For the output feedback controller the closed-loop system is given as

$$\begin{aligned} \lambda I - (A + kBC) &= 0 \\ \lambda I - A - kBC &= 0 \\ (\lambda I - A)X_1 n_1 - kBCX_1 n_1 &= 0 \\ (\lambda I - A)X_1 n_1 + BCY_1 n_1 &= 0 \\ \begin{bmatrix} \lambda I - A & BC \end{bmatrix} \begin{bmatrix} X_1 n_1 \\ Y_1 n_1 \end{bmatrix} &= 0 \end{aligned} \tag{1}$$

Setting the eigenvector gives

$$\begin{aligned} X_1 n_1 &= R_{des}(1) \\ n_1 &= X_1^{\dagger} R_{des}(1) \end{aligned} \tag{2}$$

and

$$R = X_1 n_1 \quad M = Y_1 n_1 \tag{3}$$

Noticed that

$$\begin{aligned} MR^{-1} &= -kC^T C \\ M &= -kC^T CR \\ Y_1 n_1 &= -kC^T CX_1 n_1 \\ Y_1 n_1 + kC^T CX_1 n_1 &= 0 \\ \begin{bmatrix} kC^T C & I \end{bmatrix} \begin{bmatrix} X_1 n_1 \\ Y_1 n_1 \end{bmatrix} &= 0 \end{aligned} \tag{4}$$