

A dynamical system is given as

$$\begin{cases} \dot{x} = Ax + Bu \\ x(0) = x_0 \end{cases} \quad (1)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x \in \mathbb{R}^{n \times 1}$ and $u \in \mathbb{R}^{m \times 1}$. The control signal is constrained with

$$g_i \leq u \leq h_i \quad i = 1, 2, \dots, m \quad (2)$$

Let's assume the system matrices as follows

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}, \quad (3)$$

and hence, $n = 2$ and $m = 1$. Then,

$$g \leq u \leq h \quad (4)$$

and

$$L = \begin{bmatrix} l_{11} \\ l_{21} \end{bmatrix} \quad (5)$$

can be stated. The closed-loop matrix is obtained as,

$$A + BL^T = \begin{bmatrix} a_{11} + b_{11}l_{11} & a_{12} + b_{11}l_{21} \\ a_{21} + b_{21}l_{11} & a_{22} + b_{21}l_{21} \end{bmatrix} \quad (6)$$

$E(L)$ is defined as follows

$$\begin{aligned} E(L) &= \{z | z \in \mathbb{R}^2 \text{ and } g \leq l_i^T z \leq h\} \\ &= g \leq l_{11}z_{11} + l_{21}z_{21} \leq h \end{aligned} \quad (7)$$

$F(L)$ is defined as follows

$$F(L) = \bigcap_{t \in [0, \infty]} \{(e^{A_c t})^{-1} E(L)\} \quad (8)$$

where $F(L)$ is a subset of $E(L)$.