

# **TTIC 31230, Fundamentals of Deep Learning**

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## **The Educational Framework (EDF)**

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The educational framework (EDF) is 150 lines of Python-NumPy that implement a deep learning framework.

In EDF we write

$$\begin{aligned}y &= F(p, x) \\z &= G(q, y, x) \\u &= H(z) \\\mathcal{L} &= u\end{aligned}$$

This is Python code where variables are bound to objects.

## Kinds of Nodes

There are three kinds of nodes — inputs, parameters and computed nodes (CompNodes).

Inputs and computed nodes have a batch index. Parameters do not have a batch index.

The value of inputs and parameters are provided. The value of CompNodes is computed by the forward pass.

## The EDF Framework

$$\begin{aligned}y &= F(p, x) \\z &= G(q, y, x) \\u &= H(z) \\\mathcal{L} &= u\end{aligned}$$

$x$  is an object in the class **Input**.

$p$  and  $q$  are objects in subclasses of **Parameter**.

$y$  is an object in the class  $F$ ;  $z$  is an object in the class  $G$ ; and  $u$  and  $\mathcal{L}$  are the same object in the class  $H$ .

The classes  $F$ ,  $G$ , and  $H$  are subclasses of **CompNode**.

## The Core of EDF

```
def Forward():  
    for c in CompNodes: c.forward()  
  
def Backward(loss):  
    for c in CompNodes + Parameters: c.grad = 0  
    loss.grad = 1.  
    for c in CompNodes[::-1]: c.backward()  
  
def SGD():  
    for p in Parameters:  
        p.value -= eta*p.grad
```

$$y = F(p, x)$$

```
class  $F$ (CompNode):
```

```
    def __init__(self,p,x):
```

```
        CompNodes.append(self)
```

```
        self.x = x
```

```
        self.p = p
```

```
    def forward(self):
```

```
        self.value = ... compute the value ...
```

```
    def backward(self):
```

```
        self.x.addgrad(... compute the gradient ...)
```

```
        self.p.addgrad(... compute the gradient ...)
```

## The Classes Input and CompNode

```
class Input:
    def __init__(self):
        pass
    def addgrad(self, delta):
        pass
```

```
class CompNode: #initialization is handled by the subclass
    def addgrad(self, delta):
        self.grad += delta
```

## The Class Parameter

```
class Parameter:

    def __init__(self,value):
        Parameters.append(self)
        self.value = value

    def addgrad(self, delta):
        #sums over the minibatch
        self.grad += np.sum(delta, axis = 0)/nBatch
```



## MLP in EDF

The following Python code constructs the computation graph of a multi-layer perceptron (MLP) with one hidden layer.

```
L1 = Sigmoid(Affine(Phi1,x))
Q = Softmax(Sigmoid(Affine(Phi2,L1)))
ell = LogLoss(Q,y)
```

Here **x** and **y** are input computation nodes whose value have been set. Here **Phi1** and **Phi2** are “parameter packages” (a matrix and a bias vector in this case). We have computation node classes **Affine**, **Relu**, **Sigmoid**, **LogLoss** each of which has a forward and a backward method.

## The Sigmoid Class

$$y[b, i] = \sigma(x[b, i])$$

$$y = \frac{1}{1 + e^{-x}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{e^{-x}}{(1 + e^{-x})^2} \\ &= y(1 - y) \end{aligned}$$

$$x.\text{grad}[b, i] += y.\text{grad}[b, i]y.\text{value}[b, i](1 - y.\text{value}[b, i])$$

## The Sigmoid Class

```
class Sigmoid:
    def __init__(self,x):
        CompNodes.append(self)
        self.x = x

    def forward(self):
        self.value = 1. / (1. + np.exp(-self.x.value))

    def backward(self):
        self.x.addgrad(self.grad*self.value*(1.-self.value))
```

## The Affine Class

$$\tilde{y}[b, j] = \sum_i W[i, j] x[b, i] = xW$$

$$y[b, j] = \tilde{y}[b, j] - B[j] = \tilde{y} - B \text{ (broadcasting)}$$

$$\tilde{y}.\text{grad}[b, j] += y.\text{grad}[b, j]$$

$$B.\text{grad}[j] -= \frac{1}{B} \sum_b y.\text{grad}[b, j]$$

$$x.\text{grad}[b, i] += \sum_j \tilde{y}.\text{grad}[b, j] W[i, j] = y.\text{grad} W^\top$$

$$W.\text{grad}[i, j] += \frac{1}{B} \sum_b \tilde{y}.\text{grad}[b, j] x[b, i] = ???$$

```
class Affine(CompNode):  
  
    def __init__(self, Phi, x):  
        CompNodes.append(self)  
        self.x = x  
        self.Phi = Phi  
  
    def forward(self):  
        self.value = (np.matmul(self.x.value,  
                                self.Phi.w.value)  
                      - self.Phi.b.value)
```

```
def backward(self):  
  
    self.x.addgrad(  
        np.matmul(self.grad,  
                   self.Phi.w.value.transpose()))  
  
    self.Phi.b.addgrad(- self.grad)  
  
    self.Phi.w.addgrad(self.x.value[:, :, np.newaxis]  
                       * self.grad[:, np.newaxis, :])
```

## Procedures in EDF

```
def MLP(Phi,x)

    if len(Phi) == 0
        return x

    return Sigmoid(Affine(Phi[0],MLP(Phi[1:],x)))
```

**END**