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Abstract

This work proposes a redesign method for nonlinear observers to reduce the effect of a particular class of output disturbances. Specifically, it is considered a disturbance composed by a term generated from a known system and an unstructured term. The proposed approach does not require modifying the original observer and is based on adding a simple filter that includes an internal model of the disturbance generator. Sufficient conditions for stability of the proposed filter-observer architecture are given. Moreover, the approach is validated through numerical simulations.

1. Introduction

Many control and monitoring problems require an estimate of the state variables from the available measurements. In such scenarios, it is common to design and implement an observer [1]. One of the major limiting factors of the performances of an observer is the presence of sensor noise [2], which yields a well-known trade-off between convergence rate and noise attenuation.

The effect of noise can be attenuated through a suitable tuning of the observer parameters as, for instance, in the Kalman filter approach [3], or by combining a minimization of the H_∞/L_2 input-output gain and the bounded real lemma [4]. Alternatively, one can couple the observer with a system that filters-out some spectral components of the output error signal. An example of this methodology is the Proportional-Integral observer [5–7], which combines the output-estimation error with its integral, see also the extensions [8, 9] to more general low-pass filters. Recently, [10] proposed the use of nonlinear dynamic dead-zones or dynamic saturations.

The mentioned approaches only achieve a reduction of the effect of output disturbances. A stronger result can be obtained if there is a known generating model of the output disturbance [11]. In such case, the generating model can be included in the observer in order to exactly cancel the disturbance. In the presence of a generating model, here referred to as the *exosystem*, it is common to consider an extended system composed by the original system and the exosystem, and to design an observer that jointly estimates the states of the original system and exosystem [11]. However, this approach is not easily extendable to nonlinear systems, where the existence of an observer for the original system does not guarantee that of an observer for the extended system. With this in mind, the objective of this work is to interconnect an existing observer with a system that perfectly cancels the output disturbance without requiring any redesign of the original observer.

There exists a large literature on design methodologies to compensate disturbances with a known generating model affecting the measured output. Some remarkable examples are [12–14], where adaptive controllers are proposed to solve a stabilization problem in the presence of harmonic disturbances. Alternatively, [15] presents an adaptive observer to estimate the output disturbance of a linear system. Nonetheless, to the best of the authors' knowledge, this work considers a different problem that has not been previously studied in the literature. That is, the case where a (possibly nonlinear) observer is already given and the objective is to compensate a particular disturbance without modifying the observer structure or gain tuning. Such an extra constraint is motivated by possible applications that go beyond the mere scope of observers design, such as distributed multi-agent systems [16] or cyber-security problems [17], in which completely redesigning the given feedback controller could be critical or not allowed due to practical constraints.

More precisely, this work proposes an alternative route to embed the exosystem in the observer with a modular approach. First, we assume that an observer for the system in nominal conditions, i.e. in the absence of the measurement disturbance, is known. Second, the observer is

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connected in cascade with a filter that includes the generating model of the disturbances. The design of the filter follows from an internal model based philosophy [18] in order to exactly cancel the output disturbance. We consider nonlinear plants and observers [1] satisfying an incremental passivity assumption [19]. Additionally, this work relies on the property of incremental input-to-state stability (δISS) [20]. A definition of these properties is given at the end of this section.

Notation: $|x|$ denotes the Euclidean norm; I_k denotes the identity matrix of dimension k; $\sup_{s\in[0,t]}|u(s)|$ denotes the supremum norm of $u(s)$ in the interval $[0,t]$; $\mathrm{rank}(A)$ and $\sigma(A)$ denote the rank and the spectrum of matrix A, respectively. By $\mathrm{blckdiag}(\cdot)$ we denote block-diagonal concatenations. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_{\geq 0} := [0,\infty)$ $\mathbb{C}$ denotes the of complex numbers, $\mathbb{C}^- := \{s \in \mathbb{C} : \Re(s) \leq 0\}$ $\mathbb{C}^+ := \{s \in \mathbb{C} : \Re(s) \geq 0\}$, and $\star$ denotes the off-diagonal elements of a symmetric matrix. For the definition and properties of class $\mathcal{K}$ and $\mathcal{KL}$ functions, we refer to [21].

Consider a system of the form

$$\dot{x} = f(x,d), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $d \in D \subset \mathbb{R}^{n_u}$ is a known measurable and locally essentially bounded signal taking values in a compact set D, and f is locally Lipschitz in its first argument. We have the following definition [20].

Definition 1 (Incremental input-to-state stability)**.** *System* (1) *is δISS with respect to the input d if there exist $\beta \in \mathcal{KL}$ and $\rho \in \mathcal{K}$ such that, for every two solutions x and x' to* (1) *subject to inputs d and d', the following holds for all $t \geq 0$*

$$|x(t)-x'(t)| \leq \beta(|x(0)-x'(0)|,t) + \sup_{s\in[0,t)} \rho\big(|d(s)-d'(s)|\big).$$

Consider now the system

$$\begin{aligned} \dot{x} &= f(x) + Bu \\ y &= Cx, \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $y \in \mathbb{R}^{n_y}$ is the measured output, and $u \in \mathcal{U} \subset \mathbb{R}^{n_u}$ is a bounded input. The function f is sufficiently smooth in its argument. We have the following definition [19].

Definition 2 (Incremental passivity)**.** *System* (2) *is incrementally passive from u to y if there exists a C^1 storage function $S(x,x')$ such that for every pair of inputs $u,u' \in U$ and any pair (x,x') of solutions of* (2) *corresponding to these inputs, with corresponding outputs y,y', the following is satisfied*

$$\dot{S} \leq (y-y')^\top (u-u').$$

For systems of the form (2), there exists a sufficient condition to verify incremental passivity in the form of a matrix inequality [19, Lemma 3]. Specifically, if the following holds for some symmetric positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and for all $x \in \mathbb{R}^{n_x}$

$$\begin{aligned} &P\frac{\partial f}{\partial x}(x) + \frac{\partial f}{\partial x}(x)^\top P \preceq 0 \\ &PB = C^\top, \end{aligned}$$

then, the system is incrementally passive from u to y with a storage function $S = (x-x')^\top P(x-x')$.

2. Problem Formulation

2.1. Framework

This work considers the multi-output nonlinear system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= Cx + d, \end{aligned} \tag{3}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is a known control input, $y \in \mathbb{R}^{n_y}$ is the measured output, $d \in \mathbb{R}^{n_y}$ is an unknown output disturbance, and f is a locally Lipschitz function in its first argument. A standing assumption in this work is that the states of the system and the disturbances are bounded.

Assumption 1. *The disturbance d and the control input u are Lebesgue measurable and bounded. In particular, $d(t) \in D$ and $u(t) \in U$ for all $t \geq 0$, for some compact sets $D \subset \mathbb{R}^{n_y}$ and $U \subset \mathbb{R}^{n_u}$.*

Assumption 2. *There exists a compact set $X_0 \subset \mathbb{R}^{n_x}$ that is forward invariant for system* (3) *for every input u satisfying Assumption 1.*

In this work, we restrict ourselves to initial conditions of the system (3) in $X_0 \subset \mathbb{R}^{n_x}$.

To ease certain parts of the presentation, some conditions will be particularized to the case of linear systems. In such a context, system (3) takes the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + d, \end{aligned} \tag{4}$$

for some matrices A, B, C of suitable dimension.

We suppose that we are given an observer of the form

$$\dot{\hat{x}} = f(\hat{x},u) + \kappa(\hat{x}, y - C\hat{x}), \tag{5}$$

in which $\hat{x} \in \mathbb{R}^{n_x}$ is the estimation of the state, and κ denotes the output injection term, satisfying $\kappa(\hat{x},0) = 0$ for all $\hat{x} \in \mathbb{R}^{n_x}$. Under Assumptions 1 and 2, we assume that the observer ensures global exponential stability of the error variable $\hat{x} - x$, uniformly in u, and satisfies an input-to-state stability property with respect to the output disturbance d, namely the following holds

$$|x(t) - \hat{x}(t)| \leq ke^{-\lambda t}|x(0) - \hat{x}(0)| + \sup_{s\in[0,t]} \rho(|d(s)|)$$

for all $t \geq 0$, for every pair of initial conditions $(x(0), \hat{x}(0)) \in X_0 \times \mathbb{R}^{n_x}$, for some class $\mathcal{K}$ function ρ, and for some constants $k, \lambda > 0$. Typical examples of observers that fit such a framework are the high-gain observer [22, 23], observers derived from LMIs or circle-criterion approaches [24, 25] and observers derived from a dissipativity condition [26]. See also [1, Section 4] for more references.

We assume that the plant's dynamics and the observer dynamics are expressed in the same set of coordinates. However, this is not always the case, as often the observer dynamics lives in a space of larger dimension. If this is the case, one can suppose that the plant's dynamics are expressed in the same coordinates of the observer and then develop the following result in such a coordinates. See, e.g., [1, Section 8].

We suppose that the output disturbance d is modeled as the sum of a signal composed by a finite sum of sinusoids and a term v representing unstructured bounded noise, namely

$$d_j(t) = \sum_{i=1}^{N_d} d_{j,i} \sin(\omega_i t + \varphi_{j,i}) + v_j(t), \qquad j = 1, \ldots, n_y,$$

where d_j and v_j denote the j-th components of d and v, respectively. The disturbance can be therefore assumed to be generated by an exosystem of the form

$$\dot{w} = \Phi w, \qquad d = \Gamma^\top w + v. \tag{6}$$

where $w \in \mathbb{R}^{n_w}$ is the exosystem state, the matrix Φ is such that $\sigma(\Phi) = \{\pm j\omega_i \,:\, i = 1, \ldots, N_d\}$, and the pair $(\Phi, \Gamma^\top)$ is observable. Since the particular representation of Φ, Γ does not play any role for the forthcoming results, without loss of generality we can assume that Φ and Γ have the following form

$$\Phi = \mathrm{blkdiag}(\underbrace{\phi, \ldots, \phi}_{n_y \text{ times}}), \quad \Gamma = \mathrm{blkdiag}(\underbrace{\gamma, \ldots, \gamma}_{n_y \text{ times}}), \tag{7}$$

with ϕ, γ such that

$$\begin{aligned} \phi &= \mathrm{blkdiag}(\phi_1, \ldots, \phi_{N_d}), & \phi_i &= \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}, \\ \gamma &= \begin{bmatrix} g^\top & \ldots & g^\top \end{bmatrix}^\top, & g &= \begin{bmatrix} 0 & 1 \end{bmatrix}. \end{aligned}$$

Note that if one considers $\omega_i = 0$, then, the corresponding block dimension is 1, and in this case $\phi_i = 0, g = 1$.

The objective of this article is to augment the observer (5) with a disturbance compensator to guarantee that the state estimation error converges asymptotically to a bound that only depends on the unstructured term of the noise, namely, for some class $\mathcal{K}$ function ρ, we have

$$\limsup_{t\to\infty} |x(t) - \hat{x}(t)| \leq \sup_{s\in[0,\infty)} \rho(|v(s)|). \tag{8}$$

As anticipated in the introduction, a direct method to solve this problem would be to consider an extended system that includes the disturbance dynamics

$$\begin{aligned} \dot{x} &= f(x, u) \\ \dot{w} &= \Phi w \\ y &= Cx + \Gamma^\top w + v, \end{aligned} \tag{9}$$

and design an observer for the resulting system (9). Although this is a perfectly valid approach for linear systems, it cannot be easily extended to the nonlinear case. First, the existence of an observer for the original system (3) does not guarantee, in general, the existence of an observer for the extended dynamics (9). Indeed, nonlinear observer design commonly relies on structural assumptions that may be lost in the extended dynamics (9). Therefore, even if it is possible to implement a certain nonlinear observer technique in the original system, it is not guaranteed that the same technique can be implemented for the extended system. Second, this approach does not take particular advantage of the previous existence of an observer. To overcome these limitations, this work proposes a solution following a modular perspective. Specifically, we follow a strategy based on an internal model approach [18]. The main idea is to connect in cascade the already designed observer with a filter that includes an internal model of the disturbance generator. Similar filters can be found in previous literature of control and signal processing. Some remarkable examples are the "notch filter" [27] or the "washout filter" in the context of pre-processing output regulation [28]. Nonetheless, to the best of our knowledge, these results have not been implemented in the observer case. It should be remarked that this work follows a modular philosophy. That is, we assume that the already designed observer (5), and in particular the structure of its gain κ cannot be modified.

2.2. Filter design

The proposed filter is designed as

$$\begin{aligned} \dot{\eta} &= \Psi\eta + \Gamma(y - C\hat{x}) \\ z &= (y - C\hat{x}) - \Gamma^\top \eta \end{aligned} \tag{10}$$

where $\eta(t) \in \mathbb{R}^{n_\eta}$ is the state of the filter, partitioned as $\eta = (\eta_1, \ldots, \eta_{n_y})$, with $\eta_i(t) \in \mathbb{R}^{2N_d}$, $z(t) \in \mathbb{R}^{n_y}$ is the filter's output and the matrix Ψ is selected as

$$\Psi = \Phi - \Gamma\Gamma^\top = \mathrm{blkdiag}(\underbrace{\phi - \gamma\gamma^\top, \ldots, \phi - \gamma\gamma^\top}_{n_y \text{ times}}). \tag{11}$$

The proposed internal model filter can be understood as a "bank" of identical filters, each one acting on a different output component. Specifically, consider the decomposition $y = (y_1, y_2, \ldots, y_{n_y})$, where $y_i \in \mathbb{R}$ for $i = 1, \ldots, n_y$. Then, (10) reads as

$$\begin{aligned} \dot{\eta}_i &= (\phi - \gamma\gamma^\top)\eta_i + \gamma(y_i - \hat{y}_i) \\ z_i &= (y_i - \hat{y}_i) - \gamma^\top \eta_i, \end{aligned} \qquad \forall i = 1, \ldots, n_y,$$

where $\hat{y}_i$ is the estimation of the ith component of the output produced by the observer (5).

Note that, by construction, the matrix Ψ is Hurwitz. The latter is a direct consequence of the fact that ϕ is skew-symmetric and (Φ, Γ) is controllable. To interconnect the filter (10) and the observer (5), we substitute in the output injection term of the observer (5) the error signal $\tilde{y}$ with the filtered output z. This gives the modified observer

$$\dot{\hat{x}} = f(\hat{x}, u) + \kappa(\hat{x}, z). \tag{12}$$

To better understand the motivation of the internal model filter (10), notice that such a filter is a relative degree zero linear system that has the zeros placed at the eigenvalues of Φ. Intuitively, the presence of a zero in the filter implies that the signal z cannot have any spectral component at such a frequency. Therefore, as the zeros of the filter are placed at the eigenvalues of Φ, the modelled part of the output disturbance d does not have any effect on the steady-state estimation. Hence, the desired disturbance rejection problem is solved if the overall observer (10),(12) still possesses good convergence properties. This result is formalized in the next sections.

3. Asymptotic Analysis

The overall filtered observer (10), (12) reads

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, u) + \kappa(\hat{x}, Cx - C\hat{x} - \Gamma^\top \eta + d) \\ \dot{\eta} &= \Psi\eta + \Gamma(Cx - C\hat{x} + d). \end{aligned} \tag{13}$$

The objective of this section is to prove that, under an additional assumption given below, the observer (13) rejects the disturbance generated by the exosystem (6).

Assumption 3. *Under Assumptions 1 and 2, system* (13) *is δISS with respect to the disturbance d, uniformly in u and x. Namely, there exist $\beta \in \mathcal{KL}$ and $\rho \in \mathcal{K}$ such that, for every input u and every two inputs d and d' satisfying Assumption 1, every solution x to* (3) *originating in X_0 and corresponding to u, and every two solutions $(\hat{x}, \eta)$ and $(\hat{x}', \eta')$ of* (13) *corresponding to (u, x, d) and (u, x, d'), respectively, the following holds*

$$\begin{aligned} &|(\hat{x}(t), \eta(t)) - (\hat{x}'(t), \eta'(t))| \leq \\ &\qquad \beta(|(\hat{x}(0), \eta(0)) - (\hat{x}'(0), \eta'(0))|, t) \\ &\qquad\qquad + \sup_{s\in[0,t)} \rho\big(|d(s) - d'(s)|\big) \end{aligned} \tag{14}$$

for all $t \geq 0$.

Assumption 3 is a strong incremental stability condition requiring that the gains of the filter (10) have been selected so as to strengthen the ISS properties of the original observer (5) to a stricter δISS property (see Definition 1) with respect to the disturbance d. While Assumption 3 may not be satisfied for all filter-observer interconnections [20], Section 4 provides sufficient and constructive conditions to design a filter such that Assumption 3 holds.

We now present the main result of the article, showing that the solutions to the filtered observer (13) satisfy the asymptotic bound property (8).

Theorem 1. *Consider system* (3) *and the filtered observer* (13) *subject to the input d satisfying* (6)*. Suppose that Assumptions 1-3 hold. Then, every solution of* (3)*,* (6)*,* (13) *with x originating in X_0 satisfies* (8)*, in which ρ is the same function for which Assumption 3 holds.*

Proof. System (3), (6), (13) reads as

$$\begin{aligned} \dot{w} &= \Phi w \\ \dot{x} &= f(x, u) \\ \dot{\hat{x}} &= f(\hat{x}, u) + \kappa(\hat{x}, Cx - C\hat{x} - \Gamma^\top\eta + \Gamma^\top w + v) \\ \dot{\eta} &= \Psi\eta + \Gamma(Cx - C\hat{x} + \Gamma^\top w + v). \end{aligned} \tag{15}$$

Pick arbitrarily a solution pair $((w, x, \hat{x}, \eta), v)$ to (15), and consider the signals

$$\hat{x}_{ss}(\cdot) := x(\cdot), \quad \eta_{ss}(\cdot) := w(\cdot). \tag{16}$$

Simple computations show that $((w, x, \hat{x}_{ss}, \eta_{ss}), 0)$ (namely, obtained for the same (w, x) and with $v = 0$) is a solution pair of (15) as well. Indeed for such a choice one obtains $\kappa(t, \hat{x}_{ss}, 0) = 0$ in the $\hat{x}_{ss}$-dynamics, that is $\dot{\hat{x}}_{ss} = f(\hat{x}_{ss}, u)$, while the η_{ss}-dynamics reads $\dot{\eta}_{ss} = \Phi\eta_{ss}$, due to the definition of Ψ in (11). In other words, (16) is a solution to (15) for $v = 0$. As a consequence, direct application of the inequality (14) in which $\hat{x}' = \hat{x}_{ss} = x$, $\eta' = \eta_{ss} = w$, $d = \Gamma^\top w + v$ and $d' = \Gamma^\top w$ yields

$$\begin{aligned} &|(\hat{x}(t), \eta(t)) - (x(t), w(t))| \leq \\ &\qquad \beta(|(\hat{x}(0), \eta(0)) - (x(0), w(0))|, t) + \sup_{s\in[0,t)} \rho\big(|v(s)|\big), \end{aligned}$$

from which (8) directly follows. □

To better understand the ideas underlying the proposed design, we now particularize the result to the linear case. In particular, we consider a linear system of the form (4). In this case, the observer (5) reads

$$\dot{\hat{x}} = A\hat{x} + Bu(t) + L(y - C\hat{x}) \tag{17}$$

where $L \in \mathbb{R}^{n_x \times n_y}$ is designed so that $(A - LC)$ is Hurwitz. In this linear context, the closed-loop system (13) reads

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu(t) + L(y - C\hat{x} + d) - L\Gamma^\top\eta \\ \dot{\eta} &= \Psi\eta + \Gamma(y - C\hat{x} + d). \end{aligned} \tag{18}$$

Note that due to linearity, a δISS property is equivalent to asymptotic stability. Thus, in this context, the result of Theorem 2 reads as follows.

Corollary 1. *Consider the linear system* (4) *and the filtered observer* (18)*, and suppose that the following matrix*

$$A_{\mathrm{cl}} = \begin{bmatrix} A - LC & L\Gamma^\top \\ \Gamma C & \Psi \end{bmatrix}$$

is Hurwitz. Then, every solution of (3)*,* (6)*,* (18) *satisfies* (8)*.*

The proof is omitted for reasons of space. Note that for linear systems we do not need the boundedness conditions in Assumptions 1 and 2.

4. Sufficient passivity-based stability conditions

The previous asymptotic analysis is based on the assumption that the filter-observer architecture satisfies the δISS property of Assumption 3. The aim of this section is to develop constructive conditions for filter design such that the bound in Assumption 3 is satisfied.

Note that it is not easy to study the interconnection (10), (12) and standard small-gain arguments would in general fail to work, because the H_∞ gain of the filter (10) can not be tuned, and it is equal to 1. For instance, in the context of linear systems the small-gain condition would require the matrix A in (4) to be Hurwitz[1]. As a consequence, we rely in this section on passivity arguments. To do so, it is convenient to include an additional set of assumptions. It should be remarked that the next assumptions are sufficient to obtain a constructive methodology, but are not necessary for the filter-observer stability.

4.1. Sufficient conditions

In the first assumption we provide sufficient condition for the observer (12) to be incremental passivity from z to $\hat{y}$ as presented in Definition 2.

Assumption 4. *There exists a pair of symmetric positive definite matrices $P, \bar{P} \in \mathbb{R}^{n \times n}$ such that for some $\bar{q}, \ell > 0$, system* (3) *satisfies*

$$P\frac{\partial f}{\partial x}(x,u) + \frac{\partial f}{\partial x}(x,u)^\top P \preceq 0, \tag{19}$$

$$\bar{P}\frac{\partial f}{\partial x}(x,u) + \frac{\partial f}{\partial x}(x,u)^\top \bar{P} - \ell C^\top C \preceq -\bar{q}\bar{P}, \tag{20}$$

for all $x \in \mathbb{R}^{n_x}$, and all $u \in U$.

Inequality (19) implies that the plant dynamics (3) is incrementally dissipative according to the notion introduced in [19]. Additionally, (19) is a sufficient condition to satisfy the boundedness requirement in Assumption 2, as shown in the next subsection.

[1]Note that, in this case, the overall goal could be simply solved by taking $L = 0$ as output injection gain so that to obtain an exponentially stable observer not affected by any output disturbance.

Remark 1. *In the linear case, a necessary and sufficient condition for* (19) *is that $\sigma(A) \subset \mathbb{C}^-$.*

Now consider that the output injection gain κ is selected as

$$\kappa(\hat{x}, y - C\hat{x}) = \ell P^{-1} C^\top (y - C\hat{x}). \tag{21}$$

Then, combining (21) with inequality (19), we can show that the dynamics in (12) are incrementally passive (see Definition 2) from the input z to the output y with the constant metric P. In order to ease the presentation, we assume $\ell = 1$ in the rest of the section.

In parallel, the inequality (20) is a differential detectability condition (with constant metric) as developed for instance in [29, 30], see also [1, Section 4]. Inequality (20) combined with a LaSalle-type argument (or [31]) is sufficient to show that observer (5), with the output injection gain κ selected as in (21), is a convergent observer.

We stress that Assumption 4 provides a sufficient condition to develop the next stability analysis, but it is not necessary as shown by a counterexample in Section 5.

Assumption 5. *The extended system*

$$\begin{aligned} \dot{\xi} &= F(\xi, u) = \begin{bmatrix} f(x,u) \\ \Phi\eta \end{bmatrix}, \quad \xi = \begin{bmatrix} x \\ \eta \end{bmatrix} \\ \zeta &= H\xi = \begin{bmatrix} C & \Gamma^\top \end{bmatrix} \xi, \end{aligned} \tag{22}$$

with state ξ and output ζ, is differentially detectable, namely there exists a symmetric positive definite matrix $Q = Q^\top > 0$ and $\mu, q > 0$ such that

$$Q\frac{\partial F}{\partial \xi}(\xi,u) + \frac{\partial F}{\partial \xi}(\xi,u)^\top Q - 2\mu H^\top H \preceq -qI \tag{23}$$

for all $\xi \in \mathbb{R}^{n_x + n_\eta}$ and all $u \in U$.

Assumption 5 imposes a minimal observability condition on the extended dynamics (9). Clearly, such a condition is necessary to distinguish between the state trajectories and the output disturbances; the objective of the system (10) is to filter the effect of the disturbance d without losing useful information from y needed to recover the estimate of the full state x. We stress that the knowledge of the matrix Q is needed only for analysis purposes but not for design.

To better understand the implications of Assumption 5, it is convenient to particularize it in the linear case scenario, i.e., for systems of the form (4). Indeed, in the linear case, Assumption 5 implies that the unstable modes of A and Φ are observable from y. Precisely, based on the results in [32] and standard Hautus (PBH) test, and the fact that the pair $(\Phi, \Gamma^\top)$ is observable, it can be deduced that the extended system (22) system is detectable if

$$\operatorname{rank} \begin{bmatrix} A - sI & 0 \\ 0 & \Phi - sI \\ C & \Gamma^\top \end{bmatrix} = n_x + n_\eta, \quad \forall s \in \mathbb{C}^+. \tag{24}$$

Note that equation (24) reduces to the following disjoint spectrum condition $\sigma(A) \cap \sigma(\Phi) = \emptyset$, which is similar to the non-resonance condition of other internal model based designs [18].

As a last remark, it is worth noticing that different notions of non-resonance condition (although very similar from a conceptual point of view and completely equivalent in the linear case) have been proposed in [28, Assumption 5], [33, Proposition 3] or [34].

4.2. Checking Assumptions 4 and 5

Notice that Assumptions 4 and 5 require solving a set of matrix inequalities that depend on the state x and input u. That is, they require solving an infinite set of LMIs. Nonetheless, we can obtain a finite set of LMIs through convex relaxation, see e.g. [35]. Specifically, let $\mathcal{A} := \{A_1, \dots, A_N\}$ be a family of matrices in $\mathbb{R}^{n_x}$ such that $\frac{\partial f}{\partial x}(x,u) \in \text{ConvexHull}(\mathcal{A})$ for all (x,u). Then, at each (x,u) we have that $\dfrac{\partial f}{\partial x}(x,u) = \sum_{i=1}^N \varrho_i(x,u)A_i$, for some $\varrho_i(x,u)$ such that $\sum_{i=1}^N \varrho_i(x,u) = 1$ for all (x,u). With this in mind, inequality (19) reads as

$$\sum_{i=1}^{N} \varrho_i(x,u)\left(PA_i + A_i^\top P\right) \preceq 0.$$

Consequently, any symmetric positive definite matrix P that is a solution of

$$PA_i + A_i^\top P \preceq 0, \quad i = 1, \dots, N$$

is also a solution of (19). Then, (20) and Assumption 5 can be analyzed similarly.

4.3. Stability Analysis

The main result of this section is formalized as follows.

Theorem 2. *Let Assumptions 1, 4 and 5 hold, and the observer feedback term being* (21). *Then, Assumptions 2 and 3 are satisfied.*

Proof. First, recall the following identity

$$g(1) - g(0) = \int_0^1 \frac{\partial g}{\partial s}(s)ds$$

valid for any C^1 function $g : \mathbb{R} \to \mathbb{R}$. Hence, by denoting $g(s) := f(\hat{x} + (s-1)\tilde{x}, u)$, where $\tilde{x} := x - \hat{x}$, we get

$$f(\hat{x},u) - f(\hat{x}-\tilde{x},u) = \left(\int_0^1 \frac{\partial f}{\partial x}(\hat{x}+(s-1)\tilde{x},u)ds\right)\tilde{x} \quad (25)$$

for all $\hat{x}, \tilde{x} \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$. Similar derivations will be used all throughout the proof. We start by showing that Assumption 2 holds. To this end, consider the Lyapunov function $V = x^\top P x$ with P given by (19). Differentiating along solutions and using (25) we obtain

$$\begin{aligned} \dot{V} &= 2x^\top P f(x,u) = 2x^\top P\left(\int_0^1 \frac{\partial f}{\partial x}(sx,u)ds\right)x \\ &\le x^\top \left[\int_0^1 \left(P\frac{\partial f}{\partial x}(sx,u) + \frac{\partial f}{\partial x}(sx,u)^\top P\right)ds\right]x \quad \le 0\,, \end{aligned}$$

where the last inequality comes from (19). We conclude that the set $X_0 = \{x^\top P x \le c\}$, $c = \sup_{x \in X_0} |x^\top P x|$ is forward invariant.

Next, consider any two solution pairs $(\hat{x}, \eta)$ and $(\hat{x}', \eta')$ to system (13) with the output injection gain (21), denote

$$e := \begin{bmatrix} \tilde{x} \\ \tilde{\eta} \end{bmatrix} = \begin{bmatrix} \hat{x} - \hat{x}' \\ \eta - \eta' \end{bmatrix}, \quad \delta := d - d',$$

and define $\hat{\xi} = \begin{bmatrix} \hat{x} \\ \eta \end{bmatrix}$. The e-dynamics evolves according to

$$\begin{aligned} \dot{\tilde{x}} &= \psi(\hat{x}, \tilde{x}, u) - LC\tilde{x} - L\Gamma^\top \tilde{\eta} + L\delta \\ \dot{\tilde{\eta}} &= (\Phi - \Gamma\Gamma^\top)\tilde{\eta} - \Gamma C\tilde{x} + \Gamma\delta \end{aligned} \quad (26)$$

with $L = P^{-1}C^\top$ and $\psi(\hat{x}, \tilde{x}, u) := f(\hat{x}, u) - f(\hat{x} - \tilde{x}, u)$, or, in compact notation,

$$\dot{e} = \Theta(\hat{\xi}, e, u) - KHe + K\delta \quad (27)$$

with

$$H := \begin{bmatrix} C & \Gamma^\top \end{bmatrix}, \qquad K := \begin{bmatrix} L \\ \Gamma \end{bmatrix},$$

$$\Theta(\hat{\xi}, e, u) := F(\hat{\xi}, u) - F(\hat{\xi} - e, u), \qquad F(\xi, u) := \begin{bmatrix} f(x,u) \\ \Phi\eta \end{bmatrix}.$$

Consider the Lyapunov function

$$V = \frac{1}{2}\tilde{x}^\top P \tilde{x} + \frac{1}{2}\tilde{\eta}^\top \tilde{\eta}. \quad (28)$$

The derivative of the first term $V_x = \frac{1}{2}\tilde{x}^\top P \tilde{x}$ along the solutions to (26) gives

$$\dot{V}_x = \tilde{x}^\top P\big(\psi(\hat{x}, \tilde{x}, u) - LC\tilde{x} - L\Gamma^\top\tilde{\eta} + L\delta\big).$$

Using Assumption 4, we have the following property

$$\tilde{x}^\top P \psi(\hat{x}, \tilde{x}, u) \le 0. \quad (29)$$

Indeed,

$$\begin{aligned} &2\tilde{x}^\top P[f(\hat{x},u) - f(\hat{x}-\tilde{x},u)] \\ &\quad = 2\tilde{x}^\top P\left(\int_0^1 \frac{\partial f}{\partial x}(\hat{x} + (s-1)\tilde{x}, u)ds\right)\tilde{x} \\ &\quad = \tilde{x}^\top \left[\int_0^1 \left(P\frac{\partial f}{\partial x}((\tilde{x}', s), u) + \frac{\partial f}{\partial x}^\top((\tilde{x}', s), u)P\right)ds\right]\tilde{x}, \end{aligned}$$

where in the second step we used the identity (25) and in the third step we used the compact notation $(\tilde{x}', s)$ for $\tilde{x}' = \hat{x} + (s-1)\tilde{x}$. Thus, (29) follows from (19). Then, since in view of (21), $PL = C^\top$, the derivative of V_x satisfies

$$\dot{V}_x \le -\tilde{x}^\top C^\top (C\tilde{x} + \Gamma^\top\tilde{\eta} - \delta). \quad (30)$$

Next, we compute the derivative of the second term $V_\eta = \frac{1}{2}\tilde{\eta}^\top\tilde{\eta}$. Recalling the skew-symmetric properties of the matrix Φ, we have

$$\begin{aligned} \dot{V}_\eta &= \tilde{\eta}^\top[(\Phi\tilde{\eta} - \Gamma\Gamma^\top)\tilde{\eta} - \Gamma C\tilde{x} + \Gamma\delta] \\ &= -\tilde{\eta}^\top\Gamma(\Gamma^\top\tilde{\eta} + C\tilde{x} - \delta). \end{aligned} \quad (31)$$

Combining inequalities (30) and (31), we finally obtain

$$\dot V = \dot V_x + \dot V_\eta \le -(\tilde\eta^\top\Gamma + \tilde x^\top C^\top)(\Gamma^\top\tilde\eta + C\tilde x - \delta) \\ \le -|He|^2 + |He|\,|\delta|\,. \tag{32}$$

The next steps consists on "strictifying" the Lyapunov function V in order to derive the incremental input-to-state stability condition. To derive a strict Lyapunov function, we follow the observer-based approach proposed in [31]. Specifically, consider the Lyapunov function $U(e) = e^\top Qe$ together with Assumption 5 and (23). Then, the derivative of $U(e)$ along the solutions of (27) gives

$$\begin{aligned}\dot U &= 2e^\top Q(\Theta(\hat\xi, e, u) - KHe + K\delta) \\ &= 2e^\top Q(\Theta(\hat\xi, e, u) - \mu Q^{-1}He) \\ &\qquad + 2e^\top Q(\mu Q^{-1}He - KHe + K\delta).\end{aligned}$$

Recalling the definition of Θ, the first term becomes

$$\begin{aligned}&2e^\top Q\big[F(\hat\xi, u) - F(\hat\xi - e, u) - \mu Q^{-1}He\big] \\ &= 2e^\top\left(\int_0^1 Q\frac{\partial F}{\partial\xi}(\hat\xi + (s-1)e, u)dse\right) - 2\mu e^\top H^\top He \\ &= e^\top\int_0^1\left(Q\frac{\partial F}{\partial\xi}((\hat\xi', s), u) + \frac{\partial F}{\partial\xi}((\hat\xi', s), u)^\top Q\right. \\ &\qquad\qquad \left. - 2\mu H^\top H\right)ds\, e,\end{aligned}$$

where in the second step we used similar derivations as those in (25), and in the third step we used the compact notation $(\hat\xi', s)$ for $\hat\xi' = \hat\xi + (s-1)e$. As a consequence, using (23) we obtain

$$2e^\top Q(\Theta(\hat\xi, e, u) - \mu Q^{-1}He) \le -q|e|^2$$

for all $(\hat\xi, e)$. Moreover, using the Young's inequality, the derivative of U is estimated as

$$\dot U \le -q|e|^2 + 2|He|(\mu + |QK|)|e| + 2|e||QK|\,|\delta|. \tag{33}$$

Finally, let $\nu > 0$ be a positive parameter to be fixed and define the Lyapunov function $W = \nu V + U$. Combining (32) with (33), the derivative of W is computed as

$$\begin{aligned}\dot W \le &- \begin{bmatrix}|e| & |He|\end{bmatrix}\begin{bmatrix}q & -\mu - |QK| \\ \star & \nu\end{bmatrix}\begin{bmatrix}|e| \\ |He|\end{bmatrix} \\ &+ |e|\big(\nu|H| + 2|QK|\big)|\delta|\end{aligned}$$

Hence, selecting ν such that $q\nu > (\mu + |QK|)^2$, the previous shows the existence of $\varepsilon, \rho_v > 0$ such that

$$\dot W \le -\varepsilon|e|^2 + \rho_v|\delta|^2.$$

Hence, W is a (uniform in the trajectories of the plant x) global exponential incremental ISS Lyapunov function [20], concluding the proof of the theorem. □

5. Numerical Simulations

5.1. Linear case

Consider a linear system of the form (4) with

$$A = \begin{bmatrix}0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 1 & 0 & -1\end{bmatrix},\ C = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 1\end{bmatrix},\ B = \begin{bmatrix}0 \\ 0 \\ 0\end{bmatrix}$$

where the measured output is corrupted with an additive disturbance generated by the exosystem (6) with $\omega_1 = 0$, $\omega_2 = 1$ and $\omega_3 = 2\pi$. Assume that the states of the system are estimated through a linear observer of the form (17) with the gain $L = \left[\begin{smallmatrix}0.994 & 0.093 \\ 0 & 0.704 \\ 0.094 & 1.534\end{smallmatrix}\right]$. In the simulations we considered the following set of initial conditions: $x(0) = (1, 1, 1)$, $\hat x(0) = 0$ and $w(0) = (1, 0.2, 0, 0.2, 0)$ and v is taken as a bounded realization of white noise of variance 0.01 in both outputs.

Naturally, the presence of the output disturbance prevents the convergence of the state-estimation error $\tilde x$ to zero. This fact can be seen in Fig. 1. The objective is to robustify the observer in front of the presented output disturbance. Note that the pair (A, C) is detectable, i.e., (20) is satisfied. Moreover, the matrix A is negative semidefinite. As a consequence, there exists a matrix P such that (19) holds. Moreover, the unstable modes of A are observable and are located at $\lambda = \pm 1.5\jmath$, which are disjointed from the unstable modes of Φ, that are located at $\{\pm 6.28\jmath, \pm 1\jmath, 0\}$, thus, Assumption 5 is also satisfied.

Therefore, according to Theorem 1 and Theorem 2, if the output-estimation error $\tilde y$ is filtered through an internal model filter (10), the state-estimation error converges asymptotically to zero. Fig. 1 presents the state-estimation error of an observer filtered with the internal model filter (10). The filter completely rejects the modeled part of the output disturbance.

5.2. Nonlinear case

Consider the ball-and-beam system studied in [36]

$$\begin{aligned}\dot x = f(x, u) &:= \begin{bmatrix} x_2 \\ \dfrac{x_1x_4^2 - g\sin x_3}{\mathcal{J}_b/(M\mathcal{R}^2) + 1} \\ x_4 \\ -\dfrac{x_1x_2x_4 + gx_2\cos x_3}{x_1^2 + \mathcal{J}/M + \mathcal{J}_b/M}\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ 0 \\ 1\end{bmatrix}u \\ y = Cx + d &:= \begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix}x + d\end{aligned} \tag{34}$$

where $x = (x_1, \ldots, x_4) \in \mathbb{R}^4$ is the state vector, $y = (y_1, y_2) \in \mathbb{R}^2$ is the measured output, d is an output disturbance, and the input $u \in \mathbb{R}$ is designed as

$$\begin{aligned}u = &\frac{2x_1x_2x_4 + gx_1\cos x_3}{x_1^2 + \mathcal{J}/M + \mathcal{J}_b/M} + \frac{24(\mathcal{J}_b/\mathcal{R}^2 + M)x_1}{Mg} \\ &+ \frac{50(\mathcal{J}_b/R^2 + M)x_2}{Mg} - 35x_3 - 10x_4 + \sin t.\end{aligned}$$

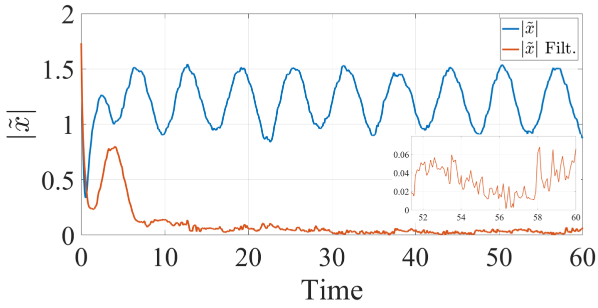

Figure 1: Evolution of the state-estimation error without (blue) and with (red) internal model filter.

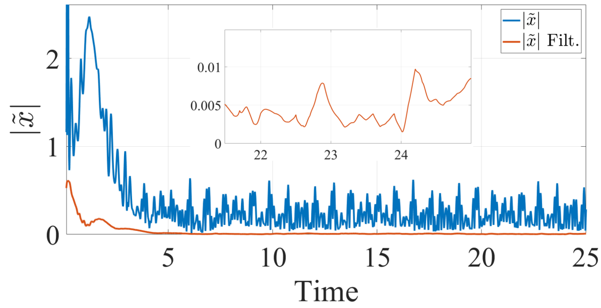

Figure 2: Evolution of the state-estimation error without (blue) and with (red) internal model filter.

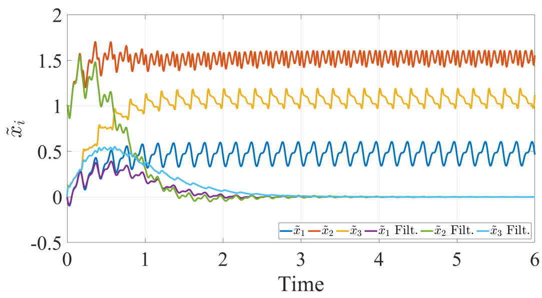

Figure 3: Evolution of the state-estimation error without (blue, red, yellow) and with (purple, green, ciano) internal model filter.

The parameters of the model are taken from [37]: $\mathcal{J} = 0.02$, $M = 0.05$, $\mathcal{J}_b = 2 \times 10^{-6}$, $\mathcal{R} = 0.01$, $g = 9.81$. According to [36], the considered model satisfies (19) with $P = \begin{bmatrix} 3.61 & 2.18 & -4.28 & -0.292 \\ 2.18 & 3.11 & -6.99 & -0.45 \\ -4.28 & -6.99 & 25.50 & 1.47 \\ -0.29 & -0.44 & 1.47 & 0.3972 \end{bmatrix}$ for $|x_1| \le 3$, $x_2 \in \mathbb{R}$, $|x_3| \le 0.65$ and $x_4 \le 0.19$. The system also satisfies (20) with the same matrix P for all $\ell > 0$. Therefore, an observer of the form (5) can be implemented with a feedback term as in (21). Now, assume that the output additive disturbance d is generated by the exosystem (6) with $\omega_k = 20\sqrt{k}$ for $k = 1, 2, 3$, initial condition $w(0) = (5, 0, 5, 0, 5, 0)$, and v is a bounded realization of a white noise of variance 0.0005 and 0.00001 in the outputs y_1 and y_2, respectively. Notice that the ball and beam system and the considered disturbance model evolve in two different time-scales. Therefore, the detectability condition in Assumption 5 is satisfied.

Due to the presence of the output disturbance d the observer does not converge to the true value. This fact can be seen in Figure 2. The objective is to redesign the proposed observer to obtain the bound in (8). Since the system satisfies Assumptions 4-5, according to Theorems 1-2 adding an internal model filter to the observer as in (12) obtains the desired bound. Indeed, the evolution of the estimation error of the filter-observer architecture in (12) is depicted in Figure 2. As it can be seen, the estimation error converges to a bound of lower magnitude. That is, the filter rejects the periodic disturbance and ensures the bound in (8), which validates the result.

5.3. Non-passive example

The objective of this example is to show that the passivity condition in Assumption 4 is only a sufficient (but not necessary) condition to ensure the δISS bound in Assumption 3. Precisely, consider the system in [38]

$$\dot{x} = Ax + G\vartheta(Hx), \quad y = Cx + d$$

where $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, $y \in \mathbb{R}$, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$, $C = [1\ 0\ 0]$, $G = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$, $H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\vartheta(x) = \begin{bmatrix} 1/3x_2^3 + x_2 x_3^2 \\ 1/3x_3^3 + x_3 x_2^2 \end{bmatrix}$. The authors in [38] propose the observer

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + G\vartheta\big(H\hat{x} + K(C\hat{x} - y)\big). \quad (35)$$

The observer gains are $L = \begin{bmatrix} 3.2 \\ 11.72 \\ 3.16 \end{bmatrix}$, $K = \begin{bmatrix} 1.92 \\ 0.61 \end{bmatrix}$. The objective is to robustify the observer in front of a disturbance of the form (6), with $\omega_1 = 10\pi$ and $\omega_2 = 20\pi$, $w_1(0) = w_3(0) = 1$ and $v = 0$. In the presence of such disturbance, the observer estimation not only converges to an oscillatory trajectory, but presents a significant bias with respect to the true value, see Fig. 3. This behaviour is a consequence of introducing the noise through the nonlinear feedback term (35). Moreover, even though the approach proposed in [38] can be implemented for the original system, it leads to infeasible linear matrix inequalities for the extended system of the form (9).

Notice that the system does not satisfy the incremental dissipativity assumption in Assumption 4, as some states drift to infinity. Nonetheless, in the considered numerical simulation, the filter-observer architecture satisfies the stability condition in Assumption 3. Thus, according to Theorem 1, the internal model filter exactly rejects the modeled part of the disturbance, as depicted in Fig. 3.

6. Conclusion

This work proposes an observer redesign method to reduce the effect of output disturbances of the form (6). The main idea is to connect in cascade the already designed observer with a filter that includes an internal model of the modeled part of the disturbance. It has been established that, if the filter-observer architecture satisfies a particular stability condition, then the state estimation error is ISS with respect to the unstructured term of the disturbance and the structured terms are exactly eliminated. Moreover, this work shows that, for any incremental dissipative system that satisfies a particular differential detectability property, it is possible to design a filter that guarantees stability of the overall observer. Numerical simulations confirm that the incremental dissipativity property is a sufficient but not necessary condition for stability.

Future works will focus on relaxing this dissipativity condition and using Riemannian metrics (see, e.g., [39] and [1, Section 4]). Additionally, future works will explore the practical application of the proposal in multi-agent distributed problems [16] and masking protocols [17]

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