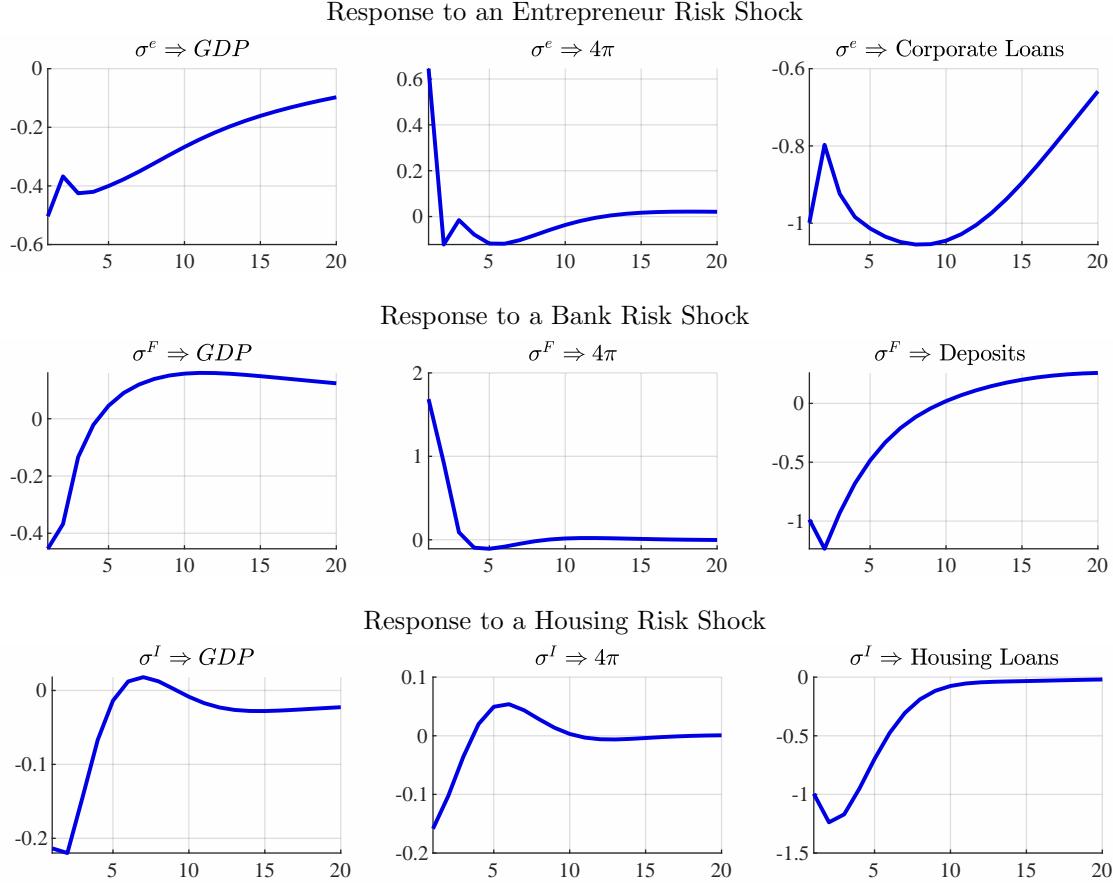
Figure 6 shows the effect of idiosyncratic risk shocks on the three agents that act as risky borrowers: households and entrepreneurs that borrow from banks, and banks that raise deposits from households. In all cases, the magnitude of the shocks is calibrated to generate a one percentage point reduction, on impact, on their respective liabilities (loans for households and entrepreneurs, and deposits for corporate banks). In all cases, the shocks lead to a contraction of the economy, as a decrease in new debt causes a reduction in aggregate demand. In the case of risk shocks to entrepreneurs and banks, because they are part of the productive chain, the increased financial burden leads to higher marginal costs and, ultimately, higher inflation. Housing risk shocks, on the other hand, have no direct influence on productive marginal costs, and therefore the demand channel, where households reduce their consumption due to a more restricted access to loans, dominates in the aggregate, leading to the initial drop in inflation shown in the lower figure's panels.

The model also features liquidity shocks that increase the required return for long-term bonds. These shocks can be helpful to analyze the consequences of exogenous changes in the yield curve. Given imperfect substitution among assets of different maturities, the reduced liquidity acts as a contractionary shock, propagating through the economy through mechanisms similar to monetary shocks. As shown in Figure 7, a liquidity shock that leads to a 100bp rise in long interest rates causes a drop in GDP, consumption, investment, and inflation. While qualitatively

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<sup>14</sup>Medina and Soto (2016) analyze the inflationary implications of shocks to commodity export prices in an emerging economy with flexible exchange rates. They find that the shocks lead to a drop in inflation even as the higher commodity prices usher in higher demand and real wages as the real depreciation they cause dominates through the imported inflation channel. The authors notice that, in the presence of a real exchange rate stabilization policy, the higher commodity price is, indeed, inflationary.

Figure 6: Risk Shocks



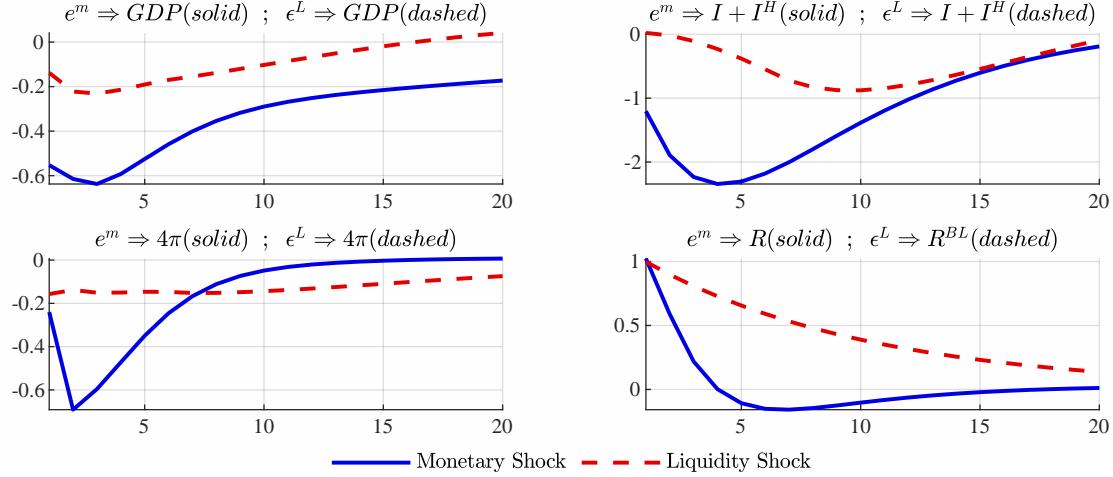
Notes.— The figure shows the impulse responses to three different agents' risk shocks. The top panels show the response to a shock to the time varying volatility of the idiosyncratic return on entrepreneurs' assets ( $\sigma_t^e$ ). The middle panels show the response to a shock to the time varying volatility of the idiosyncratic return on corporate banks' projects ( $\sigma_t^F$ ). The bottom panels show the response to a shock to the time varying volatility of the idiosyncratic valuation of households' housing assets ( $\sigma_t^I$ ). In all three cases, the shocks' sizes are calibrated to generate a one percentage point decrease, on impact, on the corresponding liabilities (corporate loans for entrepreneurs' shocks, deposits for banks' shocks, and housing loans for households' shocks). The simulations consider the estimated persistence of the respective shocks, as described in Table 5. The shocks' impulse responses for a more comprehensive set of variables can be found in the appendix.

similar, compared to a monetary policy shock that increases the policy rate by the same 100bp, the effects of a liquidity shock are much milder. This result is in line with the one obtained, with a similar setup, by Chen et al. (2012) while analyzing the quantitative effects of the Fed's LSAP II program. They show that allowing for an endogenous response of the MPR (the underlying assumption in the exogenous liquidity shock exercise conducted here), the economic effects of increased liquidity are relatively small.<sup>15</sup>

Finally, the implementation of financial frictions allows analyzing the implication of exogenous shocks to the banks' and entrepreneurs' equity decisions. The response to these shocks, with a magnitude calibrated to get a 5% drop in the respective sector equity, appears in figures 8 and 9, respectively. The qualitative results are similar in both cases: lower equity, lower investment, and lower inflation. In the case of bank dividends, an increase in

<sup>15</sup>Chen et al. (2012) do find, however, that QE programs that aim at increasing liquidity at the long end of the yield curve can be indeed effective if the MPR remains constant, a sensible assumption given those asset purchase programs were executed during a period when the FFR was fixed at the effective lower bound

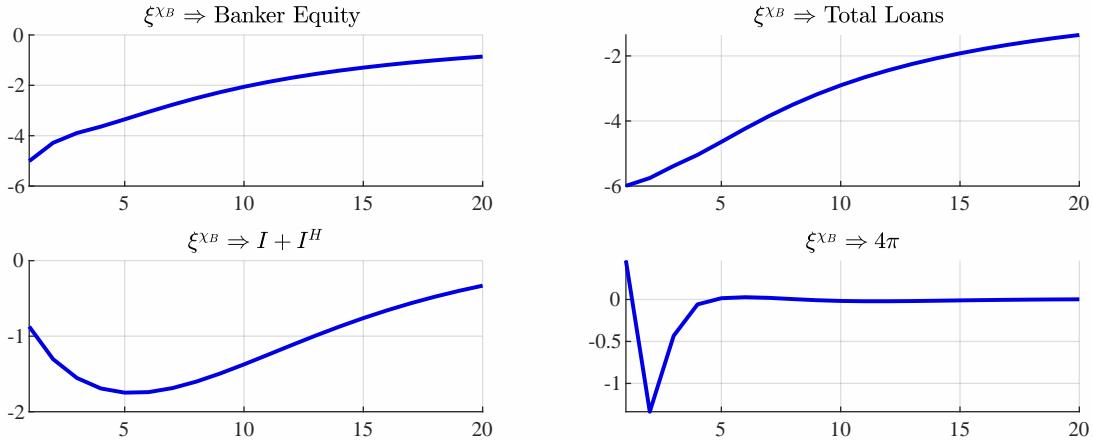
Figure 7: Response to Monetary Policy and Long Term Bonds Liquidity Shock



Notes.— The figure shows the impulse responses to a monetary policy shock ( $e_t^m$ , in blue solid lines), and a long term bond's liquidity shock ( $e_t^L$ , in red dashed lines). In both cases, the shocks' sizes are calibrated to generate a 100 bps increase, on impact, on the respective interest rate ( $R_t$ , the monetary policy rate, for the monetary shock; and  $R_t^{BL}$ , the long term government bond's gross yield to maturity, for the liquidity shock). Both interest rates are expressed in annual terms. The simulations consider the estimated persistence of the respective shocks, as described in Table 5. The shocks' impulse responses for a more comprehensive set of variables can be found in the appendix.

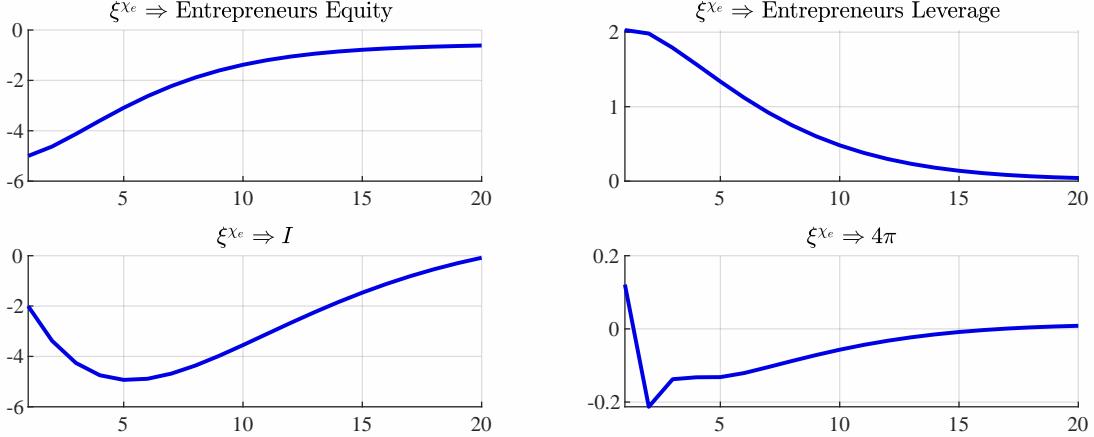
dividends leads to a rapid reduction in their equity. Due to unchanged capital requirement obligations, the equity reduction leads to a decrease in loans and, therefore, in housing and productive capital investment. An increase in entrepreneurs' dividends also reduces their equity, which increases their leverage and the borrowing rates they can access. The higher financing costs, in turn, lead to a decrease in investment, output, and inflation.

Figure 8: Response to a Banker's Dividend Policy Shock



Notes.— The figure shows the impulse responses to a shock to the bankers' dividend policy( $\xi_t^{XB}$ ). The shock's size is calibrated to generate a five percentage points drop, on impact, on the bankers' equity. The simulation considers the estimated persistence of the shock, as described in Table 5. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

Figure 9: Response to an Entrepreneur's Dividend Policy Shock



Notes.— The figure shows the impulse responses to a shock to the entrepreneurs' dividend policy ( $\xi_t^{\chi_e}$ ). The shock's size is calibrated to generate a five percentage points drop, on impact, on the entrepreneurs' equity. The simulation considers the estimated persistence of the shock, as described in Table 5. The shock's impulse responses for a more comprehensive set of variables can be found in the appendix.

### 4.3 Variance Decomposition

A brief analysis of the model's variance decomposition reaffirms the results discussed above. Table 7 shows the percentage of the variance of selected variables explained by different groups of shocks. In particular, we aggregate the shocks into productivity, demand, monetary policy, financial-risk, financial-term premia, and others. First, the results show a relevant role for financial shocks explaining the cycle, mainly through the influence of risk shocks. Second, for demand shocks, the financial multiplier mechanism dominates. Introducing financial frictions results in an amplification of the role of demand shocks explaining GDP and its components. However, as discussed in the previous sections, for productivity shocks, the Fisherian debt-deflation mechanism acquires a dominant role, leading to a dampened role for productivity shocks in the model with financial frictions. Finally, for monetary policy, we observe clear indications that incorporating financial frictions cause an increase in the sacrifice ratio. The increase is due to a financial multiplier effect combined with a financing costs channel that increases monetary policy's role in output, mainly through the investment component, with only minor changes in its role in explaining inflation.

## 5 Conclusions

This paper introduces the Central Bank of Chile's Macro Financial Model, a DSGE model with financial intermediaries and financial frictions. We build on a simplified version of the model described in Garcia et al. (2019) by introducing a financial system with financial frictions, following Clerc et al. (2014), long term bonds, in the spirit of Woodford (2001), preferred habitat theory of the term structure, as in Vayanos and Vila (2009), and imperfect

Table 7: Unconditional Variance Decomposition, selected variables (Percent)

		Productivity <sup>a</sup>	Demand <sup>b</sup>	Monet. Policy <sup>c</sup>	Financial: Risk <sup>d</sup>	Financial: TP <sup>e</sup>	Others <sup>f</sup>
GDP Growth <sup>g</sup>	FF	25.8	54.3	8.8	5.7	0.8	4.6
	No FF	62.1	27.8	5.3	0	0	4.7
Inflation <sup>g</sup>	FF	32.8	24.1	8.9	7.3	2.2	24.7
	No FF	60.4	12.4	10.7	0	0	16.5
Consumption Growth <sup>g</sup>	FF	13.2	80.0	2.6	0.8	2.4	1.0
	No FF	19.9	72.0	5.5	0	0	2.5
Investment Growth <sup>g</sup>	FF	76.8	5.9	5.8	8.7	0.5	2.3
	No FF	95.8	1.9	0.5	0	0	1.9

<sup>a</sup> Include shocks to transitory and permanent productivity, mining productivity, and marginal efficiency of investment in the housing and productive capital sectors.

<sup>b</sup> Include shocks to households' preference for present consumption, households' preference for housing consumption, government spending, foreign demand and commodity prices.

<sup>c</sup> Include monetary policy shocks.

<sup>d</sup> Include risk shocks to entrepreneurs, households and banks, shocks to bank portfolio assignment between housing and corporate credit, and dividend policies of banks and entrepreneurs.

<sup>e</sup> Include liquidity shocks to the holding of long term bonds.

<sup>f</sup> Include shocks to foreign inflation and interest rates, import prices, preference for leisure, premium on government bonds, and to the interest rate parity.

<sup>g</sup> Inflation and growth corresponds to the the seasonally adjusted log-difference of core CPI and corresponding real variables relative to the previous quarter.

asset substitution as in Andres et al. (2004). The model is estimated for the Chilean economy, using Bayesian techniques, incorporating real, financial, and price data between 2001 and 2019.

On the one hand, we describe how the existence of a financial system, financial frictions, and defaults affect the propagation of shocks through different channels. And on the other hand, how introducing financial frictions also for the analysis of shocks that would otherwise had no real effects.

The model features a financial multiplier mechanism that, similar to Coenen et al. (2018), amplifies the effects on output, but with proportionally less inflationary pressures. In addition, we found a Fisherian debt-deflation type of mechanism, as in Christiano et al. (2010), that further amplifies the financial multiplier facing shocks that move output and inflation in the same direction, but leads to a dampened multiplier following shocks that push output and inflation in opposing directions.

Finally, we find a relevant role for financial and risk shocks in explaining the business cycle through a variance decomposition analysis.

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## A Stationary Equilibrium Conditions

In the model described in the previous sections, real variables in uppercase contain a unit root in equilibrium due to the presence of the non-stationary productivity vector  $A_t$ . Uppercase nominal variables contain an additional unit root given by the non-stationarity of the price level. In this section we show the stationary version of the model, where we define  $a_t = A_t/A_{t-1}$ , and all lowercase variables denote the stationary counterpart of the original variables, obtained by dividing them by its co-integration vector ( $A_t$  or  $P_t$ ).

The rational expectations equilibrium of the stationary version of the model is then the set of sequences for the endogenous variables such that for a given set of initial values and exogenous processes the following conditions are satisfied:

### A.1 Patient Households

#### A.1.1 Unrestricted (U)

$$\hat{c}_t^U = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \xi_t^h \left( \frac{h_{t-1}^U}{a_t} - \phi_{hh} \frac{h_{t-2}^U}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \quad (1)$$

$$\lambda_t^U = (\hat{c}_t^U)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{c}_t^U}{(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t})} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (2)$$

$$\varrho_t \lambda_t^U q_t^H = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ (\hat{c}_{t+1}^U a_{t+1})^{-\sigma} \xi_{t+1}^h \left( \frac{o_{\hat{C}} \hat{c}_{t+1}^U a_{t+1}}{\xi_{t+1}^h (h_t^U - \phi_{hh} \frac{h_{t-1}^U}{a_t})} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1 - \delta_H) \lambda_{t+1}^U a_{t+1}^{-\sigma} q_{t+1}^H \right\} \quad (3)$$

$$\varrho_t \lambda_t^U = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \quad (4)$$

$$\varrho_t \lambda_t^U = \beta_U \mathbb{E}_t \left\{ \frac{\tilde{R}_{t+1}^D}{\pi_{t+1}} \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \right\} \quad (5)$$

$$\varrho_t \lambda_t^U = \beta_U R_t^* \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \quad (6)$$

$$\varrho_t \lambda_t^U (1 + \zeta_t^L) q_t^{BL} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} R_{t+1}^{BL} q_{t+1}^{BL} \right\} \quad (7)$$

$$\varrho_t \lambda_t^U (1 + \zeta_t^L) q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} q_{t+1}^{BB} \right\} \quad (8)$$

### A.1.2 Restricted (R)

$$\hat{c}_t^R = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \xi_t^h \left( \frac{h_{t-1}^R}{a_t} - \phi_{hh} \frac{h_{t-2}^R}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \quad (9)$$

$$\lambda_t^R = (\hat{c}_t^R)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{c}_t^R}{\left( c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (10)$$

$$\varrho_t \lambda_t^R q_t^H = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ (\hat{c}_{t+1}^R a_{t+1})^{-\sigma} \left( \frac{o_{\hat{C}} \hat{c}_{t+1}^R a_{t+1}}{\xi_{t+1}^h \left( h_t^R - \phi_{hh} \frac{h_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + (1 - \delta_H) \lambda_{t+1}^R a_{t+1}^{-\sigma} q_{t+1}^H \right\} \quad (11)$$

$$\varrho_t \lambda_t^R q_t^{BL} = \beta_R \mathbb{E}_t \{ \varrho_{t+1} \lambda_{t+1}^R q_{t+1}^{BL} R_{t+1}^{BL} a_{t+1}^{-\sigma} \} \quad (12)$$

$$q_t^{BL} b_t^R + c_t^R + q_t^H h_t^R = q_t^{BL} R_t^{BL} \frac{b_{t-1}^R}{a_t} + w_t n_t^R + q_t^H (1 - \delta_H) \frac{h_{t-1}^R}{a_t} \quad (13)$$

### A.2 Impatient Households

$$\frac{R_t^H}{\pi_t} = \frac{q_t^H (1 - \delta_H)}{q_{t-1}^H} \quad (14)$$

$$\hat{c}_t^I = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \xi_t^h \left( \frac{h_{t-1}^I}{a_t} - \phi_{hh} \frac{h_{t-2}^I}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \quad (15)$$

$$\lambda_t^I = (\hat{c}_t^I)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{c}_t^I}{\left( c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad (16)$$

$$\bar{\omega}_t^I = \frac{R_t^I q_t^{\hat{L}} l_{t-1}^H}{R_t^H q_{t-1}^H h_{t-1}^I} \pi_t \quad (17)$$

$$\log q_t^{\hat{L}} \equiv \alpha_{Q^L}^1 (\alpha_{Q^L}^2 \log q_{t-1}^{\hat{L}} + (1 - \alpha_{Q^L}^2) \log q^L) + (1 - \alpha_{Q^L}^1) \log q_t^L \quad (18)$$

$$R_t^I = \frac{1}{q_t^L} + \kappa \quad (19)$$

$$\varrho_t \lambda_t^I q_t^H = \mathbb{E}_t \left\{ \begin{aligned} & \beta_I \varrho_{t+1} \left( (\hat{c}_{t+1}^I a_{t+1})^{-\sigma} \left( \frac{o_{\hat{C}} \hat{c}_{t+1}^I a_{t+1}}{\xi_{t+1}^h \left( h_t^I - \phi_{hh} \frac{h_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + \lambda_{t+1}^I a_{t+1}^{-\sigma} [1 - \Gamma_I(\bar{\omega}_{t+1}^I)] \frac{R_{t+1}^H}{\pi_{t+1}} q_t^H \right) \\ & + \varrho_t \lambda_t^I [1 - \Gamma_H(\bar{\omega}_{t+1}^H)] [\Gamma_I(\bar{\omega}_{t+1}^I) - \mu_I G_I(\bar{\omega}_{t+1}^I)] \frac{R_{t+1}^H q_t^H}{\bar{\rho}_{t+1}^H \phi_H} \end{aligned} \right\} \quad (20)$$

$$\beta_I \mathbb{E}_t \{ \tilde{\rho}_{t+1}^H \} = \mathbb{E}_t \left\{ \frac{\varrho_t \lambda_t^I \pi_{t+1}}{\phi_H \varrho_{t+1} \lambda_{t+1}^I a_{t+1}^{-\sigma}} [1 - \Gamma_H(\bar{\omega}_{t+1}^H)] \frac{[\Gamma'_I(\bar{\omega}_{t+1}^I) - \mu_I G'_I(\bar{\omega}_{t+1}^I)]}{\Gamma'_I(\bar{\omega}_{t+1}^I)} \right\} \quad (21)$$

$$c_t^I + q_t^H h_t^I - q_t^L l_t^H = \frac{w_t n_t}{2} + [1 - \Gamma_I(\bar{\omega}_t^I)] \frac{R_t^H q_t^H h_{t-1}^I}{a_t \pi_t} \quad (22)$$

$$PD_t^I = F_I(\bar{\omega}_t^I) \quad (23)$$

### A.3 Entrepreneurs

$$q_t^K k_t = n_t^e + l_t^F \quad (24)$$

$$\frac{R_t^e}{\pi_t} = \frac{r_t^k + (1 - \delta_K) q_t^K}{q_{t-1}^K} \quad (25)$$

$$\bar{\omega}_t^e = \frac{R_{t-1}^L l_{t-1}^F}{R_t^e q_{t-1}^K k_{t-1}} \quad (26)$$

$$c_t^e = \chi_e \xi_t^{\chi_e} \psi_t^e \quad (27)$$

$$n_t^e = (1 - \chi_e \xi_t^{\chi_e}) \psi_t^e \quad (28)$$

$$\psi_t^e a_t \pi_t = [1 - \Gamma_e(\bar{\omega}_t^e)] R_t^e q_{t-1}^K k_{t-1} \quad (29)$$

$$(1 - \Gamma_{t+1}^e) = \lambda_t^e \left( \frac{\rho_{t+1}^F \phi_t^F}{R_{t+1}^e} - (1 - \Gamma_{t+1}^F) [\Gamma_{t+1}^e - \mu^e G_{t+1}^e] \right) \quad (30)$$

$$\Gamma_{t+1}^{e'} = \lambda_t^e (1 - \Gamma_{t+1}^F) \left[ \Gamma_{t+1}^{e'} - \mu^e G_{t+1}^{e'} \right] \quad (31)$$

$$PD_t^e = F_e(\bar{\omega}_t^e) \quad (32)$$

### A.4 Bankers and Banking System

$$\mathbb{E}[\rho_{t+1}^F] = \xi_t^{b, roe} \mathbb{E}[\tilde{\rho}_{t+1}^H] \quad (33)$$

$$c_t^b = \xi_t^{\chi_b} \chi_b \psi_t^b \quad (34)$$

$$n_t^b = (1 - \xi_t^{\chi_b} \chi_b) \psi_t^b \quad (35)$$

$$\psi_t^b a_t \pi_t = \rho_t^F e_{t-1}^F + \tilde{\rho}_t^H e_{t-1}^H \quad (36)$$

$$n_t^b = e_t^F + e_t^H \quad (37)$$

$$PD_t^D = \frac{Q_{t-1}^{BB} BB_{t-1} PD_t^H + d_{t-1}^{Tot} PD_t^F}{Q_{t-1}^{BB} BB_{t-1} + d_{t-1}^{Tot}} \quad (38)$$

### A.5 F Banks

$$d_t^F + e_t^F = l_t^F \quad (39)$$

$$\bar{\omega}_t^F = (1 - \phi_F) \frac{R_{t-1}^D}{\tilde{R}_t^F} \quad (40)$$

$$e_t^F = \phi_F l_t^F \quad (41)$$

$$\rho_t^F = [1 - \Gamma_F(\bar{\omega}_t^F)] \frac{\tilde{R}_t^F}{\phi_F} \quad (42)$$

$$\tilde{R}_t^F = [\Gamma_e(\bar{\omega}_t^e) - \mu_e G_e(\bar{\omega}_t^e)] \frac{R_t^e q_{t-1}^K k_{t-1}}{l_{t-1}^F} \quad (43)$$

$$PD_t^F = F_F(\bar{\omega}_t^F) \quad (44)$$

## A.6 H Banks

$$\tilde{\rho}_t^H = (1 - \kappa) \rho_t^H + \kappa \mathbb{E}[\tilde{\rho}_{t+1}^H] \quad (45)$$

$$\bar{\omega}_t^H = (1 - \phi_H) \frac{R_t^{BB} q_t^{\overline{BB}}}{\tilde{R}_t^H q_{t-1}^{\overline{BB}}} \pi_t \quad (46)$$

$$\log q_t^{\overline{BB}} \equiv \alpha_{Q^{BB}}^1 (\alpha_{Q^{BB}}^2 \log q_{t-1}^{\overline{BB}} + (1 - \alpha_{Q^{BB}}^2) \log q_t^{BB}) + (1 - \alpha_{Q^{BB}}^1) \log q_t^{BB} \quad (47)$$

$$q_t^{BB} bb_t^{Pr} + e_t^H = q_t^L l_t^H \quad (48)$$

$$e_t^H = \phi_H q_t^L l_t^H \quad (49)$$

$$\rho_t^H = [1 - \Gamma_H(\bar{\omega}_t^H)] \frac{\tilde{R}_t^H}{\phi_H} \quad (50)$$

$$\tilde{R}_t^H = [\Gamma_I(\bar{\omega}_t^I) - \mu_I G_I(\bar{\omega}_t^I)] \frac{R_t^H q_{t-1}^H h_{t-1}^I}{q_{t-1}^L l_{t-1}^H} \quad (51)$$

$$PD_t^H = F_H(\bar{\omega}_t^H) \quad (52)$$

## A.7 Capital and Housing Goods

$$k_t = (1 - \delta_K) \frac{k_{t-1}}{a_t} + \left[ 1 - \frac{\gamma_K}{2} \left( \frac{i_t}{i_{t-1}} a_t - a \right)^2 \right] \xi_t^i i_t \quad (53)$$

$$\begin{aligned} 1 &= q_t^K \left[ 1 - \frac{\gamma_K}{2} \left( \frac{i_t}{i_{t-1}} a_t - a \right)^2 - \gamma_K \left( \frac{i_t}{i_{t-1}} a_t - a \right) \frac{i_t}{i_{t-1}} a_t \right] \xi_t^i \\ &+ \beta_P \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P}{\varrho_t \lambda_t^P} a_{t+1}^{-\sigma} q_{t+1}^K \gamma_K \left( \frac{i_{t+1}}{i_t} a_{t+1} - a \right) \left( \frac{i_{t+1}}{i_t} a_{t+1} \right)^2 \xi_{t+1}^i \right\} \end{aligned} \quad (54)$$

$$h_t = (1 - \delta_H) \frac{h_{t-1}}{a_t} + \left[ 1 - \frac{\gamma_H}{2} \left( \frac{i_{t-N_H}^{AH}}{i_{t-N_H-1}^{AH}} a_t - a \right)^2 \right] \xi_{t-N_H}^{ih} \frac{i_{t-N_H}^{AH}}{\prod_{i=0}^{N_H-1} a_{t-j}} \quad (55)$$

$$\begin{aligned}
0 &= E_t \sum_{j=0}^{N_H} \beta_P^j \varrho_{t+j} \lambda_{t+j}^P \varphi_j^H \prod_{i=j+1}^{N_H} (a_{t+i}^\sigma) \\
&- E_t \beta_P^{N_H} \varrho_{t+N_H} \lambda_{t+N_H}^P q_{t+N_H}^H \left\{ \left[ 1 - \frac{\gamma_H}{2} \left( \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right)^2 \right] - \gamma_H \left( \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right) \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t \right\} \xi_t^{ih} \\
&- E_t \beta_P^{N_H+1} \varrho_{t+N_H+1} \lambda_{t+N_H+1}^P q_{t+N_H+1}^H a_{t+N_H+1}^{-\sigma} \left\{ \gamma_H \left( \frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} - a \right) \left( \frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} \right)^2 \xi_{t+1}^{ih} \right\}
\end{aligned} \tag{56}$$

$$i_t^H = \sum_{j=0}^{N_H} \varphi_j^H \frac{i_{t-j}^{AH}}{\prod_{i=0}^{j-1} a_{t-i}} \tag{57}$$

## A.8 Final Goods

$$y_t^C = \left[ \omega^{1/\eta} (x_t^H)^{1-1/\eta} + (1-\omega)^{1/\eta} (x_t^F)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{58}$$

$$x_t^F = (1-\omega) (p_t^F)^{-\eta} y_t^C \tag{59}$$

$$x_t^H = \omega (p_t^H)^{-\eta} y_t^C \tag{60}$$

## A.9 Home Goods

$$f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} (\tilde{p}_t^H)^{1-\epsilon_H} y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} (\pi_{t+1}^H)^{\epsilon_H} f_{t+1}^H \right\} \tag{61}$$

$$f_t^H = (\tilde{p}_t^H)^{-\epsilon_H} m c_t^H y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left( \frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} (\pi_{t+1}^H)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{62}$$

$$1 = (1 - \theta_H) (\tilde{p}_t^H)^{1-\epsilon_H} + \theta_H \left( \frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \tag{63}$$

$$\pi_t^{I,H} = (\pi_{t-1}^H)^{\kappa_H} (\pi^T)^{1-\kappa_H} \tag{64}$$

$$m c_t^H = \frac{p_t^Z}{p_t^H} \tag{65}$$

## A.10 Wholesale Domestic Goods

$$mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha}{z_t} \left\{ w_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) y_t^Z p_t^Z \right. \\ \left. - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \right\}^{1-\alpha} \quad (66)$$

$$\frac{k_{t-1}}{\tilde{n}_t} = \frac{\alpha}{(1-\alpha) r_t^k} \left\{ w_t + \gamma_n \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right) \left( \frac{1}{\tilde{n}_{t-1}} \right) y_t^Z p_t^Z \right. \\ \left. - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t} - 1 \right) \left( \frac{\tilde{n}_{t+1}}{\tilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \right\} a_t \quad (67)$$

$$p_t^Z = mc_t^Z \quad (68)$$

## A.11 Foreign Goods

$$p_t^F mc_t^F = rer_t \xi_t^m \quad (69)$$

$$f_t^F = \frac{\epsilon_F - 1}{\epsilon_F} (\tilde{p}_t^F)^{1-\epsilon_F} y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left( \frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} (\pi_{t+1}^F)^{\epsilon_F} f_{t+1}^F \right\} \quad (70)$$

$$f_t^F = (\tilde{p}_t^F)^{-\epsilon_F} mc_t^F y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left( \frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} (\pi_{t+1}^F)^{1+\epsilon_F} f_{t+1}^F \right\} \quad (71)$$

$$1 = (1 - \theta_F) (\tilde{p}_t^F)^{1-\epsilon_F} + \theta_F \left( \frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \quad (72)$$

$$\pi_t^{I,F} = (\pi_{t-1}^F)^{\kappa_F} (\pi^T)^{1-\kappa_F} \quad (73)$$

## A.12 Wages

$$\lambda_t^W = \frac{\lambda_t^P + \lambda_t^I}{2} \quad (74)$$

$$\lambda_t^P = \wp_U \lambda_t^U + (1 - \wp_U) \lambda_t^R \quad (75)$$

$$\Theta_t = \frac{(\wp_U \Theta_t^U + (1 - \wp_U) \Theta_t^R) + \Theta_t^I}{2} \quad (76)$$

$$mc_t^W = \Theta_t \frac{\xi_t^n (\tilde{n}_t)^\varphi}{\lambda_t^U w_t} \quad (77)$$

$$\Theta_t^i = \tilde{\chi}_t^i (\hat{c}_t^i)^{-\sigma} \quad \forall \quad i = \{U, R, I\} \quad (78)$$

$$\tilde{\chi}_t^i = \left( \tilde{\chi}_{t-1}^i \right)^{1-v} \left( \hat{c}_t^i \right)^{\sigma v} \quad \forall \quad i = \{U, R, I\} \quad (79)$$

$$\begin{aligned} f_t^W &= \left( \frac{\epsilon_W - 1}{\epsilon_W} \right) \tilde{w}_t^{1-\epsilon_W} \tilde{n}_t \\ &+ \left( \frac{(\omega_{UP} \beta^{UP} + (1 - \omega_{UP}) \beta^{RP}) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1} \lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left( \frac{\pi_{t+1}^W}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^W \right\} \end{aligned} \quad (80)$$

$$\begin{aligned} f_t^W &= \tilde{w}_t^{-\epsilon_W(1+\varphi)} m c_t^W \tilde{n}_t \\ &+ \left( \frac{(\omega_{UP} \beta^{UP} + (1 - \omega_{UP}) \beta^{RP}) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1} \lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left( \frac{\pi_{t+1}^W}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^W \right\} \end{aligned} \quad (81)$$

$$1 = (1 - \theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left( \frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W} \quad (82)$$

$$\pi_t^{I,W} = a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W} \quad (83)$$

### A.13 Fiscal Policy

$$\begin{aligned} \tau_t + R_{t-1} \frac{bs_{t-1}^G}{a_t \pi_t} + q_t^{BL} R_t^{BL} b l_{t-1}^G \frac{1}{a_t} + \chi s_t p_t^{Co\star} y_t^{Co} &= g_t + bs_t^G + q_t^{BL} b l_t^G + \gamma_D \frac{PD_t^D R_{t-1}^D d_{t-1}^F}{a_t \pi_t} \\ &+ \gamma_{BH} \frac{PD_t^H R_t^{BB} q_t^{BB} b b_{t-1}^{Pr}}{a_t} \end{aligned} \quad (84)$$

$$\tau_t = \alpha^T gdpn_t + \epsilon_t (bs^G - bs_t^G + q^{BL} b l^G - q_t^{BL} b l_t^G) \quad (85)$$

### A.14 Monetary Policy and Rest of the World

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\alpha_R} \left[ \left( \frac{(1 - \alpha_E) \pi_t + \alpha_E \mathbb{E}_t \{ \pi_{t+4} \}}{\pi_t^T} \right)^{\alpha_\pi} \left( \frac{gdp_t}{gdp_{t-1}} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \quad (86)$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \quad (87)$$

$$R_t^\star = R_t^W \exp \left\{ \frac{-\phi^\star}{100} \left( \frac{rer_t b_t^\star}{gdpn_t} - \frac{rer b^\star}{gdpn} \right) \right\} \xi_t^R z_t^\tau \quad (88)$$

$$x_t^{H\star} = \left( \frac{p_t^H}{rer_t} \right)^{-\eta^\star} y_t^\star \quad (89)$$

## A.15 Aggregation and Market Clearing

$$y_t^C = c_t^P + c_t^I + i_t^K + i_t^H + g_t + v_t \quad (90)$$

$$c_t^P = \varphi_U c_t^U + (1 - \varphi_U) c_t^R \quad (91)$$

$$\begin{aligned} v_t a_t \pi_t = & \gamma_D P D_t^D R_{t-1}^D d_{t-1}^F + \gamma_{BH} P D_t^H R_t^{BB} q_t^{BB} b b_{t-1}^{Pr} + \mu_e G_e (\bar{\omega}_t^e) R_t^e q_{t-1}^K k_{t-1} + \mu_I G_I (\bar{\omega}_t^I) R_t^H q_{t-1}^H h_{t-1}^I \\ & + \mu_H G_H (\bar{\omega}_t^H) \tilde{R}_t^H l_{t-1}^H q_{t-1}^L + \mu_F G_F (\bar{\omega}_t^F) \tilde{R}_t^F l_{t-1}^F + \frac{\gamma_n}{2} \left( \frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1 \right)^2 y_t^Z p_t^Z \end{aligned} \quad (92)$$

$$y_t^H = x_t^H + x_t^{H*} \quad (93)$$

$$y_t^F = x_t^F \quad (94)$$

$$h_t = h_t^P + h_t^I \quad (95)$$

$$h_t^P = \varphi_U h_t^U + (1 - \varphi_U) h_t^R \quad (96)$$

$$bl_t^{Pr} = \varphi_U bl_t^U + (1 - \varphi_U) bl_t^R \quad (97)$$

$$bs_t^{Pr} = \varphi_U bs_t^U \quad (98)$$

$$bb_t^{Tot} = \varphi_U bb_t^U \quad (99)$$

$$b_t^{*Tot} = \varphi_U b_t^{*U} \quad (100)$$

$$bl_t^{Pr} + bl_t^{CB} + bl_t^G = 0 \quad (101)$$

$$bs_t^{Pr} + bs_t^G = 0 \quad (102)$$

$$d_t^F = \varphi_U d_t^U \quad (103)$$

$$\zeta_t^L = \left( \frac{q_t^{BL} bl_t^U + q_t^{BB} b b_t^U}{bs_t^U + rer_t b_t^{*,U} + d_t^U} \right)^{\eta_\zeta} \epsilon_t^{L,S} \quad (104)$$

$$\tilde{R}_t^D = R_{t-1}^D (1 - \gamma_D P D_t^D) \quad (105)$$

$$\tilde{R}_t^{BB} = R_t^{BB} (1 - \gamma_{BH} P D_t^H) \quad (106)$$

$$R_t^{BL} = \frac{1}{q_t^{BL}} + \kappa_{BL} \quad (107)$$

$$R_t^{BB} = \frac{1}{q_t^{BB}} + \kappa_{BB} \quad (108)$$

$$R_t^{Nom, BL} = R_t^{BL} \pi_t \quad (109)$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \quad (110)$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \quad (111)$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} a_t \pi_t \quad (112)$$

$$\pi_t^{\widetilde{W}} = \frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \pi_t^W \quad (113)$$

$$y_t^H \Xi_t^H = x_t^Z \quad (114)$$

$$y_t^Z = z_t \left( \frac{k_{t-1}}{a_t} \right)^\alpha \tilde{n}_t^{1-\alpha} \quad (115)$$

$$y_t^Z = x_t^Z \quad (116)$$

$$\Xi_t^H = (1 - \theta_H) (\tilde{p}_t^H)^{-\epsilon_H} + \theta_H \left( \frac{\pi_t^{I,H}}{\pi_t^H} \right)^{-\epsilon_H} \Xi_{t-1}^H \quad (117)$$

$$m_t = y_t^F \Xi_t^F \quad (118)$$

$$\Xi_t^F = (1 - \theta_F) (\tilde{p}_t^F)^{-\epsilon_F} + \theta_F \left( \frac{\pi_t^{I,F}}{\pi_t^F} \right)^{-\epsilon_F} \Xi_{t-1}^F \quad (119)$$

$$n_t = \tilde{n}_t \Xi_t^W \quad (120)$$

$$\Xi_t^W = (1 - \theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_W \left( \frac{\pi_t^{I,W}}{\pi_t^W} \right)^{-\epsilon_W} \Xi_{t-1}^W \quad (121)$$

$$n_t = n_t^P + n_t^I \quad (122)$$

$$n_t^P = n_t^I \quad (123)$$

$$n_t^P = \wp_U n_t^U + (1 - \wp_U) n_t^R \quad (124)$$

$$n_t^U = n_t^R \quad (125)$$

$$gd p_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + x_t^{H\star} + y_t^{Co} - m_t \quad (126)$$

$$gd p_n_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + tb_t \quad (127)$$

$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \quad (128)$$

$$rer_t b_t^\star = \frac{rer_t}{a_t \pi_t^\star} b_{t-1}^\star R_{t-1}^\star + tb_t + rer_t ren^* - (1 - \chi) rer_t p_t^{Co\star} y_t^{Co} \quad (129)$$

The exogenous processes are:

$$\log(z_t/z) = \rho_z \log(z_{t-1}/z) + u_t^z$$

$$\log(a_t/a) = \rho_a \log(a_{t-1}/a) + u_t^a$$

$$\log(\xi_t^n/\xi^n) = \rho_{\xi^n} \log(\xi_{t-1}^n/\xi^n) + u_t^{\xi^n}$$

$$\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + u_t^{\xi^h}$$

$$\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + u_t^{\xi^i}$$

$$\log(\xi_t^{ih}/\xi^{ih}) = \rho_{\xi^{ih}} \log(\xi_{t-1}^{ih}/\xi^{ih}) + u_t^{\xi^{ih}}$$

$$\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + u_t^{\xi^R}$$

$$\log(e_t^m/e^m) = \rho_{e^m} \log(e_{t-1}^m/e^m) + u_t^{e^m}$$

$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + u_t^g$$

$$\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + u_t^{y^{Co}}$$

$$\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + u_t^{\pi^\star}$$

$$\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + u_t^{R^W}$$

$$\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + u_t^{y^\star}$$

$$\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + u_t^{p^{Co\star}}$$

$$\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + u_t^{\xi^m}$$

$$\log(\sigma_t^I/\sigma^I) = \rho_{\sigma^I} \log(\sigma_{t-1}^I/\sigma^I) + u_t^{\sigma^I}$$

$$\log(\sigma_t^e/\sigma^e) = \rho_{\sigma^e} \log(\sigma_{t-1}^e/\sigma^e) + u_t^{\sigma^e}$$

$$\log(\sigma_t^F/\sigma^F) = \rho_{\sigma^F} \log(\sigma_{t-1}^F/\sigma^F) + u_t^{\sigma^F}$$

$$\log(\sigma_t^H/\sigma^H) = \rho_{\sigma^H} \log(\sigma_{t-1}^H/\sigma^H) + u_t^{\sigma^H}$$

$$\log(\epsilon_t^{L,S}/\epsilon^{L,S}) = \rho_{\epsilon^{L,S}} \log(\epsilon_{t-1}^{L,S}/\epsilon^{L,S}) + u_t^{\epsilon^{L,S}}$$

$$\log(bl_t^G/bl^G) = \rho_{bl^G} \log(bl_{t-1}^G/bl^G) + u_t^{bl^G}$$

$$\log(bl_t^{CB}/bl^{CB}) = \rho_{bl^{CB}} \log(bl_{t-1}^{CB}/bl^{CB}) + u_t^{bl^{CB}}$$

$$\log(\varrho_t/\varrho) = \rho_\varrho \log(\varrho_{t-1}/\varrho) + u_t^\varrho$$

$$\log(\xi_t^{\chi^b}/\xi^{\chi^b}) = \rho_\xi^{\chi^b} \log(\xi_{t-1}^{\chi^b}/\xi^{\chi^b}) + u_t^{\xi^{\chi^b}}$$

$$\log(\xi_t^{\chi^e}/\xi^{\chi^e}) = \rho_\xi^{\chi^e} \log(\xi_{t-1}^{\chi^e}/\xi^{\chi^e}) + u_t^{\xi^{\chi^e}}$$

$$\log(\xi_t^{roe}/\xi^{roe}) = \rho_\xi^{roe} \log(\xi_{t-1}^{roe}/\xi^{roe}) + u_t^{\xi^{roe}}$$

$$\log(z_t^\tau/z^\tau) = \rho_{z^\tau} \log(z_{t-1}^\tau/z^\tau) + u_t^{z^\tau}$$

Where all disturbances  $u_t^j$  are normally distributed with zero mean and  $\sigma^j$  standard deviation:  $u_t^j \sim \mathcal{N}(0, (\sigma^j)^2)$

## B Steady State Computation

In this section we show how to compute the steady state for a given value of most of the parameters and all exogenous variables in the long run, except for  $o_{\hat{C}}$ ,  $R^W$ ,  $\pi^*$ ,  $g$ ,  $y^{Co}$ ,  $y^*$ ,  $ren^*$ ,  $\xi^n$ ,  $\sigma^F$ ,  $\sigma^H$ ,  $\sigma^e$  and  $\sigma^I$  that are determined endogenously by imposing values for the steady state of the following endogenous variables:  $r^{h,k} = q^H h / q^K k$ ,  $\pi^s$ ,  $p^H$ ,  $s^g = g / gdpn$ ,  $s^{Co} = p^{Co*} y^{Co} rer / gdpn$ ,  $s^{tb} = tb / gdpn$ ,  $s^{b*} = b^* rer / gdpn$ ,  $n$ ,  $R^D$ ,  $R^L$  and  $R^{LG}$ , while also imposing  $PD^F = PD^H$ .

Use (86), (4), (5), (6), (88) and (87):

$$\pi = \pi^T, \quad R = \frac{\pi a^\sigma}{\beta_{UP}}, \quad \tilde{R}^D = R, \quad R^* = \frac{R}{\pi^s}, \quad R^W = \frac{R^*}{\xi^R}, \quad \pi^* = \frac{\pi}{\pi^s}$$

From (110), (111), (64) and (73) and using  $\pi = \pi^T$ :

$$\pi^H = \pi^F = \pi^{I,H} = \pi^{I,F} = \pi$$

From (112), (113) and (83):

$$\pi^W = \pi^{\tilde{W}} = \pi^{I,W} = a\pi$$

From (63), (72), (82), (62), (61), (70), (71), (80), (81), (117), (119) and (121):

$$\tilde{p}^H = \tilde{p}^F = \tilde{w} = 1, \quad mc^H = \frac{\epsilon_H - 1}{\epsilon_H}, \quad mc^F = \frac{\epsilon_F - 1}{\epsilon_F}, \quad mc^W = \frac{\epsilon_W - 1}{\epsilon_W}, \quad \Xi^H = \Xi^F = \Xi^W = 1$$

From (54) and (56):

$$q^K = 1/\xi^i, \quad q^H = \frac{a^{N_H} \sigma \varphi_0^H}{\beta_{UP}^{N_H} \xi^{ih}} \left( \frac{1 - \left( \frac{\beta_{UP} \rho^{\varphi_H}}{a^\sigma} \right)^{N_H+1}}{1 - \frac{\beta_P \rho^{\varphi_H}}{a^\sigma}} \right)$$

From (14) and (120):

$$R^H = \pi (1 - \delta_H), \quad \tilde{n} = n$$

From (45), (33), (35), (36) and (37):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (38), (105) and using  $PD^F = PD^H$ :

$$PD^D = \frac{1}{\gamma_D} \left( 1 - \frac{\tilde{R}^D}{R^D} \right) = PD^H = PD^F$$

From (109)

$$R^{BL} = \frac{R^{Nom, BL}}{\pi}$$

From (12)

$$\beta_{RP} = \frac{a^\sigma}{R^{BL}}$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (106)

$$R^{BB} = \frac{\tilde{R}^{BB}}{1 - \gamma_D PD^H}$$

From (18), (19), (47), (107), and (109)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}, \quad q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}, \quad q^L = \frac{1}{R^I - \kappa_L}, \quad q^{\widehat{BB}} = q^{BB}, \quad q^{\widehat{L}} = q^L$$

From (40) and (42) we obtain:

$$\bar{\omega}^F = [1 - \Gamma_F(\bar{\omega}^F)] \left( \frac{1 - \phi_F}{\phi_F} \right) \frac{R^D}{\tilde{\rho}^F}$$

Which we solve numerically jointly with (44):

$$PD^F = F_F(\bar{\omega}^F)$$

to obtain  $\bar{\omega}^F$  and  $\sigma^F$ .

Similarly, from (46) and (50) we obtain:

$$\bar{\omega}^H = [1 - \Gamma_H(\bar{\omega}^H)] \left( \frac{1 - \phi_H}{\phi_H} \right) \frac{R^{BB}}{\rho^H \pi}$$

Which we solve numerically jointly with (52):

$$PD^H = F_H(\bar{\omega}^H)$$

To obtain  $\bar{\omega}^H$  and  $\sigma^H$ .

Then, from (42) and (50):

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F(\bar{\omega}^F)}, \quad \tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H(\bar{\omega}^H)}$$

From (43) and (26) we obtain:

$$\tilde{R}^F = [\Gamma_e(\bar{\omega}^e) - \mu_e G_e(\bar{\omega}^e)] \frac{R^L}{\bar{\omega}^e}$$

And from (28), (24), (29), (30), (31), (42) and (43) we obtain:

$$\frac{\Gamma'_e(\bar{\omega}^e) - \mu_e G'_e(\bar{\omega}^e)}{\Gamma'_e(\bar{\omega}^e)} = \frac{(1 - \chi_e) \tilde{R}^F}{a\pi}$$

Which we solve numerically, jointly with the previous equation, to obtain  $\bar{\omega}^e$  and  $\sigma^e$ .

Analogously, from (51) and (17) we obtain:

$$\tilde{R}^H = [\Gamma_I(\bar{\omega}^I) - \mu_I G_I(\bar{\omega}^I)] \frac{R^I \pi}{\bar{\omega}^I}$$

Using (50) and (21) we can obtain

$$\frac{\Gamma'_I(\bar{\omega}^I) - \mu_I G'_I(\bar{\omega}^I)}{\Gamma'_I(\bar{\omega}^I)} = \frac{\beta_I \tilde{R}^H}{a\pi}$$

which we solve together to find  $\sigma_I$  and  $\bar{\omega}^I$ .

From (32) and (23):

$$PD^e = F_e(\bar{\omega}^e), \quad PD^I = F_I(\bar{\omega}^I)$$

From (28), (24), (29) and (43):

$$R^e = \frac{\tilde{R}^F a\pi}{a\pi [\Gamma_e(\bar{\omega}^e) - \mu_e G_e(\bar{\omega}^e)] + [1 - \Gamma_e(\bar{\omega}^e)] (1 - \chi_e) \tilde{R}^F}$$

From (25):

$$r^K = q^K \left[ \frac{R^e}{\pi} - (1 - \delta_K) \right]$$

From (66), (67), (53) and (115):

$$w = \left[ \frac{\alpha^\alpha (1-\alpha)^{1-\alpha} p^H m c^Z z}{(r^k)^\alpha} \right]^{\frac{1}{1-\alpha}}$$

$$k = \frac{\alpha}{1-\alpha} \tilde{n} \frac{w}{r^k} a$$

$$i = k \left[ \frac{1 - (1 - \delta_K)/a}{\xi^i} \right]$$

$$y^Z = z \left( \frac{k}{a} \right)^\alpha \tilde{n}^{1-\alpha}$$

Also, from (114) and (116)

$$x^Z = y^Z, \quad y^H = \frac{x^Z}{\Xi^H}$$

From (29), (28), (27), (31) and (24):

$$\psi^e = [1 - \Gamma_e(\bar{\omega}^e)] \frac{R^e q^K k}{a \pi}$$

$$n^e = (1 - \chi_e) \psi^e, \quad c^e = \chi_e \psi^e$$

$$\lambda^e = \frac{\Gamma^{e'}(\bar{\omega}^e)}{(1 - \Gamma^F(\bar{\omega}^F)) [\Gamma^{e'}(\bar{\omega}^e) - \mu^e G^{e'}(\bar{\omega}^e)]}$$

$$l^F = q^K k - n^e$$

From (41) and (39):

$$e^F = \phi_F l^F, \quad d^F = l^F - e^F$$

From  $r^{h,k} = q^H h / q^K k$ , (55) and (57):

$$h = \frac{r^{h,k} q^K k}{q^H}, \quad i^{AH} = \frac{h a^{N_H}}{\xi^{ih}} \left[ 1 - \left( \frac{1 - \delta_H}{a} \right) \right], \quad i^H = i^{AH} \varphi_0^H \left[ \frac{1 - \left( \frac{\rho^{\varphi_H}}{a} \right)^{N_H+1}}{1 - \frac{\rho^{\varphi_H}}{a}} \right]$$

From (58), (59) and (60):

$$p^F = \left[ \frac{1 - \omega(p^H)^{1-\eta}}{1 - \omega} \right]^{\frac{1}{1-\eta}}$$

From (69):

$$rer = mc^F p^F / \xi^m$$

Obtain a numerical solution for  $l^H$  from several model equations. See Appendix B.1. for details. Then, from (17) solve for  $h^I$ :

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

and from (48) and (49):

$$bb^{Tot} = (1 - \phi_H) \frac{q^L l^H}{q^{BB}}; \quad e_h = \phi_H q^L l^H$$

From (37), (35) and (34):

$$n^b = e^F + e^H, \quad \psi^b = \frac{n^b}{1 - \chi_b \xi^{\chi_b}}, \quad c^b = \chi_b \xi^{\chi_b} \psi^b$$

Then, from (92):

$$v = \frac{1}{a\pi} \begin{pmatrix} \gamma_D P D^D R^D d^F + \gamma_{BB} P D^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e (\bar{\omega}^e) R^e q^K k \\ + \mu_I G_I (\bar{\omega}^I) R^H q^H h^I + \mu_H G_H (\bar{\omega}^H) \tilde{R}^H q^L l^H + \mu_F G_F (\bar{\omega}^F) \tilde{R}^F l^F \end{pmatrix}$$

From (127), (90), (128), (58), (59), (60), (118), (93) and (94):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta} (p^F - rer \xi^m \Xi^F) (1 - \omega) v - v}{1 - s^{Co} - (1 - s^{tb}) (p^F)^{-\eta} (p^F - rer \xi^m \Xi^F) (1 - \omega)}$$

Thus, from their definitions:

$$tb = s^{tb} gdpn, \quad g = s^g gdpn, \quad y^{Co} = \frac{s^{Co} gdpn}{p^{Co*} rer}, \quad b^* = \frac{s^{b*} gdpn}{rer}$$

From (90), (127), (59), (60), (93), (89), (94) and (118):

$$\begin{aligned} y^C &= gdpn + v - tb, & x^F &= (1 - \omega)(p^F)^{-\eta} y^C, \\ x^H &= \omega(p^H)^{-\eta} y^C, & x^{H\star} &= y^H - x^H, \\ y^\star &= x^{H\star} \left( \frac{p^H}{rer} \right)^{\eta\star}, & y^F &= x^F, \\ m &= y^F \Xi^F \end{aligned}$$

From (95):

$$h^P = h - h^I$$

From (22):

$$c^I = \frac{wn}{2} + q^H h^I \left[ (1 - \Gamma_I) \frac{R^H}{a\pi} - 1 \right] + q^L l^H$$

From (20) and (16):

$$o_{\hat{C}} = \left\{ (a)^{-\sigma\eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}}-1} \left( \frac{ac^I \left( 1 - \frac{\phi_c}{a} \right)}{h^I \left( 1 - \frac{\phi_{hh}}{a} \right)} \right) \left( \frac{1}{\beta_I} \left[ q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1 - \Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( c^I \left( 1 - \frac{\phi_c}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \frac{\xi^h h^I}{a} \left( 1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (16):

$$\lambda^I = \left\{ (\hat{c}^I)^{-\sigma} \right\} \left( \frac{(1 - o_{\hat{C}}) \hat{c}^I}{c^I \left( 1 - \frac{\phi_c}{a} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

Use ratios  $\alpha_{BLG} = \frac{bl^G}{gdpnq^{BL}}$  and  $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$

$$bs^G = \alpha_{SG} gdpn$$

Then from (101) and (102), and normalizing  $bl^{CB} = 0$

$$bl^{Pr} = -bl^G$$

$$bs^{Pr} = -bs^G$$

We can solve for bond holdings of the unrestricted households Also, from (98) and (99)

$$bs^U = \frac{bs^{Pr}}{\wp^U}, \quad bb^U = \frac{bb^{tot}}{\wp^U}$$

Use ratio  $s^{b*} = b^*rer/gdpn$ , and (100)

$$b^{*Tot} = s^{b*} * gdpn/rer, \quad b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household,  $\omega_{BL}$

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}};$$

We can then, using (97) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U};$$

Next, we solve for  $h^R$ ,  $c^R$ ,  $\hat{c}^R$ ,  $\lambda^R$ . From (10) and (11) and the restricted household budget constraint (13)

$$h^R = \frac{q^{BL} bl^R \left( \frac{R^{BL}}{a} - 1 \right) + \frac{wn}{2}}{q^H - \frac{q^H}{a} (1 - \delta_H) + aux_1}$$

with  $aux_1$

$$aux_1 = (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^h)^{1 - \eta_{\hat{C}}} \left( \frac{q^H}{\beta_P} - (1 - \delta_H) a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left( 1 - \frac{\phi_{hh}}{a} \right)}{o_{\hat{C}} \left( 1 - \frac{\phi_c}{a} \right)}$$

and

$$c^R = h^R aux_1$$

From (9):

$$\hat{c}^R = \left[ \left( 1 - o_{\hat{C}} \right)^{\frac{1}{\eta_{\hat{C}}} \left( c^R \left( 1 - \frac{\phi_c}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} + \left( o_{\hat{C}} \right)^{\frac{1}{\eta_{\hat{C}}} \left( \xi^h \frac{h^R}{a} \left( 1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}}} \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}} - 1}}$$

From (10):

$$\lambda^R = \left\{ \left( \hat{c}^R \right)^{-\sigma} \right\} \left( \frac{\left( 1 - o_{\hat{C}} \right) \hat{c}^R}{c^R \left( 1 - \frac{\phi_c}{a} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

Also, from (96) we get

$$h^U = \frac{h^P - (1 - \wp_U)h^R}{\wp_U}$$

which together with (2) and (3) lets us solve for  $c^U$

$$c^U = h^U(a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}} \left( \frac{q^H}{\beta_P} - (1 - \delta_H) a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

From (1) we solve for  $\hat{c}^U$

$$\hat{c}^U = \left[ (1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( c^U \left(1 - \frac{\phi_c}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left( \xi^h \frac{h^U}{a} \left(1 - \frac{\phi_{hh}}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (2) we obtain  $\lambda^U$

$$\lambda^U = (\hat{c}^U)^{-\sigma} \left( \frac{(1 - o_{\hat{C}}) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (91):

$$c^P = \wp_U c^U + (1 - \wp_U) c^R$$

From (75):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U) \lambda^R$$

From (79):

$$\tilde{\chi}^U = (\hat{c}^U)^\sigma, \quad \tilde{\chi}^R = (\hat{c}^R)^\sigma, \quad \tilde{\chi}^I = (\hat{c}^I)^\sigma$$

From (78):

$$\Theta^U = 1, \quad \Theta^U = 1, \quad \Theta^I = 1, \quad \Theta = \frac{(\omega_{UP} \Theta^U + (1 - \omega_U) \Theta^R) + \Theta^I}{2} = 1$$

From (74) and (77):

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}, \quad \xi^n = \frac{mc^W \lambda^W w}{\Theta \tilde{n}^\varphi}$$

From (122), (123), (124), (125)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

From (126) and (129):

$$gdः = c + i + i^h + g + x^{H\star} + y^{Co} - m$$

$$ren^* = b^* \left( 1 - \frac{R^*}{a\pi^*} \right) - \frac{tb}{rer} + (1 - \chi) p^{Co\star} y^{Co}$$

From (7) and (104)

$$\epsilon^{L,S} = \frac{\beta_U R^{BL} a^{-\sigma} - 1}{\left( \frac{q^{BL} b l^U + q_t^{BB} b b_t^U}{bs^U + rer b^{*U} + d^U} \right)^{\rho_\zeta}}$$

From (104) :

$$\zeta^L = \left( \frac{q^{BL} b l^U + q_t^{BB} b b_t^U}{bs^U + rer b^{*U} + d^U} \right)^{\rho_\zeta} \epsilon^{L,S}$$

From (84):

$$\tau = g + dia - bs^G \left( \frac{R}{a\pi} - 1 \right) - q^{BL} bl^G \left( \frac{R^{BL}}{a} - 1 \right) - \chi rerp^{Co*} y^{Co}$$

From (85):

$$\alpha^T = \frac{\tau}{gdpm}$$

Finally, from (62), (71) and (81):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H m c^H}{1 - \beta_{UP} \theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F m c^F}{1 - \beta_{UP} \theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W (1+\varphi)} m c^W \tilde{n}}{1 - \left( \frac{(\omega_{UP} \beta^{UP} + (1-\omega_{UP}) \beta^{RP}) + \beta_I}{2} \right) \theta_W a^{1-\sigma}}$$

## B.1 Numerical solution for $l^H$

First, guess  $l^H$ . Then, from (17) solve for  $h^I$ :

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

From (49) and (48):

$$bb^{Tot} = (1 - \phi_H) \frac{q^L l^H}{q^{BB}}$$

Then, from (92):

$$v = \frac{1}{a\pi} \begin{pmatrix} \gamma_D P D^D R^D d^F + \gamma_{BB} P D^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e (\bar{\omega}^e) R^e q^K k \\ + \mu_I G_I (\bar{\omega}^I) R^H q^H h^I + \mu_H G_H (\bar{\omega}^H) \tilde{R}^H q^L l^H + \mu_F G_F (\bar{\omega}^F) \tilde{R}^F l^F \end{pmatrix}$$

From (127), (90), (128), (58), (59), (60), (118), (93) and (94):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta} (p^F - rer\xi^m \Xi^F) (1 - \omega) v - v}{1 - s^{Co} - (1 - s^{tb}) (p^F)^{-\eta} (p^F - rer\xi^m \Xi^F) (1 - \omega)}$$

From (95):

$$h^P = h - h^I$$

From (22):

$$c^I = \frac{wn}{2} + q^H h^I \left[ (1 - \Gamma_I) \frac{R^H}{a\pi} - 1 \right] + q^L l^H$$

From (20) and (16):

$$o_{\hat{C}} = \left\{ (a)^{-\sigma \eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}} - 1} \left( \frac{ac^I \left( 1 - \frac{\phi_c}{a} \right)}{h^I \left( 1 - \frac{\phi_{hh}}{a} \right)} \right) \left( \frac{1}{\beta_I} \left[ q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1 - \Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Use ratios  $\alpha_{BLG} = \frac{bl^G}{gdpn q^{BL}}$  and  $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$

$$bs^G = \alpha_{SG} gdpn$$

Then from (101) and (102), and normalizing  $bl^{CB} = 0$

$$bl^{Pr} = -bl^G$$

$$bs^{Pr} = -bs^G$$

Also, from (98) and (99)

$$bs^U = \frac{bs^{Pr}}{\wp^U}, \quad bb^U = \frac{bb^{tot}}{\wp^U}$$

Use ratio  $s^{b*} = b^*rer/gdpn$ , and (100)

$$b^{*Tot} = s^{b*} * gdpn/rer, \quad b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the ratio of long to short term instruments held by the unrestricted patient household,  $\omega_{BL}$

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}};$$

which using (97) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U};$$

From (10) and (11) and the restricted household budget constraint (13)

$$h^R = \frac{q^{BL} bl^R \left( \frac{R^{BL}}{a} - 1 \right) + \frac{wn}{2}}{q^H - \frac{q^H}{a} (1 - \delta_H) + aux_1}$$

with  $aux_1$

$$aux_1 = (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^h)^{1 - \eta_{\hat{C}}} \left( \frac{q^H}{\beta_P} - (1 - \delta_H) a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left( 1 - \frac{\phi_{hh}}{a} \right)}{o_{\hat{C}} \left( 1 - \frac{\phi_c}{a} \right)}$$

and

$$c^R = h^R aux_1$$

Also, from (96) we get

$$h^U = \frac{h^P - (1 - \wp_U) h^R}{\wp_U}$$

which together with (2) and (3) lets us solve for  $c^U$

$$c^U = h^U(a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}} \left( \frac{q^H}{\beta_P} - (1 - \delta_H) a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

From (91):

$$c^P = \wp_U c^U + (1 - \wp_U) c^R$$

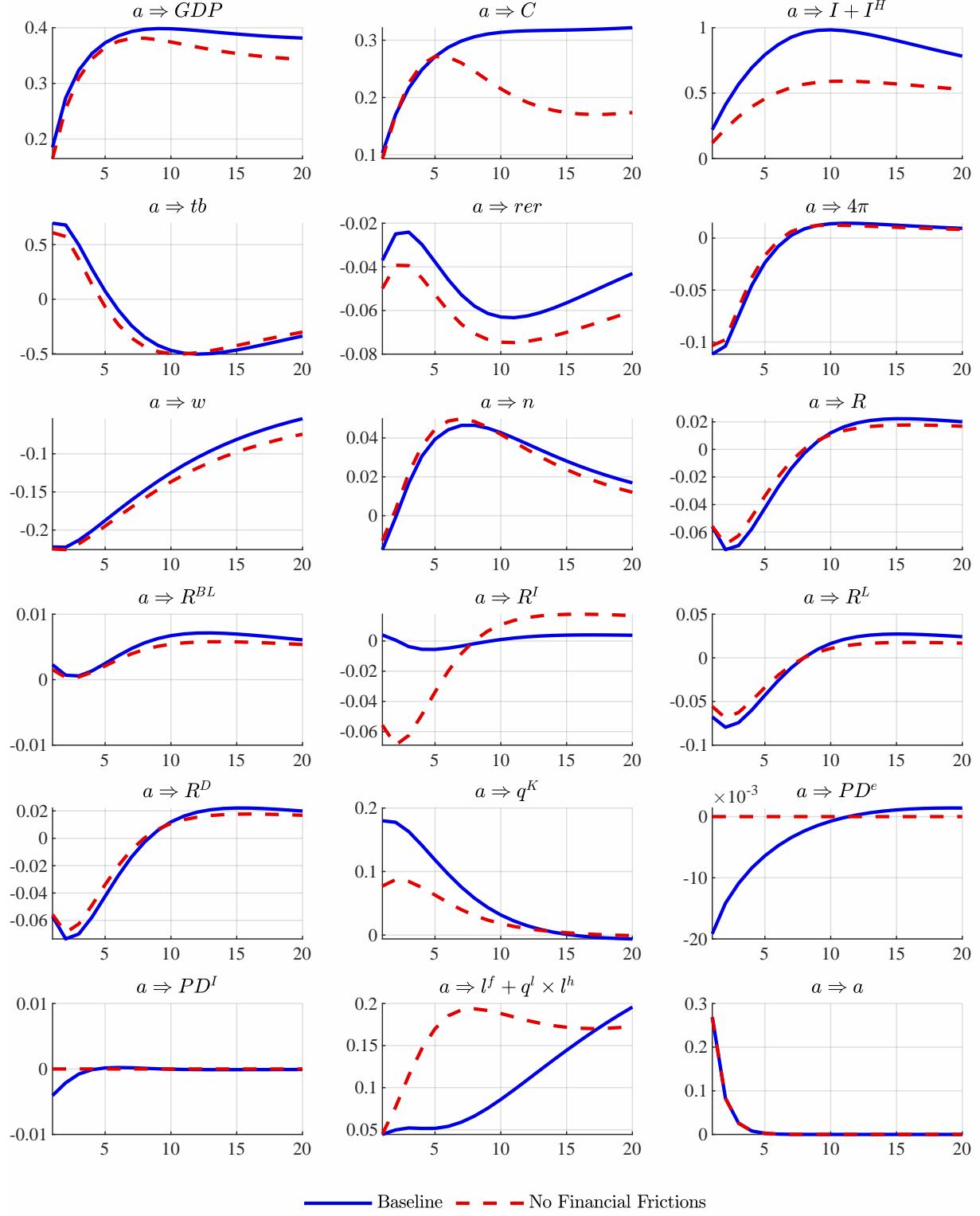
Then, the following equation must hold:

$$gdpn = c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn$$

If it does not, update guess of  $l^H$  and repeat.

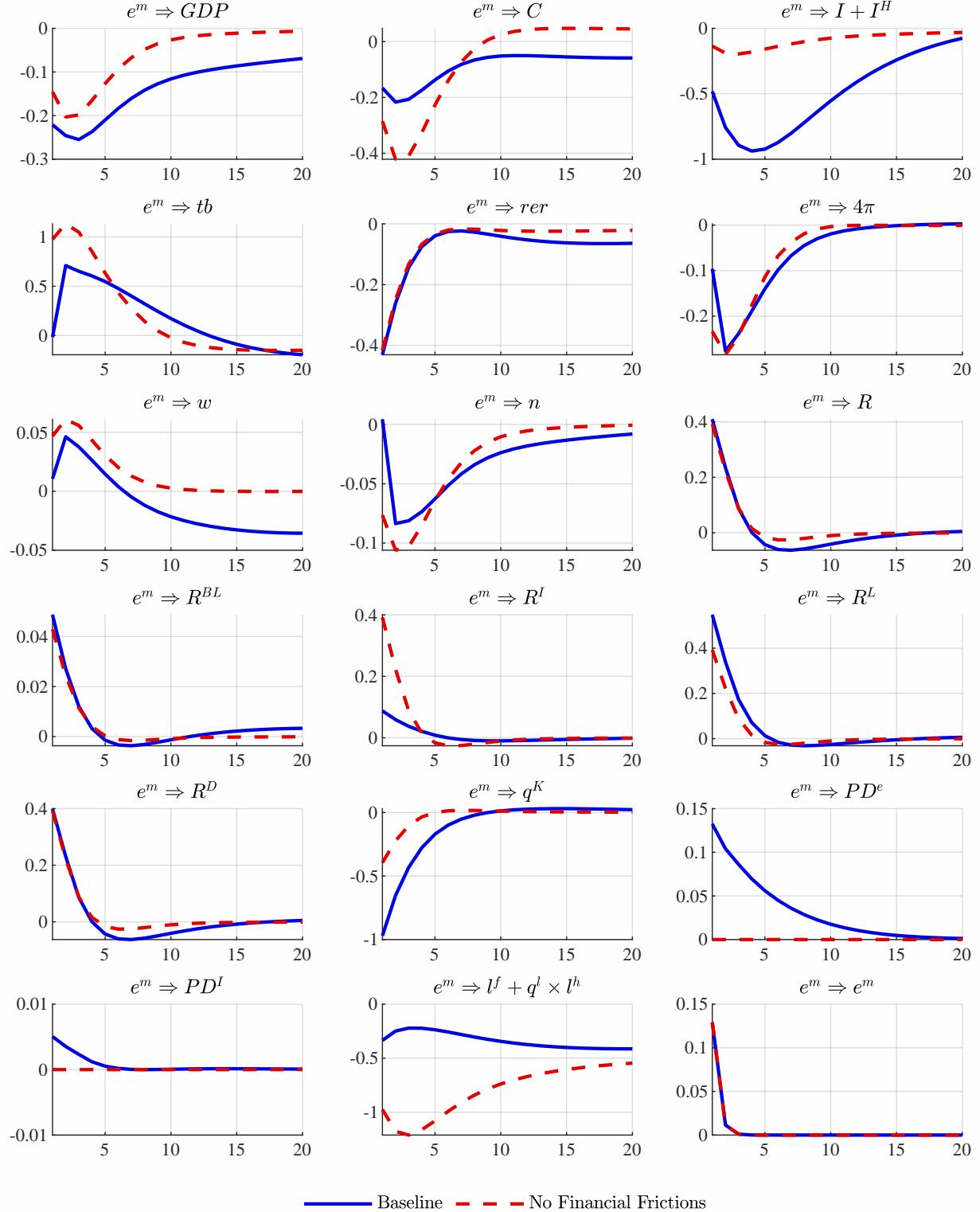
## C Impulse responses

Figure 10: Response to a non-stationary productivity shock



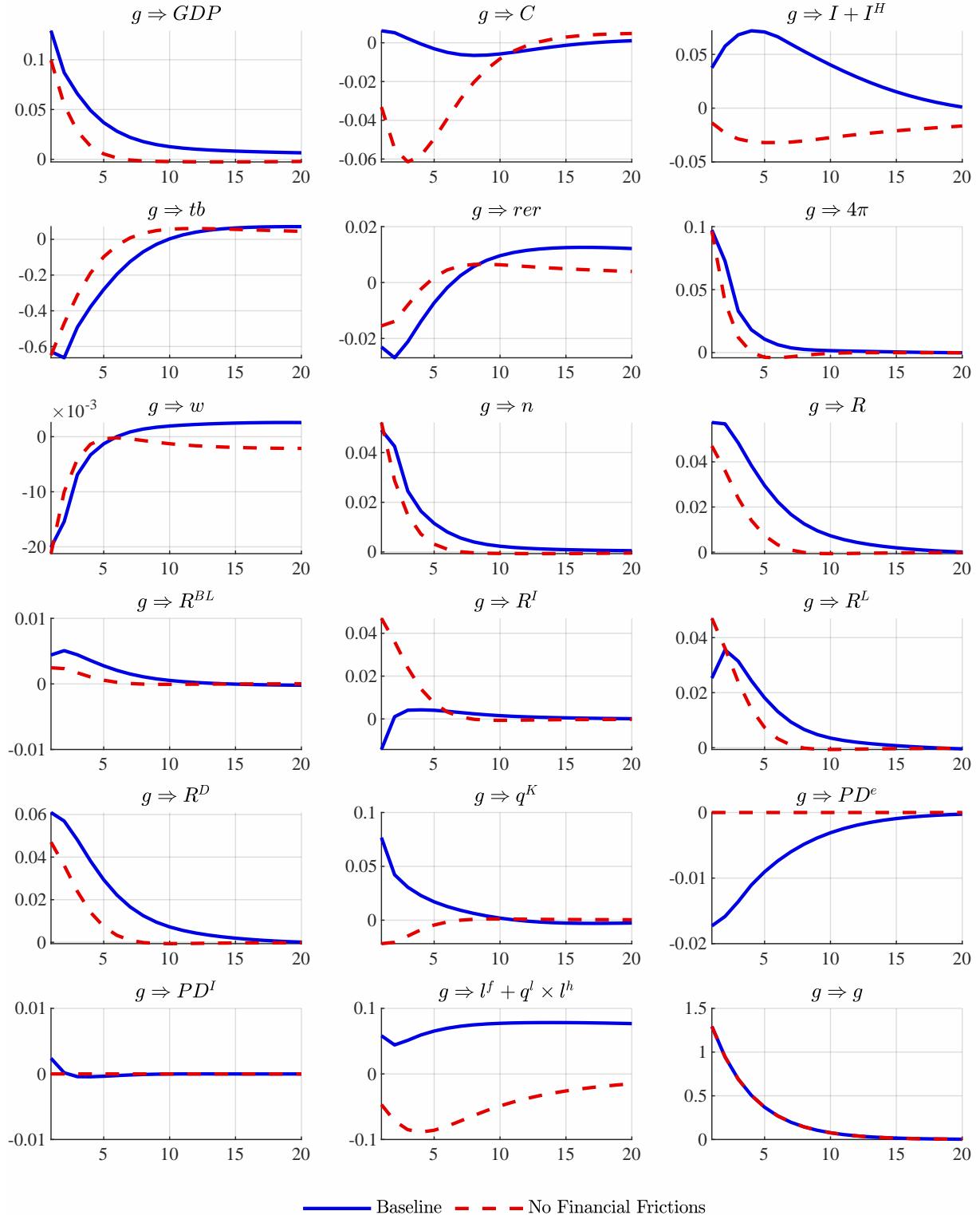
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 11: Response to a monetary policy shock



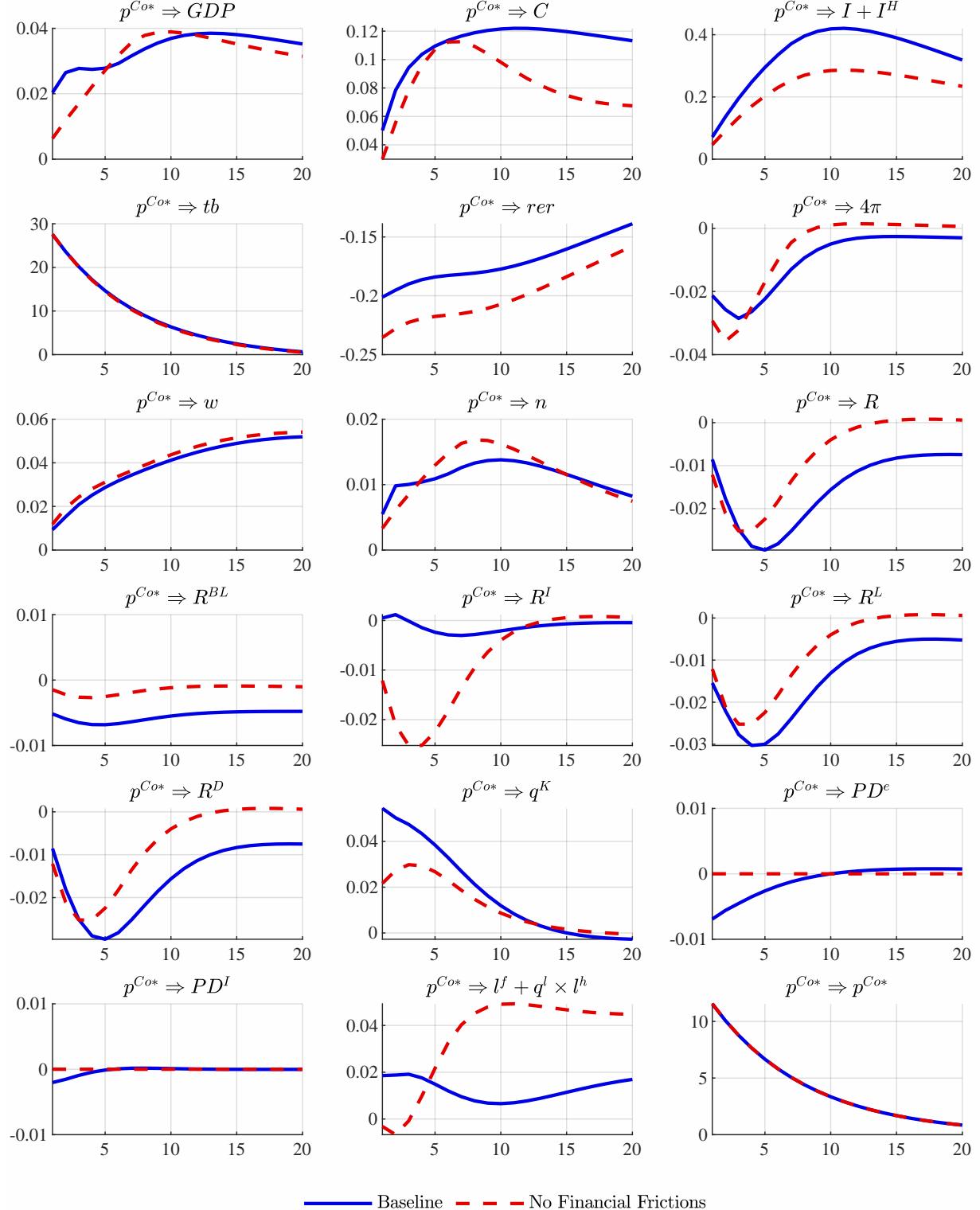
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 12: Response to a government spending shock



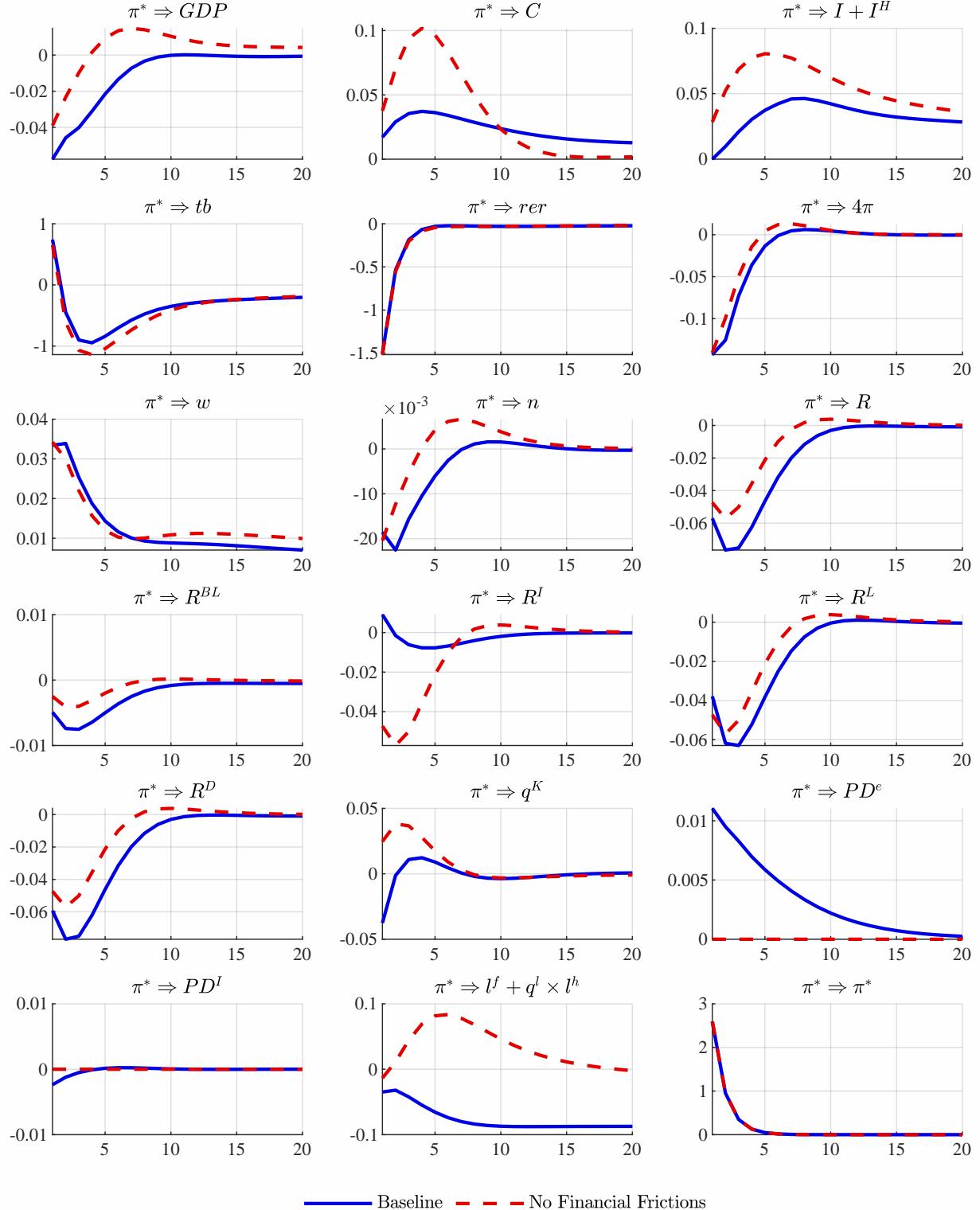
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 13: Response to a shock to the international price of the exported commodity



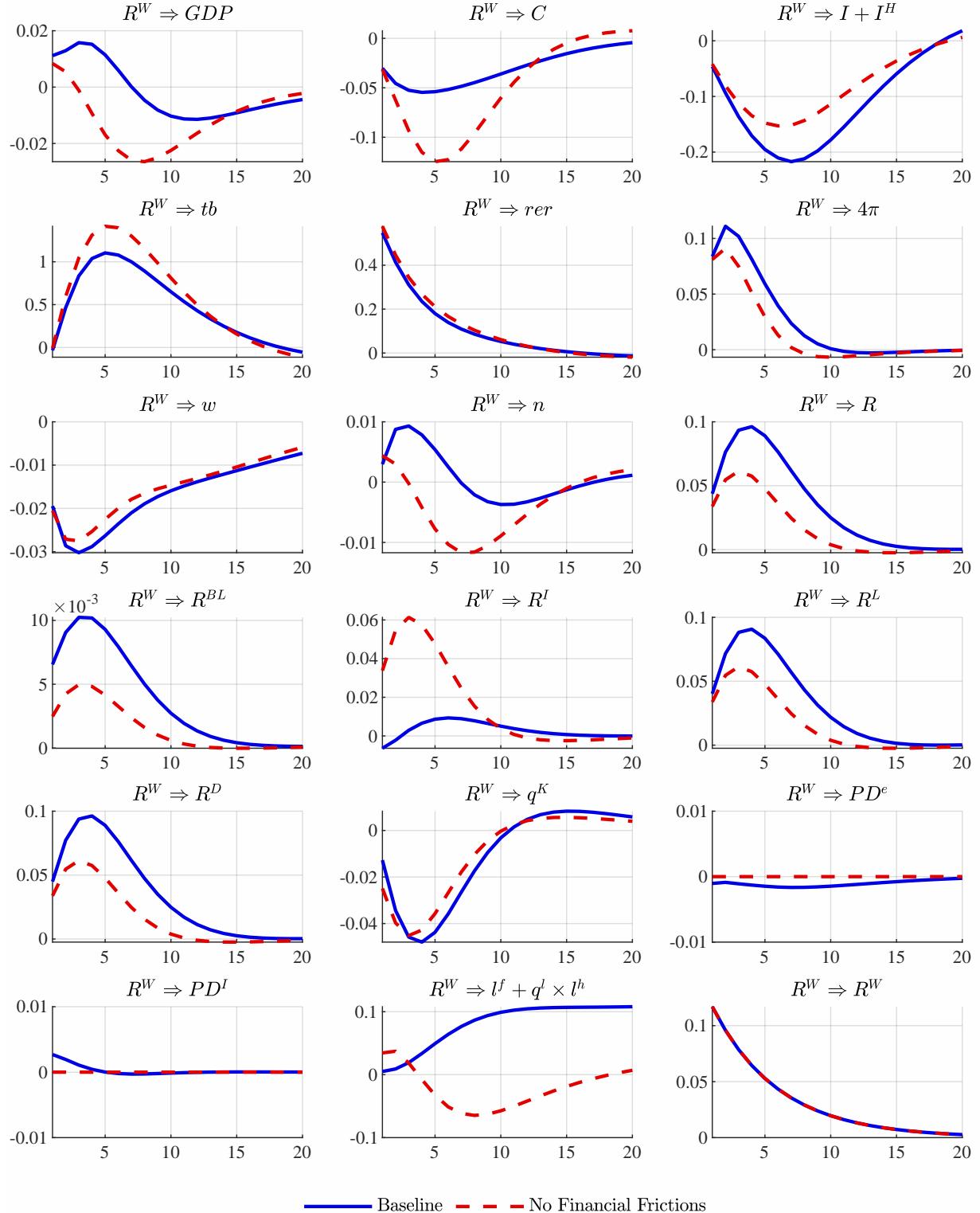
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 14: Response to a foreign inflation shock



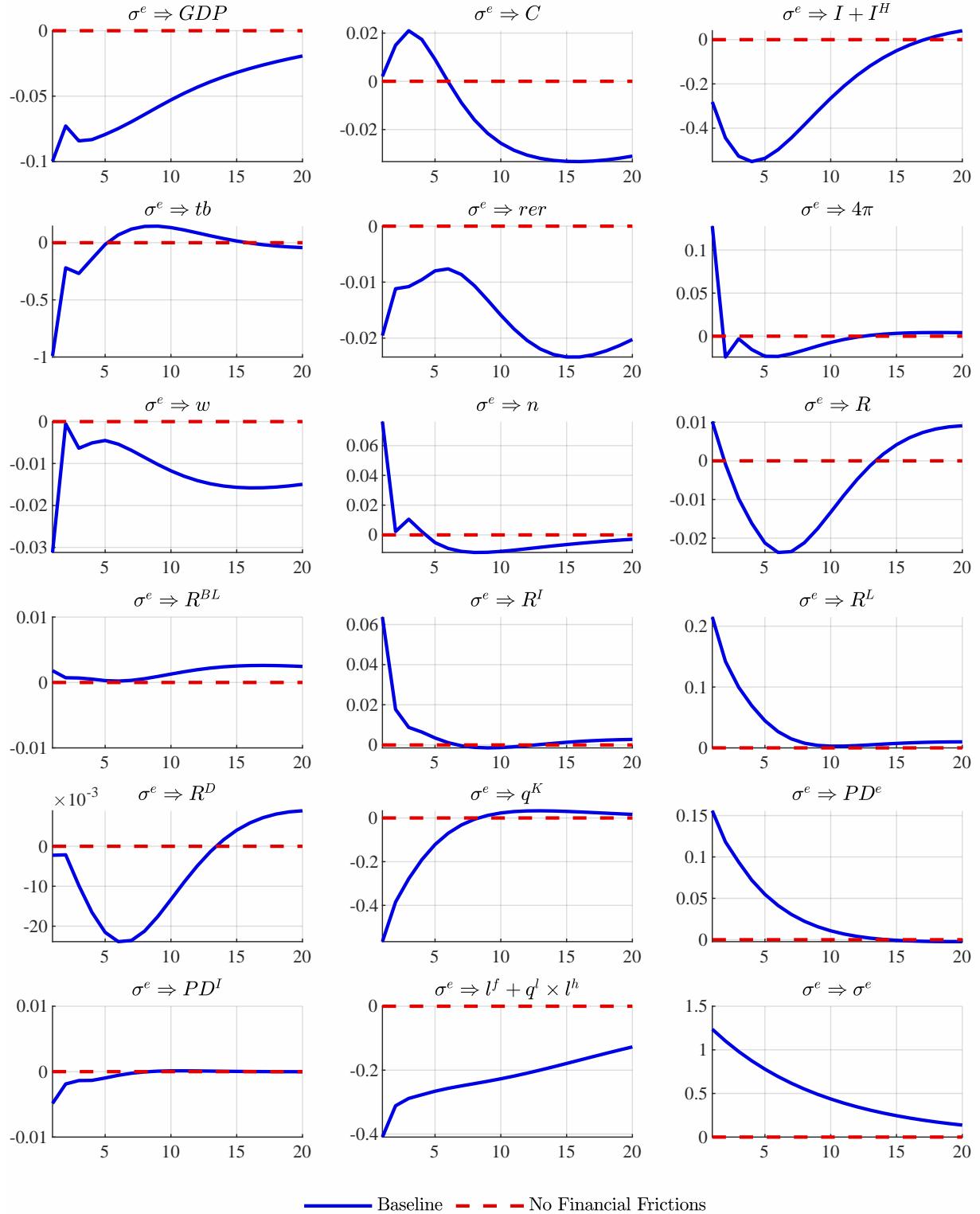
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 15: Response to a foreign interest rate shock



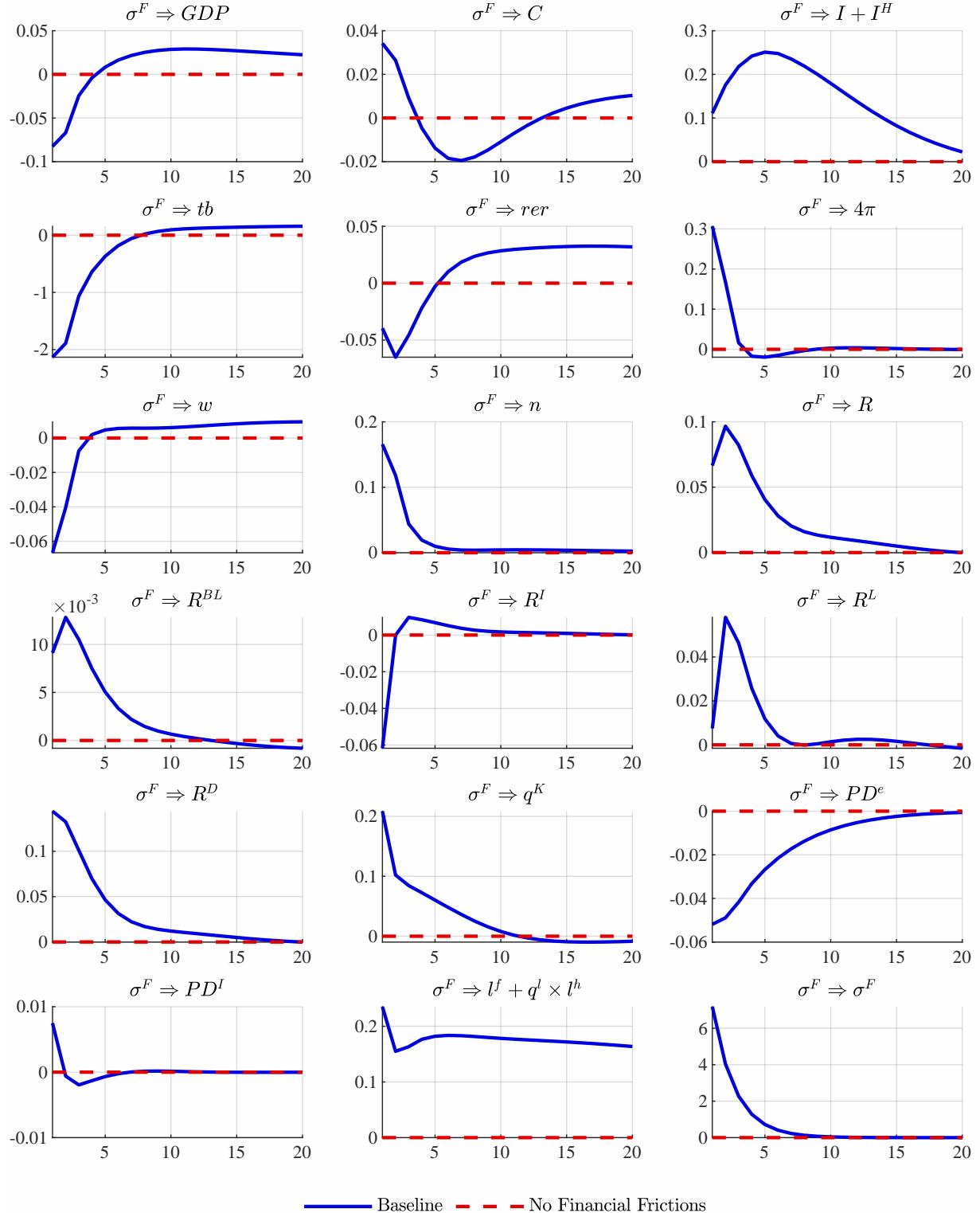
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 16: Response to a shock to entrepreneurs' risk



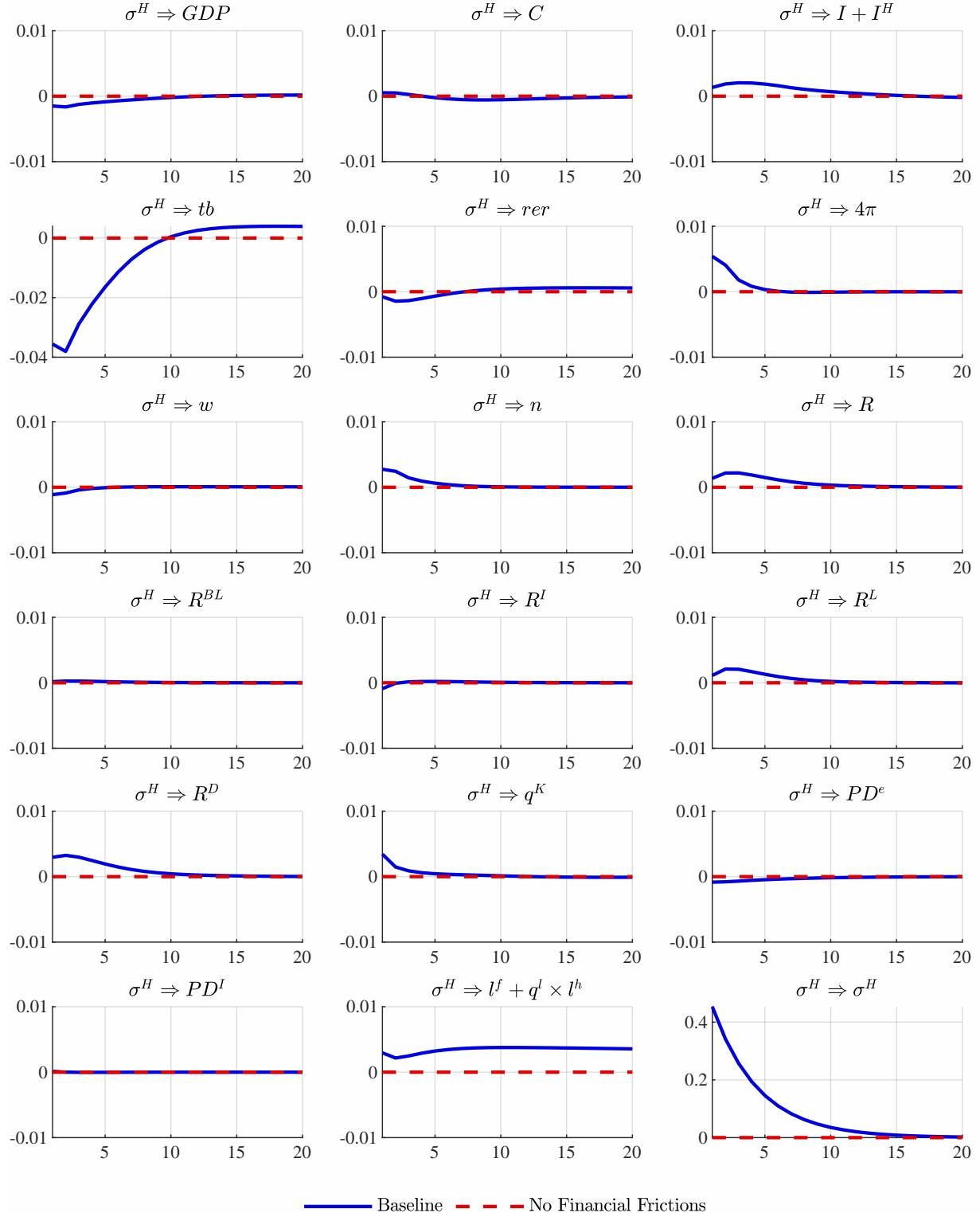
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 17: Response to a corporate banks' risk shock



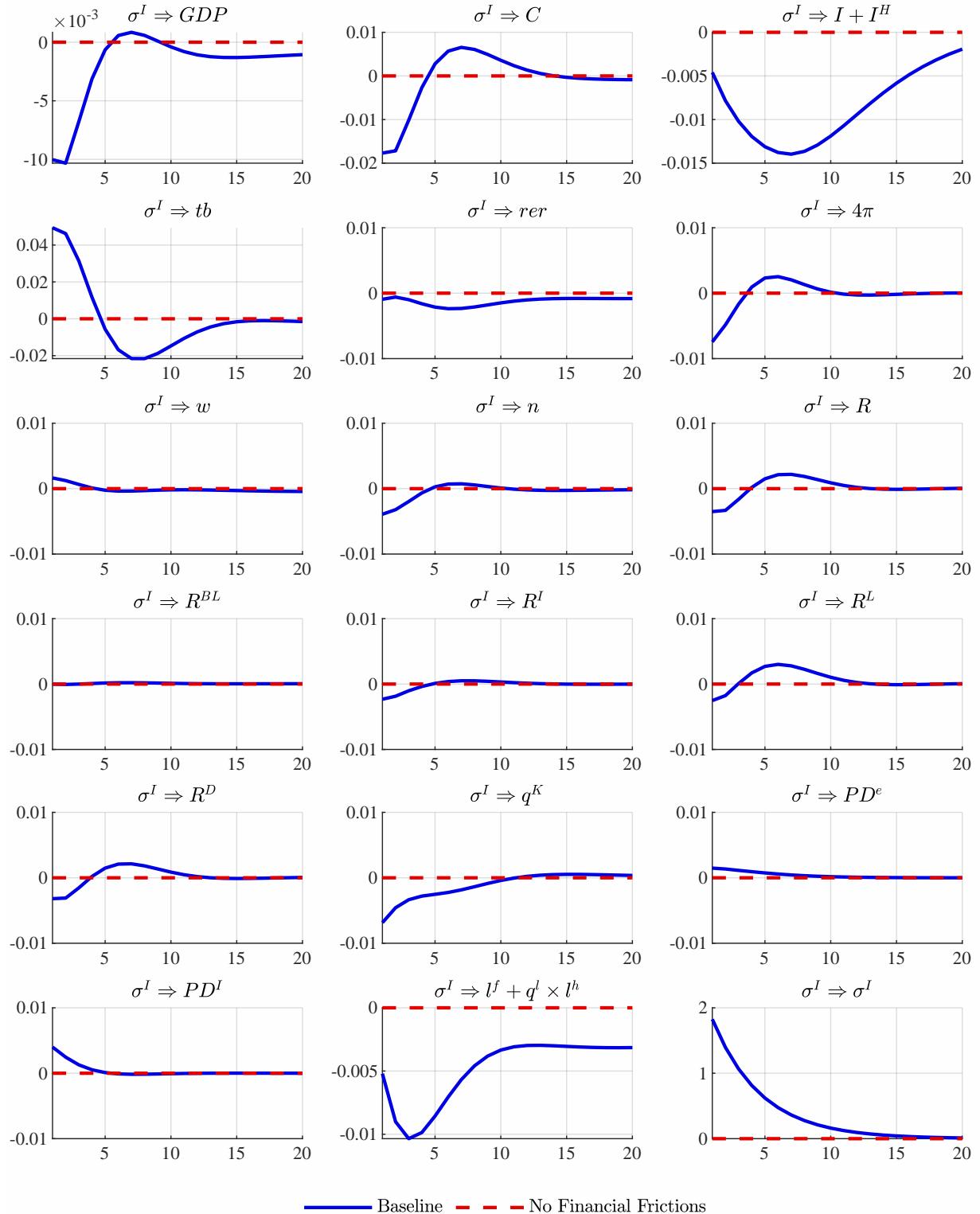
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 18: Response to a housing banks' risk shock



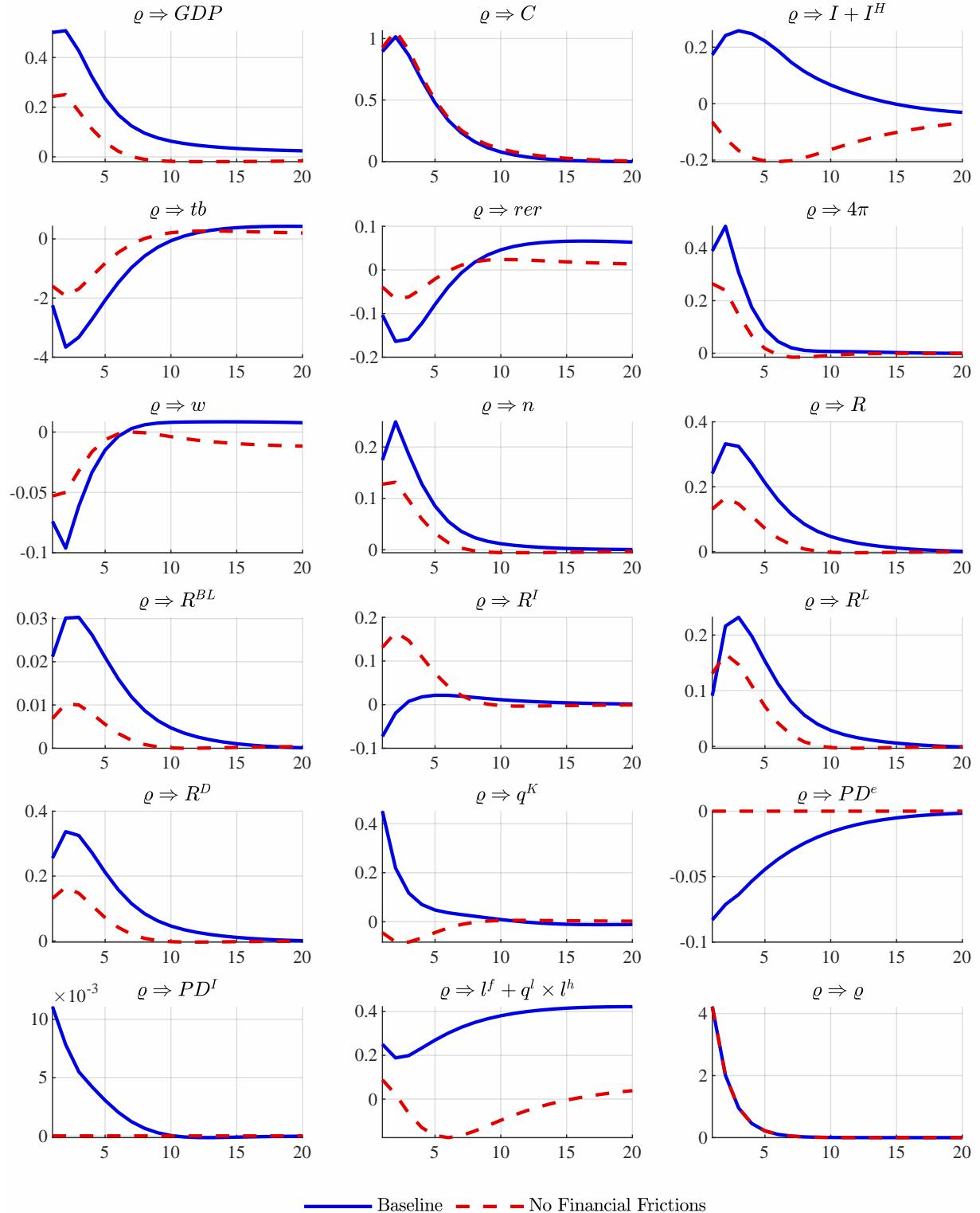
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 19: Response to a housing valuation risk shock



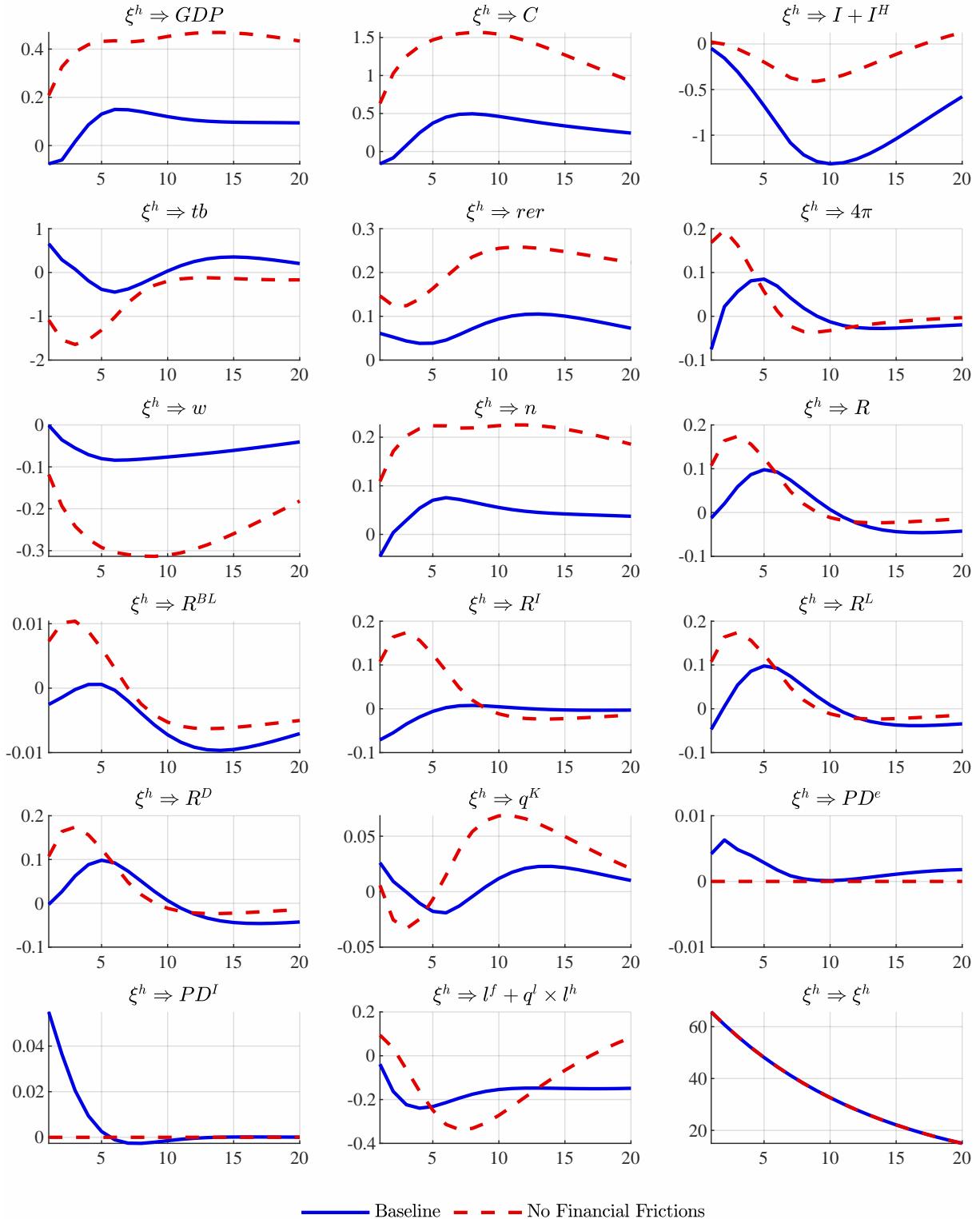
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 20: Response to a shock to households' preference for present consumption



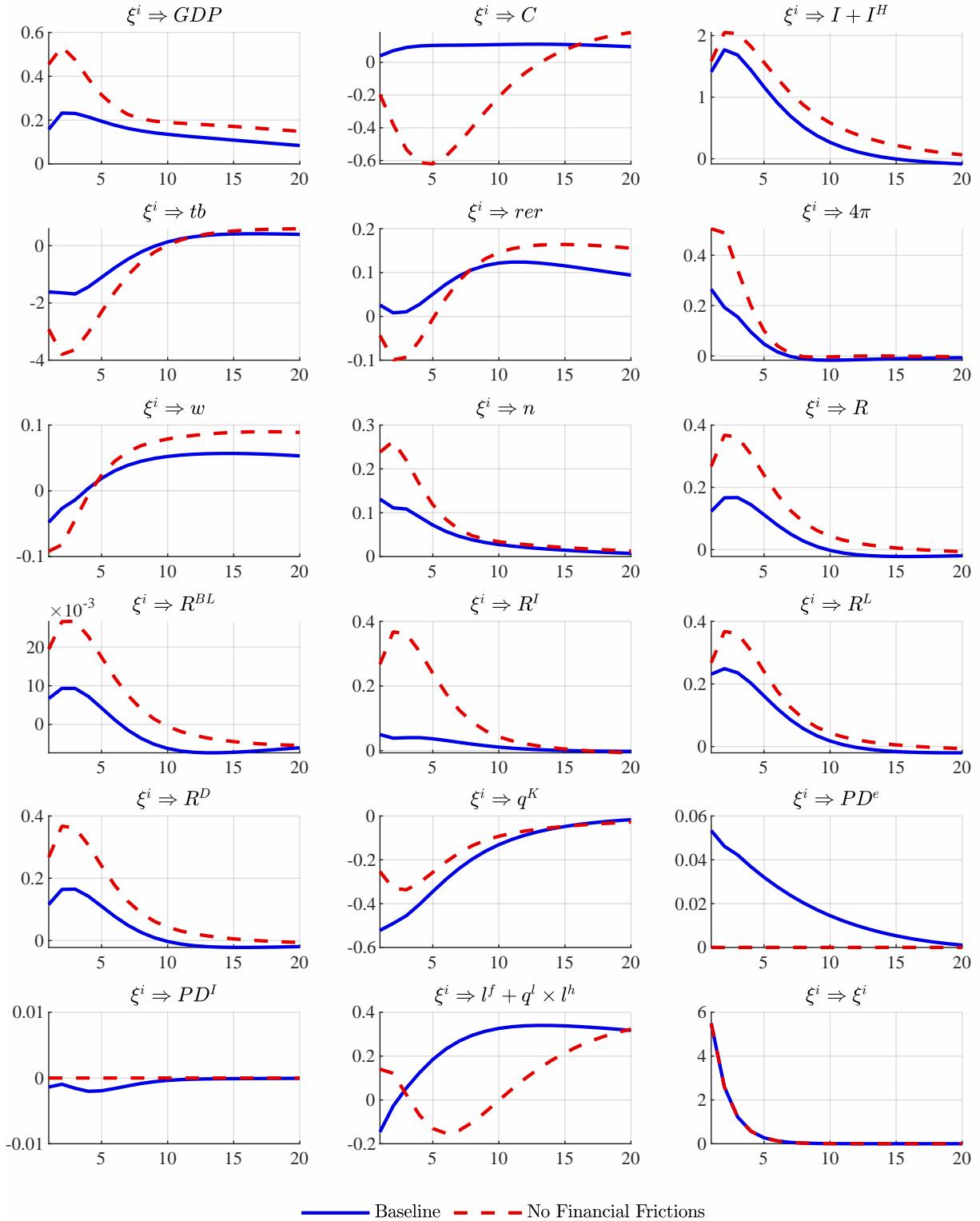
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 21: Response to a shock to households' preference for housing



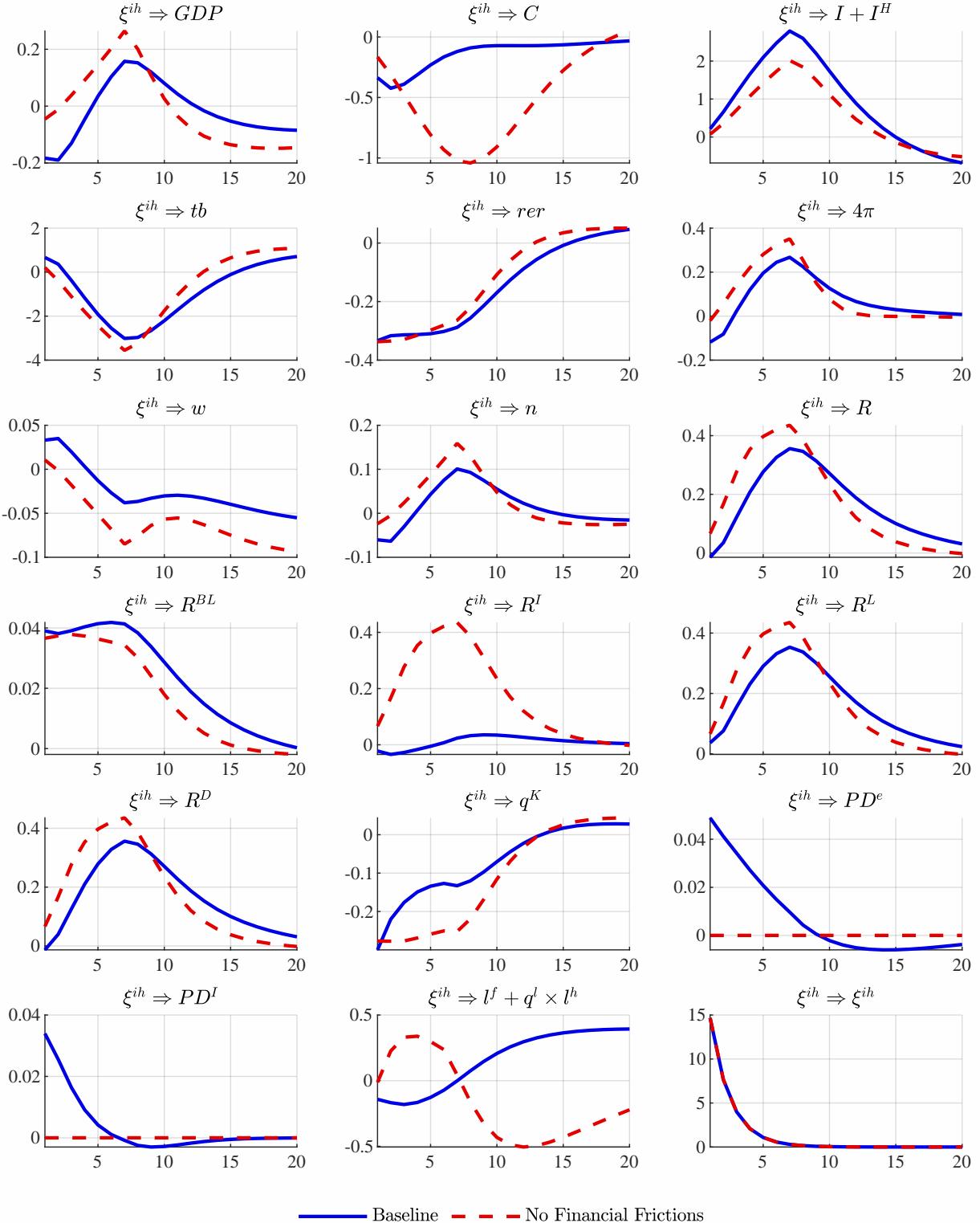
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 22: Response to a shock to the marginal efficiency of productive investment



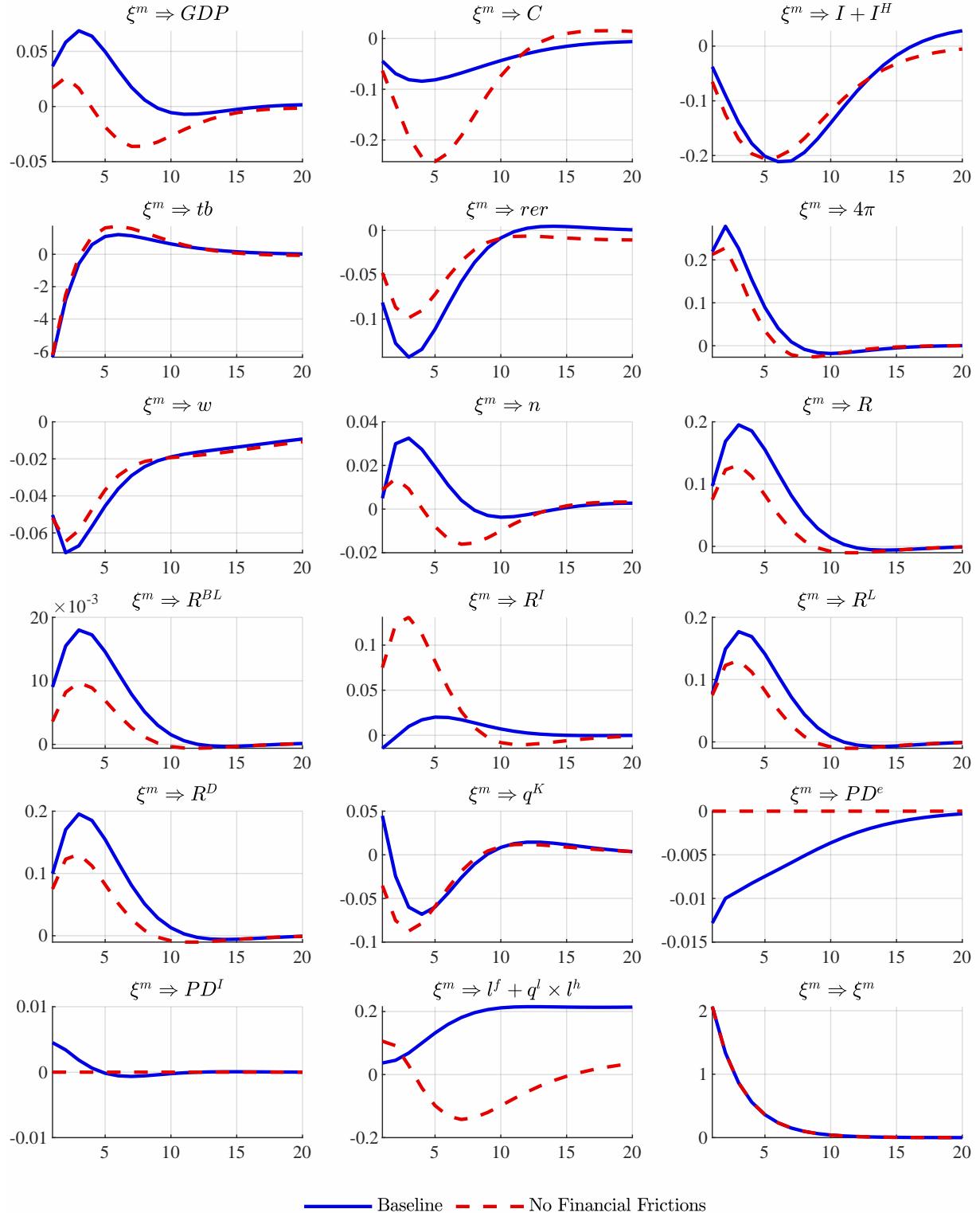
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 23: Response to a shock to the marginal efficiency of housing investment



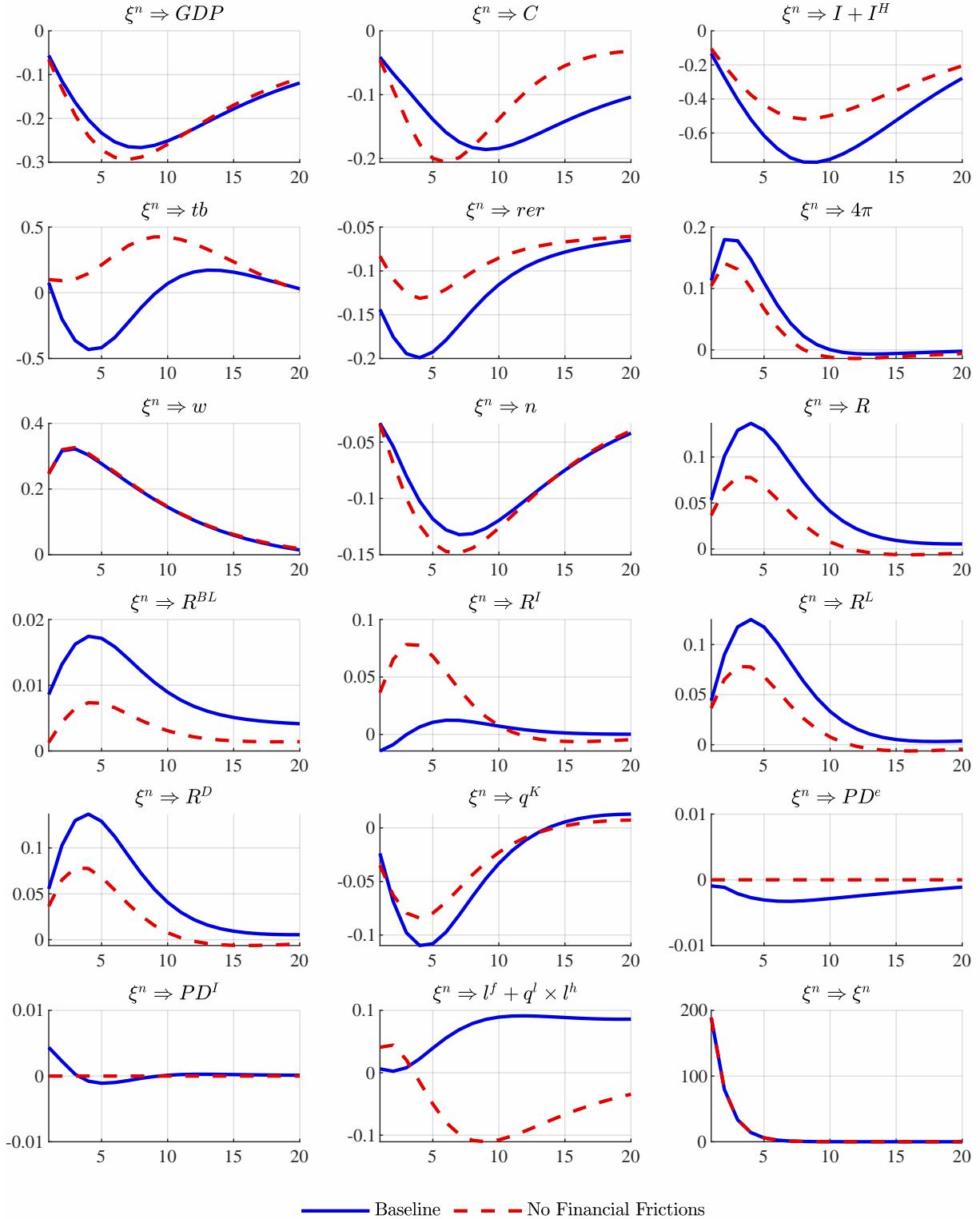
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 24: Response to a shock to the international price of imported goods



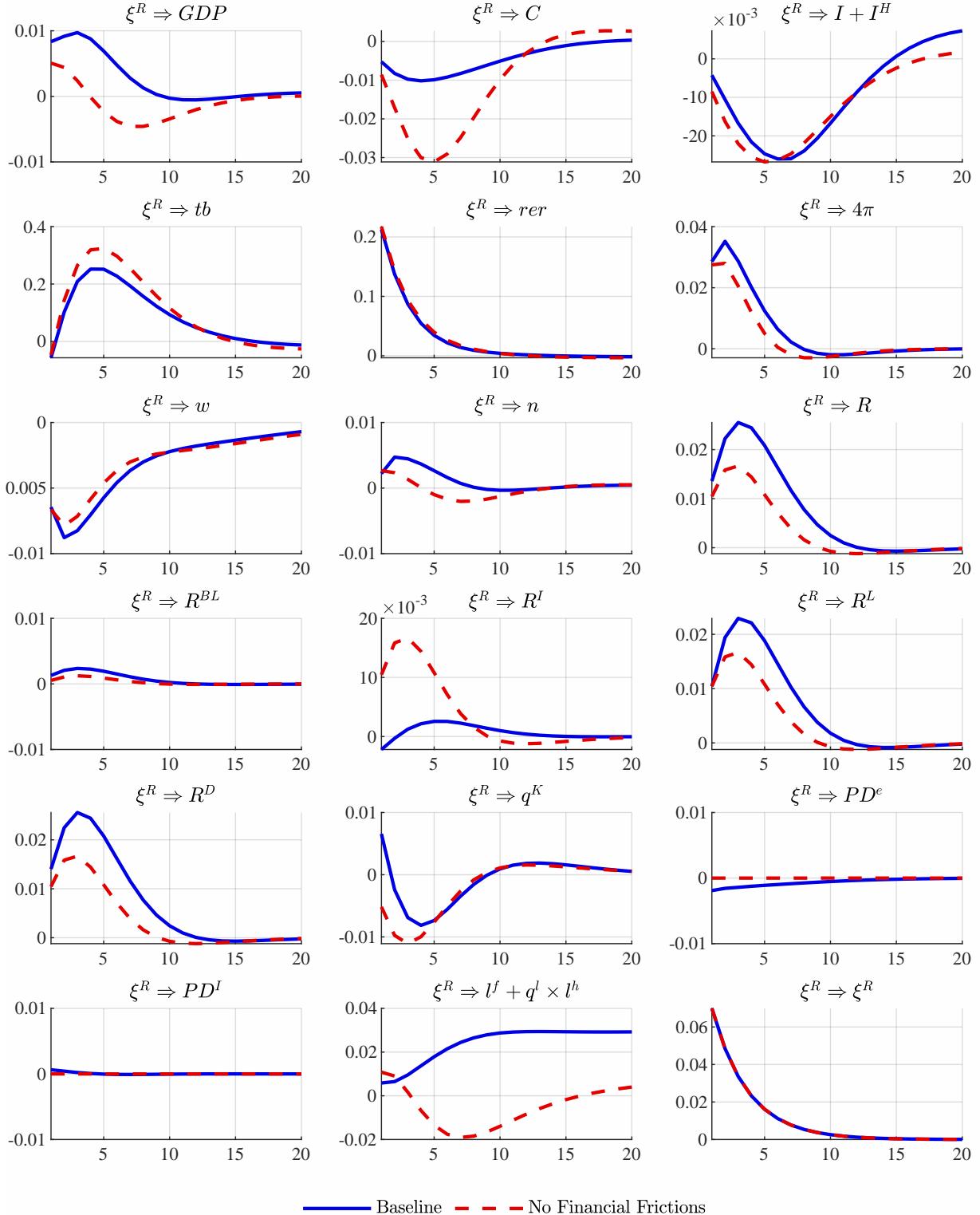
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 25: Response to a shock to households' labor disutility



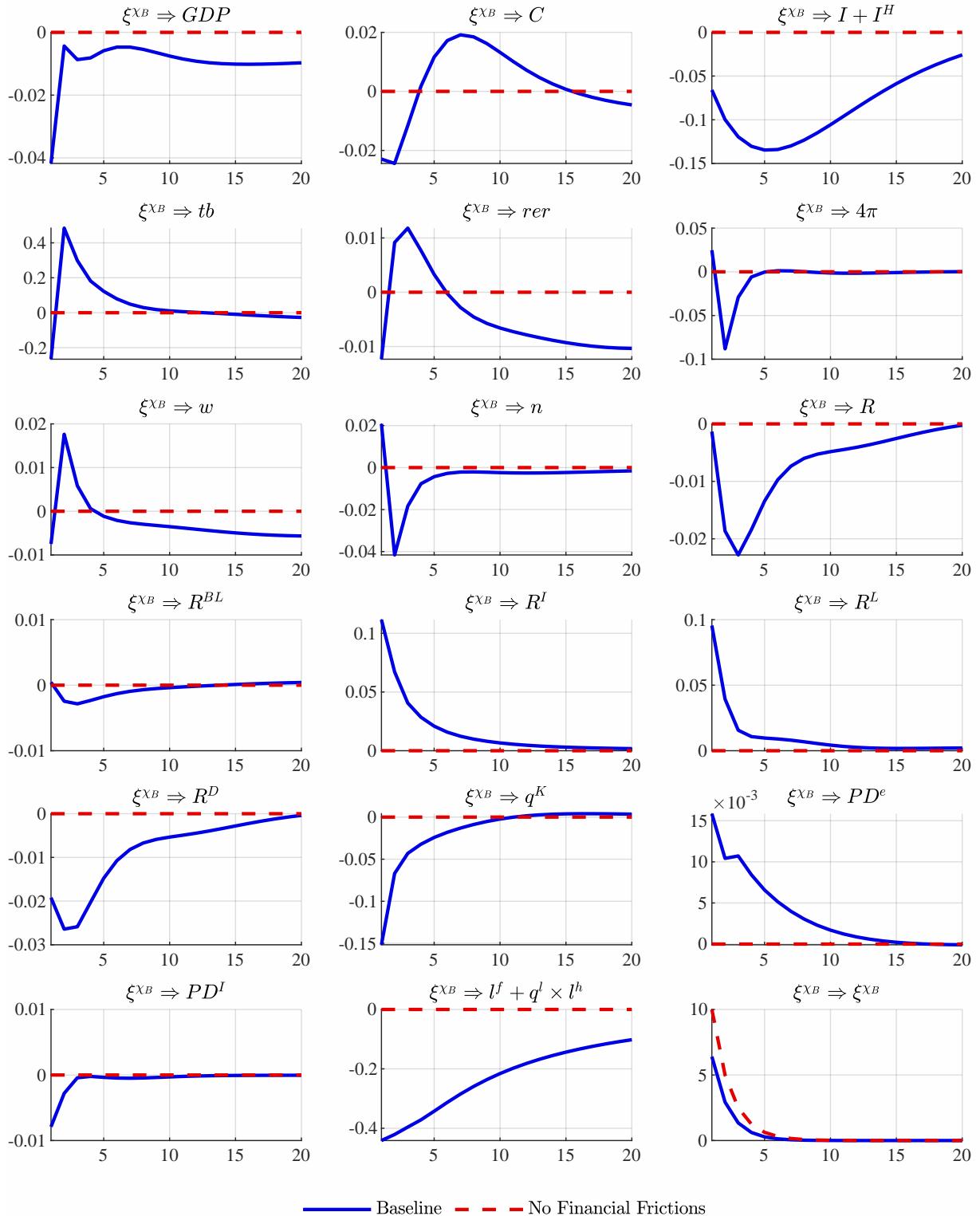
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 26: Response to a shock to the country risk premium



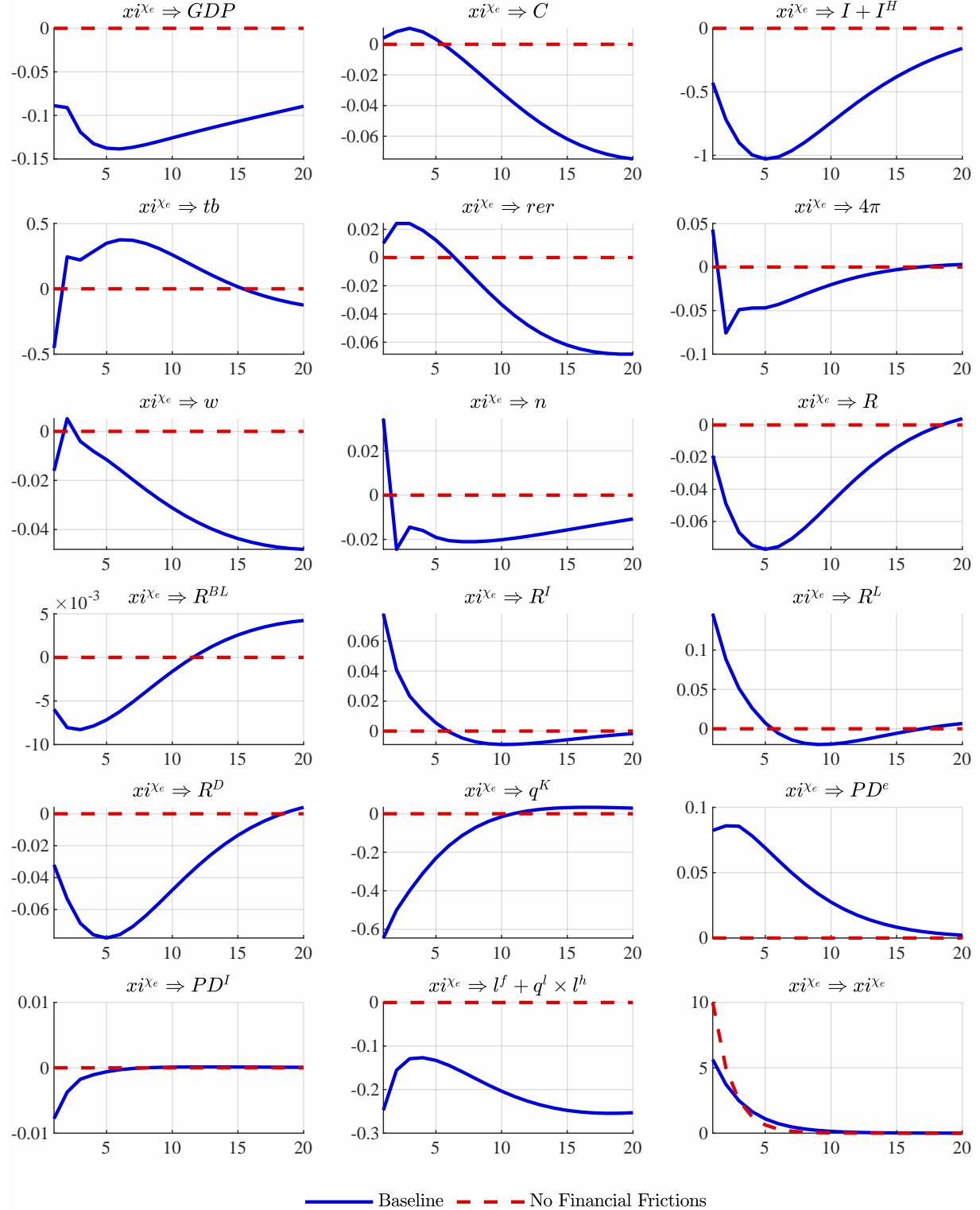
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 27: Response to a shock to bankers' dividend policy



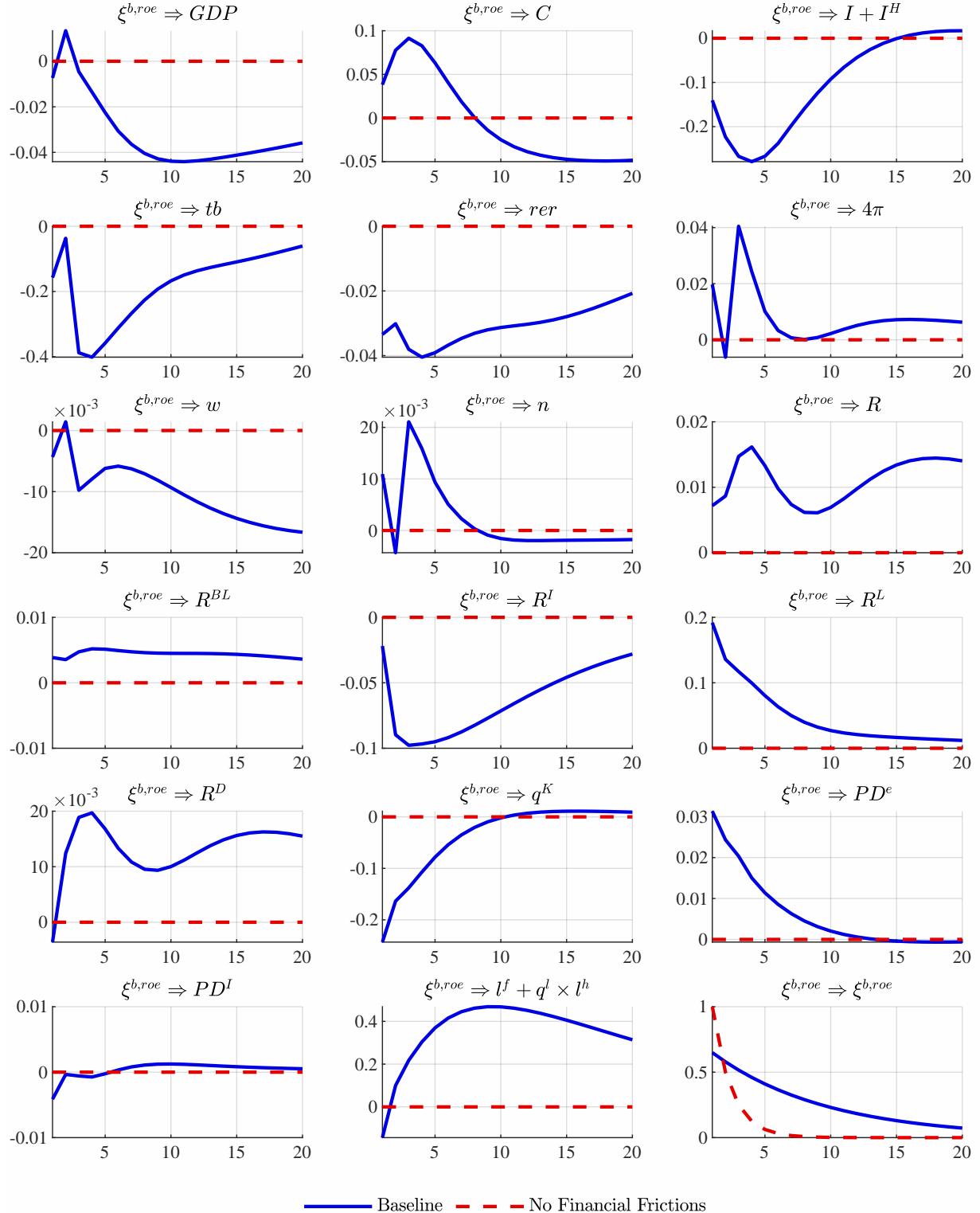
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 28: Response to a shock to entrepreneurs' dividend policy



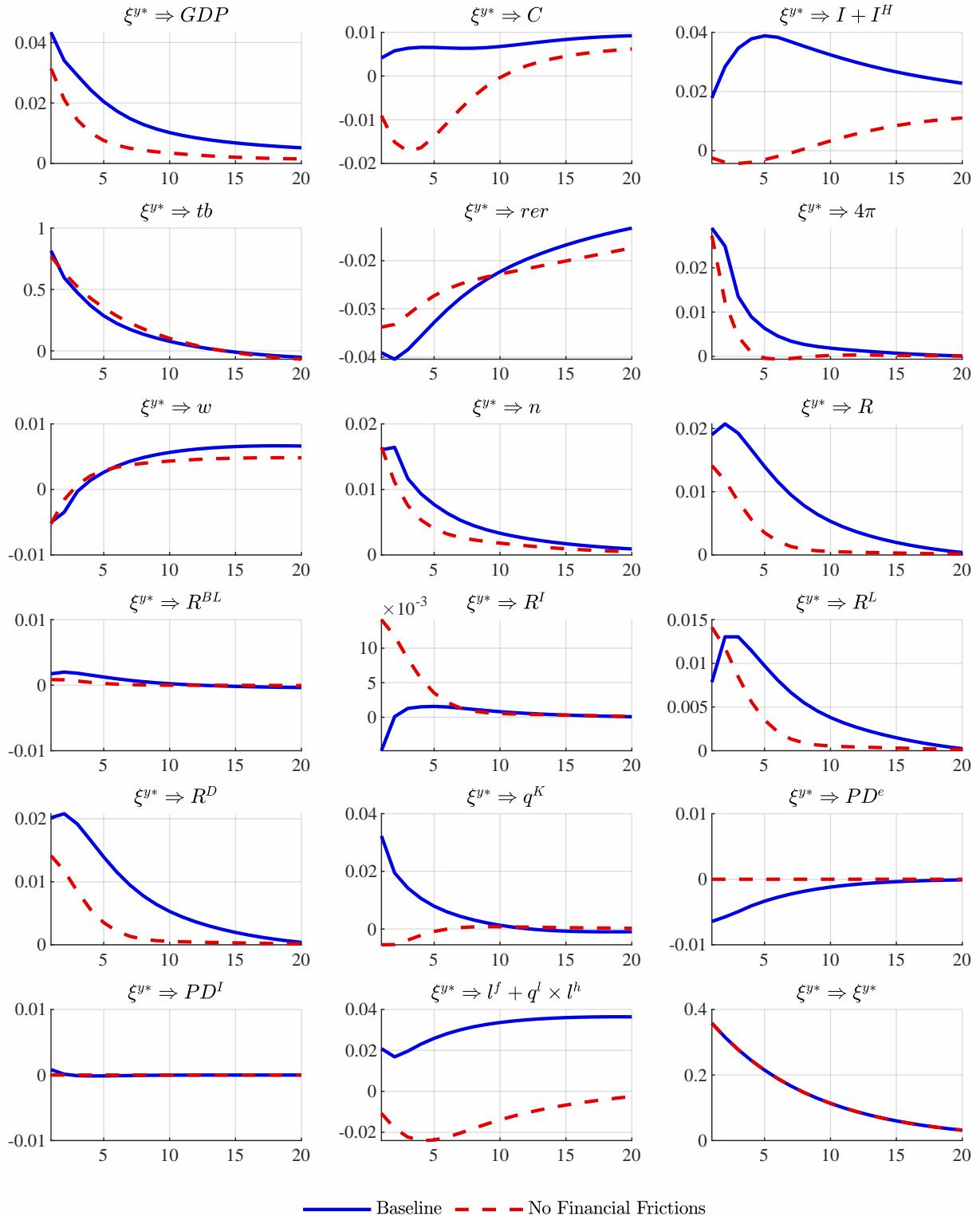
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 29: Response to a shock to bankers' required return to equity invested in H banks



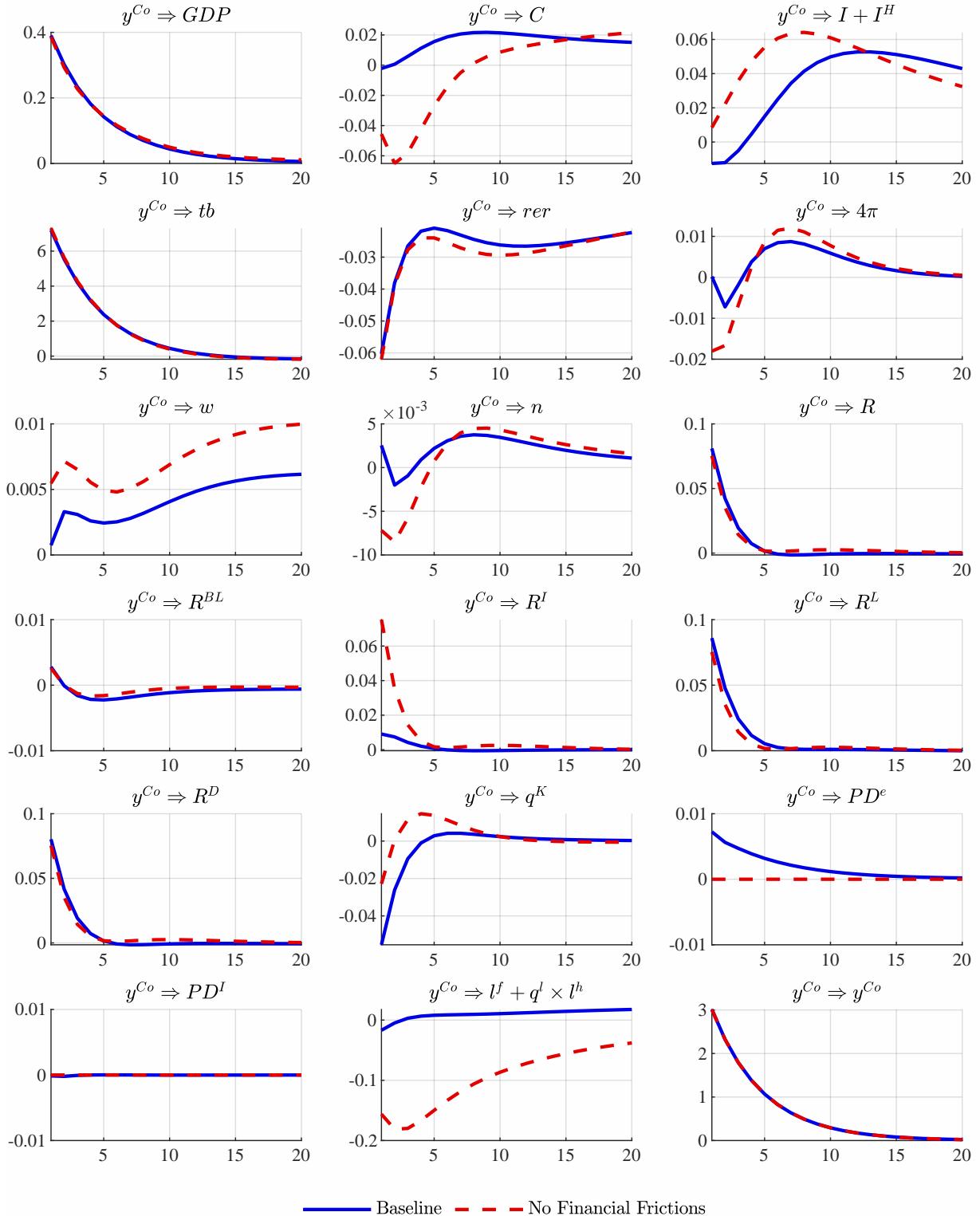
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 30: Response to a foreign demand shock



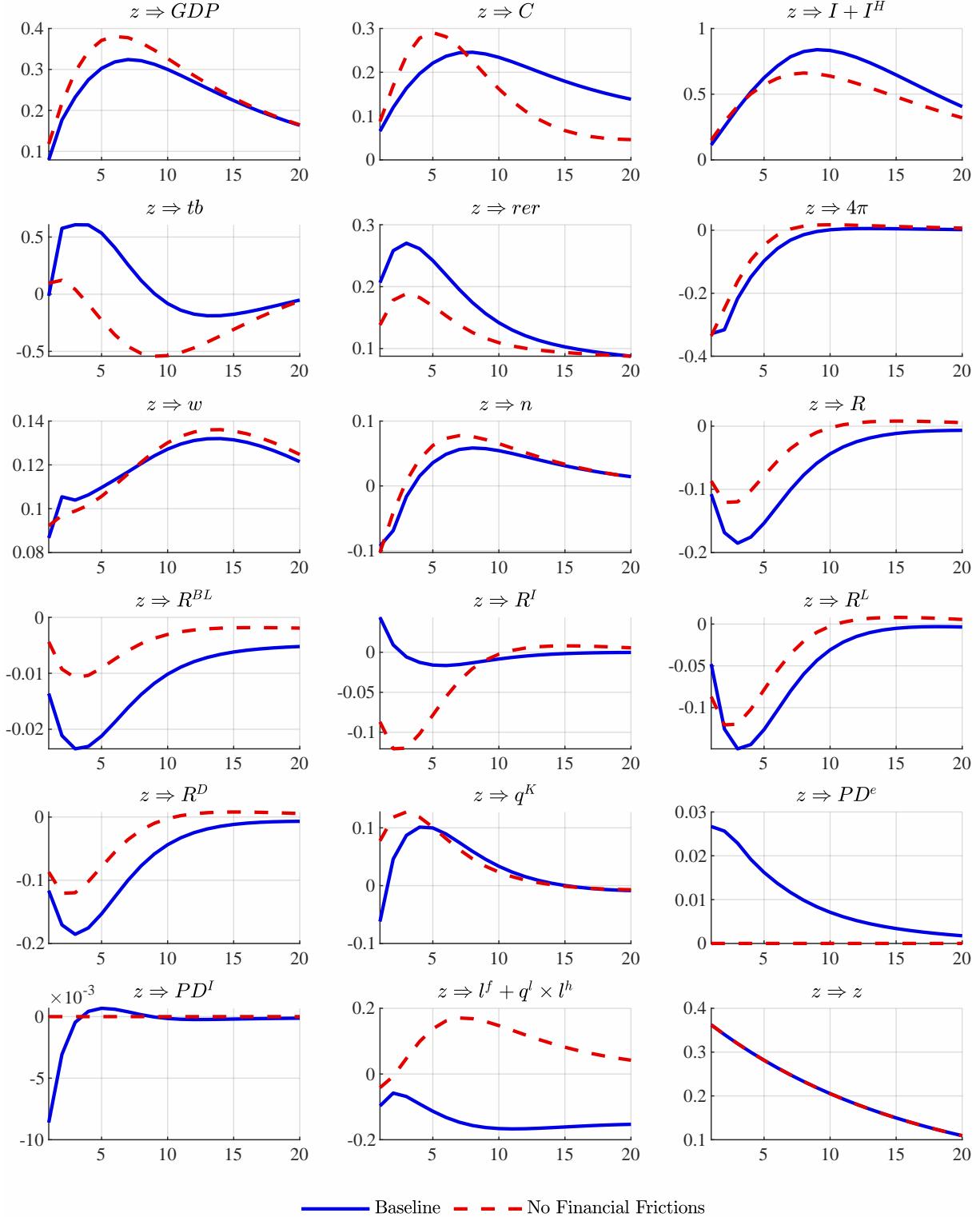
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 31: Response to a shock to the commodity sector's productivity



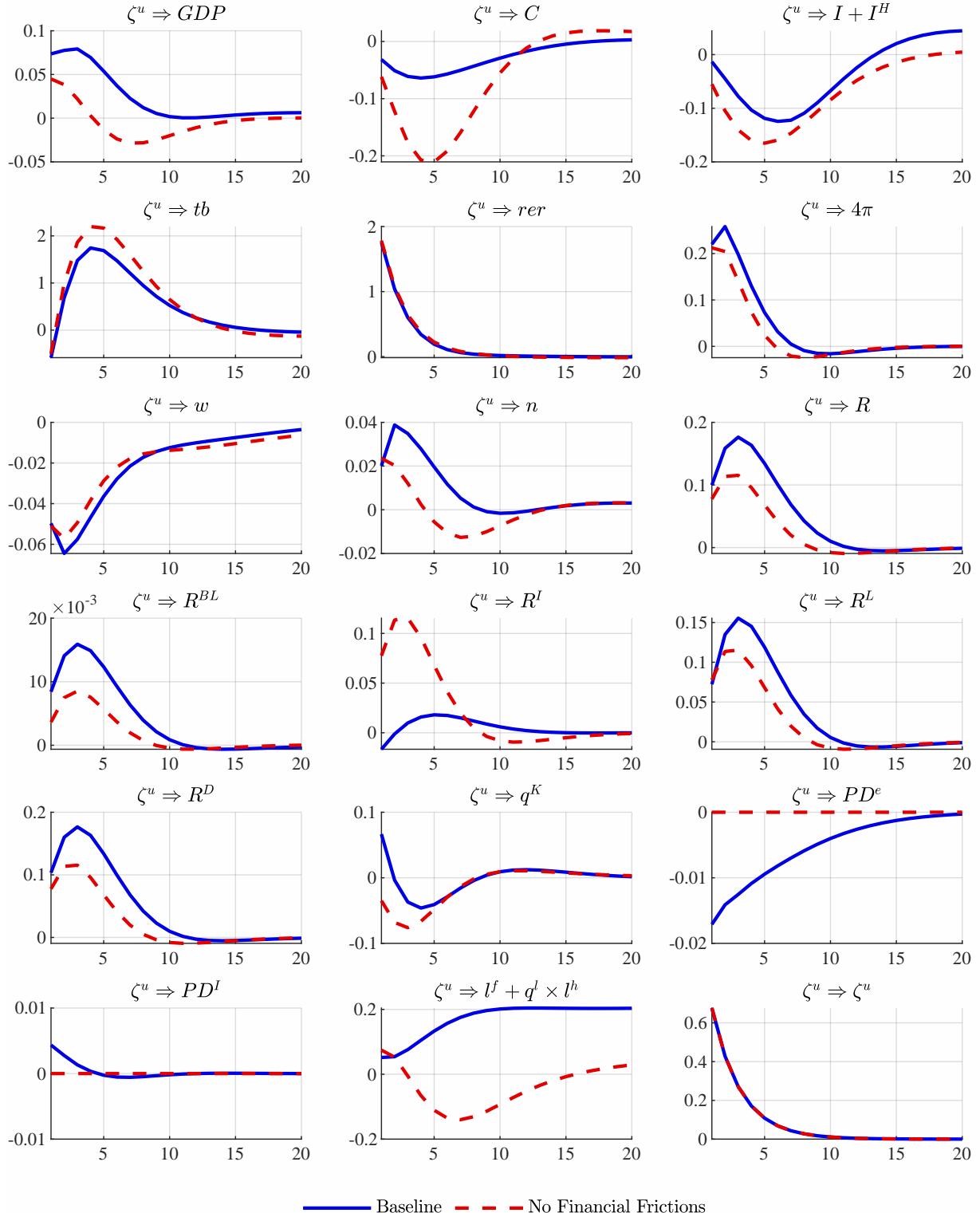
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 32: Response to a stationary productivity shock



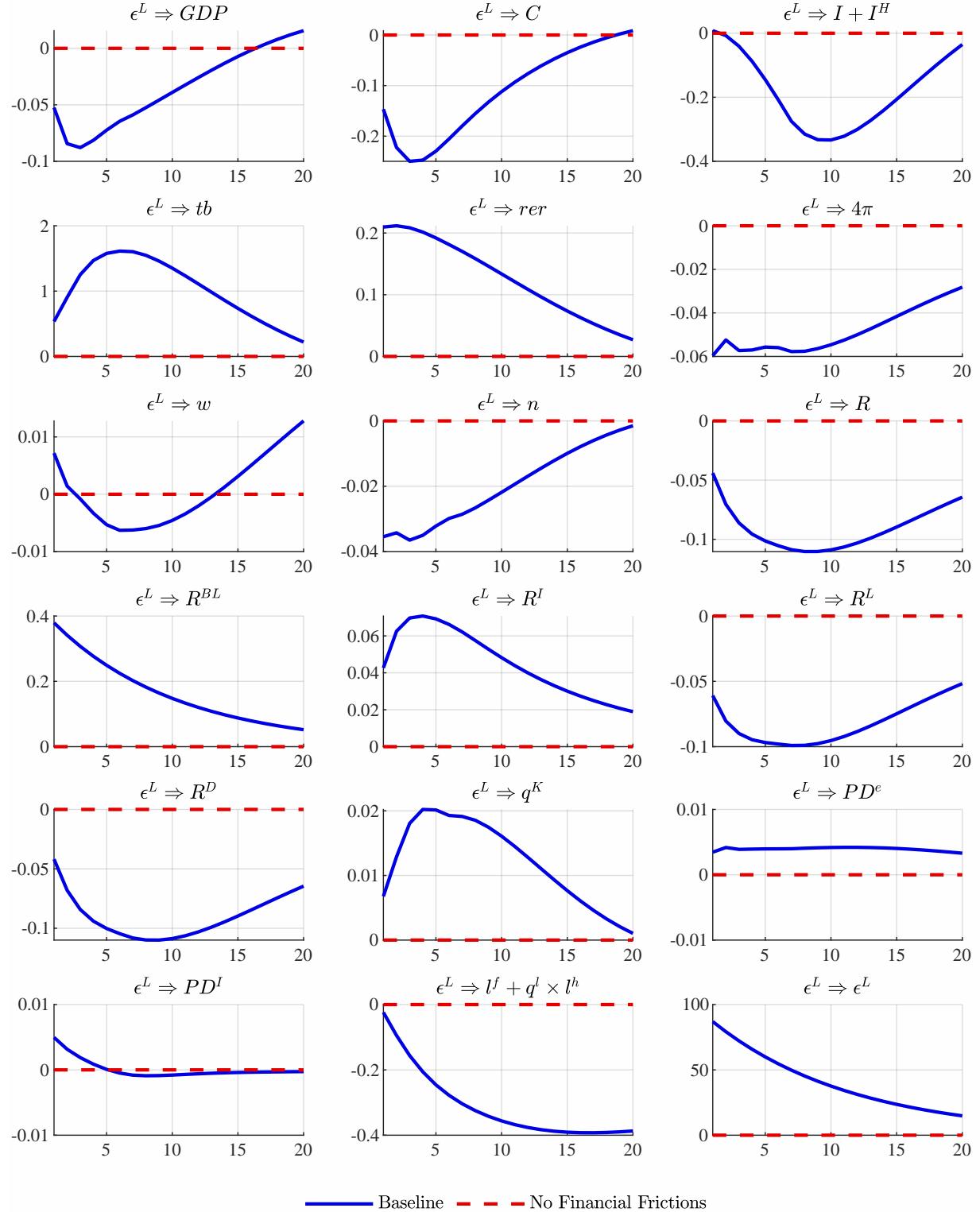
Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 33: Response to a shock to the uncovered interest rate parity



Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

Figure 34: Response to a shock to long maturity bonds' transaction costs



Notes.—The simulation considers the shock's estimated s.d. and persistence, as described in Table 5. All interest rates are expressed in annual terms. The blue solid lines show the responses in the baseline model. The red dashed lines show the responses in an alternative version of the model without financial frictions.

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