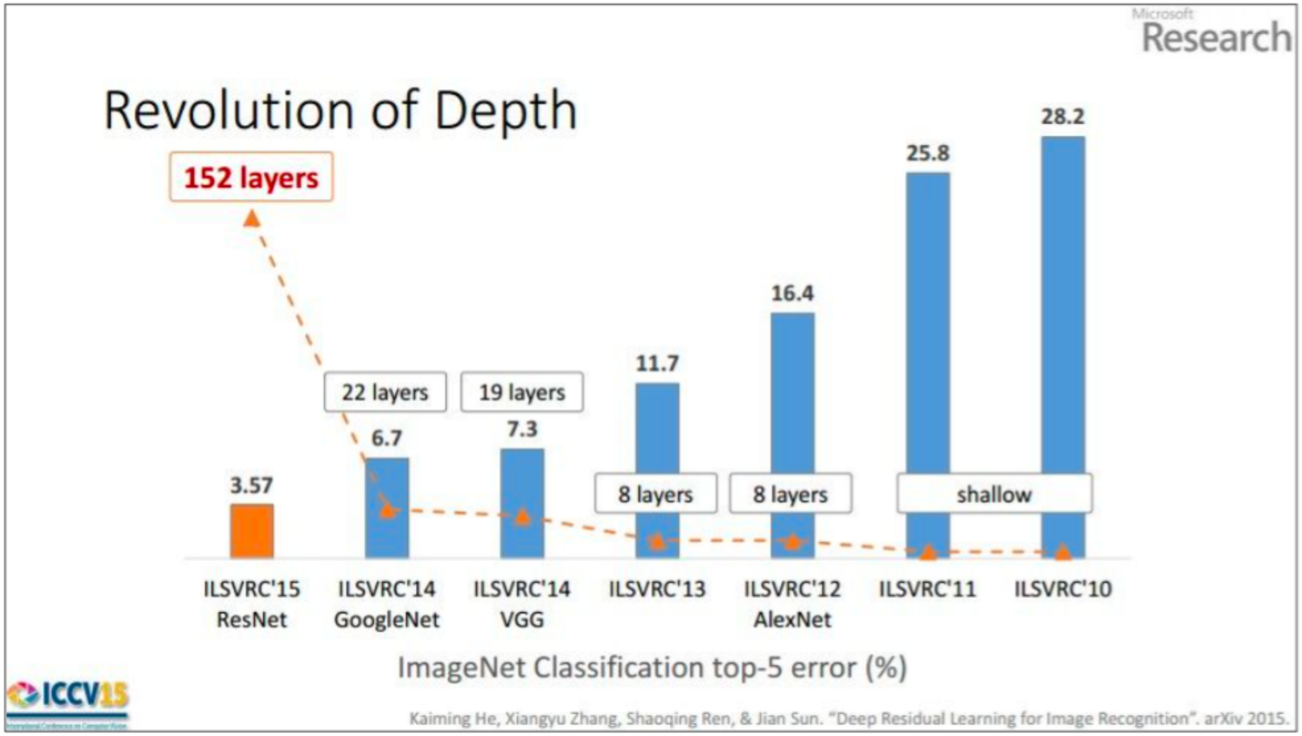
TTIC 31230, Fundamentals of Deep Learning

David McAllester, Autumn 2020

Convolutional Neural Networks (CNNs)

Imagenet Classification

1000 kinds of objects.



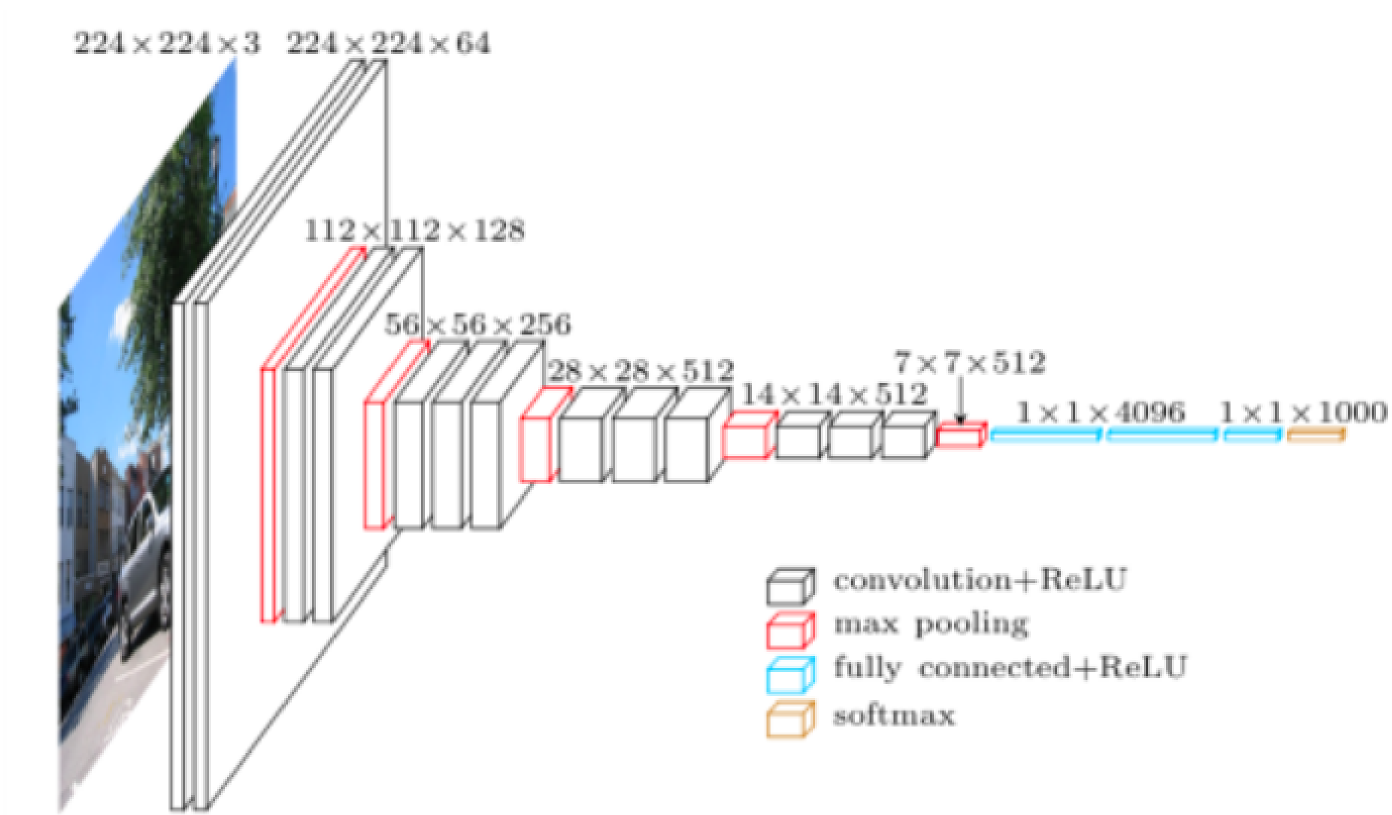
(slide from Kaiming He's recent presentation)

2016 is 3.0%, is 2017 2.25%

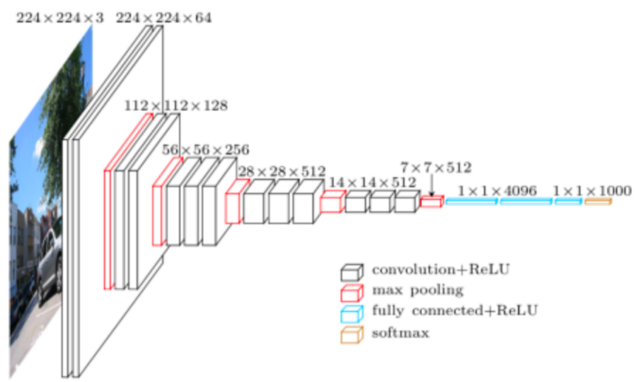
SOTA as of January 2020 is 1.3%

What is a CNN?

VGG, Zisserman, 2014



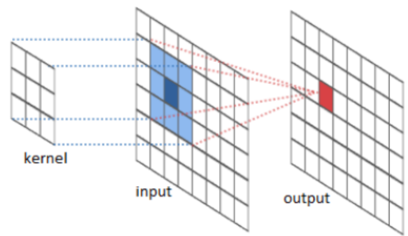
A Convolution Layer



Each box is a tensor $L_\ell[b, x, y, i]$

Each value $L_\ell[b, x, y, i]$ (for $\ell > 0$) is the output of a single linear threshold unit.

A Convolution Layer



$$W[\Delta x, \Delta y, i, j] \qquad L_\ell[b, x, y, i] \qquad L_{\ell+1}[b, x, y, j]$$

River Trail Documentation

$$L_{\ell+1}[b, x, y, j]$$

$$= \sigma\left(W[\Delta X, \Delta Y, I, j]\; L_\ell[b, x + \Delta X, y + \Delta Y, I] - B[j]\right)$$

2D CNN in PyTorch

conv2d(**input, weight, bias, stride, padding, dilation, groups**)

input – tensor (minibatch,in-channels,iH,iW)

weight – filters (out-channels, in-channels/groups,kH,kW)

bias – tensor (out-channels) . Default: None

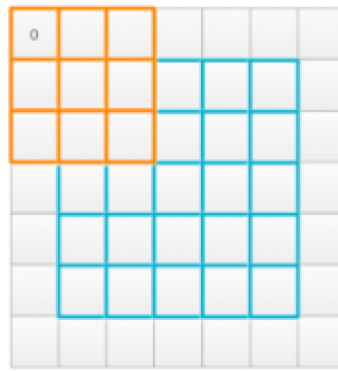
stride – Single number or (sH, sW). Default: 1

padding – Single number or (padH, padW). Default: 0

dilation – Single number or (dH, dW). Default: 1

groups – split input into groups. Default: 1

Padding



Jonathan Hui

If we pad the input with zeros then the input and output can have the same spatial dimensions.

Zero Padding in NumPy

In NumPy we can add a zero padding of width p to an image as follows:

```
padded = np.zeros(W + 2*p,  H + 2*p)

padded[p:W+p, p:H+p] = x
```

Padding

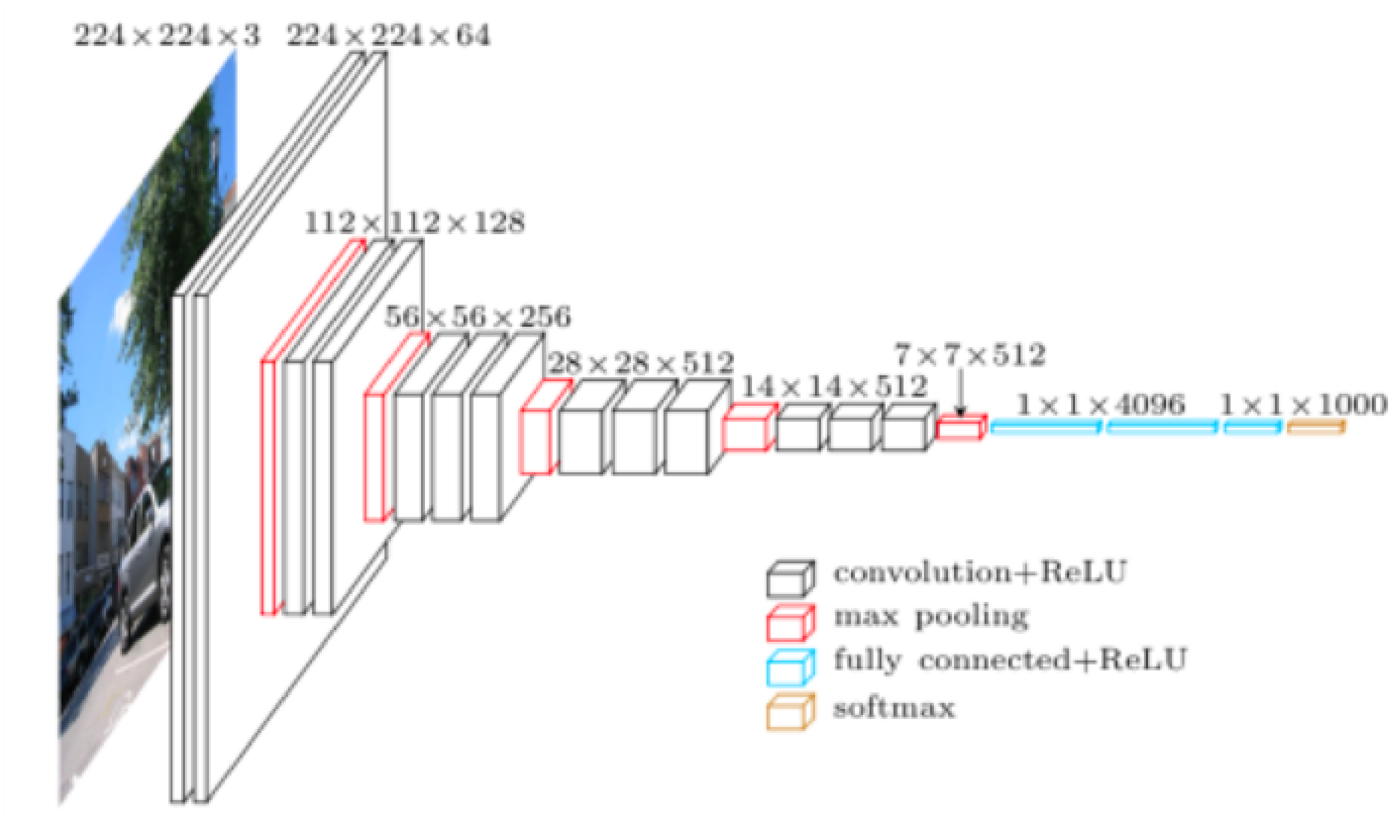
$$L'_\ell = \mathrm{Padd}(L_\ell,\ p)$$

$$L_{\ell+1}[b, x, y, j] =$$

$$\sigma\left(W[\Delta X, \Delta Y, I, j]\ L'_\ell[b, x + \Delta X, y + \Delta Y, I] - B[j]\right)$$

If the input is padded but the output is not padded then Δx and Δy are non-negative.

Reducing Spatial Dimention



Reducing Spatial Dimensions: Strided Convolution

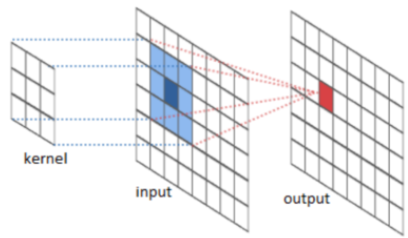
We can move the filter by a “stride” s for each spatial step.

$$L_{\ell+1}[b, x, y, j] =$$

$$\sigma\left(W[\Delta X, \Delta Y, I, j] L_\ell[b, s * x + \Delta X, s * y + \Delta Y, I] - B[j]\right)$$

For strides greater than 1 the spatial dimention is reduced.

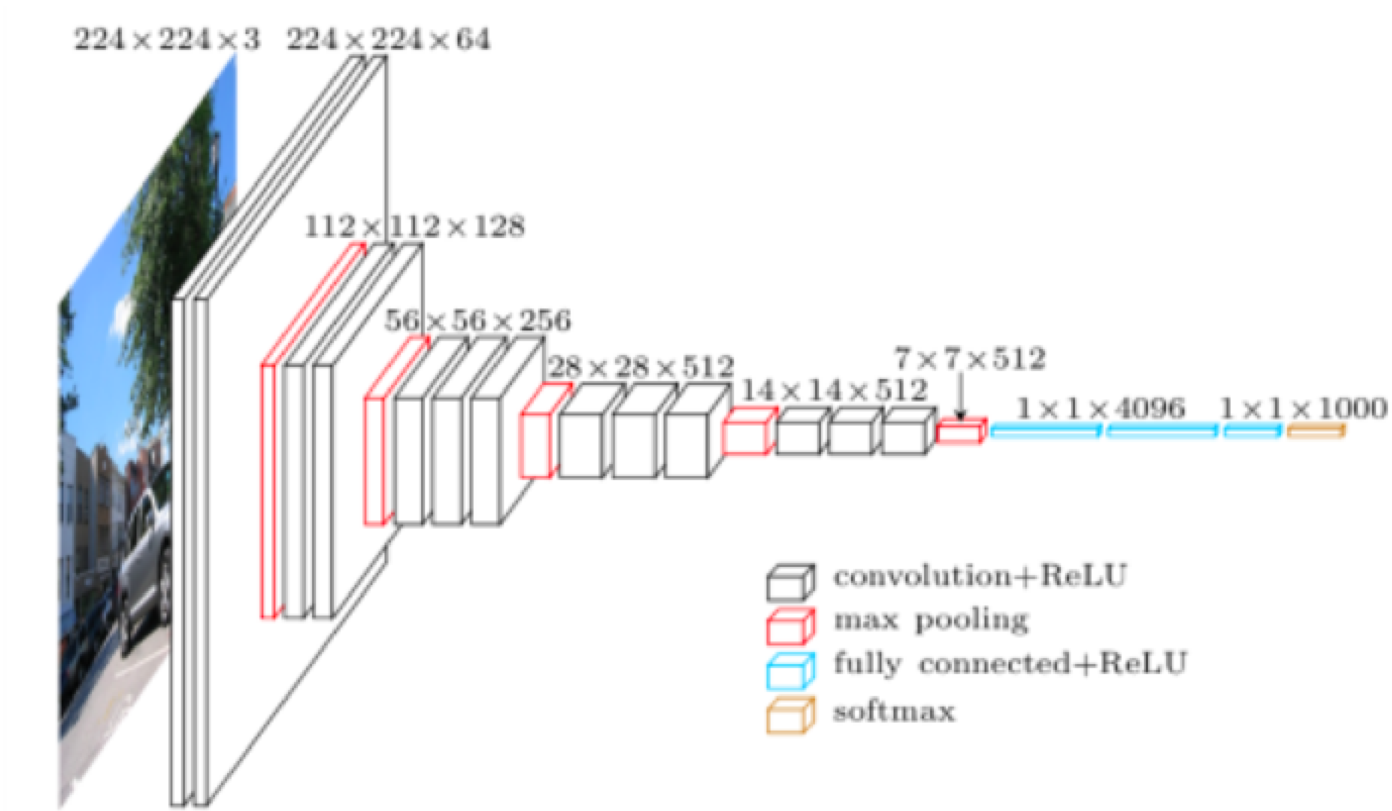
Reducing Spatial Dimensions: Max Pooling



$$L_{\ell+1}[b, x, y, i] = \max_{\Delta x, \Delta y} L_\ell[b, s * x + \Delta x,\ s * y + \Delta y,\ i]$$

This is typically done with a stride greater than one so that the image dimension is reduced.

Fully Connected (FC) Layers



Fully Connected (FC) Layers

We reshape $L_\ell[b,x,y,i]$ to $L_\ell[b,i']$ and then

$$L_{\ell+1}[b,j] = \sigma\left(W[j,I]\; L_\ell[b,I] - B[j]\right)$$

2D CNN in PyTorch

conv2d(**input, weight, bias, stride, padding, dilation, groups**)

input – tensor (minibatch,in-channels,iH,iW)

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bias – tensor (out-channels) . Default: None

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dilation – Single number or (dH, dW). Default: 1

groups – split input into groups. Default: 1

Dilation and Grouping

Dilation is used for “hypercolumns” where higher layers have the same spatial dimension as the input but each spatial location in a higher layer is a whole-image representation of a region of the input image.

Grouping reduces the computation by limiting the inputs to a feature to be values in the same “group” as the input.

Dilation and grouping are rarely used today.

Modern Trends

Modern Convolutions use 3X3 filters. This is faster and has fewer parameters. Expressive power is preserved by increasing depth with many stride 1 layers.

Max pooling has disappeared.

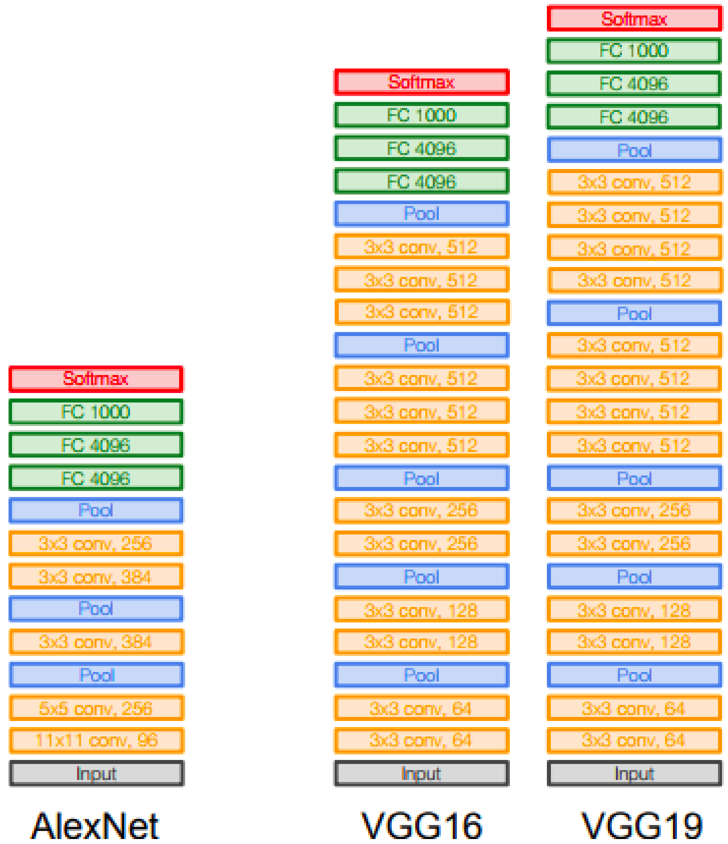
ResNet and resnet-like architectures are now dominant.

Alexnet, 2012

Given Input$[227, 227, 3]$

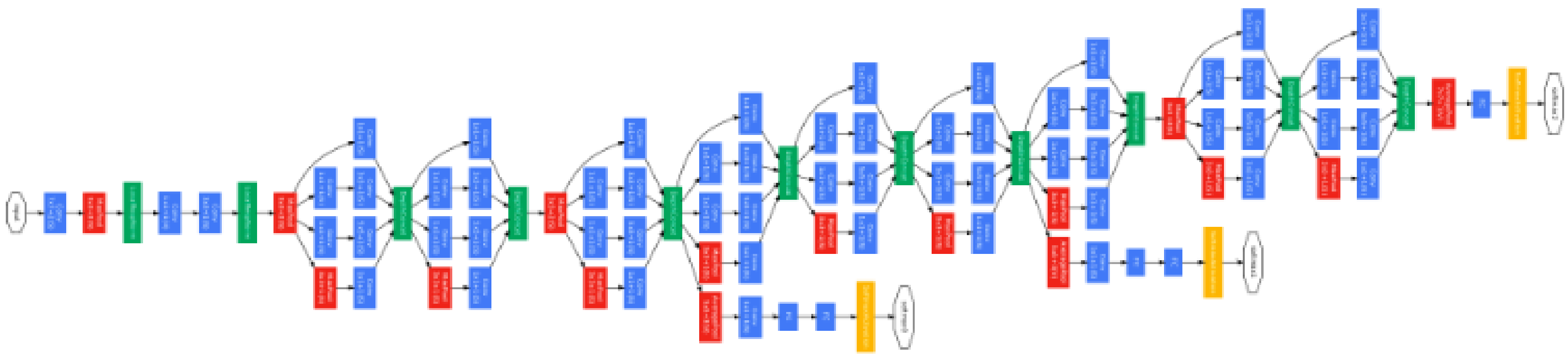
$$\begin{aligned}
L_1[55 \times 55 \times 96] &= \mathrm{ReLU}(\mathrm{CONV}(\mathrm{Input}, \Phi_1, \text{width } 11, \text{pad } 0, \text{stride } 4)) \\
L_2[27 \times 27 \times 96] &= \mathrm{MaxPool}(L_1, \text{width } 3, \text{stride } 2)) \\
L_3[27 \times 27 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_2, \Phi_3, \text{width } 5, \text{pad } 2, \text{stride } 1)) \\
L_4[13 \times 13 \times 256] &= \mathrm{MaxPool}(L_3, \text{width } 3, \text{stride } 2)) \\
L_5[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_4, \Phi_5, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_6[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_5, \Phi_6, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_7[13 \times 13 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_6, \Phi_7, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_8[6 \times 6 \times 256] &= \mathrm{MaxPool}(L_7, \text{width } 3, \text{stride } 2)) \\
L_9[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_8, \Phi_9)) \\
L_{10}[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_9, \Phi_{10})) \\
s[1000] &= \mathrm{ReLU}(\mathrm{FC}(L_{10}, \Phi_s)) \;\; \text{class scores}
\end{aligned}$$

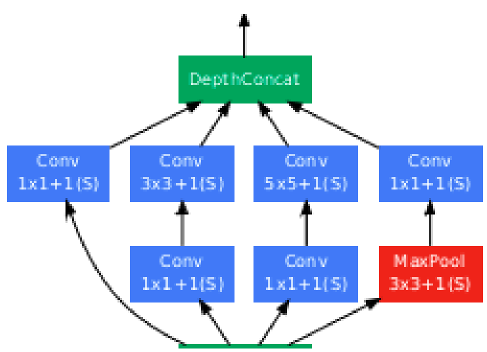
VGG, 2014



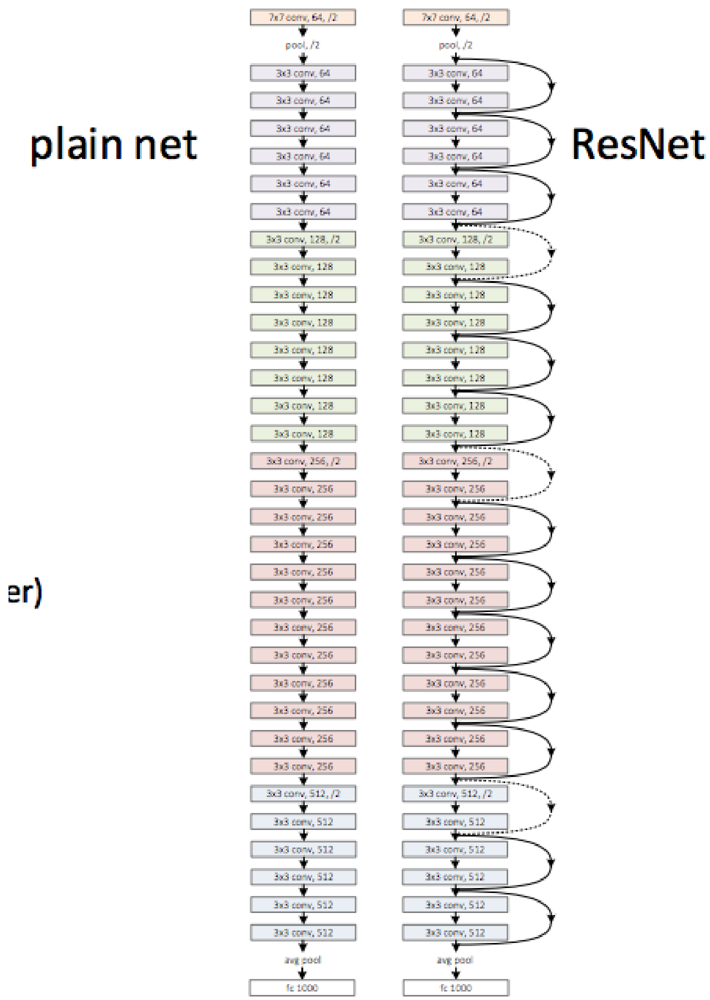
Stanford CS231

Inception, Google, 2014





ResNet, 2015



[Kaiming He]

END

Image to Column (Im2C)

Reduce convolution to matrix multiplication

more space but faster.

$$\tilde{L}_{\ell+1}[b,x,y,j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j] * L_\ell[b, x+\Delta x,\ y+\Delta y,\ i]\right) + B[j]$$

We make a bigger tensor $\tilde{L}$ with two additional indeces.

$$\tilde{L}_\ell[b,x,y,\Delta x,\Delta y,i] = L_\ell[b,x+\Delta x,y+\Delta y,i]$$

Image to Column (Im2C)

$$\tilde{L}_{\ell+1}[b, x, y, j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j] * L_\ell[b, x + \Delta x,\ y + \Delta y,\ i] \right) + B[j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} \tilde{L}_\ell[b, x, y, \Delta x, \Delta y, i] * W[\Delta x, \Delta y, i, j] \right) + B[j]$$

$$= \left(\sum_{(\Delta x, \Delta y, i)} \tilde{L}_\ell[(b, x, y), (\Delta x, \Delta y, i)] * W[(\Delta x, \Delta y, i), j] \right) + B[j]$$