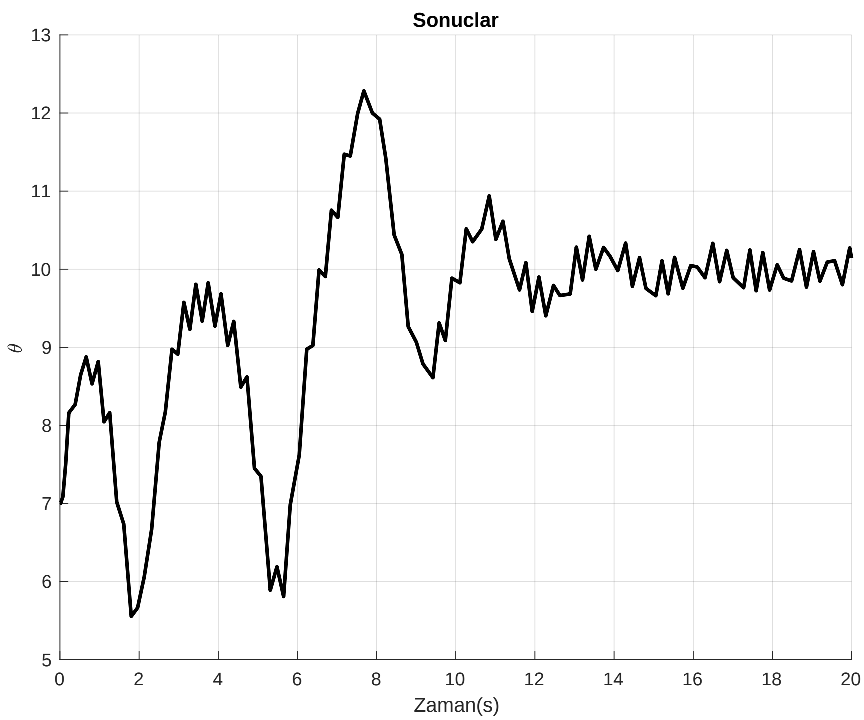
1 Ötelemesiz PLL

$$
\begin{aligned}
\dot{\theta}_r &= \omega_r, \quad v_r = \sin(\theta_r) \\
\dot{\theta} &= \omega_0 + k_v u, \quad v = \sin(\theta) \\
\bar{e} &= v \cdot v_r \\
e &= \frac{w_c}{s + w_c} \bar{e}, \quad \dot{e} = -w_c e + w_c \bar{e} \\
u &= F(s) e
\end{aligned}
$$

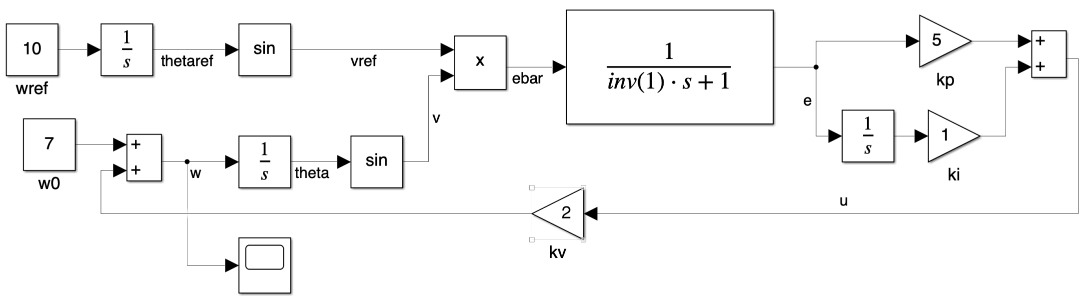
$$
\begin{aligned}
\bar{e} &= v \cdot v_r \\
&= \sin(\theta) \sin(\theta_r) \\
&= \frac{1}{2} \left[\cos(\theta_r - \theta) - \cos(\theta_r + \theta) \right], \quad \text{LPF} \\
&\approx \frac{1}{2} \cos(\theta_r - \theta)
\end{aligned}
$$

$k_v = 2$ olsun,

$$
\begin{aligned}
\dot{\theta} &= \omega_0 + k_v u \\
\dot{\theta} &= \omega_0 + k_v F(s) e \\
\dot{\theta} &= \omega_0 + F(s) \cdot 2e \\
\dot{\theta} &\approx \omega_0 + F(s) \cdot 2 \frac{1}{2} \cos(\theta_r - \theta) \\
\dot{\theta} &\approx \omega_0 + F(s) \cdot \cos(\theta_r - \theta)
\end{aligned}
$$



Şekil 1: Sonuçlar-1



Şekil 2: Simulink modeli

2 LMI

Sistem denklemleri

$$\dot{x} = Ax + Bu, \quad y = Cx + b + n$$

Gözleyici denklemleri

$$\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}), \quad y = C\hat{x} + \hat{b} \\
\dot{\hat{b}} &= -\Gamma(y - \hat{y})
\end{aligned}$$

Hatalar $e = x - \hat{x}$ ve $e_b = b - \hat{b}$ olmak üzere,

$$\begin{aligned}
\dot{e} &= \dot{x} - \dot{\hat{x}} \\
&= Ax + Bu - A\hat{x} - Bu - L(Cx + b + n - C\hat{x} - \hat{b}) \\
&= Ax - A\hat{x} - LCx - Lb - Ln + LC\hat{x} + L\hat{b} \\
\dot{e} &= (A - LC)e - Le_b - Ln
\end{aligned}$$

ve

$$\begin{aligned}
\dot{e_b} &= \dot{\overline{b - \hat{b}}} \\
&= \dot{b} - \dot{\hat{b}} \\
&= -\dot{\hat{b}} \\
&= \Gamma(y - \hat{y}) \\
&= \Gamma(Cx + b + n - C\hat{x} - \hat{b}) \\
&= \Gamma(Ce + e_b + n) \\
&= \Gamma Ce + \Gamma e_b + \Gamma n
\end{aligned}$$

Hata dinamikleri,

$$\begin{bmatrix} \dot{e} \\ \dot{e_b} \end{bmatrix} = \begin{bmatrix} A - LC & -L \\ \Gamma C & \Gamma \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix} + \begin{bmatrix} -L \\ \Gamma \end{bmatrix} n$$

Lyapunov fonksiyonu $V = e^T Pe + e_b^T e_b/\gamma^2$,

$$V = \begin{bmatrix} e \\ e_b \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix}$$

Türevi,

$$
\begin{aligned}
\dot{V} &= \begin{bmatrix} \dot{e} \\ \dot{e}_b \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix} + \begin{bmatrix} e \\ e_b \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} \dot{e} \\ \dot{e}_b \end{bmatrix} \\
&= \begin{bmatrix} e \\ e_b \end{bmatrix}^T \begin{bmatrix} (A-LC)^T & C^T \Gamma^T \\ -L^T & \Gamma^T \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix} + n^T \begin{bmatrix} -L^T & \Gamma^T \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix} \\
&+ \begin{bmatrix} e \\ e_b \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} A-LC & -L \\ \Gamma C & \Gamma \end{bmatrix} \begin{bmatrix} e \\ e_b \end{bmatrix} + \begin{bmatrix} e \\ e_b \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \gamma^{-2} I \end{bmatrix} \begin{bmatrix} -L \\ \Gamma \end{bmatrix} n \\
&= \begin{bmatrix} e \\ e_b \\ n \end{bmatrix}^T \begin{bmatrix} P(A-LC) + (A-LC)^T P & \gamma^{-2} C^T \Gamma^T - PL & -PL \\ \gamma^{-2} \Gamma C - L^T P & \gamma^{-2} \Gamma + \gamma^{-2} \Gamma^T & \gamma^{-2} \Gamma \\ -L^T P & \gamma^{-2} \Gamma^T & 0 \end{bmatrix} \begin{bmatrix} e \\ e_b \\ n \end{bmatrix}
\end{aligned}
$$

H_∞ problemi

$$
\dot{V} + \begin{bmatrix} e & e_b & n \end{bmatrix}^T \begin{bmatrix} e & e_b & n \end{bmatrix} - \gamma_n^2 n^T n < 0
$$

ile

$$
\begin{bmatrix} P(A-LC) + (A-LC)^T P + I & \gamma^{-2} C^T \Gamma^T - PL & -PL \\ \gamma^{-2} \Gamma C - L^T P & \gamma^{-2} \Gamma + \gamma^{-2} \Gamma^T & \gamma^{-2} \Gamma \\ -L^T P & \gamma^{-2} \Gamma^T & -\gamma_n^2 I \end{bmatrix} \prec 0
$$

kullanılarak optimizasyon problemi,

$$
\begin{aligned}
&\min\,(\rho_n) \quad \text{s.t.} \\
&P \succ 0 \\
&\begin{bmatrix} A^T P + PA - YC - C^T Y^T + I & \rho C^T \Gamma^T - Y & -Y \\ \rho \Gamma C - Y^T & \rho \Gamma + \rho \Gamma^T & \rho \Gamma \\ -Y^T & \rho \Gamma^T & -\rho_n I \end{bmatrix} \prec 0
\end{aligned}
$$

olarak elde edilir ve $Y \triangleq PL$, $\rho_n \triangleq \gamma_n^2$, $\rho \triangleq \gamma^{-2}$ $\rho = 1$ için

$$
\begin{aligned}
&\min\,(\rho_n) \quad \text{s.t.} \\
&P \succ 0 \\
&\begin{bmatrix} A^T P + PA - YC - C^T Y^T + I & C^T \Gamma^T - Y & -Y \\ \Gamma C - Y^T & \Gamma + \Gamma^T & \Gamma \\ -Y^T & \Gamma^T & -\rho_n I \end{bmatrix} \prec 0
\end{aligned}
$$

olarak elde edilir.