In Catilinam IV ⋆

Marcus Tullius Cicero [a], Julius Caesar [b], Publius Maro Vergilius [c]

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Abstract

Cum M. Cicero consul Nonis Decembribus senatum in aede Iovis Statoris consuleret, quid de iis coniurationis Catilinae sociis fieri placeret, qui in custodiam traditi essent, factum est, ut duae potissimum sententiae proponerentur, una D. Silani consulis designati, qui morte multandos illos censebat, altera C. Caesaris, qui illos publicatis bonis per municipia Italiae distribuendos ac vinculis sempiternis tenendos existimabat.

Key words: Cicero; Catiline; orations.

1 Introduction

Gauge invariance is a central organizing principle of modern physics, expressing the idea that observable quantities should remain unchanged under certain local transformations of the mathematical description. In classical field theories and quantum electrodynamics, this concept ensures that different representations related by gauge transformations correspond to the same physical reality [3,5]. From an abstract viewpoint, gauge invariance provides a systematic way to separate physically meaningful dynamics from redundant coordinate or potential offsets. This symmetry-based perspective has proven essential for constructing robust and consistent models in physics, and it motivates the extension of invariance principles beyond their traditional domain toward engineered dynamical systems subject to measurement distortions and unknown offsets.

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2 Work

2.1 The System Model

Let $x \in \mathbb{R}^{n\times 1}$, $y \in \mathbb{R}^{p\times 1}$ and $w \in \mathbb{R}^{r\times 1}$ be the state, measurement, exogenous vectors, respectively, the SIMO/MIMO LTI system addressed in this paper is stated as,

$$\dot{x} = Ax + B_w w + B_u u,\ y = Cx + D_w w \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_w \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{p\times n}$ and $D_w \in \mathbb{R}^{p\times r}$. The following additional rank condition

$$\operatorname{rank}(C) = r < p \tag{2}$$

arises in sparse sensor applications[4], topologies used in Multi Agen Systems(MAS)[2] and distributed networks [7], and is also called Strictly Output Redundant(SOR) system [6]. Here, the system output is overdetermined, therefore,

$$\dim \mathcal{N}(C^T) = p - r \geq 1 \tag{3}$$

or using orthogonality $C^T C^{\perp} = 0$,

$$\dim \mathcal{N}(C^{\perp}) = p - r \geq 1 \tag{4}$$

which ensures nontrivial orthogonal component in the output space.

2.2 The Observer Models

The classical Luenberger Observer is defined as,

$$\dot{\hat{x}} = A\hat{x} + B_u u + L(y - \hat{y}),\ \hat{y} = C\hat{x} \tag{5}$$

where $L \in \mathbb{R}^{n\times p}$ is the observer gain. Assuming (A, C)-pair observable, the error dynamics are obtained for the error $e \triangleq x - \hat{x}$ as,

$$\dot{e} = (A - LC)e + (B_w - LD_w)w \tag{6}$$

The $\mathbb{H}_\infty$ optimal observer for objective function $z = C_z e$ is designed using the following LMI problem[1],

$$\min_{Y}\,(\gamma) \quad \text{s.t.}$$

$$\begin{bmatrix} (PA - YC) + (PA - YC)^T & PB_w - YD_w & C_z^T \\ \star & -\gamma I & 0 \\ \star & 0 & -\gamma I \end{bmatrix} \prec 0$$

$$P \succ 0$$

where the observer gain is recovered with $L = P^{-1}Y$.

2.3 The Projection

The projection defined using Gauge-Invariance principle is "proposed" as follows,

$$\Pi \triangleq I - C(C^TC)^{-1}C^T \tag{7}$$

where the following properties are satisfied,

$$\begin{aligned} &\Pi C = (I - C(C^TC)^{-1}C^T)C = 0 \\ &\Pi C^\perp = (I - C(C^TC)^{-1}C^T)C^\perp = C^\perp,\ C^TC^\perp = 0 \\ &\Pi^2 = \Pi,\ \Pi \in \mathbb{R}^{p\times p} \end{aligned} \tag{8}$$

Using the projection on the measured output gives,

$$\Pi y = \Pi(Cx + D_w w) = \Pi D_w w \tag{9}$$

The projection eliminates the state dependency and utilizing

$$D_w w = C\alpha + C^\perp \beta \tag{10}$$

the projection gives,

$$\Pi y = \Pi C\alpha + \Pi C^\perp \beta = C^\perp \beta \tag{11}$$

which is the orthogonal component.

2.4 The GI-Luenberger Observer

The classical Luenberger observer is fed with this orthogonal component forming the GI-Luenberger observer, the observer expression becomes,

$$\dot{\hat{x}} = A\hat{x} + B_u u + L(y - \hat{y}) + \Lambda\Pi y,\ \hat{y} = C\hat{x} + \Pi y \tag{12}$$

where $\Lambda \in \mathbb{R}^{n\times p}$ and the error dynamics are obtained as,

$$\dot{e} = (A - LC)e + (B_w - LD_w)w - (L - \Lambda)\Pi y \tag{13}$$

For

$$\begin{aligned} A &= \begin{bmatrix} 0.1495 & 0.5832 \\ 0.8922 & 0.8997 \end{bmatrix},\ B_w = \begin{bmatrix} 0.3489 \\ 0.1484 \end{bmatrix} \\ C &= \begin{bmatrix} 0.1184 & 0.9606 \\ 0.4265 & 0.3471 \\ 0.7257 & 0.3820 \end{bmatrix},\ D_w = \begin{bmatrix} 0.8844 \\ 0.9317 \\ 0.0529 \end{bmatrix} \end{aligned} \tag{14}$$

the following is obtained,

$$\begin{aligned} &B_w - LD_w = \\ &\begin{bmatrix} 0.3489 - 0.0529l_{13} - 0.9317l_{12} - 0.8844l_{11} \\ 0.1484 - 0.0529l_{23} - 0.9317l_{22} - 0.8844l_{21} \end{bmatrix} \end{aligned} \tag{15}$$

a solution is picked as

$$L = \begin{bmatrix} 0.3945 & 0 & 0 \\ 0.1678 & 0 & 0 \end{bmatrix} \tag{16}$$

3 Numerical Example

3.1 Example 1

References

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A A summary of Latin grammar

B Some Latin vocabulary