# Cryptography 2

**INGI2347: COMPUTER SYSTEM SECURITY (Spring 2015)**

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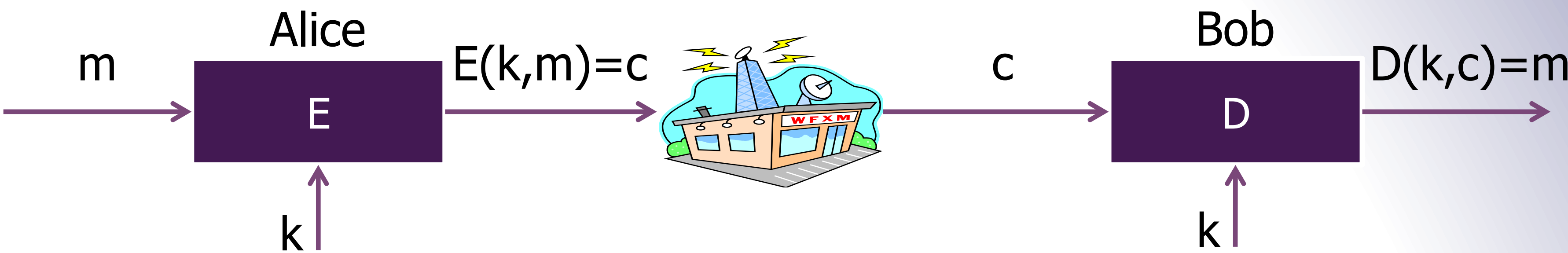
Lecture slides adapted from UCL INGI2347 by Gildas Avoine, Upenn CSE331 by Steve Zdancewic, and Dan Boneh, Stanford Cryptography I@Coursera. Reproduced with permission

# Plan for today

## Lecture 7

- Recap on Symmetric vs Public Key Crypto NEXT
- RSA
- Diffie-Hellman
- Authentication
- Generic collision resistance attack
- Integrity

# Symmetric Key Encryption




E, D: cipher k: secret key (e.g., 128 bits)

m, c: plaintext, ciphertext

- Same secret key for both encryption and decryption
- Stream ciphers
  - Act on the plaintext one symbol at a time
- Block ciphers
  - Act on the plaintext in blocks of symbols

# Stream Ciphers: The One Time Pad (Vernam 1917)

First example of a "secure" cipher

$$M = C = \{0,1\}^n, \quad K = \{0,1\}^n$$

key = (random bit string as long the message)

$$E(k,m) = k \oplus m$$

$$D(k,c) = k \oplus c$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| msg: | 0 | 1 | 1 | 0 | 1 | 1 | 1 | $\oplus$ |
| key: | 1 | 0 | 1 | 1 | 0 | 1 | 0 | |
| CT: | 1 | 1 | 0 | 1 | 1 | 0 | 1 | |

# One-time vs Many-time Security

**Never use stream cipher key more than once !!**

$$C_1 \leftarrow m_1 \oplus k$$
$$C_2 \leftarrow m_2 \oplus k$$

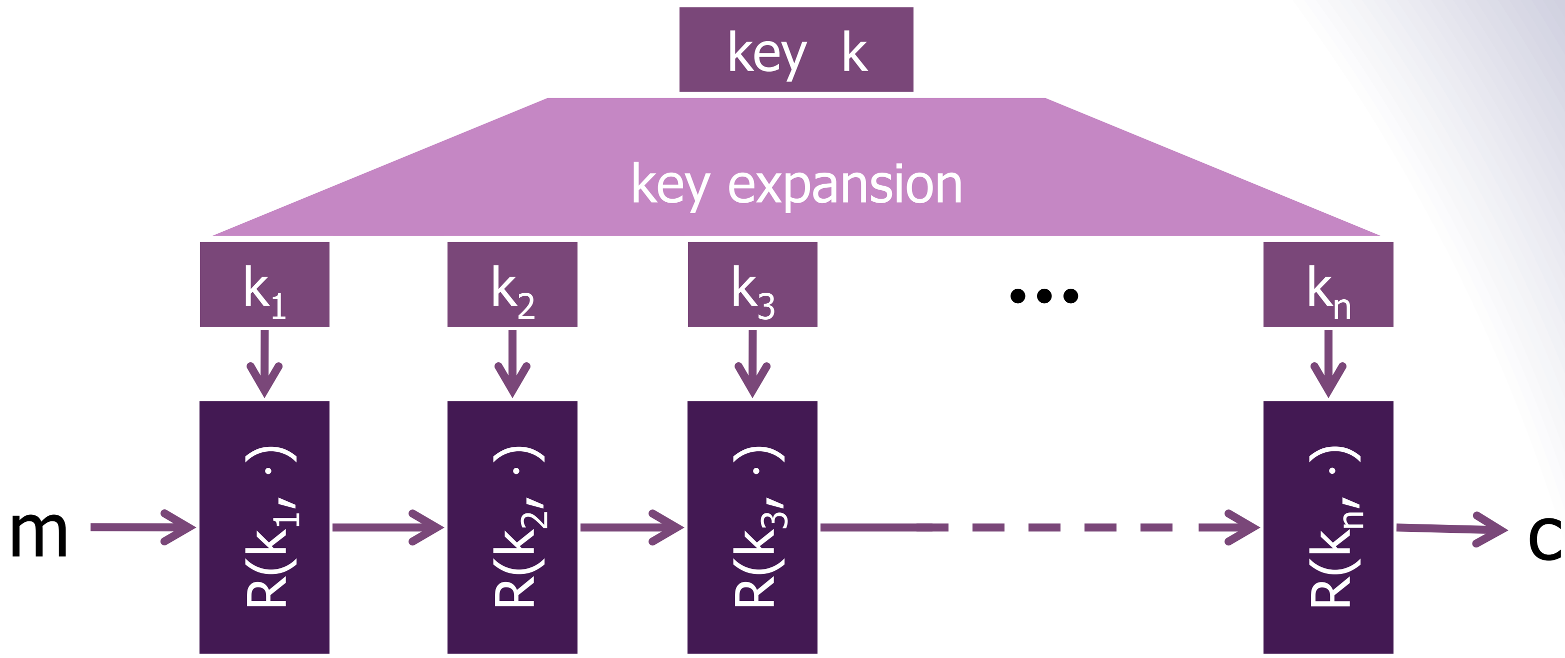
Eavesdropper does:

$$C_1 \oplus C_2 \rightarrow m_1 \oplus m_2$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \rightarrow m_1 , m_2$$

# Block Ciphers Built by Iteration




R(k,m) is called a round function

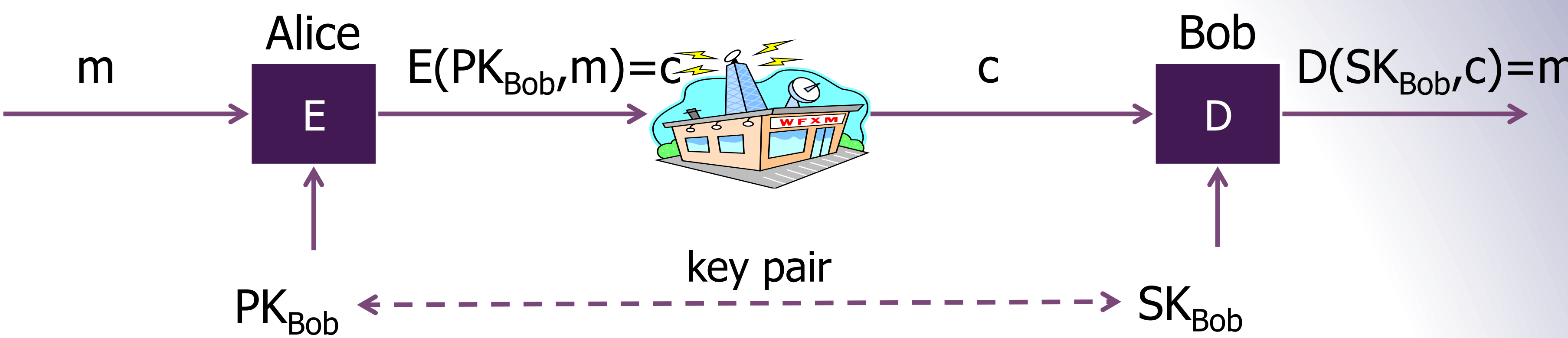
**for 3DES (n=48), for AES-128 (n=10)**

# Problems with Shared Key Crypto

- Compromised key means interceptors can decrypt any ciphertext they've acquired
  - Change keys frequently to limit damage
- Distribution of keys is problematic
  - Keys must be transmitted securely
  - Use couriers?
  - Distribute in pieces over separate channels?
- $O(n)$ keys per user ; $O(n^2)$ keys in the system
- Online TTP not an ideal solution

# Public Key Encryption






PK: public key ,     SK: secret key (e.g., 1024 bits)

Example: Bob generates ($PK_{Bob}$, $SK_{Bob}$) and gives $PK_{Bob}$ to Alice

- Only the private key must be kept secret!
- Interactive applications: session setup
- Non-interactive applications: e.g., email

# Establishing a shared secret

**Alice** **Bob**

(pk, sk) ⟵ G()

"Alice", pk ⟶

choose random
$x \in \{0,1\}^{128}$

⟵ "Bob", c⟵E(pk,x)

D(sk,c) ⟶ x shared secret

Note: protocol is vulnerable to man-in-the-middle

# Insecure against man in the middle

The protocol is insecure against **active** attacks

**Alice** **MiTM** **Bob**

$(pk, sk) \leftarrow G()$ $(pk', sk') \leftarrow G()$

"Alice", pk

**"Alice", pk'**

choose random
$x \in \{0,1\}^{128}$

"Bob", $E(pk', x)$

$x \leftarrow D(sk', E(pk', x))$

**"Bob", $E(pk, x)$**

# Trade-offs for Public Key Crypto

- More computationally expensive than symmetric (shared) key crypto
  - Algorithms are harder to implement
  - Require more complex machinery
- More formal justification of difficulty
  - Hardness based on complexity-theoretic results
- A principal needs 1 private key and 1 public key
  - Number of keys for pair-wise communication is O(n)

# Model of the attacker

- Ciphertext-only attack
  - Attacker has access to cipher test of one or more messages, all of which were encrypted with the same key K
  - His goal is to find the corresponding plaintext, or even better K
- Known-plaintext attack
  - Attacker has access to one or more plaintext-ciphertext pairs, encrypted with the same key K
  - His goal is to determine K
    - An example of this is the DES challenge

# Model of the attacker

- Chosen-ciphertext attack (CCA)
  - The attacker has access to a **decryption** oracle: he can choose ciphertexts (based on the same key K) and get their corresponding plaintext
- Chosen-plaintext attack (CPA)
  - The adversary has access to an **encryption** oracle: he can choose plaintexts and get their corresponding ciphertexts, based on the same key K
  - More powerful than CCA

# RSA

# RSA Algorithm

- Ron **R**ivest, Adi **S**hamir, Leonard **A**dleman
  - Proposed in 1979
  - They won the 2002 Turing award for this work
- Has withstood years of cryptanalysis
  - Not a guarantee of security!
  - But a strong vote of confidence
    - Further reading: Twenty years of attacks on the RSA cryptosystem, D. Boneh, Notices of the AMS, 1999
- Hardware implementations:
  - 1000 x slower than DES

# RSA at a High Level

- Public and private key are derived from secret prime numbers
  - Today at least 2048 bits to ensure security (4096 bits is better)
- Plaintext message (a sequence of bits)
  - Treated as a (large!) binary number
- Encryption is modular exponentiation
- To break the encryption, conjectured that one must be able to factor large numbers
  - Not known to be in P (polynomial time algorithms)

# RSA Details: Key Generation

- Choose two distinct random prime numbers $p$ and $q$
- Compute the **modulus**: $n = p \cdot q$
- Compute $\varphi(n) = (p-1)(q-1)$
  - $\varphi$ is Euler's totient function
  - $\varphi(n)$ counts the positive integers $\leq n$ that are relatively prime to $n$
  - $a$ and $b$ are relatively prime iff their greatest common divisor = 1
    - $\mathrm{GDC}(a, b) = 1$

Euler's theorem:

- $a^{\varphi(n)} \equiv 1 \bmod n$, for any $a$ relatively prime with $n$

# RSA Details: Key Generation

- Choose $e$ such that $1 < e < \varphi(n)$ with $e$ and $\varphi(n)$ relatively prime
  - $e$ is the **public key exponent** (public key = $(e, n)$)
  - Typically small: e.g. $e = 2^{16} + 1 = 65537$
- Determine $d \equiv e^{-1} \cdot \bmod \varphi(n)$, that is the multiplicative inverse of $e$
  - $d$ is the **private key exponent** (private key = $(d, n)$)
  - We have that $d \cdot e \equiv 1 \bmod \varphi(n)$

# RSA Details: Key Generation

- Publish $(e, n)$ as the public key
- Keep $(d, n)$ as the private key
- $p$, $q$, and $\varphi(n)$ must also be kept secret or even thrown away altogether!
  - Why?

# RSA Details: Encryption

- Message M is turned to an integer $m$ s.t. $0 \leq m < n$
- We use the **recipient's public key** $(e, n)$ to compute:

$$c \equiv m^e \bmod n$$

- We use exponentiation by squaring to perform this quickly:

$$m^e \equiv (m^2 \bmod n)^{(e/2)} \bmod n \qquad \text{, if } e \equiv 0 \bmod 2$$

$$m^e \equiv m\,(m^2 \bmod n)^{((e\text{-}1)/2)} \bmod n \qquad \text{, else}$$

# RSA Details: Encryption Example



- Scaled-down example
  - (explicit form of the one on Wikipedia):

$$
\begin{aligned}
65^{17} &\equiv 65\,(65^2 \bmod 3233)^8 && \equiv 65 \cdot 992^8 \\
&\equiv 65\,(992^2 \bmod 3233)^4 && \equiv 65 \cdot 1232^4 \\
&\equiv 65\,(1232^2 \bmod 3233)^2 && \equiv 65 \cdot 1547^2 \\
&\equiv 65\,(1547^2 \bmod 3233) && \equiv 65 \cdot 789 \\
&\equiv 2790 \pmod{3233}
\end{aligned}
$$

# RSA Details: Decryption



- The recipient uses its private key $(d, n)$ to compute:

$$m \equiv c^d \bmod n$$

- This works. Why?

$$\begin{aligned} c^d \bmod n &\equiv (m^e \bmod n)^d \bmod n \\ &\equiv m^{(e \cdot d)} \bmod n \\ &\equiv m^1 \bmod n \end{aligned}$$

  - Last step works thanks to Euler's theorem and Fermat's Little Theorem

# RSA Details: Miscellaneous

- How to encrypt long messages ($m > n$)?
  - Use a mode of encryption such as CBC?
  - **Too expensive!**
  - Use hybrid encryption: encrypt a symmetric key with RSA, then use this to **encrypt the bulk data**
- How would one do signature with RSA?
  - Sign the message by applying the decryption alg. with the private key
  - For long messages, hash the message first, then sign the hash value

# RSA Details: Miscellaneous

- The "1024" bits (or 2048, or 4096, ...) is the size of the **modulus** $n$
- Does that mean "1024-bit security", like with block ciphers?
- **No!**

| cipher key size | modulus size |
|---|---|
| 80 bits | 1024 bits |
| 128 bits | 3072 bits |
| 256 bits (AES) | **15360** bits |

- RSA is not CCA-secure (see exercises), but it is never used as explained here!

# Trapdoor functions (TDF)

**Def**: a trapdoor func. $X \longrightarrow Y$ is a triple of efficient algs. $(G, F, F^{-1})$

- $G()$: randomized alg. outputs key pair $(pk, sk)$
- $F(pk, \cdot)$: det. alg. that defines a func. $X \longrightarrow Y$
- $F^{-1}(sk, \cdot)$: defines a func. $Y \longrightarrow X$ that inverts $F(pk, \cdot)$

Security: $F(pk, \cdot)$ is one-way without $sk$

# Public-key encryption from TDFs

- $(G, F, F^{-1})$: secure TDF $X \longrightarrow Y$
- $(E_s, D_s)$ : symm. auth. encryption with keys in K
- $H: X \longrightarrow K$ a hash function

We construct a pub-key enc. system (G, E, D):

Key generation G: same as G for TDF

# Public-key encryption from TDFs

- $(G, F, F^{-1})$: secure TDF $X \longrightarrow Y$
- $(E_s, D_s)$ : symm. auth. encryption with keys in K
- H: $X \longrightarrow K$ a hash function

**E( pk, m) :**

$x \xleftarrow{R} X, \quad y \leftarrow F(pk, x)$

$k \leftarrow H(x), \quad c \leftarrow E_s(k, m)$

output $(y, c)$

**D( sk, (y,c) ) :**

$x \leftarrow F^{-1}(sk, y),$

$k \leftarrow H(x), \quad m \leftarrow D_s(k, c)$

output m

# Diffie-Hellman Key Exchange

# Diffie-Hellman Key Exchange

- Problem with shared-key systems:

  Distributing the shared key

- Suppose that Alice and Bob want to agree on a secret (i.e. a key)
  - Communication link is public
  - They don't already share any secrets

# Diffie-Hellman Key Exchange

- Choose a prime p (publicly known)
  - Should be about 512 bits or more
- Pick g < p (also public)
  - g must be a *primitive root* of p
  - A primitive root generates the finite field p
  - Every n in {1, 2, ..., p-1} can be written as $g^k$ mod p
  - Example: 2 is a primitive root of 5
  - $2^0 = 1$ $2^1 = 2$ $2^2 = 4$ $2^3 = 3$ (mod 5)
  - Intuitively means that it's hard to take logarithms base g because there are many candidates

# Diffie-Hellman Protocol

**Alice** **Bob**

"Let's use (p, g)" →

← "OK"

$g^A \bmod p$ →

← $g^B \bmod p$

1. Alice & Bob decide on a public prime p and primitive root g
2. Alice chooses secret number A Bob chooses secret number B
3. Alice sends Bob $\mathbf{g^A \bmod p}$ Bob sends Alice $\mathbf{g^B \bmod p}$
4. The shared secret is $\mathbf{g^{AB} \bmod p}$

Note: security against eavesdropping only (vulnerable to man-in-the-middle)

# Diffie-Hellman Details

- Alice computes $g^{AB}$ mod p because she knows A:

$$g^{AB} \bmod p = (g^B \bmod p)^A \bmod p$$

- An eavesdropper gets $g^A$ mod p and $g^B$ mod p
  - They can easily calculate $g^{A+B}$ mod p but that doesn't help
  - The problem of computing discrete logarithms (to recover A from $g^A$ mod p) is hard

# Diffie-Hellman Example

- Alice and Bob agree that p=71 and g=7
- Alice selects a private key A=5 and calculates a public key

  $g^A \equiv 7^5 \equiv 51 \pmod{71}$ ; she sends this to Bob
- Bob selects a private key B=12 and calculates a public key

  $g^B \equiv 7^{12} \equiv 4 \pmod{71}$ ; he sends this to Alice
- Alice calculates the shared secret:

  $S \equiv (g^B)^A \equiv 4^5 \equiv 30 \pmod{71}$
- Bob calculates the shared secret:

  $S \equiv (g^A)^B \equiv 51^{12} \equiv 30 \pmod{71}$

# Why Does It Work?

- Security is provided by the difficulty of calculating discrete logarithms
- Feasibility is provided by
  - The ability to find large primes
  - The ability to find primitive roots for large primes
  - The ability to do efficient modular arithmetic
- Correctness is an immediate consequence of basic facts about modular arithmetic

+

# Authentication

# Authenticated channel

- You should always expect a **man-in-the-middle**
  - e.g. on the internet, your messages go through many intermediaries

- Solution: Use an authenticated channel
  - For instance, Alice and Bob have certificates that contain a public key, and exchange them prior to the DH exchange
  - They use them to authenticate the values in the DH phase
  - More on that in the SSL/TLS lecture

# Collision resistance

Generic birthday attack

# Cryptographic Hashes

- Create a hard-to-invert summary of input data

$$h : \{0,1\}^* \xrightarrow{\text{hash}} \{0,1\}^n$$

- Sometimes called a Message Digest
- Examples:
  - Secure Hash Algorithm (SHA)
  - Message Digest (MD4, MD5)

# Desired Properties

- One way hash function
  - Given a hash value $y$, it should be infeasible to find $m$ s.t. $\mathrm{h}(m)=y$
- Collision resistance
  - It should be infeasible to find two different messages $m_1$ and $m_2$ s.t. $\mathrm{h}(m_1)=\mathrm{h}(m_2)$
- Random oracle property
  - $\mathrm{h}(m)$ is indistinguishable from a random n-bit value
  - Attacker must spend a lot of effort to be able to modify the message without altering the hash value

# Generic attack on C.R. functions

Let $H: M \to \{0,1\}^n$ be a hash function ( $|M| >> 2^n$ )

Generic alg. to find a collision **in time** $\mathbf{O(2^{n/2})}$ hashes

Algorithm:

1. Choose $\mathbf{2^{n/2}}$ random messages in M: $m_1, \ldots, m_{2^{n/2}}$
   (distinct w.h.p )
2. For $i = 1, \ldots, 2^{n/2}$ compute $t_i = H(m_i) \in \{0,1\}^n$
3. Look for a collision $(t_i = t_j)$.
   If not found, got back to step 1.

How well will this work?

# The birthday paradox

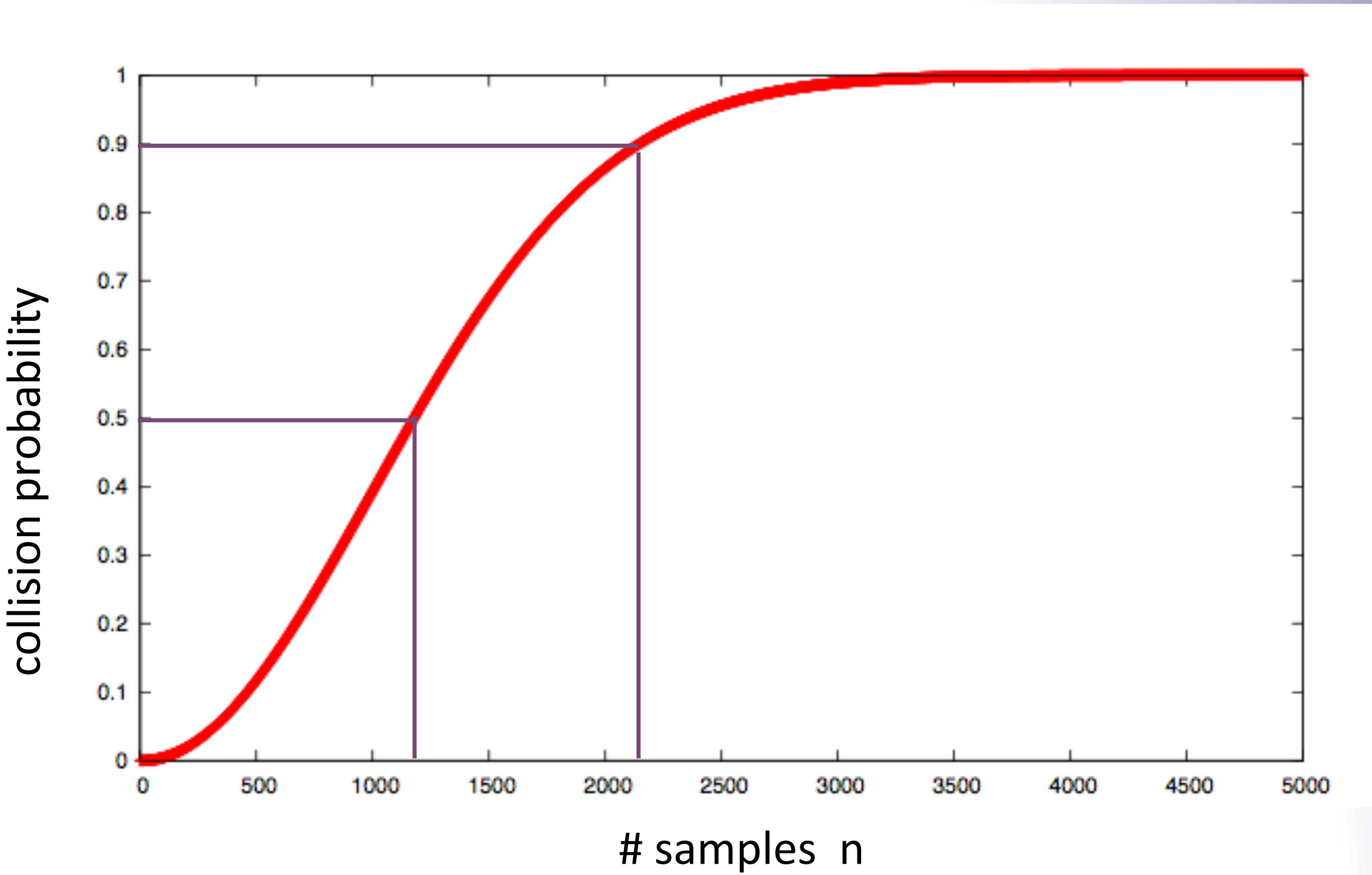
- In a group of **23** people, the probability to have at least two people with the same birthday is about **50%**

- Theorem: If we pick $\theta\sqrt{N}$ independently and uniformly distributed random numbers in {1,2,…,N}, we get at least two occurrences of the same number with probability:

$$1-\frac{N!}{N^{\theta\sqrt{N}}(N-\theta\sqrt{N})!} \xrightarrow[N\to+\infty]{} 1-e^{-\frac{\theta^2}{2}}$$

N=$10^6$




collision probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
# samples n

# Generic attack

H: M $\rightarrow \{0,1\}^n$ .      Collision finding algorithm:

1. Choose $\mathbf{2^{n/2}}$ random elements in M:   $m_1, \ldots, m_{2^{n/2}}$
2. For i = 1, …, $2^{n/2}$ compute   $t_i = H(m_i)$   $\in \{0,1\}^n$
3. Look for a collision ($t_i = t_j$).
   If not found, got back to step 1.

Expected number of iteration ≈ 2

Running time: $\mathbf{O(2^{n/2})}$    (space $O(2^{n/2})$ )

# Integrity

# Message Integrity

Goal: **integrity**, no confidentiality

Examples:

- Protecting public binaries on disk
- Protecting banner ads on web pages

# Message Integrity: MAC


**Generate tag:**
**tag ← S(k, m)**

**Verify tag:**
**V(k, m, tag) $\stackrel{?}{=}$ 'yes'**

Def: **MAC** $I=(S,V)$ defined over $(K,M,T)$ is a pair of algs

- $S(k,m)$ outputs $t$ in $T$
- $V(k,m,t)$ outputs 'yes' or 'no'

Consistency: $\forall (\mathrm{k}PK,\ SK)$ output by $G$ :

$$\forall k \in K,\ \forall m \in M: \quad V(k, \mathrm{m}, S(k, m)\,) = \text{'}yes\text{'}$$

# Secure MACs

- Attacker information: chosen message attack
  - for $m_1, m_2, \ldots, m_q$ attacker is given $t_i \leftarrow S(k, m_i)$

- Attacker's goal: existential forgery.
  - produce some **new** valid message/tag pair $(m,t)$.

$$(m,t) \notin \{ (m_1,t_1), \ldots, (m_q,t_q) \}$$

---

$\Rightarrow$ attacker cannot produce a valid tag for a new message

$\Rightarrow$ given $(m,t)$ attacker cannot even produce $(m,t')$ for $t' \neq t$

# Secure PRF $\Rightarrow$ Secure MAC

For a Pseudo Random Function **$F: K \times X \longrightarrow Y$**

define a MAC $I_F = (S,V)$ as:

- $S(k,m)$ := $F(k,m)$
- $V(k,m,t)$: output `yes' if $t = F(k,m)$ and `no' otherwise.

$\Rightarrow$ $I_F$ is secure as long as $|Y|$ is large, say $|Y| = 2^{80}$


**tag $\leftarrow$ F(k,m)**

accept msg if

**tag = F(k,m)**

# Standardized method: HMAC (Hash-MAC)

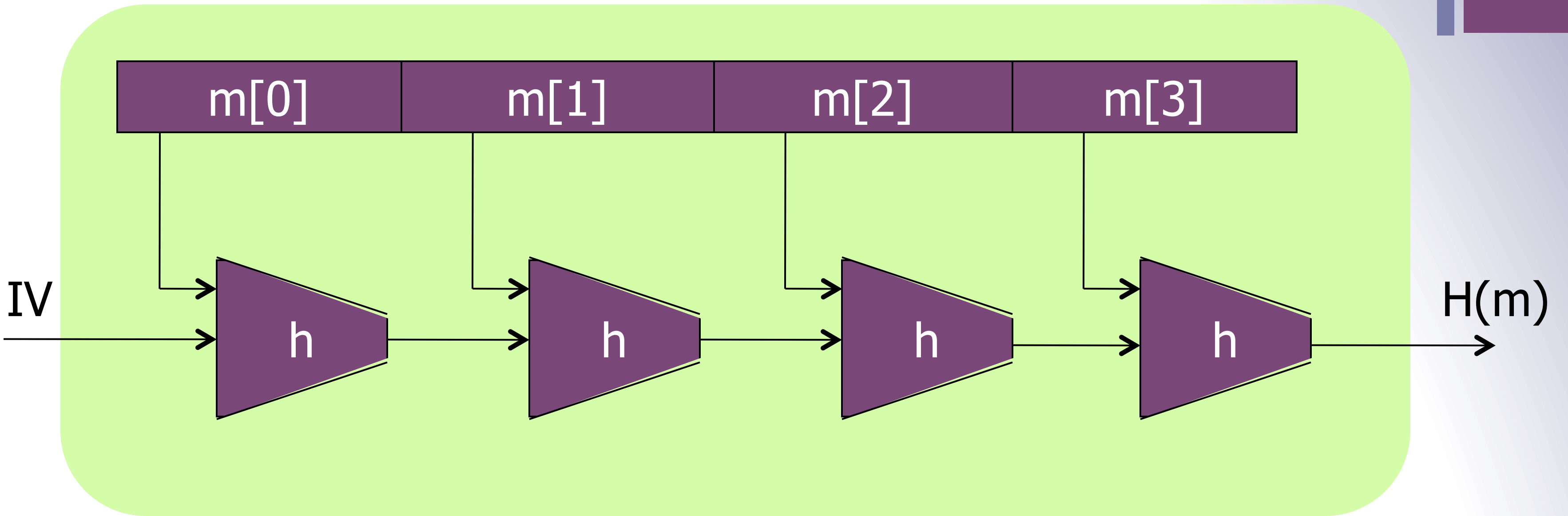
- Most widely used MAC on the Internet
  - Proposed by Bellare, Canetti, Krawczyk in 1996
  - Provably secure
  - Standards: FIPS 198-1, RFC 2104, ISO 9797-2

- Builds a MAC out of a hash function

HMAC: $S(k, m) = H\big(\, k \oplus \text{opad} \;||\; H(\, k \oplus \text{ipad} \,||\, m \,)\,\big)$

  - Maintains performance of the original hash function
  - Examples:
    - HMAC-SHA256: H = SHA256 ; output is 256 bits
    - HMAC-SHA1-96: H = SHA1 ; output truncated to 96 bits

# SHA-256: Merkle-Damgard




h(t, m[i]): compression function

Thm 1: if h is collision resistant then so is H

"Thm 2": if h is a PRF then HMAC is a PRF

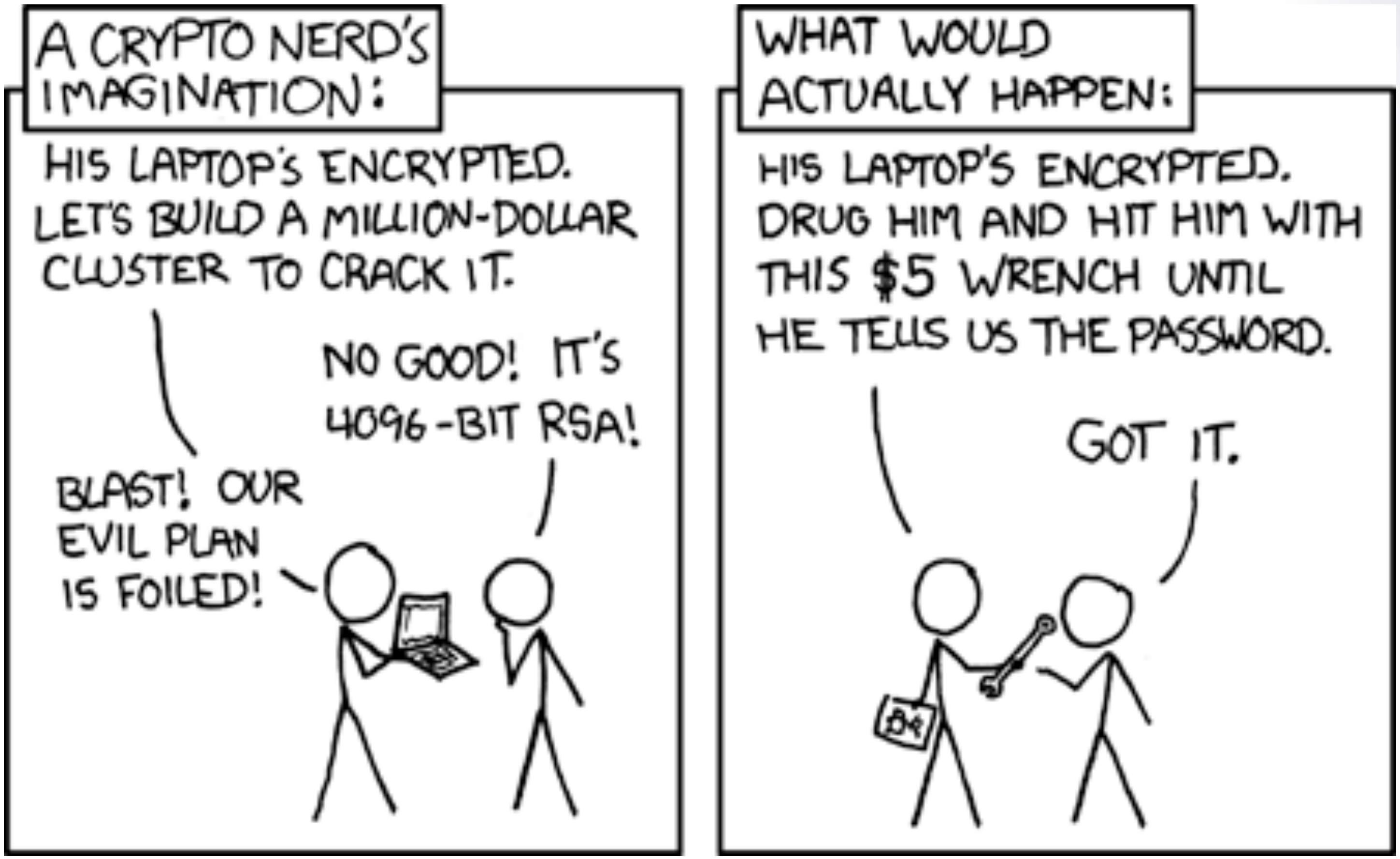
# An Insecure MAC Construction

- Let us define $t = S(m, k) = H(k \,||\, m)$
- Because of the way typical hash function are implemented (up to SHA-2), the "Merkle-Damgård" construction, an attack is possible
- An adversary can compute $t' = H(k \,||\, m \,||\, \text{padding} \,||\, m')$ without knowing $m$
- She can therefore send $m'$, $t'$ instead of $m$, $t$

# Things To Remember

- Cryptography is:
  - A tremendous tool
  - The basis for many security mechanisms
- Cryptography is **NOT**:
  - The solution to all security problems
  - Reliable unless implemented and used properly
  - Something you should try to invent yourself

# Any questions?



# Stay tuned


Next time you will learn about

# Network vulnerabilities