

TTIC 31230 Fundamentals of Deep Learning

Problems for GANs.

Problem 1. Conditional GANs In a conditional GAN we model a conditional distribution $\text{Pop}(y|x)$ defined by a population distribution on pairs $\langle x, y \rangle$. For conditional GANs we consider the probability distribution over triples $\langle x, y, i \rangle$ defined by

$$\begin{aligned}\tilde{P}_{\Phi}(i = 1) &= 1/2 \\ \tilde{P}_{\Phi}(y|x, i = 1) &= \text{pop}(y|x) \\ \tilde{P}_{\Phi}(y|x, i = -1) &= p_{\Phi}(y|x)\end{aligned}$$

(a) Write the conditional GAN adversarial objective function for this problem in terms of $\tilde{P}(x, y, i)$, $P_{\Phi}(y|x)$ and $P_{\Psi}(i|y, x)$.

Solution:

$$\Phi^* = \underset{\Phi}{\operatorname{argmax}} \min_{\Psi} E_{x, y, i \sim \tilde{P}(x, y, i)} - \ln P_{\Psi}(i|x, y)$$

Problem 2. GAN instability

Consider the following adversarial objective where x and y are scalars (real numbers).

$$\max_x \min_y xy$$

(a) Write the differential equation for gradient flow of this adversarial objective.

Solution:

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -x\end{aligned}$$

(b) Give a general solution to your differential equation. (Hint: It goes in a circle). Your solution should have parameters allowing for any given initial value of x and y .

Solution:

$$x = r_0 \sin(t + \Theta_0)$$

$$y = r_0 \cos(t + \Theta_0)$$

Problem 3. Contrastive GANs.

A GAN can be built with a “contrastive” discriminator. Rather than estimate the probability that y is from the population, the discriminator must select which of y_1, \dots, y_N is from the population.

More formally, for $N \geq 2$ let $\tilde{P}_\Phi^{(N)}$ be the distribution on tuples $\langle i, y_1, \dots, y_N \rangle$ defined by drawing one “positive” from Pop and $N - 1$ IID negatives from P_Φ ; then inserting the positive at a random position among the negatives; and returning (i, y_1, \dots, y_N) where i is the index of the positive.

$$\Phi^* = \operatorname{argmax}_{\Phi} \min_{\Psi} E_{(i, y_1, \dots, y_{N+1}) \sim \tilde{P}_\Phi^{(N)}} - \ln p_\Psi(i | y_1, \dots, y_{N+1}) \quad (1)$$

Restate the above definition of $\tilde{P}_\Phi^{(N)}$ and the GAN adversarial objective for the case of conditional contrastive GANs.

Solution:

$$\Phi^* = \operatorname{argmax}_{\Phi} \min_{\Psi} E_{(i, y_1, \dots, y_{N+1}, x) \sim \tilde{P}_\Phi^{(N)}} \ln -P_\Psi(i | y_1, \dots, y_{N+1}, x)$$