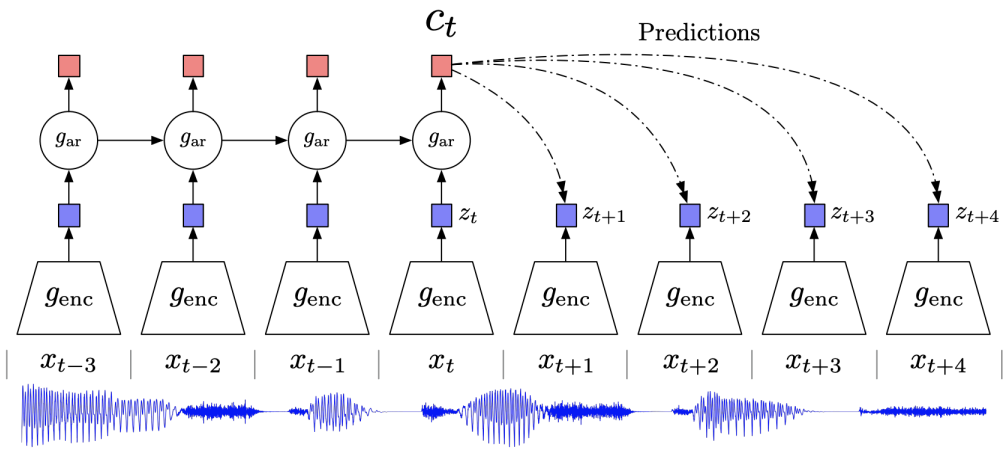
TTIC 31230, Fundamentals of Deep Learning

David McAllester, Autumn 2021

Mutual Information Coding

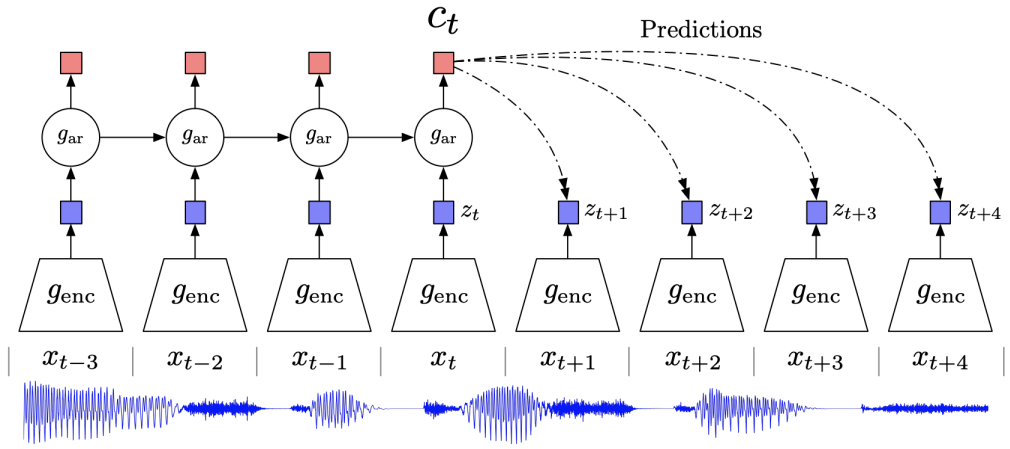
Mutual Information Coding



van den ORD et al. 2018

Consider the problem of learning phonetic representations of speech sounds. In the figure each z_t is a symbol representing the sound at time t.

Mutual Information Coding



van den ORD et al. 2018

Unlike VAEs, mutual information coding is about **predicting latent variables**. There is no attempt to model the input speech sound. Intuitively we want to separate signal from noise and avoid modeling noise.

wav2vec 2.0

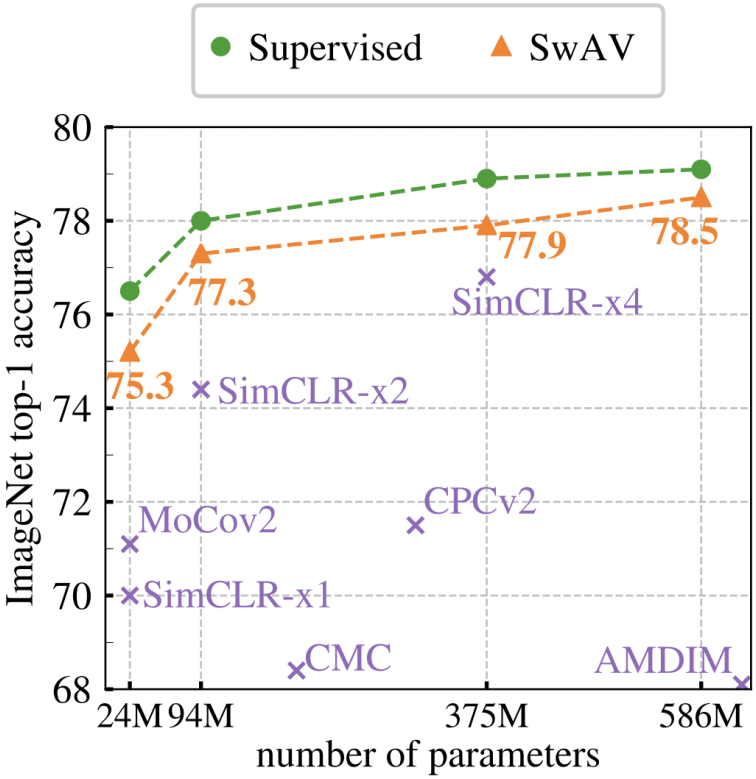
Trained on 53k hours of **unlabeled** audio they convert speech to a sequence of symbols they call "pseudo-text units".

Using this pre-trained transcription of speech into pseudo-text they reduce the amount of labeld data needed for a given accuracy in speech recognition by a factor of 100.

Baevski et al., 2020

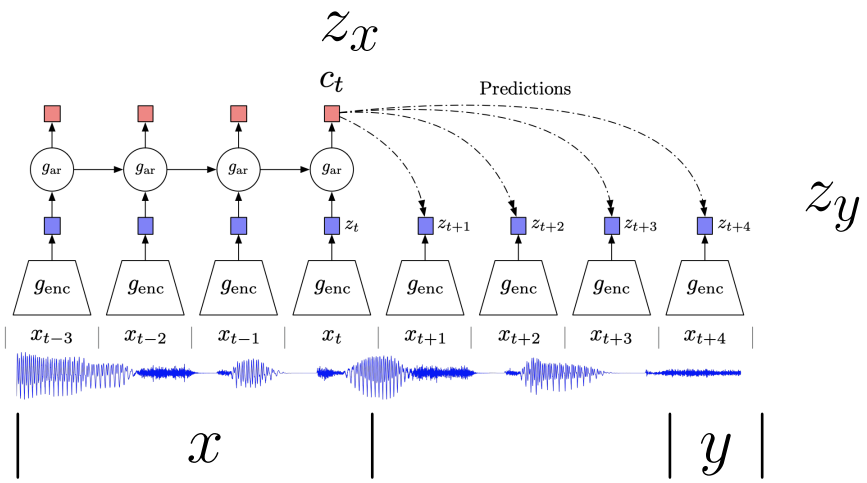
SwAV

Mutual information coding as pretraining of image features.



Caron et al. 2021

Mutual Information Coding: General Formulation



Consider a population distribution on pairs $\langle x, y \rangle$.

We are interested in extracting latent variables z_x and z_y from x and y respectively that preserve the mutual information between x and y.

Mutual Information Coding General Formulation

For a population on $\langle x, y \rangle$ we introduce latent variables z_x and z_y defined by mappings $z_x(x)$ and $z_y(y)$ where these mappings are defined by parameters Φ_x and Φ_y respectively.

$$\Phi_x^*, \Phi_y^* = \underset{\Phi_x, \Phi_y}{\text{argmax}} \; I(z_x, z_y)$$

This has a degenerate solution of $z_x(x) = x$ and $z_y(y) = y$ but this can be avoided with various types of restrictions on z_x and z_y as described below.

Contrastive Predictive Coding (CPC)

For z_x and z_y vectors, and for $N \geq 2$, we define a distribution on tuples $(z_x, z_y^1, \ldots, z_y^N, i)$ by drawing pairs $(x_1, y_1), \ldots (x_n, y_n)$ from the population, and i uniformly from 1 to N, and constructing

$$(z_x(x_i), z_y(y_1), \ldots, z_y(y_N), i).$$

We then train a model to predict i.

$$\Phi^* \;=\; \operatorname*{argmin}_{\Phi} \; E_{(z_x, z_y^1, \ldots, z_y^N, i)} \left[-\ln P_\Phi(i | (z_x, z_y^1, \ldots, z_y^N) \right]$$

$$P_\Phi(i | z_x, z_y^1, \ldots z_y^N) \;=\; \operatorname*{softmax}_i \; z_x^\top z_y^i$$

The CPC Theorem

For any distribution on pairs (z_x, z_y), if CPC probabilities are computed by

$$P_\Phi(i|z_x, z_y^1, \ldots z_y^N) \;=\; \underset{i}{\text{softmax}}\; s(z_x, z_y^i)$$

then

$$I(z_x, z_y) \;\geq\; \ln N - \;\; E_{(z_x, z_y^1, \ldots, z_y^N, i)} \left[-\ln P_\Phi(i|(z_x, z_y^1, \ldots, z_y^N) \right]$$

Chen et al., On Variational Bounds of Mutual Information, May 2019.

The CPC Restriction on $z_x(x)$ and $x_y(y)$

$$P_\Phi(i|z_x, z_y^1, \dots z_y^N) \;=\; \underset{i}{\text{softmax}}\; z_x^\top z_y^i$$

By only using z_x and z_y in inner products at the final layer we force the features to carry information in a shallow (linear) representation.

Contrastive Predictive Coding for Images

(SimCLR:) A Simple Framework for Contrastive Learning of Visual Representations, Chen et al., Feb. 2020 (self-supervised leader as of February, 2020).

They construct a distribution on pairs $\langle x, y \rangle$ defined by drawing an image from ImageNet and then drawing x and y as random "augmentations" (modifications) of the image.

The training maximizes the contrastive lower bound on $I(x, y)$.

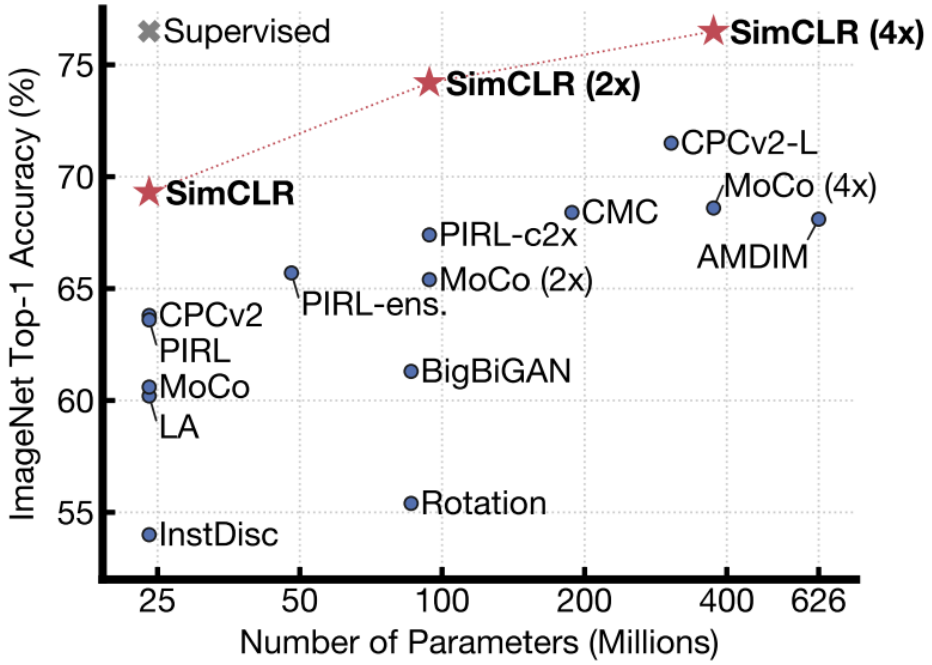
Contrastive Predictive Coding for Images

A resulting feature map z_Φ on images is extracted from this training.

The feature map z_Φ is tested by using a linear classifier for ImageNet based on these features.

This is called linear probing.

SimCLR



Chen et al. 2020

A Weakness of CPC

$$I(z_x, z_y) \geq \ln N - \quad E_{(z_x, z_y^1, \ldots, z_y^N, i)} \left[-\ln P_\Phi(i | (z_x, z_y^1, \ldots, z_y^N) \right]$$

The discrimination problem is too easy.

The guarantee can never be stronger than $\ln N$ where N is the number of choices in the discrimination task.

Direct Mutual Information (MI) Coding

For a population on $\langle x, y \rangle$ we introduce latent variables z_x and z_y defined by mappings $z_x(x)$ and $z_y(y)$ where these mappings are defined by parameters Φ_x and Φ_y respectively.

$$\begin{aligned}
\Phi_x^*, \Phi_y^* &= \underset{\Phi_x, \Phi_y}{\text{argmax}} \; I(z_x, z_y) \\
&= \underset{\Phi_x, \Phi_y}{\text{argmax}} \; H(z_y) - H(z_y|z_x) \\
&= \underset{\Phi_x, \Phi_y}{\text{argmin}} \; H(z_y|z_x) - H(z_y)
\end{aligned}$$

Direct MI Coding

$$\Phi_x^*, \Phi_y^* = \underset{\Phi_x, \Phi_y}{\operatorname{argmin}} \; H(z_y|z_x) - H(z_y)$$

We can block the solution of $z_x(x) = x$ and $z_y(y) = y$ by requiring that z_y is a symbol from a limited finite vocabulary.

This typically ensures $H(z_y) << H(y)$.

We can then allow $H(z_x)$ to be large, for example z_x might be a vector representation of the a symbol sequence $z_1, \ldots, z_t$.

Direct MI Coding

$$\Phi_x^*, \Phi_y^* = \underset{\Phi_x, \Phi_y}{\operatorname{argmin}} \; H(z_y|z_x) - H(z_y)$$

If z_y is a symbol from a limited vocabulary we can estimate $H(z_y)$ from an empirical histogram over the symbols.

We can use a model Ψ to predict z_y from z_x and train on cross-entropy loss.

Direct MI Coding

$$\Phi_x^*, \Phi_y^*, \Psi^* = \underset{\Phi_x, \Phi_y, \Psi}{\operatorname{argmin}} \; E_{(x,y)\sim \mathrm{Pop}} \left[-\ln P_\Psi(z_y | z_x) + \ln \hat{P}(z_y) \right]$$

where $\hat{P}(z_y)$ is an estimate (perhaps an exponential moving average) of the probability of z_y over the draw of $(x, y) \sim$ Pop.

Direct MI Coding Theorem

$$\Phi_x^*, \Phi_y^*, \Psi^* = \underset{\Phi_x, \Phi_y, \Psi}{\operatorname{argmin}} \; E_{(x,y)\sim \mathrm{Pop}} \left[-\ln P_\Psi(z_y | z_x) + \ln P(z_y) \right]$$

$$I(z_x, z_y) \geq H(z_y) - \hat{H}(z_y | z_x)$$

$$\hat{H}(z_y | z_x) = E_{(x,y)\sim \mathrm{Pop}} \left[-\ln P_\Psi(z_y(y) | z_x(x) \right]$$

Direct MI Coding Theorem

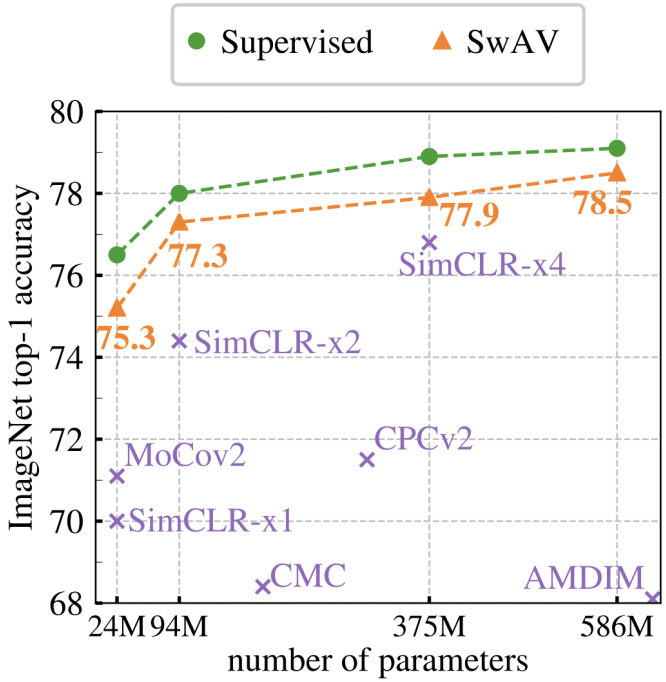
For $H(z_y) = \ln K$ where K is the number of symbols, and $\hat{H}(z_y|z_x) = 0$ we get

$$I(z_x, z_y) \geq \ln K$$

Which typically improves significantly on the best possible bound $I(z_x, z_y) \geq \ln N$ from CPC.

SwAV

SwAV uses direct MI coding rather than SimCLR's CPC.



Caron et al. 2021

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