# TTIC 31230, Fundamentals of Deep Learning

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# Contrastive Predictive Coding

# Maximizing Mutual Information

We consider the distribution on $x$, $y$, $z_x$ and $z_y$ defined by drawing $\langle x, y\rangle \sim \mathrm{Pop}$, $z_x \sim P_\Phi(z_x|x)$ and $z_y \sim P_\Phi(z_y|y)$.

We are interested in optimizing $P_\Phi(z_x|x)$ and $P_\Phi(z_y|y)$ under the following objective.

$$\Phi^* = \operatorname*{argmax}_\Phi \; I_{\mathrm{Pop},\Phi}(z_x, z_y) - \beta(H_{\mathrm{Pop},\Phi}(z_x) + H_{\mathrm{Pop},\Phi}(z_y))$$

# Maximizing Mutual Information

$$\begin{aligned}\Phi^* &= \underset{\Phi}{\mathrm{argmax}}\; I_{\mathrm{Pop},\Phi}(z_x, z_y) - \beta(H_{\mathrm{Pop},\Phi}(z_x) + H_{\mathrm{Pop},\Phi}(z_y)) \\ &\geq \underset{\Phi}{\mathrm{argmax}}\; I_{\mathrm{Pop},\Phi}(z_x, z_y) - \beta(\hat{H}_{\mathrm{Pop},\Phi}(z_x) + \hat{H}_{\mathrm{Pop},\Phi}(z_y))\end{aligned}$$

$$H_{\mathrm{Pop},\Phi}(z_x) = E_{\mathrm{Pop},\Phi}\; - \ln P_{\mathrm{Pop},\Phi}(z_x)$$

$$\hat{H}_{\mathrm{Pop},\Phi}(z_x) = E_{\mathrm{Pop},\Phi}\; - \ln P_{\Phi}(z_x)$$

$$H_{\mathrm{Pop},\Phi}(z_x) \leq \hat{H}_{\mathrm{Pop},\Phi}(z_x)$$

## Maximizing Mutual Information

$$\Phi^* = \underset{\Phi}{\operatorname{argmax}} \; I_{\mathrm{Pop},\Phi}(z_x, z_y) - \beta(\hat{H}_\Phi(z_x) + \hat{H}_\Phi(z_y))$$

It turns out that we can give a lower bound on the mutual information term using **noise contrastive estimation**.

## A Contrastive Lower Bound

We now give a contrastive lower bound for general mutual information $I(z, w)$ given only the ability to sample from the joint distribution on $z$ and $w$.

For $N \geq 2$ let $c_{z,w}$ be the density defined by drawing pairs $(z_1, w_1), \ldots (z_n, w_n)$ from the population and then constructing the tuple $(i, z_1, \ldots, z_N, w)$ where $i$ is drawn uniformly from 1 to $N$ and $w = w_i$ is the value of $w$ paired with $z_i$.

## A Constrastive Lower Bound

$$\begin{aligned}
\Phi^* &= \operatorname*{argmin}_{\Phi} \; E_{(i,z_1\ldots,z_N,w)\sim c_{z,w}} \; -\ln P_\Phi(i|z_1,\ldots,z_n,w) \\
&= \operatorname*{argmin}_{\Phi} \; \mathcal{L}(\Phi) \\
P_\Phi(i|z_1,\ldots z_n,w) &= \operatorname*{softmax}_{i} \; s_\Phi(z_i,w) \quad \text{(required)} \\
I(z,w) &\geq \ln N - \mathcal{L}(\Phi)
\end{aligned}$$

See Chen et al., On Variational Bounds of Mutual Information, May 2019.

## Forcing $z_x$ and $z_y$ to be Useful

In the objective

$$\Phi^* = \operatorname*{argmax}_{\Phi} \; I_{\mathrm{Pop},\Phi}(z_x, z_y) - \beta(H_{\mathrm{Pop},\Phi}(z_x) + H_{\mathrm{Pop},\Phi}(z_y))$$

the limitation on the entropy of $z_x$ and $z_y$ block the trivial solution of $z_x = x$ and $z_y = y$.

CPC applications have used an alternative.

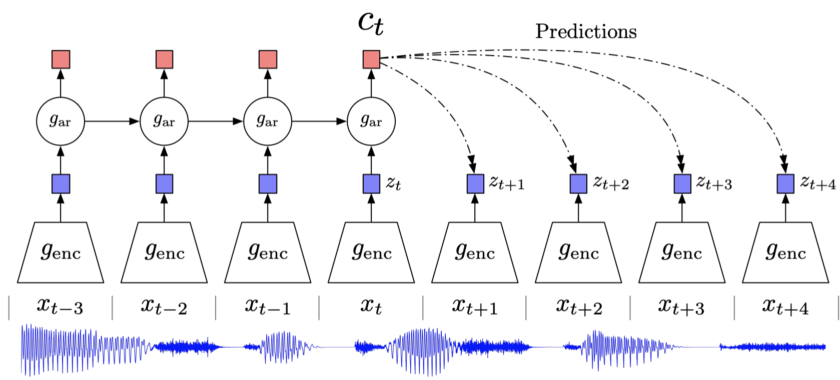
## Forcing $z_x$ and $z_y$ to be Useful

$$\Phi^* = \operatorname*{argmin}_{\Phi} \; E_{(i,z_x^1,\ldots,z_x^n,z_y)\sim c_{z_x,z_y}} \; -\ln P_\Phi(i|z_x^1,\ldots,z_x^n,z_y)$$

$$P_\Phi(i|z_x^1,\ldots,z_x^n,z_y) = \operatorname*{softmax}_i \; z_y^\top z_x^i$$

Requiring that the score be a simple inner product blocks $z_x = x$ and $z_y = y$ and forces $z_x$ and $z_y$ to carry the information in a linearly extractible way.

# Contrastive Predictive Coding for Speech




van den Oord et al., 2018

We seek to train an auto-regressive $g_{ar}$ and encoder $g_{env}$ by

$$g_{ar}^*, g_{enc}^* = \underset{g_{ar}, g_{enc}}{\operatorname{argmax}} \; E_t \sum_{k=1}^{K} I(c_t, z_{t+k})$$

The training maximizes the contrastive lower bound on $I(c_t, z_{t+k})$

# Contrastive Predictive Coding for Images

(SimCLR:) A Simple Framework for Contrastive Learning of Visual Representations, Chen et al., Feb. 2020 (self-supervised leader as of February, 2020).

They construct a distribution on pairs $\langle x, y \rangle$ defined by drawing an image from ImageNet and then drawing $x$ and $y$ as random "augmentations" (modifications) of the image.

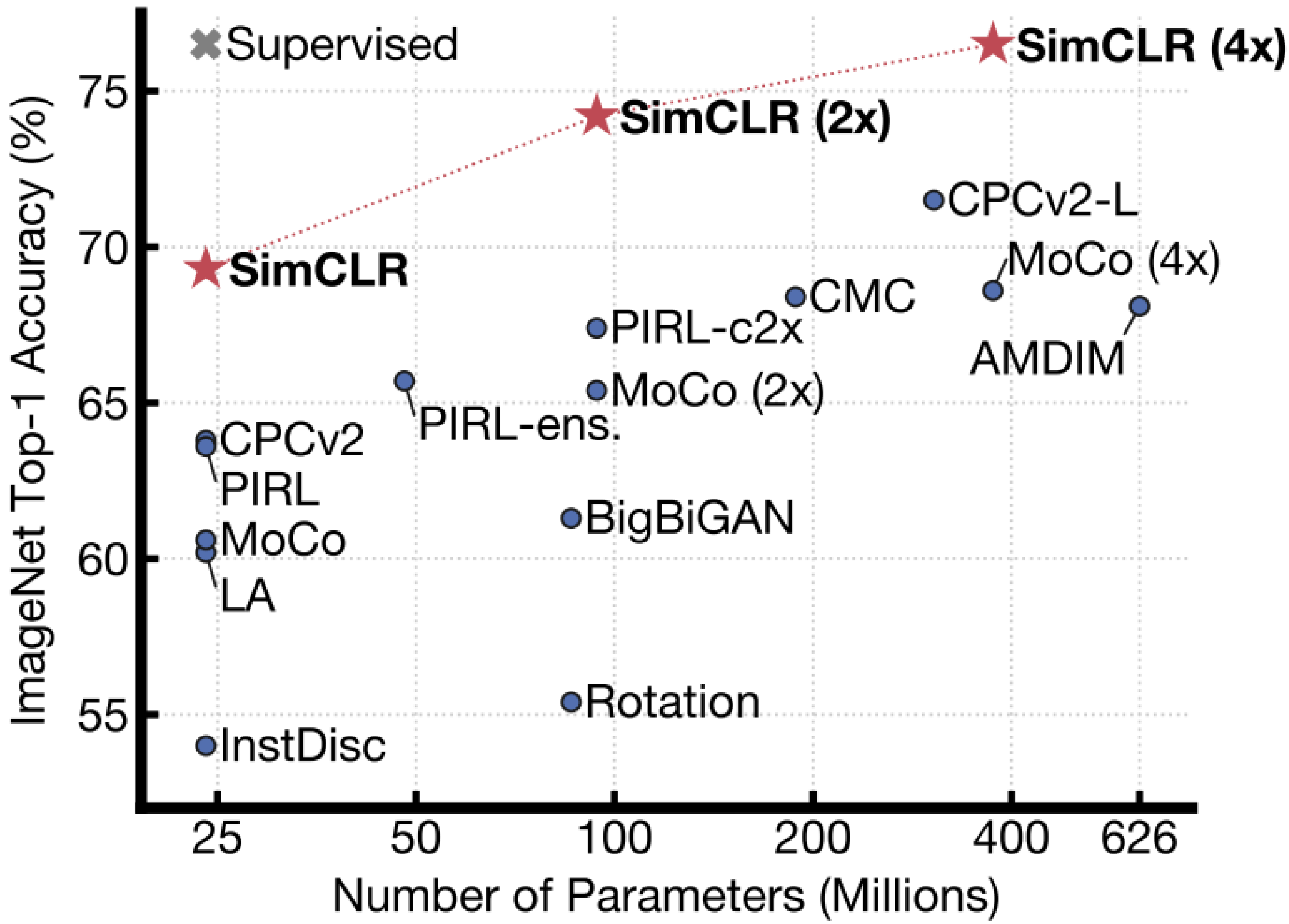
The training maximizes the contrastive lower bound on $I(x, y)$.

## Contrastive Predictive Coding for Images

A resulting feature map $z_\Phi$ on images is extracted from this training.

The feature map $z_\Phi$ is tested by using a linear classifier for ImageNet based on these features.

# SimCLR


Supervised
SimCLR (4x)
SimCLR (2x)
SimCLR
CPCv2-L
MoCo (4x)
CMC
AMDIM
PIRL-c2x
MoCo (2x)
PIRL-ens.
CPCv2
PIRL
MoCo
LA
BigBiGAN
Rotation
InstDisc
75
70
65
60
55
25
50
100
200
400
626
ImageNet Top-1 Accuracy (%)
Number of Parameters (Millions)

# END