System Dynamics & Controls

Chapter 2 – Laplace Transforms and Modeling

Laplace Transforms

- One of the most efficient ways to solve differential equations
- Laplace transforms convert differential equations into algebraic equations
- Solution to algebraic equation gives solution to differential equation when transformation is reversed
- Laplace transform of $f(t)$ given by:

- Finding the function $f(t)$ from the Laplace transform $F(s)$called taking the inverse Laplace

Example 1

Let $f(t) = e^{at}$ where a = constant. Find $\mathcal{L}[f(t)]$.

Linearity Property

Example 2

Find $\mathcal{L}[3t^5 - t^8 + 4 - 5e^{2t} + 6\cos(3t)]$.

Shifting Property

- Used when multiplying $f(t)$ by $e^{-\alpha t}$

Example 3

Find $\mathcal{L}[e^{2t}\cos(3t)]$.

Differentiation Theorem

Example 4

Find $\mathcal{L}[sin^2(at)]$.

Integration Theorem

Inverse Laplace Transformations

- Finding $f(t)$ from corresponding $F(s)$
    1. Methods
        - Use Table of Laplace Transforms
        - Use partial fraction expansion (PFE)
            - Partial fractions simplify the problem so we can get to the table
- Partial Fraction Review
    1. Distinct roots in denominator
        - See example 5
    2. Repeated roots in denominator
        - See example 6
    3. Complex roots in denominator
        - See example 7

Example 5

Find the partial fraction expansion for:

$$\frac{1}{s^2 - 5s + 6}$$

Example 6

Find the partial fraction expansion for:

$$\frac{5s^2 + 20s + 6}{s^3 + 2s^2 + s}$$

Example 7

Find the partial fraction expansion for:

$$\frac{2s^3 - 4s - 8}{(s^2 - s)(s^2 + 4)}$$

Solving Linear Differential Equations

- Initial value problems can be solved with the differentiation theorem
  - Steps:
    1. Take Laplace transform of each term
    2. Solve for dependent variable – will be fraction that's function of s
    3. If form not found in Laplace tables find partial fractions
    4. Take inverse Laplace

Example 8

Find the solution to the initial value problem:

$$\ddot{x} + 4x = 0 \qquad x(0) = 1 \quad \dot{x}(0) = 2$$

Example 9

Find the solution to the initial value problem:

$$\ddot{x} - 3\dot{x} + 2x = 4t - 6 \qquad x(0) = 1 \qquad \dot{x}(0) = 3$$

Solving Systems of Equations Using Laplace Transforms

Example 10

Solve the system of equations:

$$\dot{x}_1 + 2x_1 - x_2 = 2e^{-3t} \qquad x_1(0) = 0$$

$$\dot{x}_2 - x_1 + 3x_2 = 0 \qquad x_2(0) = 0$$

Mechanical Systems

- Mass

- Force

Mechanical Elements

- Mechanical system is made up of mechanical elements
  - 3 types of mechanical elements

1. Inertia elements
   - If something has a finite mass, it's an inertia element
     - Particles
       - Kinetic energy caused by motion of the particle

- Rigid Bodies
    - Fixed distribution of mass about center of gravity
    - Planar motion – motion lies within a plane
        - Axis of rotation is ⊥ to the plane
    - Center of mass defined as:
    - Moment of inertia – measure of how mass is distributed about the axes
    - Polar moment of inertia
    - Parallel axis theorem – let's you find moment of inertia about axis parallel to centroidal axis

Example 11

Hollow cylinder is made of steel with a mass density of 7600 kg/m^3. Find the moment of inertia about the x axis.

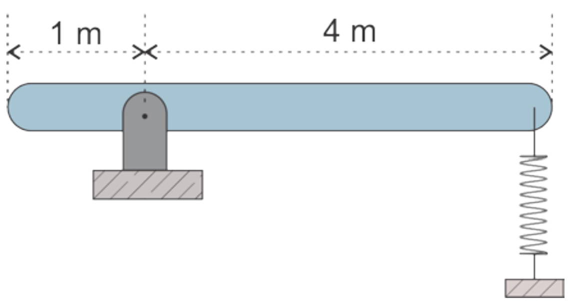
- For a rigid body undergoing planar motion

- Linear momentum of rigid body

- Angular momentum of rigid body about an axis O

Example 12

At the moment shown, the angular velocity of the 6 kg bar is 10 rad/s CCW and its angular acceleration is 3 rad/s² CW. Determine a) velocity of the center of the bar, b) the acceleration of the center of the bar, c) kinetic energy of the bar, d) angular momentum of the bar about an axis through the center of the bar.



2. Springs
   - Under tension – spring stretches
   - Under compression – spring shortens
   - Assume weight of spring is negligible
   - Springs store energy
   - In equilibrium, spring has unstretched length
   - Applied force causes change in spring length
     - Force displacement relationship

   - Work done by spring dependent on initial and final conditions

   - Torsional spring – for rotation

3. Friction elements
   - Friction used to dissipate energy
   - Damping = dissipation of energy from mechanical system
   - Damper = device that dissipates energy
     - Converts energy to heat
     - Heat released to surroundings

   <u>Viscous Damping</u>

   - Damping force is proportional to the velocity of the component

- Dashpot – device used to provide viscous damping
    - Cylinder filled with oil
    - Piston moves in/out of cylinder
    - Fluid resists motion
        - Pressure difference between the faces of the piston
            - Resisting force on piston
    - Piston moving with velocity v and cylinder is stationary:
    - If cylinder has velocity
- Rotation – torsional viscous damper
    - Creates resisting moment that's proportional to angular velocity of resisting component

Friction

- Assume viscous friction between surfaces
  - Keeps things from becoming non-linear

Mechanical Input

- External forces and torques

Free Body Diagrams (FBDs)

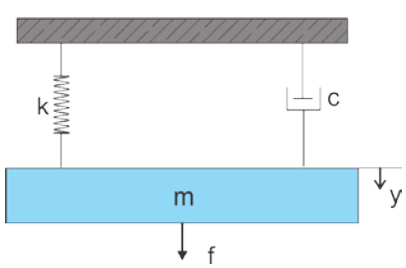
- Use FBDs to get equations of motion
- Show forces/torques acting on inertia elements
- We'll:
  - Draw FBDs
  - Find equations of motion for each inertia element
  - Solve equations of motion (do this in next section)

Example 13

Find the equations of motion.

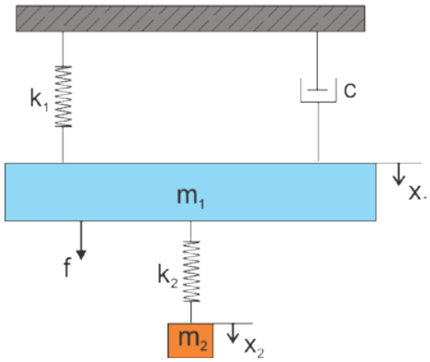
Example 14

Find the equations of motion.



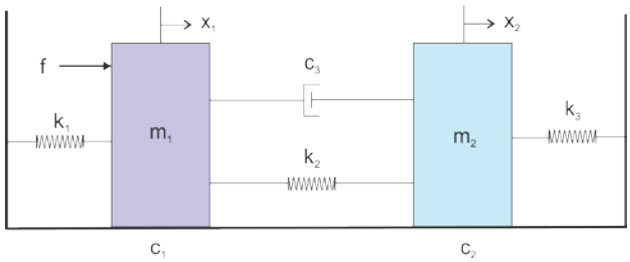
Example 15

Find the equations of motion.



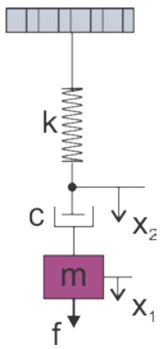
Example 16

Find the equations of motion for the masses shown.



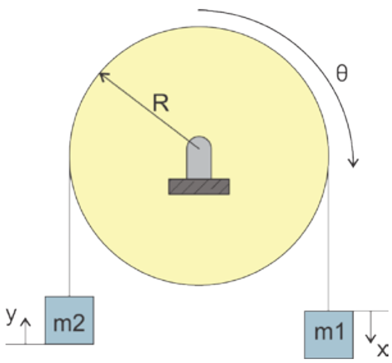
Example 17

Find the equations of motion.



Example 18

Find the math model in terms of the dependent variable x.



- Small angle approximation
    - Used to linearize equations whose dependent variables are angular displacements
    - Approximations:
        - $\sin\theta \approx \theta$
        - $\cos\theta \approx 1$
        - $\tan\theta \approx \theta$
        - $1 - cos\theta \approx \frac{1}{2}\theta^2$

Example 18

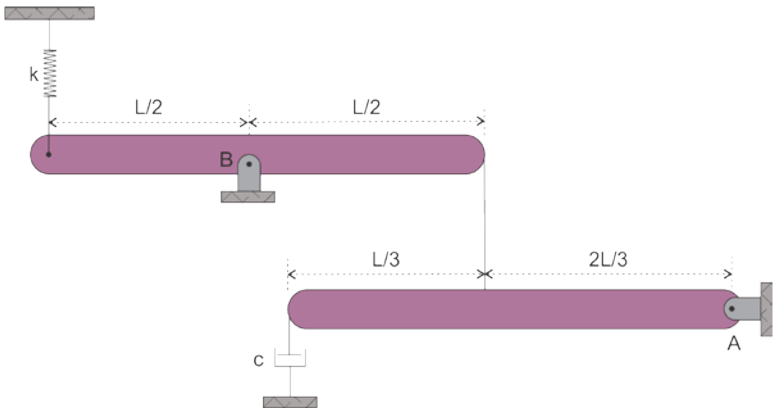
The differential equation for a system is

$$\frac{1}{3}mL^2\ddot{\theta} + L\ddot{y}sin\theta + L\ddot{x}cos\theta + mg\frac{L}{2}sin\theta = 0$$

where $x(t)$ and $y(t)$ are known and θ is the dependent variable. Derive a linearized equation governing the motion using small angles.

Example 19

Derive the math model using θ as the dependent variable assuming small θ. The bars are identical slender rods of mass m and connected by a rigid massless rod.

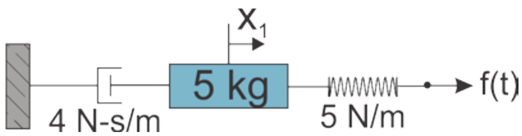


Transfer Functions

- Transient response – system's response due to changes in system input
    - Look at transfer functions to study transient response
- Transfer Functions
    - Algebraically relates input to output
    - Defined as ratio of output (response function) to input (driving function)
    - Assume initial conditions are zero
    - Look at n^{th} order differential equation

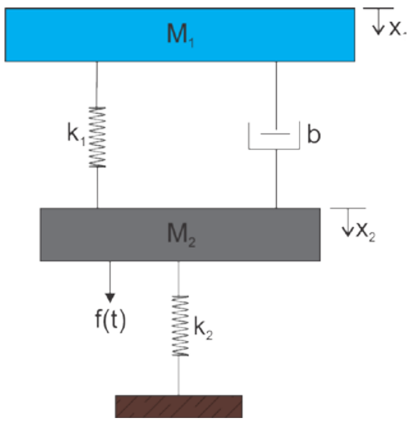
Example 20

Find $G(s) = {X_1(s)}/{F(s)}$



Example 21

The system shown represents a car driving down a bumpy road.



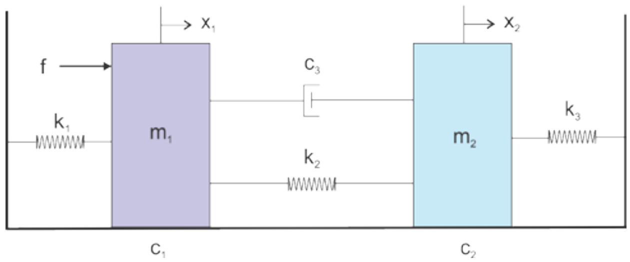
Impedence Method

- Allows for quicker solutions
- Can skip the FBDs, differential equations, etc

- Find equations using impedence

Example 22

Find the Laplace transforms of the equations of motion using the impedence method.



Rotational Systems

- Done like translational systems
- Use torque instead of force, angular displacement instead of displacement

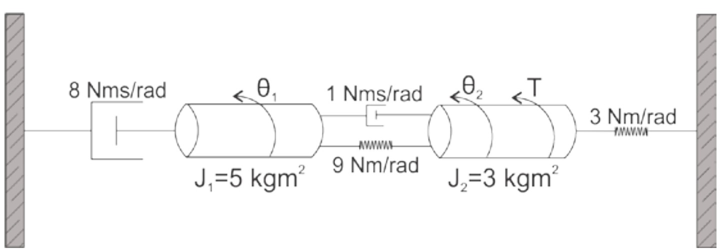
Torsional spring:

Torsional damper:

Inertia:

Example 23

Find the transfer function $\theta_1(s)/T(s)$



Transfer Functions for Multiple Inputs and Outputs

- Find matrix that consists of transfer functions

Chapter 3 – State Space Representation

- Satellite Example

- System dynamics are described by a state space model instead of transfer functions
- State space model = description in terms of a set of first order differentials written in matrix form
- Used with numerical techniques
- Easy to code up

- State space equations – if system is linear and described by n state variables, r input variables, and m output variables we get the form:

Example 24

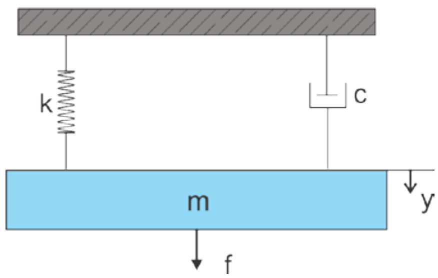
Letting $x_1 = y$ and $x_2 = \dot{y}$, write the state space representation for the system described by:

$$\ddot{y} + 4\dot{y} + 3y = 2u$$

- Steps to get state space representation
    1. Find equations of motion
    2. Choose state variables (position and velocity for each mass)
    3. Take derivative of equations from step 2
    4. Write in state space form
    5. Write output equation

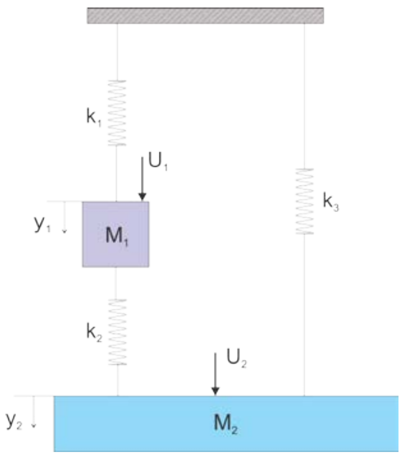
Example 25

Find the state space representation for the system shown. Output is displacement of the mass.



Example 26

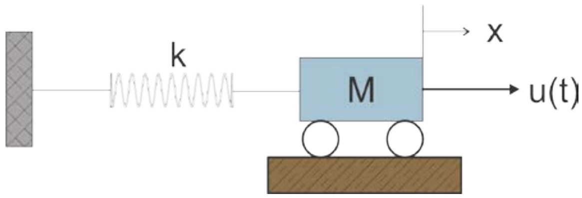
Find the state space representation. Outputs are the displacements of the two masses.



- To numerically integrate in MATLAB:

Example 27

Find the state space representation. Input is the applied force u(t) and output is the displacement x.



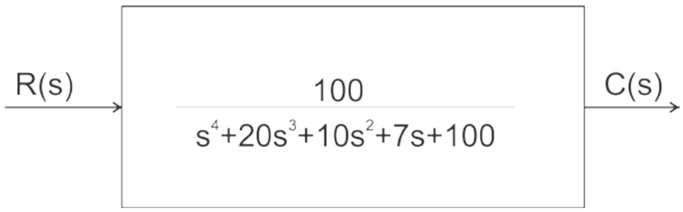
Converting a Transfer Function to State Space

- Consider the differential equation:

- For the state variable choose output y and its $n-1$ derivatives

Example 28

Find the state space representation for the system shown.



Find Transfer Functions from State Space Equations

- Standard state space equations are:

Example 29

Find the transfer function given the following:

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Chapter 4 – Time Response

- Output response is made up of 2 responses
    1. Forced response
    2. Natural response
- Start by looking at poles and zeros

Example 30

Find the poles, zeros and output time response for the system shown. The input is a unit step input.

$$R(s) \longrightarrow \boxed{\frac{s+2}{s+5}} \longrightarrow C(s)$$

- Can make some conclusions

Response Types

- Free Response
  - Nonzero initial conditions
  - No input

- Impulsive Response
  - Input is a unit impulse $\delta(t)$
  - Leads to sudden change in response

- Step Response
  - Input is a unit step function u(t)

- Ramp Response
  - Input is a unit ramp function

First Order Systems

- General form for first order differential equation:

- Free Response

- Impulsive Response

- Step Response

   - Look at the time constant

   - Time constant is time it takes for e^{-at} to decay to 37% of its initial value
   - Time constant is time it takes for step response to rise to 63% of its final value

- Rise time – time for c(t) to go from 0.1 to 0.9 of its final value

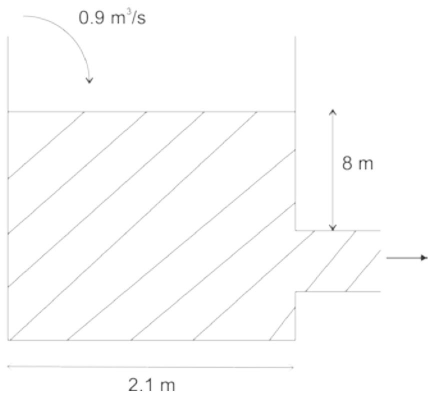
- Settling time – time for response to reach and stay within 2% of its final value

Example 31

System is at steady state when exit pressure changes to a gage pressure of 2.5 kPa. The first order equation below can be used to model the system with perturbation.

$$A\frac{dh}{dt}+\frac{1}{R}h=\frac{P}{\rho g R}$$

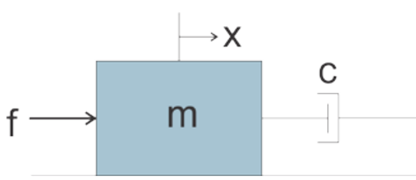
Find a) the new level of liquid when new steady state is reached, b) time response, c) settling time



- Ramp Response

Example 32

Find an equation for velocity as a function of time.



Example 33

The equation shown can be used to model temperature change with respect to time. If we want to cool 100℃ soup by putting it in a sink that's constantly being filled with water that's 5℃. How long does it take to cool the soup down to 60℃ ?

$$\frac{dT}{dt} = -k(T - T_m)$$

Second Order Systems

- Changing parameter of 1st order system only changes speed of response
- Changing parameter of 2nd order system can change type of response
- 4 types of responses
    - Undamped
    - Underdamped
    - Critically damped
    - Overdamped

Undamped Response

- Recall:

- Now add damping

Response	ξ	Poles

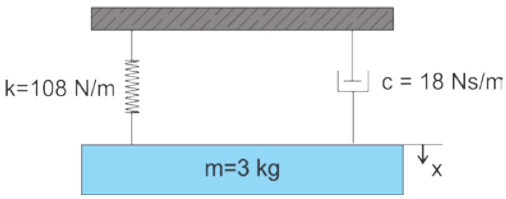
Underdamped

Overdamped

Critically Damped

Example 34

The system shown is released from rest from an initial x_o. Determine the overshoot displacement when $t = 0.605\ sec$.



Underdamped 2nd Order Systems

- Recall:

- For step response:

- Peak time - time it takes to get to first (max) peak.
    - Motion starts to settle after this first peak
    - Found by:

- <u>Percent Overshoot (% OS)</u> – amount the response overshoots the final (steady-state) value at peak time
    - Expressed as percentage of the steady state value

- <u>Settling time</u> – time for which response function $c(t)$ stays within $\pm 2\%$ of c_{final}

- Rise time – time it takes to go from 0.1 c_{final} to 0.9 c_{final}
  - No standard equation
  - Book gives:

- T_r, T_p, T_s give information about response speed
- Now relate parameters to pole location

- Look at how moving poles affects response

- Moving poles vertically

- Moving poles horizontally

- Moving poles along radial lines

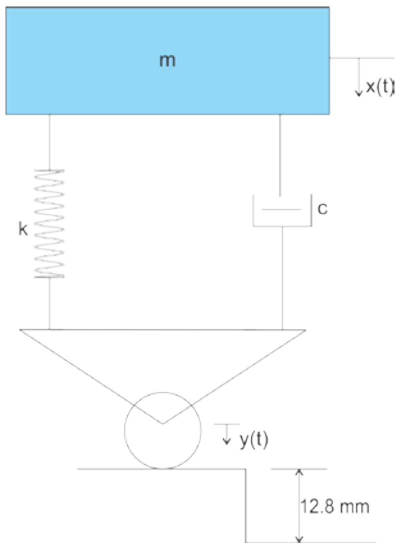
Example 35

Find the location of a second order system's pair of poles given a percent overshoot of 12% and a settling time of 0.6 sec.

Example 36

Shown below is a simplified model of suspension system for a car. Let y(t) = displacement of the wheel as it traverses the road and x(t) = displacement of the vehicle.

Consider a vehicle with $m = 500\ kg, k = 3.2 \times \frac{10^5 N}{m}$ and $c = 10000 \frac{Ns}{m}$. Vehicle travels with a constant velocity of 80 km/hr. Determine the response of the vehicle after it encounters a dip of 12.8 mm in the road.



Chapter 5 – Block Diagrams

- Let's look at an example of a heating system

- Block diagrams use the transfer functions of a model to construct a visual representation of the system
- Can be used to find the transfer functions for a system when equations of motion aren't available
- Simulation diagrams can be developed in Simulink and LabView
- Simple block diagram

- More complex systems may require multiple interconnected subsystems
- Need new schematic elements:
  - Summing junction

- Pickoff point

- Block diagram forms:
  - Cascade (aka series) form

- Parallel Form

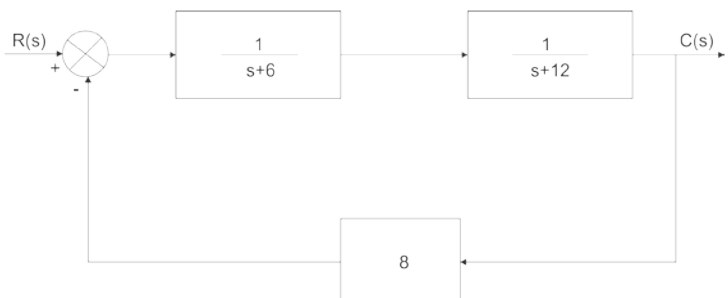
- Feedback Form
  - Basis for controls engineering
  - Feedback Controls
  - Closed-loop system
    - Used to remove disturbances from system

- So, what are we going to do?
  - Reduce the complex, interconnected block diagrams down to one block
  - Final block is equivalent to the original

- Equivalent Diagrams
  - Need to know how to simplify the diagrams

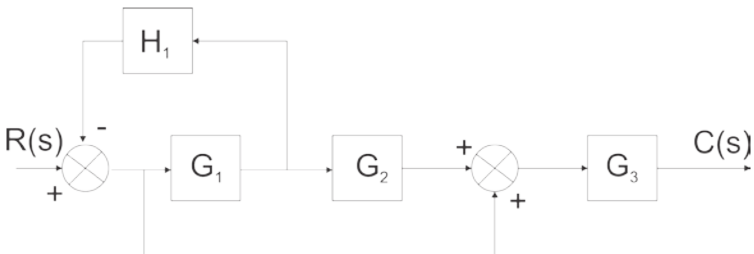
Example 37

Find the equivalent transfer function.



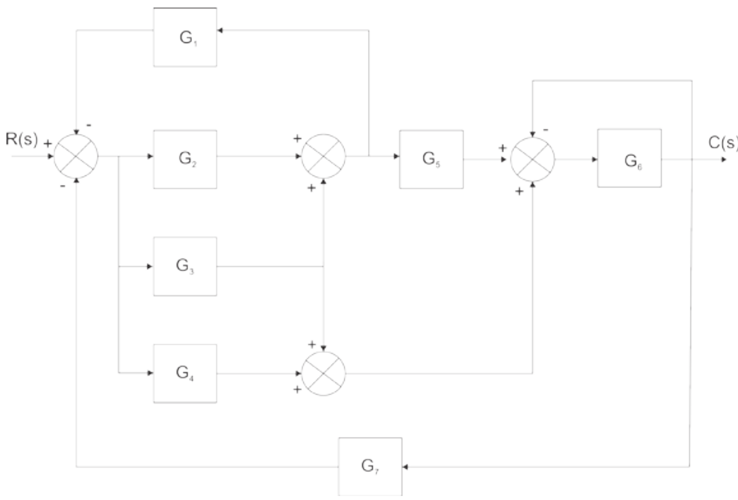
Example 38

Find the equivalent transfer function.



Example 39

Find the equivalent transfer function.



Chapter 6 – Stability Analysis

- Want stable systems
- Stable system will respond in an appropriate manner to the applied input
    - Predictable behavior
- Unstable systems have little relation between the input and output
    - Unpredictable behavior

- Use bounded input bounded output (BIBO) for analysis
    - System is BIBO stable if for every bounded input, the output remains bounded with increasing time
- Stability of linear closed system determined by location of poles
    - Stable systems – all poles must lie on left hand side of s plane
        - Natural response approaches zero as $t \to \infty$
    - Unstable systems – at least one pole on right hand side
        - Output becomes unbounded for any input
        - Response would keep growing without bound
    - Marginally Stable systems – poles on the imaginary axis with all other poles on the left
        - System stable for some inputs, unstable for others

Example 40

For each closed-loop transfer function T(s), find the poles and response terms. Discuss stability.

a) $T(s) = \frac{2}{(s+1)(s+2)}$

b) $T(s) = \frac{10s+24}{s^3+2s^2-11s-12}$

c) $T(s) = \frac{s}{(s^2+1)^2}$

- Method to determine system stability:

    - Determines is any roots lie outside of the left half plane
    - Doesn't give actual pole locations

- Routh's criterion applied to Characteristic equation (denominator of T.F.)

- Couple of things to check for first:
    - Are any coefficients are equal to zero?
        - Yes - Not all roots lie on left hand side
    - Are any coefficients negative?
        - Yes – at least one root is on right hand side
            - Unstable

- Unstable system possible with all positive coefficients
    - Use Routh's

Routh's Criterion

- Form the Routh array
    - First two rows are determined by coefficients of characteristic equation

- Count sign changes in first column
    - # of sign changes = # of roots on right hand side
    - No sign changes?
        - Stable!!

Example 41

Given the characteristic equation below determine stability.

$$s^3 + s^2 + 2s + 8 = 0$$

Example 42

Given the characteristic equation below determine stability.

$$s^3 + 2s^2 + 3s - 1 = 0$$

- Shortcut!! - last element on even exponent rows is always the same

- From calculations you can see that the array can't be completed if the first element in a row is zero

    - Special Cases

Case I

- If first element in a row is zero but there is at least one non-zero element in the same row
    - Replace the first zero with a small number ε
        - ε can be + or −
    - Calculations continue as normal
        - Some elements that follow will be functions of ε
    - Complete the array
    - Count sign changes in first column
        - Allow ε to be a very, very tiny number (almost zero)
        - # of sign changes = # roots on right hand side

Example 43

Given the characteristic equation below determine stability.

$$s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

Special Case II

- ALL elements in a row are zero
- In this case:
    - There are two real roots of equal magnitude and opposite sign and/or two conjugate imaginary roots
    - Characteristic polynomials contain an even polynomial as a factor
        - Even polynomials only have even powers
        - Even polynomial also called auxiliary polynomial

- Coefficients of auxiliary polynomial will always be the elements in the row directly above the row of zeros
- Exponent of row containing coefficients is the highest power in auxiliary polynomial
    - Rows of zeros only found in odd exponent rows!!

- To analyze:
    - Differentiate the auxiliary polynomial with respect to s
        - Coefficients of resulting polynomial replace zeros in zero row
    - Continue calculations as normal
    - Count sign changes in first column

Example 44

Given the characteristic equation below determine stability.

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$$

- Row of zeros caused by auxiliary polynomial
- Aux. polynomials only have roots that are symmetrical about the origin
- Roots can be:
    - Symmetric and real
    - Symmetric and imaginary
    - Symmetric and quadrantal
- No $j\omega$ roots unless we have a row of zeros
    - Recall $j\omega$ roots are symmetric about the origin

- Every entry in Routh array from aux. polynomial row down deals with the aux. polynomial
    - For row of zeros:
        - When counting sign changes break Routh array into two parts and analyze separately
            - Aux. polynomial row and up
                - # sign changes = # right hand side roots (just like always)
                - No $j\omega$ roots possible here!
            - Aux. polynomial row and down
                - # sign changes = # of right hand roots AND # of left hand side roots (think symmetry!!)
                - Any left over roots on $j\omega$ axis

Example 45

Tell how many closed-loop poles are in the right half plane, left half plane, and on the $j\omega$ axis. Is the system stable?

$$T(s) = \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18}$$

Example 46

Tell how many closed-loop poles are in the right half plane, left half plane, and on the $j\omega$ axis. Is the system stable?

$$G(s) = \frac{84}{s(s^7 + 5s^6 + 12s^5 + 25s^4 + 45s^3 + 50s^2 + 82s + 60)}$$

- Use Routh's Criterion to design systems

Example 47

Ensure the system with the following characteristic equation is stable.

$$s^3 + 4s^2 + 5s + 2 + 2k = 0 \text{ where } k = constant$$

Stability in State Space

- Location of poles determines stability
  - poles are easy to find with transfer functions
- How to find poles in state space?
  - Could convert state space to T.F.
    OR
  - Use eigenvalues

Recall that the number λ is an eigenvalue iff there exists a non-zero vector x such that:

- Rearrange and solve for x

- All solutions will be null vector except when denominator = 0

- Use above equation to check stability
  - Results in polynomial
    - Use Routh's Criterion

Example 48

Determine stability. How many poles are on the left hand side, right hand side and $j\omega$ axis?

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & -4 \\ -1 & 1 & 8 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \bar{x}$$

Chapter 8 – Root Locus Techniques

- Root locus gives graphical presentation of a closed-loop system
    - Shows the closed-loop poles as a system parameter is varied
- Technique can be used on higher order systems
- Also shows stability

- Root locus shows a visual representation of how poles change with k

Sketching the Root Locus

Steps to get a general sketch (Note: there are more advanced steps but we won't cover them here)

- Will not get actual values

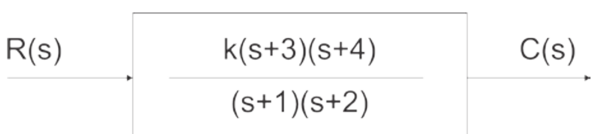
1. Number of branches
    - Each pole moves as k is varied
        - One branch per pole
        - Branch = path a pole takes
    - # of branches = # of closed-loop poles
2. Symmetry
    - Root locus is symmetric about the real axis
        - Have complex conjugates or real poles
            - Must be symmetric about real axis
3. Real Axis Segments
    - On the real axis, the root locus exists to the left of the odd number finite poles/zeros
4. Starting and Ending Points
    - Begins at poles
    - Ends at zeros or by leaving plot (infinite zeros)
5. Behavior at ∞

- Can have infinite zeros
    - If function approaches 0 as $s \to \infty$ then there is a zero at ∞
    - Check for infinite zeros
        - If # of poles $\neq$ # of finite zeros then you have infinite zeros
- Root locus approaches straight lines as asymptotes as locus approaches ∞

- The equation of asymptotes is given by the real axis intercept σ_a and angle θ_a

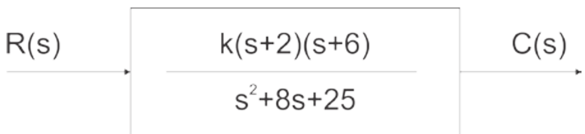
Example 49

Sketch the root locus for the system shown.



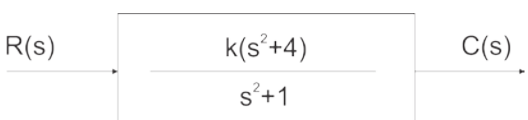
Example 50

Sketch the root locus for the system shown.



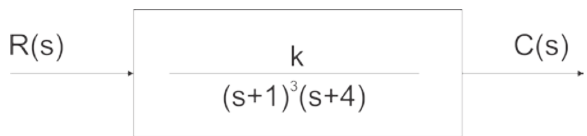
Example 51

Sketch the root locus for the system shown.



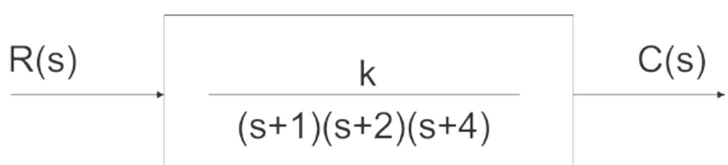
Example 52

Sketch the root locus for the system shown.



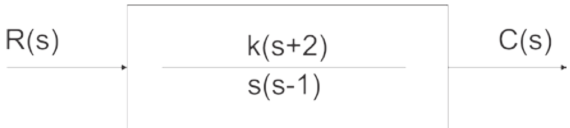
Example 53

Sketch the root locus for the system shown.



Example 54

Sketch the root locus for the system shown.



Chapter 7 – Steady State Errors

- Control system design focuses on:
  - Response (Ch 4)
  - Stability (Ch 6)
  - Steady state error (Ch 7)
- **Steady state error** – difference between the input and the desired output as $t \to \infty$
- Tradeoffs between desired response and steady state errors
- Does steady state error make sense for unstable systems? NO!!!
  - Check for stability first!

- 3 types of test inputs
  1. Step

  2. Ramp

  3. Parabolic

Consider

- Open loop transfer function

- Closed loop transfer function

Steady state error can be found from T(s) or G(s)

- Look at closed loop case:

- Want error as $t \to \infty$
    - Gives $e(\infty)$
- Use final value theorem to get $e(\infty)$

- Steady state error in terms of G(s):

- 3 test inputs:
    - Step input:

    - How do we get zero e_{ss}?

- Ramp input:

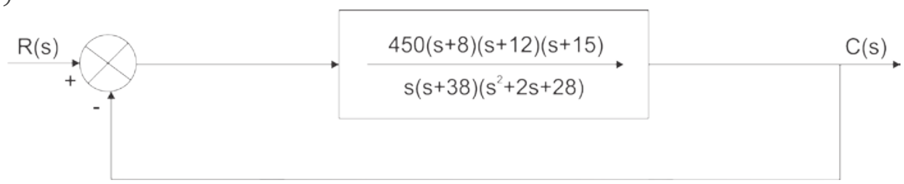
    - How to get zero e_{ss}?

- Parabolic Input:

Example 55

For the unity feedback system shown, find the steady state errors for the following test inputs.

a) $25u(t)$
b) $37tu(t)$
c) $4t^2u(t)$



Static Error Constants and System Type

- Used settling time, etc. to look at performance before
- Now define steady state error performance specifications
    - Called static error constants

Static Error Constants

- Step inputs

- Ramp input

- Parabolic input

- In ALL cases, e_{ss} decreases as static error constant increases

- Define 3 system types:

	General Equations		Type 0		Type 1		Type 2	
Input	**Static Error Constant**	e_{ss}	**Static Error Constant**	e_{ss}	**Static Error Constant**	e_{ss}	**Static Error Constant**	e_{ss}
Step	lim G(s)	1/(1+Kp)	Kp=const.	1/(1+Kp)	Kp=Inf.	0	Kp=Inf.	0
Ramp	lim sG(s)	1/Kv	Kv=0	Inf.	Kv=const.	1/Kv	Kv=Inf.	0
Parabolic	lim s^2G(s)	1/Ka	Ka=0	Inf.	Ka=0	Inf.	Ka=const.	1/Ka

Example 56

A unity feedback system with $G(s) = \frac{k(s^2+3s+30)}{s^n(s+5)}$ is to have 1/6000 error between input of $10tu(t)$ and the output in steady state.

a) Find k and n to meet the error specification
b) What are K_P, K_v, and K_a?

Modeling Electrical Circuits (Chapter 2 in Nise book)

- Electrical current
  - Flow of charged particles
  - Positive when in direction of positively charged particles or opposite the direction of negatively charged particles
- Electric potential (voltage)
  - Work required to move a 1-Coulomb charge against an electric current
- Electric power
  - Rate at which energy is transferred

<u>Components</u>

- Passive Components
  - Resistor
    - Dissipates energy by converting electrical energy to heat
    - Voltage decreases (voltage drop) across the resistor in the direction of positive current

- Capacitor
  - Device stores energy in presence of electric field

- Inductor
  - Induction – process by which current is developed due to a moving magnetic field
  - Store energy in presence of magnetic field

- Active Components
  - Voltage and Current sources

Kirchoff's Laws

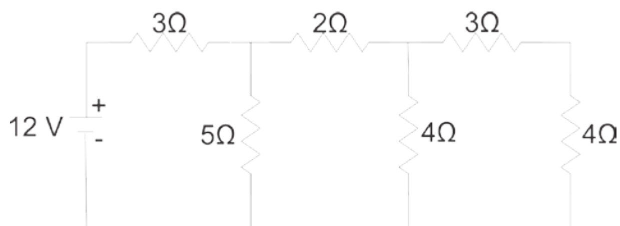
1. Current Law (KCL)– sum of currents at a node = 0

2. Voltage Law (KVL) – total voltage change in loop = 0

Example 57

For the circuit shown, find

a) Current through each resistor
b) Voltage drop across each resistor



Circuit Reduction

- Ohm's Law – current in a circuit is proportional to the voltage acting in the circuit and inversely proportional to the resistance of the circuit

- Find series and parallel components and simplify
  - Series circuits

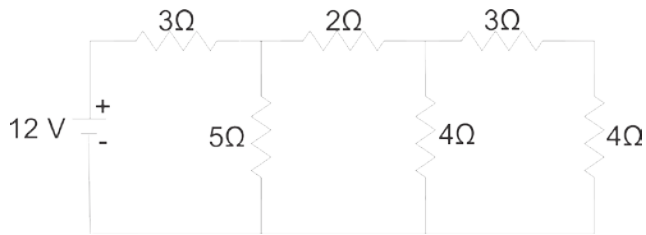
- Series Equivalents:
  - Resistors:
  - Capacitors:
  - Inductors:

- Parallel circuits

- Parallel Equivalents:
  - Resistors:
  - Capacitors:
  - Inductors:

Example 58

Replace the resistors in the circuit with an equivalent resistor.



Modeling Circuits and Complex Impedences

- Voltage rise – going from the negative to positive terminal or going through a resistance in opposition to current flow
- Voltage drop – going from positive to negative terminal or going through a resistance in the direction of current flow

- Derive transfer function for circuits:

  - All initial conditions are zero

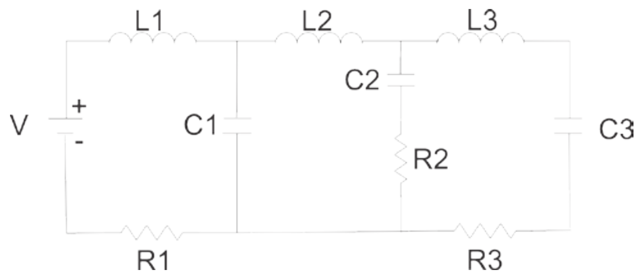
- For passive elements
  - Take the Laplace transform

Example 59

Obtain the impedence for the circuit.

Example 60

Find the math model for the circuit shown.



Example 61

Find the transfer function $V_o/_V$.