The following can be stated for the closed-loop system

$$(\lambda_1 I - (A - BK))R_1 = 0 \tag{1}$$

where R_1 is the eigenvector for eigenvalue λ_1. Let K be given as

$$K = -MR^{-1} \tag{2}$$

then

$$\begin{aligned}(\lambda_1 I - (A - BM_1R_1^{-1}))R_1 &= 0\\ \lambda_1 R_1 - AR_1 + BM_1 &= 0\\ (\lambda_1 I - A)R_1 + BM_1 &= 0\end{aligned} \tag{3}$$

Let $R_1 \triangleq X_1 n_1$ and $M_1 \triangleq Y_1 n_1$,

$$\begin{aligned}(\lambda_1 I - A)X_1 n_1 + BY_1 n_1 &= 0\\ \begin{bmatrix}\lambda_1 I - A & B\end{bmatrix}\begin{bmatrix}X_1 n_1\\ Y_1 n_1\end{bmatrix} &= 0\end{aligned} \tag{4}$$

where n_1 is the linear combination vector. Choosing

$$X_1 n_1 = R_{des}(1) \tag{5}$$

hence,

$$n_1 = X_1^{-1} R_{des}(1) \tag{6}$$

and $\mathcal{N}$ is the null space in

$$\begin{bmatrix}\lambda_1 I - A & B\end{bmatrix}\mathcal{N} = 0 \tag{7}$$

The algorithm is as follows:

1. Calculate the null space($\mathcal{N}$) of

$$\begin{bmatrix}\lambda_1 I - A & B\end{bmatrix} \tag{8}$$

2. Partition the null space matrix $\mathcal{N}$ as

$$\mathcal{N} = \begin{bmatrix}X_1 n_1\\ Y_1 n_1\end{bmatrix} \tag{9}$$

3. Calculate $n_1 = X_1^{-1} R_{des}(1)$ and set $M_1 = Y_1 n_1$ and $R_1 = X_1 n_1$.

4. Repeat steps for each eigenvalue.

5. Determine controller gain $K = -MR^{-1}$.

Since R can be used to diagonalize the closed-loop matrix

$$A + BK = R \Lambda R^{-1} \tag{10}$$

this fact can be used to evaluate the following

$$e^{(A+BK)t} = R e^{\Lambda t} R^{-1} \tag{11}$$

where R and Λ are closed-loop parameters and are known. Thereby,

$$x(t) = R e^{\Lambda t} R^{-1} x_0 \tag{12}$$

Defining $z = Rx$ and using the relation $x = R^{-1} z$ gives,

$$\begin{aligned} x(t) &= R e^{\Lambda t} R^{-1} x_0 \\ R^{-1} x(t) &= e^{\Lambda t} R^{-1} x_0 \\ z(t) &= e^{\Lambda t} z_0 \end{aligned} \tag{13}$$

and the control law is updated as

$$\begin{aligned} u &= Kx \\ u &= KR^{-1} z \end{aligned} \tag{14}$$