Let the following system be given,

$$\dot{x} = Ax + B_w(\alpha C + \beta C^{\perp}),\ y = Cx + (\alpha C + \beta C^{\perp})$$

The classical Luenberger observer is given by,

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y}),\ \hat{y} = C\hat{x}$$

Using the projection

$$\Pi = I - \frac{CC^T}{C^T C}$$

the GI-Luenberger observer can be defined as,

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y}) + B_w \Pi y,\ \hat{y} = C\hat{x} + \Pi y$$

which is in explicit form,

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y}) + B_w \beta C^{\perp},\ \hat{y} = C\hat{x} + \beta C^{\perp}$$

where $\Pi C = 0$, $\Pi C^{\perp} = C^{\perp}$ and $C^T C^{\perp} = 0$. The error dynamics of the classical Luenberger observer is obtained as,

$$\begin{aligned}
\dot{e} &= Ax + \alpha B_w C + \beta B_w C^{\perp} - A\hat{x} - L(Cx + \alpha C + \beta C^{\perp} - C\hat{x}) \\
\dot{e} &= (A - LC)e + \alpha(B_w C - LC) + \beta(B_w C^{\perp} - LC^{\perp}) \\
\dot{e} &= (A - LC)e + \alpha(B_w - L)C + \beta(B_w - L)C^{\perp}
\end{aligned}$$

where the GI-Luenberger observer is obtained as,

$$\begin{aligned}
\dot{e} &= Ax + \alpha B_w C + \beta B_w C^{\perp} - A\hat{x} - L(Cx + \alpha C + \beta C^{\perp} - C\hat{x} - \Pi y) - B_w \Pi y \\
\dot{e} &= Ax + \alpha B_w C + \beta B_w C^{\perp} - A\hat{x} - L(Cx + \alpha C + \beta C^{\perp} - C\hat{x} - \beta C^{\perp}) - B_w \beta C^{\perp} \\
\dot{e} &= (A - LC)e + \alpha(B_w - L)C
\end{aligned}$$

From the error dynamics of GI-Luenberger observer stating,

$$||B_w - L|| = 0$$

the ideal case is obtained as

$$B_w = L$$

for complete invariance respect to α, but practically the following $\mathcal{H}_\infty$ problem is more applicable,

$$\begin{aligned} \min_L \gamma \, s.t. \\ P \succ 0 \\ \begin{bmatrix} (A-LC)^T P + P(A-LC) & P(B_w - L) \\ P(B_w - L)^T & -\gamma I \end{bmatrix} \prec 0 \end{aligned}$$

which is results in the following LMI problem,

$$\begin{aligned} \min_L \rho \, s.t. \\ P \succ 0 \\ \begin{bmatrix} A^T P + PA - YC - C^T Y^T & PB_w - Y \\ (PB_w)^T - Y^T & -\rho I \end{bmatrix} \prec 0 \\ L = P^{-1} Y, \, \gamma = \sqrt{\rho} \end{aligned}$$