In Catilinam IV ⋆

Marcus Tullius Cicero [a], Julius Caesar [b], Publius Maro Vergilius [c]

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Abstract

Cum M. Cicero consul Nonis Decembribus senatum in aede Iovis Statoris consuleret, quid de iis coniurationis Catilinae sociis fieri placeret, qui in custodiam traditi essent, factum est, ut duae potissimum sententiae proponerentur, una D. Silani consulis designati, qui morte multandos illos censebat, altera C. Caesaris, qui illos publicatis bonis per municipia Italiae distribuendos ac vinculis sempiternis tenendos existimabat.

Key words: Cicero; Catiline; orations.

1 Introduction

Video, patres conscripti, in me omnium vestrum ora atque oculos esse conversos, video vos non solunn de vestro ac rei publicae, verum etiam, si id depulsum sit, de meo periculo esse sollicitos. Est mihi iucunda in malis et grata in dolore vestra erga me voluntas, sed eam, per deos inmortales, deponite atque obliti salutis meae de vobis ac de vestris liberis cogitate. Mihi si haec condicio consulatus data est, ut omnis acerbitates, onunis dolores cruciatusque perferrem, feram non solum fortiter, verum etiam lubenter, dum modo meis laboribus vobis populoque Romano dignitas salusque pariatur.

2 Work

2.1 The System Model

Let $x \in \mathbb{R}^{n\times 1}$, $y \in \mathbb{R}^{p\times 1}$, $w \in \mathbb{R}^{r\times 1}$ be the state, measurement, exogenous vectors, respectively, the SIMO/MIMO LTI system addressed in this paper is stated as,

$$\dot{x} = Ax + B_w w + B_u u,\; y = Cx + D_w w \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_w \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{p\times n}$ and $D_w \in \mathbb{R}^{p\times r}$. The following additional rank condition

$$\mathrm{rank}(C) = r < p$$

arises in sparse sensor applications[3], topologies used in Multi Agen Systems(MAS)[2] and distributed networks [5], and is also called Strictly Output Redundant(SOR) system [4]. Here, the system output is overdetermined, therefore,

$$\dim \mathcal{N}(C^T) = p - r \geq 1$$

or using orthogonality $C^T C^{\perp} = 0$,

$$\dim \mathcal{N}(C^{\perp}) = p - r \geq 1$$

which ensures nontrivial orthogonal component in the output space.

2.2 The Observer Models

The classical Luenberger Observer, in this context also called P Observer(PO) is defined as,

$$\dot{\hat{x}} = A\hat{x} + B_u u + L(y - \hat{y}),\; \hat{y} = C\hat{x} \tag{2}$$

Assuming (A, C)-pair observable, the error dynamics are obtained for the error $e \triangleq x - \hat{x}$ as,

$$\dot{e} = (A - LC)e + (B_w - D_w L)w \tag{3}$$

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The $\mathbb{H}_\infty$ optimal observer for objective function $z = C_z e$ is designed using the following LMI problem[1],

$$\min_{Y} (\gamma) \quad \text{s.t.}$$
$$\begin{bmatrix} (PA - YC) + (PA - YC)^T & PB_w - YD_w & C_z^T \\ \star & -\gamma I & 0 \\ \star & 0 & -\gamma I \end{bmatrix} \prec 0$$
$$P \succ 0$$

where the observer gain is recovered with $L = P^{-1}Y$. The P Observer utilizes a proportional term to correct its estimate which results in steady-state error in its estimate when exogenous input is present. The Proportional-Integral Observer(PIO) is defined as follows,

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B_u u + L(y - \hat{y}) + L_i q \\ \dot{q} &= y - \hat{y},\, \hat{y} = C\hat{x} \end{aligned} \tag{4}$$

which is used to compansate for steady state errors in the estimate for constant exogenous inputs. The corresponding error dynamics are obtained as,

$$\begin{bmatrix} \dot{e} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} A - LC & -L_i \\ C & 0 \end{bmatrix} \begin{bmatrix} e \\ q \end{bmatrix} + \begin{bmatrix} B_w - D_w L \\ D_w \end{bmatrix} w \tag{5}$$

The integral term L_i is chosen for fixed L for the sake of simplicity.

2.3 The Projection

The projection defined using Gauge-Invariance principle is "proposed" as follows,

$$\Pi \triangleq I - C(C^T C)^{-1} C^T \tag{6}$$

where the following properties are satisfied,

$$\begin{aligned} &\Pi C = (I - C(C^T C)^{-1} C^T)C = 0 \\ &\Pi C^\perp = (I - C(C^T C)^{-1} C^T)C^\perp = C^\perp,\, C^T C^\perp = 0 \\ &\Pi^2 = \Pi \end{aligned} \tag{7}$$

Using the projection on the measured output gives,

$$\Pi y = \Pi(Cx + D_w w) = \Pi D_w w = \Pi \tilde{w} \tag{8}$$

The projection eliminates the components in the output space and the orthogonal components remain. The exogenous signal $\tilde{w}$ is defined to deal with the D_w matrix. The exogenous signal is decomposed as follows,

$$\tilde{w} = C\alpha + C^\perp \beta \tag{9}$$

After this decomposition the projection on the output gives,

$$\Pi y = \Pi(Cx + C\alpha + C^\perp \beta) = \Pi C^\perp \beta = C^\perp \beta \tag{10}$$

which is the orthogonal component. The classical Luenberger Observer is fed with this orthogonal component naming it GI-Luenberger observer, the observer expression becomes,

$$\dot{\hat{x}} = A\hat{x} + B_u u + L(y - \hat{y}) + B_w \Pi y,\, \hat{y} = C\hat{x} + \Pi y \tag{11}$$

The error dynamics,

$$\dot{e} = (A - LC)e + (B_w - L)C\alpha + (B_w - L)C^\perp \beta \tag{12}$$

are converted into the following using the GI-term,

$$\dot{e} = (A - LC)e + (B_w - L)C\alpha \tag{13}$$

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A A summary of Latin grammar

B Some Latin vocabulary