1 State Feedback Case

The eigenvalue equation is given as

$$(A - BK)V = \lambda V \tag{1}$$

or

$$\begin{aligned} \lambda V - (A + BK)V &= 0 \\ \lambda V - AV - BKV &= 0 \\ (\lambda I - A)X + BY &= 0 \\ \begin{bmatrix} \lambda I - A & B \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} &= 0 \end{aligned} \tag{2}$$

where $K = -YX^{-1}$.

2 Output Feedback Case

Let $k \in \mathbb{R}$, then

$$\begin{aligned} -YX^{-1} &= kC \\ -Y &= kCX \\ kCX + Y &= 0 \\ \begin{bmatrix} kC & I \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} &= 0 \end{aligned} \tag{3}$$

or even

$$\begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} kX \\ Y \end{bmatrix} = 0 \tag{4}$$

Using the following

$$\begin{bmatrix} \lambda_1 I - A & B \end{bmatrix} \begin{bmatrix} X_1 n_1 \\ Y_1 n_1 \end{bmatrix} = 0 \tag{5}$$

the following needs to be satisfied

$$\begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} kX_1 n_1 \\ Y_1 n_1 \end{bmatrix} = 0 \tag{6}$$