



$$\begin{aligned} &^2(y f(2) * 2012) y_1 + e_2(x) y_2 + e_3(x) y_3 \\ &(x+1) = \left(\frac{x(x-2)}{2} \right) 1 + (x(x-1)) 0 + \left(\frac{x(x-1)}{2} \right) \\ &)^2 = \left(\frac{(x-1)(x-2)}{2} \right) 1 + (x(x-1)) 0 + \left(\frac{x(x-1)}{2} \right) \\ &f_P(x, y) \\ &)^2(y + 6x + 2)^4 - (y + 7x + 8)^2(y + 9x + 6)^4(x + 1) \\ &1)(x + 6)^4(x + 9)^4 \quad x(x + 2)^4(x + 2)^4 \\ &-9b + \sqrt{3} \sqrt{4a^3 + 27b^2} \sqrt[4]{3} + 6x)^2(y + 10x + 8)^4 x + 1 \\ &\frac{2^{1/3} 3^{2/3}}{x(x + 6)^2} \quad (y + 9x + \\ &\frac{(y + 8x)^2}{(1 - i\sqrt{3})(-9b + \sqrt{3} \sqrt{4a^3 + 27b^2})^{1/3}} \quad (y + 8x + \\ &1/3 + \frac{2^{4/3} 3^{2/3} x + 9}{(y + 8x)^2(y + 7x + 4)^4(y + 5} \end{aligned}$$

Cryptography 1

INGI2347: COMPUTER SYSTEM SECURITY (Spring 2015)

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Plan for today

Lecture 4

- What's crypto? 
- Symmetric key encryption
- Trusted 3rd parties
- Public key encryption
- Crypto hash functions



κρυπτο γραφη (Cryptography)

- Greek for “secret writing”
- Confidentiality
 - Obscure a message from eaves-droppers
- Integrity
 - Assure recipient that the message was not altered
- Authentication
 - Verify the identity of the source of a message
- Non-repudiation
 - Convince a 3rd party that what was said is accurate

Terminology



■ Encryption algorithm

- Transforms a plaintext into a ciphertext that is unintelligible for non-authorized parties
- Usually parametrized with a cryptographic key

■ Asymmetric (Public) key cryptography

- Crypto system: encryption + decryption algorithms + key generation

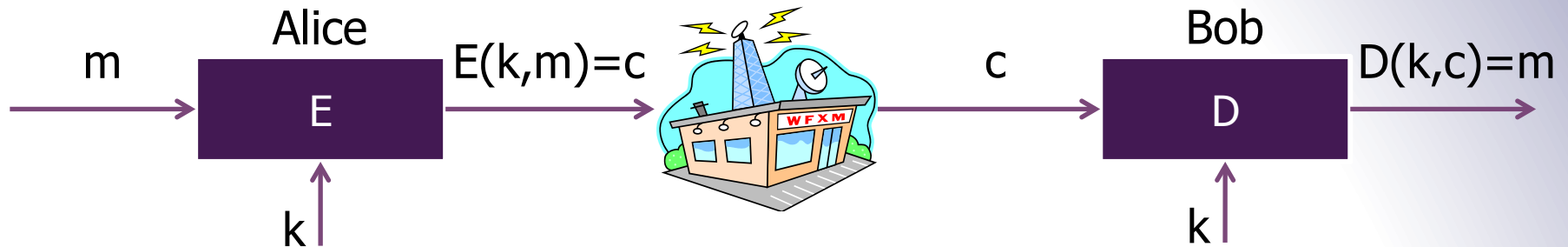
■ Symmetric (Shared) key cryptography

- Cipher/decipher: symmetric-key encryption/decryption algorithms

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Build block: sym.-key encryption



E, D : cipher

k : secret key (e.g., 128 bits)

m, c : plaintext, cipher text

- Same secret key for both encryption and decryption
- Encryption algorithm is publicly known
 - Never use a proprietary algorithm
- Fast in software or hardware implementations



Use Cases

- **Single use key:** (one time key)
 - Key is only used to encrypt one message
 - encrypted email: new key generated for every email
- **Multi use key:** (many time key)
 - Key used to encrypt multiple messages
 - encrypted files: same key used to encrypt many files
 - SSL: same key used to encrypt many packets
 - Need more machinery than for one-time key



Things To Remember

■ Cryptography is:

- A tremendous tool
- The basis for many security mechanisms

■ Cryptography is **NOT**:

- The solution to all security problems
- Reliable unless implemented and used properly
- Something you should try to invent yourself
 - many many examples of broken ad-hoc designs
- Privacy | Steganography | (Encoding|Decoding)



Cryptography is NOT

■ Privacy

- Ability to control how personal information spreads in a community

■ Steganography

- Science of information hiding

■ Encoding | Decoding

- Code is a system of symbols which represent information
- Encoding transforms information into a codeword
- Decoding recovers information from a codeword

Cryptanalysis: a rigorous science

Proves or disproves the security of a cryptosystem

■ The three steps:

- Precisely specify threat model
- Propose a construction
- Prove that breaking construction under threat mode will solve an underlying hard problem

■ Definition of break:

- Decrypting (totally or partially) a given ciphertext
- Recovering the key of the cryptosystem
- Proving a cryptosystem is less secure than what is claimed



Symmetric-key cryptography

Assumes parties already share a secret key

Symmetric Ciphers: definition

Def: a **cipher** defined over (K, M, C)

is a pair of “efficient” algs (E, D) where

$$E : K \times M \rightarrow C, \quad D : K \times C \rightarrow M$$

$$\text{s.t. } \forall m \in M, k \in K : D(k, E(k, m)) = m$$

E is often randomized | D is always deterministic

Stream vs. Block Ciphers

■ Stream ciphers

- Act on the plaintext one symbol at a time
- **Examples:** RC4, GSM A5-1, Bluetooth E0, CSS, ...
- High speed rate, hardware implementations very cheap
- Security analysis is not well established

■ Block ciphers

- Act on the plaintext in blocks of symbols
- **Examples:** DES, 3DES, AES, IDEA, Blowfish, RC5, Kasumi, Safer, ...
- Suited to software implementations on various systems
 - (e.g., 8-bit, 32-bit, 64-bit processors)
- Security analysis is well established

Stream Ciphers: The One Time Pad (Vernam 1917)

First example of a “secure” cipher

$$M = C = \{0,1\}^n, \quad K = \{0,1\}^n$$

key = (random bit string as long the message)

$$E(k, m) = k \oplus m$$

$$D(k, c) = k \oplus c$$

msg: 0 1 1 0 1 1 1

key: 1 0 1 1 0 1 0

\oplus

CT: 1 1 0 1 1 0 1

Indeed:

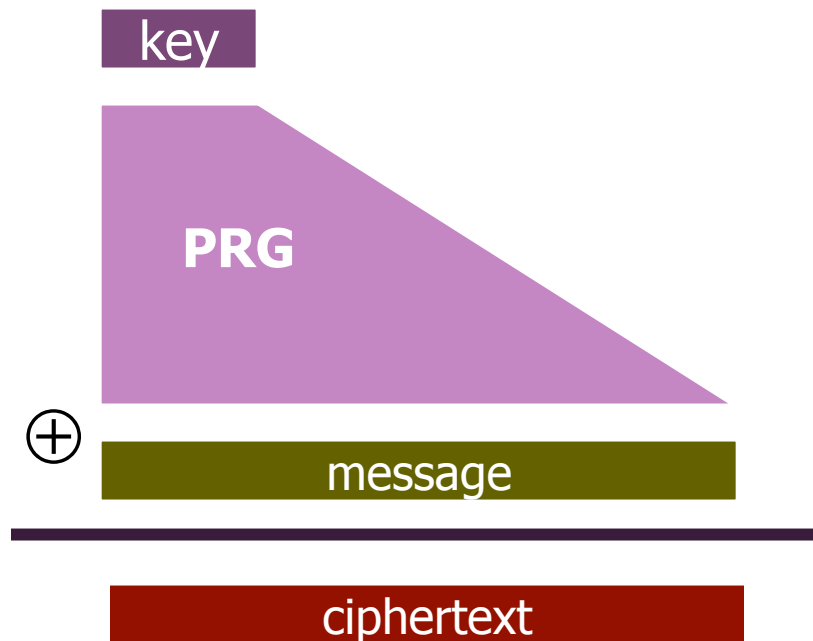
$$D(k, E(k, m)) = D(k, k \oplus m) = k \oplus (k \oplus m) = (k \oplus k) \oplus m = 0 \oplus m = m$$



Stream ciphers (single key use)

Problem: OTP key is as long the message

Solution: Pseudo random key – stream ciphers



Dangers in using stream ciphers

One time key !! “Two time pad” is insecure:

$$\begin{cases} C_1 \leftarrow m_1 \oplus \text{PRG}(k) \\ C_2 \leftarrow m_2 \oplus \text{PRG}(k) \end{cases}$$

Eavesdropper does:

$$C_1 \oplus C_2 \rightarrow m_1 \oplus m_2$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \rightarrow m_1, m_2$$



What is a secure cipher?

Attacker's abilities: **obtains one ciphertext** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$$E(k, m) = m$$

Insecure: **leaks m**

attempt #2: **attacker cannot recover all of plaintext**

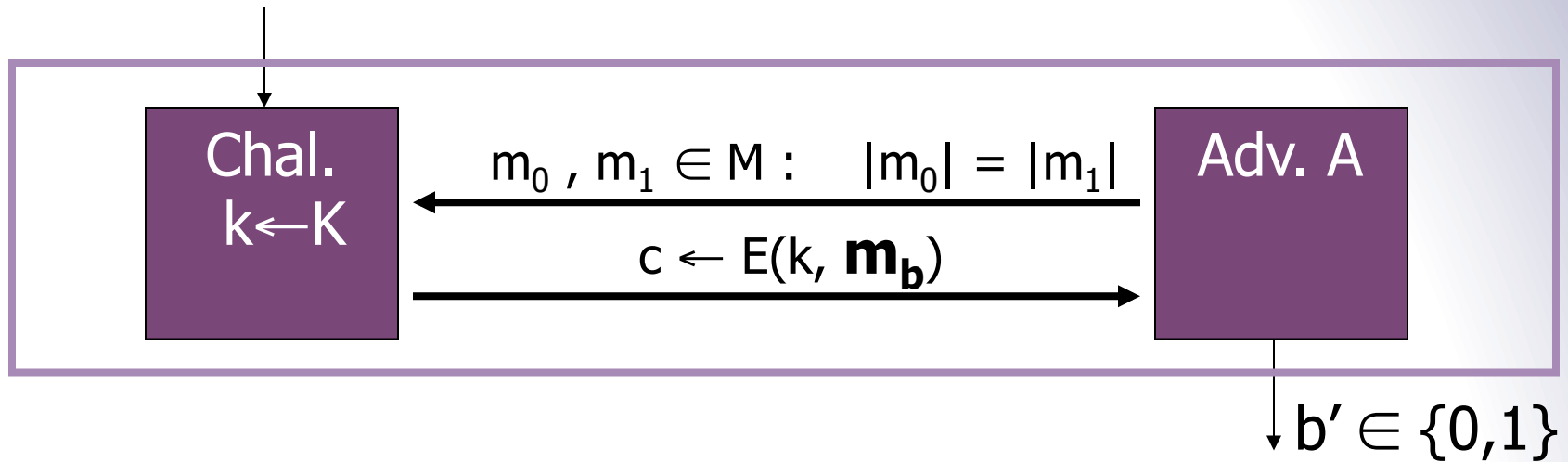
$$E(k, m_0 || m_1) = m_0 || E(k, m_1) \quad \text{Insecure: } \mathbf{leaks\ } m_0$$

Shannon's information-theoretic perfect secrecy:

ciphertext should reveal no "info" about plaintext

Semantic Security (one-time key)

For $b=0,1$ define experiments $\text{EXP}(0)$ and $\text{EXP}(1)$ as:



for $b=0,1$: $W_b := [\text{event that } \text{EXP}(b)=1]$

$$\text{Adv}_{\text{SS}}[A,E] := \left| \Pr[W_0] - \Pr[W_1] \right| \in [0,1]$$

Sematic Security Advantage of A against E



Semantic Security (one-time key)

Def: E is **semantically secure** if for all efficient A

$$\text{Adv}_{\text{SS}}[A, E] \quad \text{is negligible}$$

Model of the attacker (also for PK)

■ Chosen-ciphertext attack (CCA)

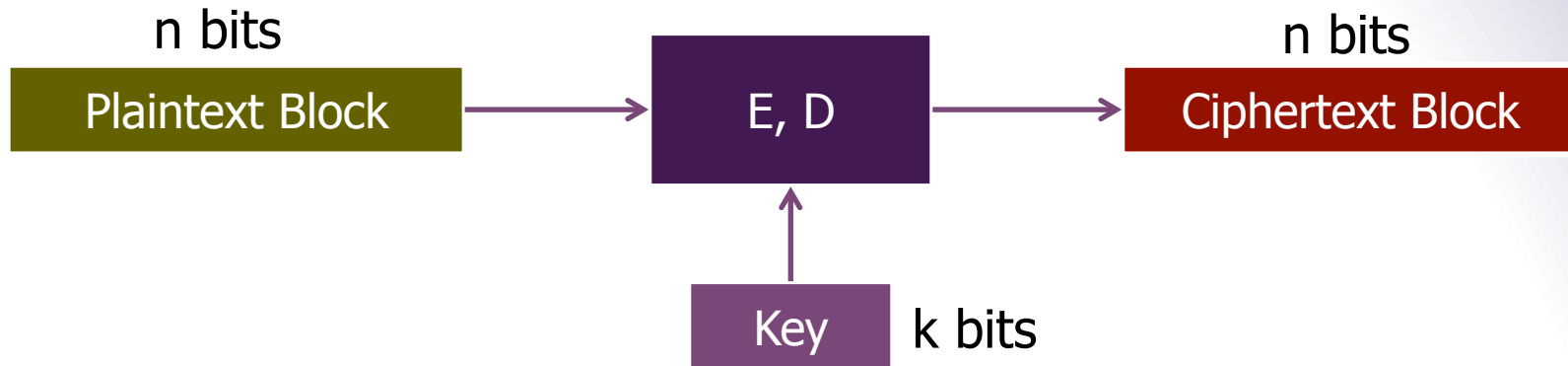
- The attacker has access to a **decryption** oracle: he can choose ciphertexts (other than the ciphertext he is challenged with) and get their corresponding plaintext

■ Chosen-plaintext attack (CPA)

- The adversary has access to an **encryption** oracle: he can choose plaintexts and get their corresponding ciphertexts
- More powerful than CCA



Block Ciphers

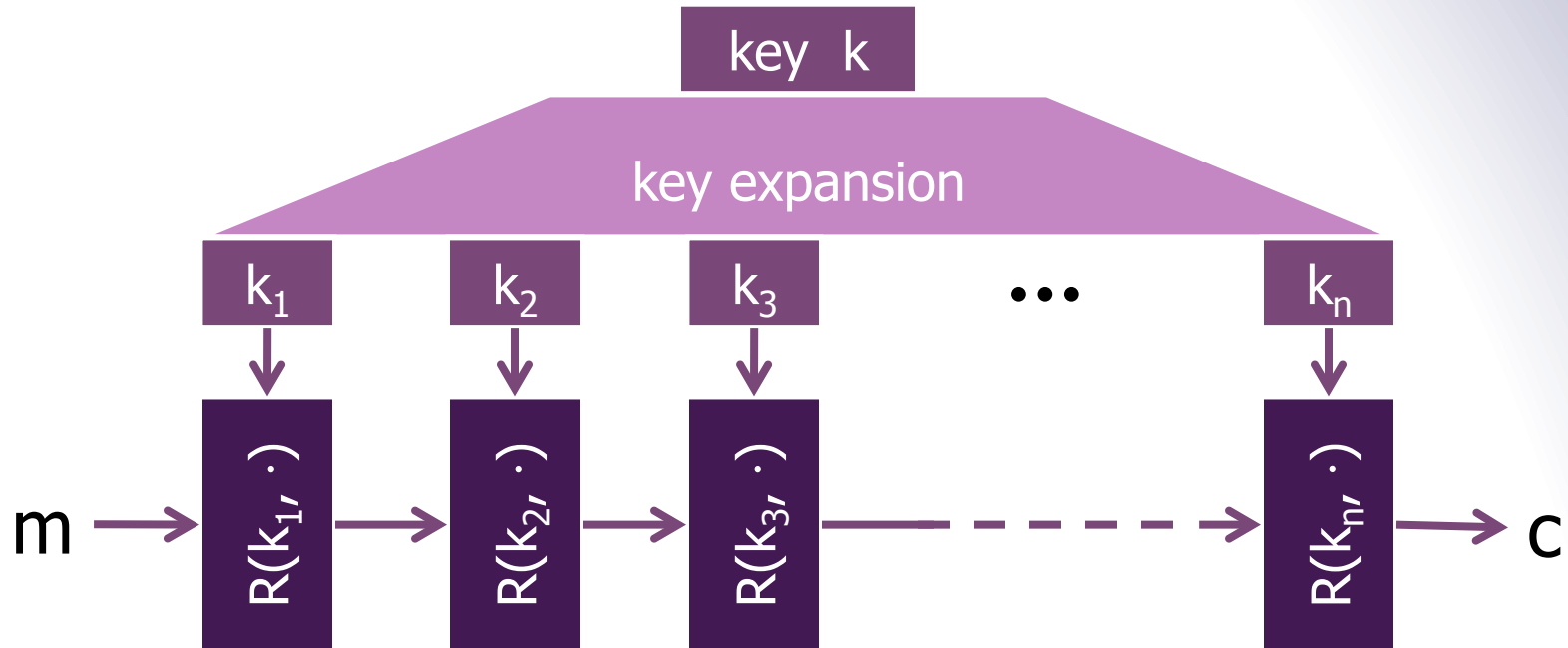


■ Canonical examples:

1. 3DES: $n = 64$ bits, $k = 168$ bits

2. AES: $n = 128$ bits, $k = 128, 192, 256$ bits

Block Ciphers Built by Iteration



$R(k, m)$ is called a round function

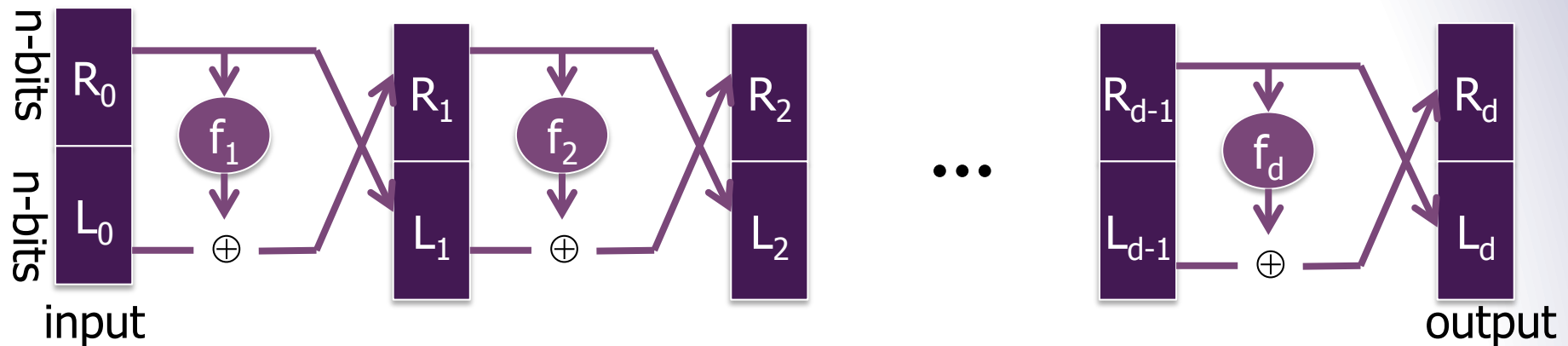
for 3DES ($n=48$), for AES-128 ($n=10$)

Block Ciphers: DES

(Feistel Network)

Given functions $f_1, \dots, f_d: \{0,1\}^n \rightarrow \{0,1\}^n$

Goal: build invertible function $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$

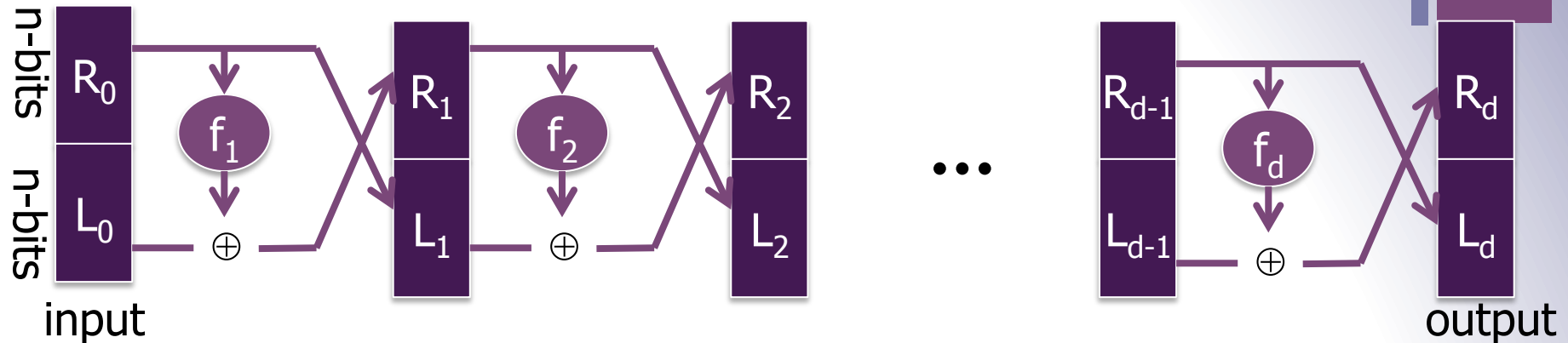


In symbols:

$$\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1} \\ L_i = R_{i-1} \end{cases}$$

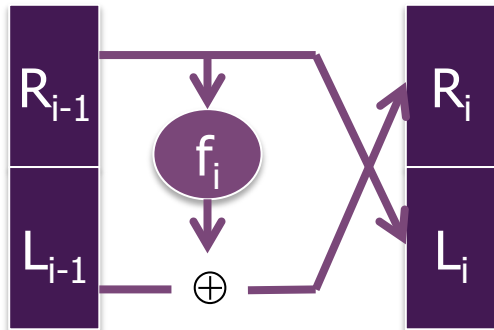
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Feistel Network



Feistel network $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$ is invertible

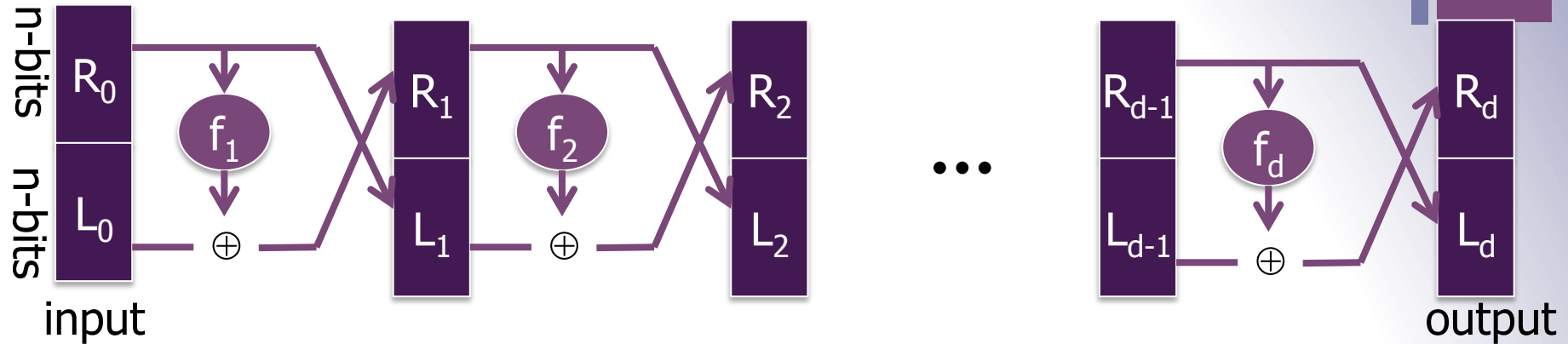
Construct inverse:



$$\left\{ \begin{array}{l} R_{i-1} = L_i \\ L_{i-1} = \end{array} \right.$$

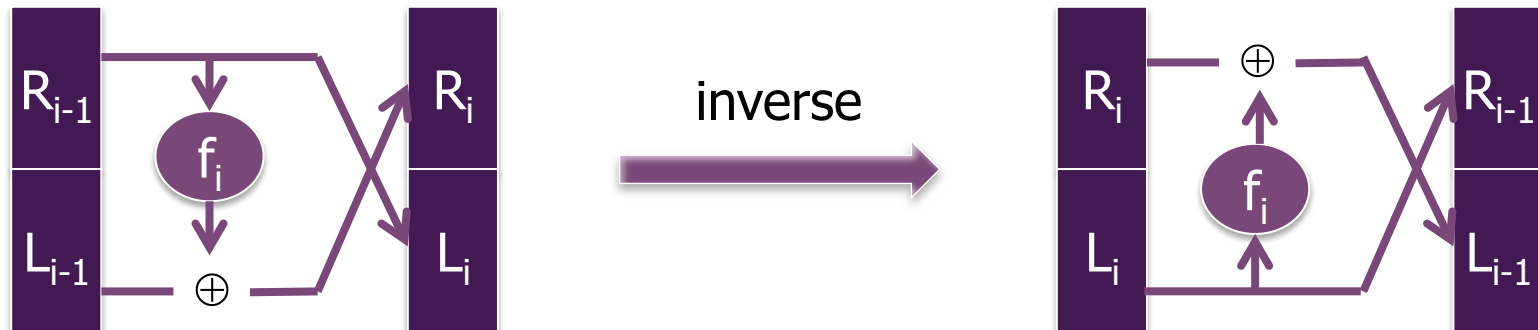
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Feistel Network inverse construction



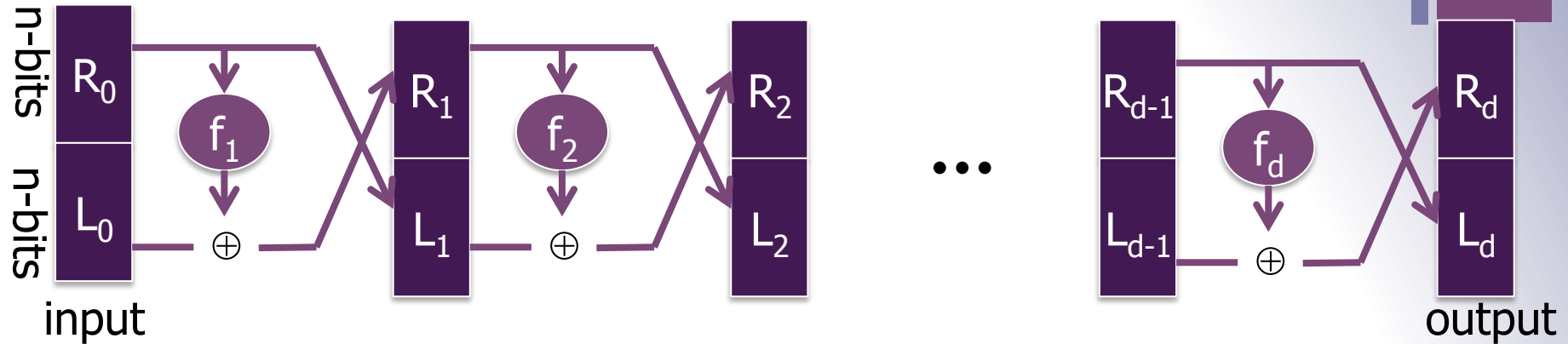
Feistel network $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$ is invertible

Construct inverse:



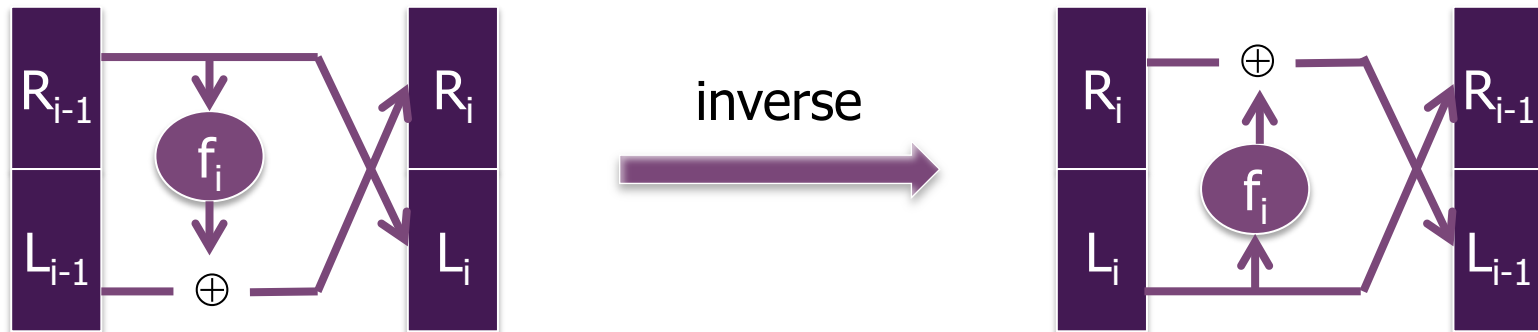
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Feistel Network Decryption



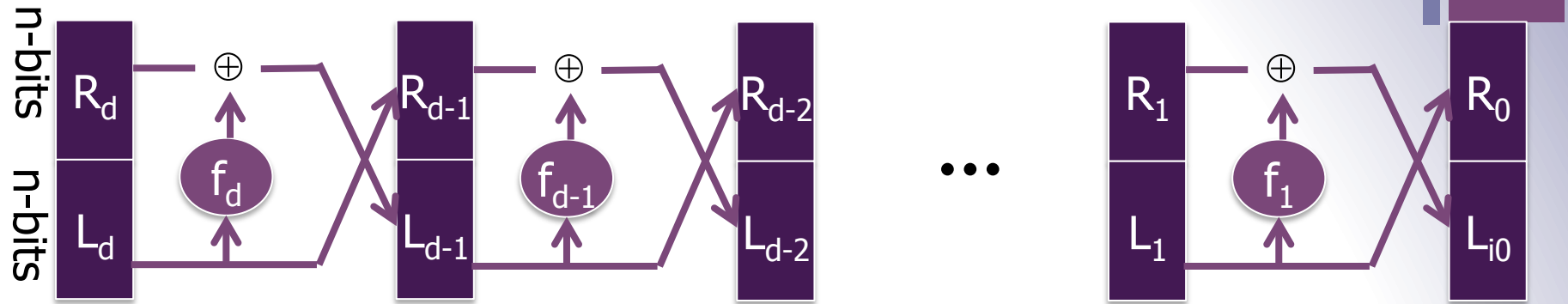
Feistel network $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$ is invertible

Construct inverse:





Feistel Network Decryption



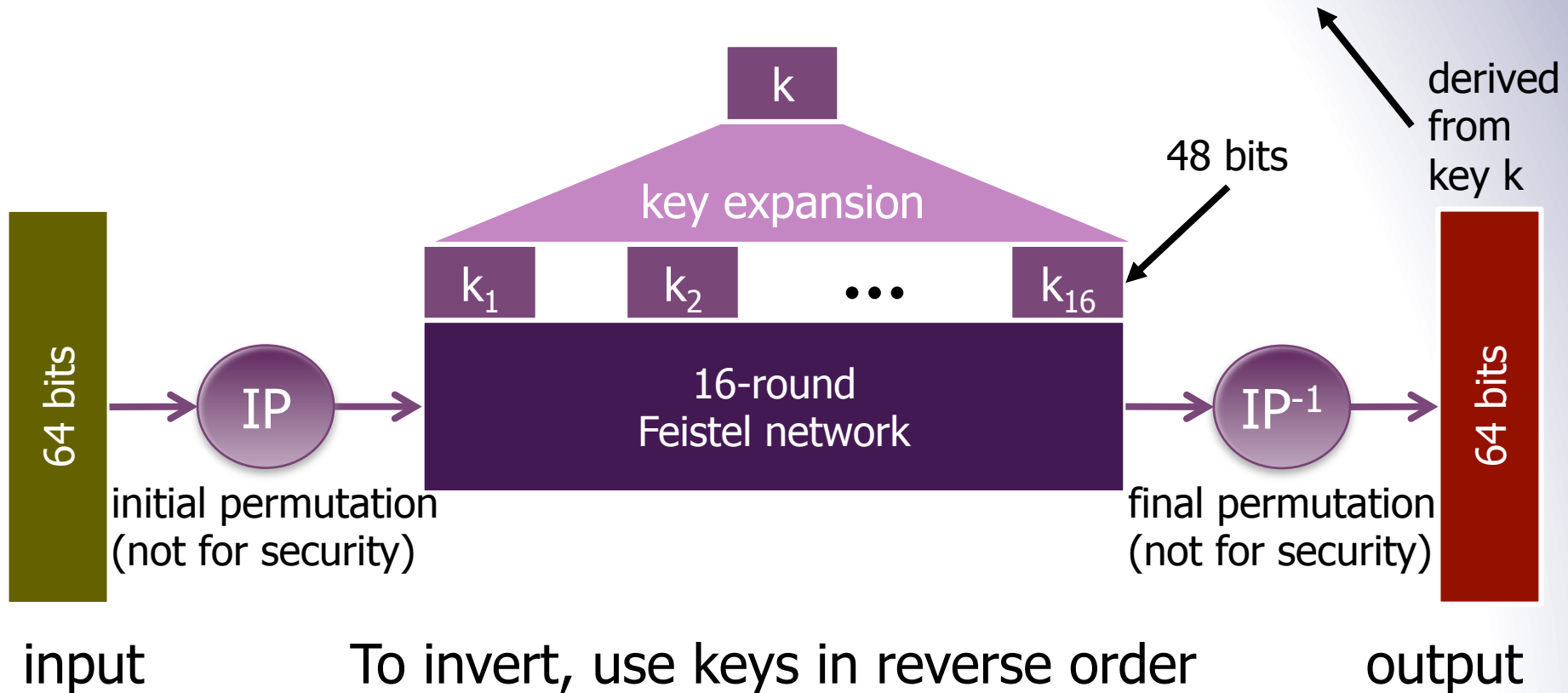
- Inversion is basically the same circuit, with f_1, \dots, f_d applied in reverse order
- General method for building invertible functions (block ciphers) from arbitrary functions
- Used in many block ciphers ... but not AES

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DES: 16-round Feistel Network

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$$f_1, \dots, f_{16}: \{0,1\}^{32} \rightarrow \{0,1\}^{32}, \quad f_i(x) = \mathbf{F}(k_i, x)$$





DES challenge

msg = "The unknown messages is: XXXX ..."

CT = c_1 c_2 c_3 c_4 ...

Goal: find $k \in \{0,1\}^{56}$ s.t. $\text{DES}(k, m_i) = c_i$ for $i=1,2,3$

1997: Internet search -- **3 months**

1998: EFF machine (deep crack) -- **3 days** (250K \$)

1999: combined search -- **22 hours**

2006: COPACOBANA (120 FPGAs) -- **7 days** (10K \$)

\Rightarrow 56-bit ciphers should not be used !! (128-bit key $\Rightarrow 2^{72}$ days)

Triple-DES

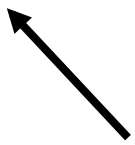
Strengthening DES against exhaustive search

- Let $E : K \times M \rightarrow M$ be a block cipher
- Define $\mathbf{3E} : K^3 \times M \rightarrow M$ as

$$\mathbf{3E}((k_1, k_2, k_3), m) = E(k_1, D(k_2, E(k_3, m)))$$

For 3DES: key-size = $3 \times 56 = 168$ bits.
3×slower than DES.

$k_1 = k_2 = k_3$
 \Rightarrow single DES



Advanced Encryption Standard (AES)

- National Institute of Standards & Technology NIST
 - Computer Security Research Center (CSRC)
 - <http://csrc.nist.gov/>
- Uses the Rijndael algorithm
 - Invented by Belgian researchers
Dr. Joan Daemen & Dr. Vincent Rijmen
 - http://jda.noekeon.org/JDA_VRI_Rijndael_V2_1999.pdf
 - Adopted May 26, 2002
 - Key length: 128, 192, or 256 bits
 - Block size: 128 bits
 - If DES could be broken in 1 second, then AES would require
149 trillion years to be broken



Performance

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	<u>Cipher</u>	<u>Block/key size</u>	<u>Speed</u> (MB/sec)
stream	RC4		126
	Salsa20/12		643
	Sosemanuk		727
block	3DES	64/168	13
	AES-128	128/128	109

Problems with Shared Key Crypto

- Compromised key means interceptors can decrypt any ciphertext they've acquired
 - Change keys frequently to limit damage
- Distribution of keys is problematic
 - Keys must be transmitted securely
 - Use couriers?
 - Distribute in pieces over separate channels?



Trusted 3rd Parties

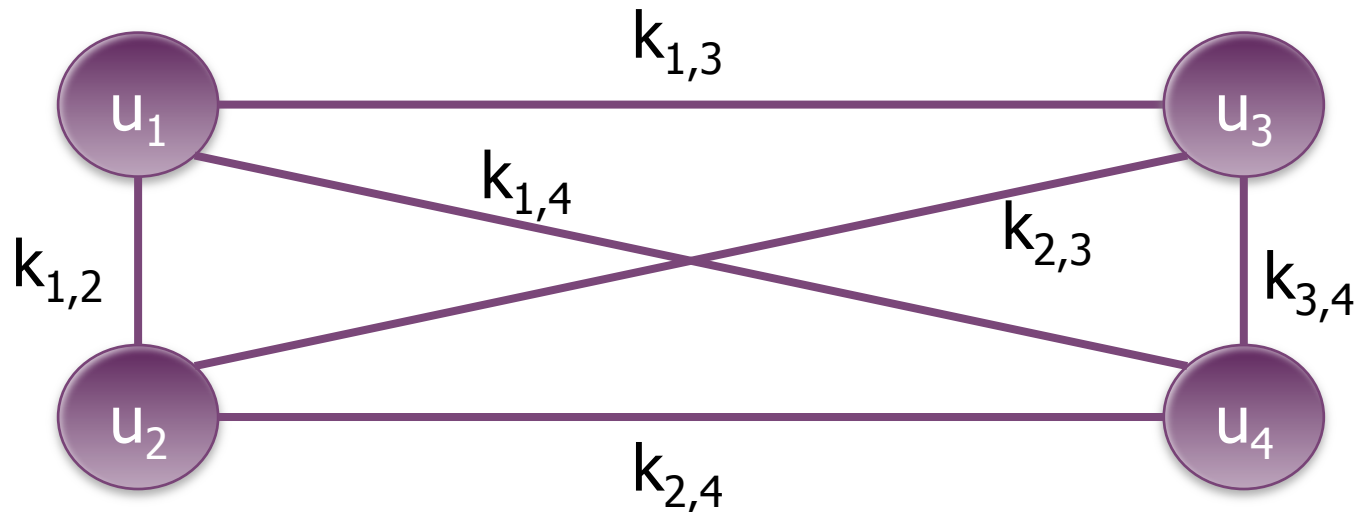
Basic key exchange



Key Management

Problem: n users

- Storing mutual secret keys is difficult

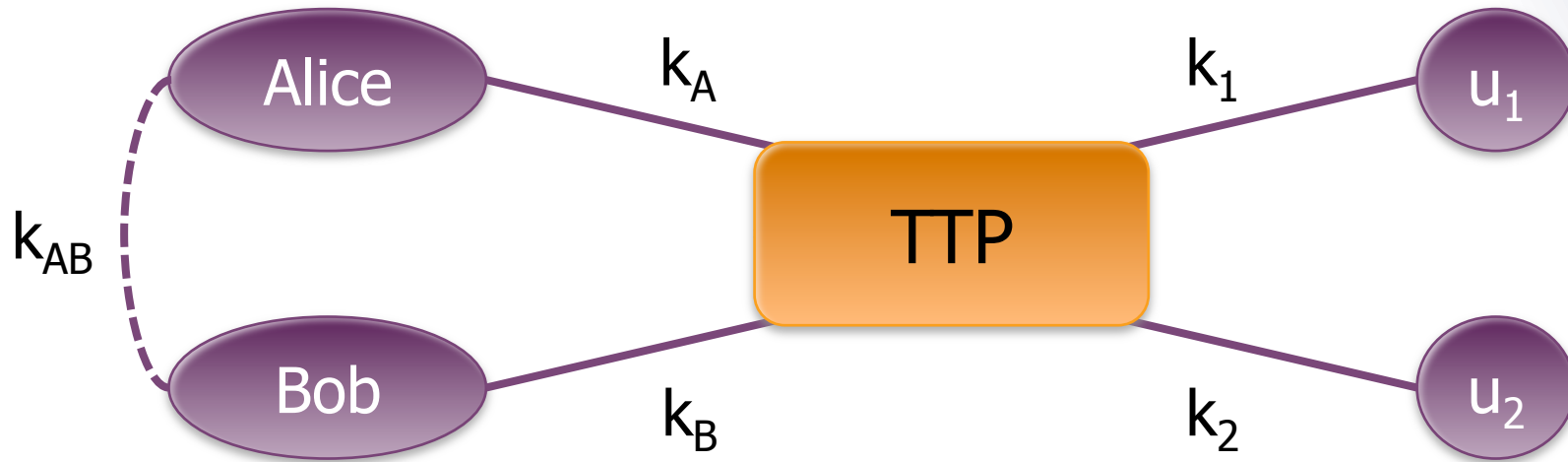


Total: $O(n)$ keys per user ; $O(n^2)$ keys in the system

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A better solution

■ Onlined Trusted 3rd Party (TTP)



■ Every user only remembers one key

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Generating keys: a toy protocol

Alice wants a shared key with Bob

- Eavesdropping security only

Bob (k_B)

Alice (k_A)

TTP

"Alice wants key with Bob"

$E(k_A, "A,B" || K_{AB})$

choose
random k_{AB}

$ticket \leftarrow E(k_B, "A,B" || K_{AB})$

ticket

k_{AB}

k_{AB}

(E,D) a CPA-secure cipher:
attacker cannot distinguish between
an encrypted value and a random one

Generating keys: a toy protocol

Alice wants a shared key with Bob

- Eavesdropping security only

Eavesdropper sees:

$$E(k_A, \text{"A, B"} \parallel k_{AB}) \ ; \ E(k_B, \text{"A, B"} \parallel k_{AB})$$

(E,D) is CPA-secure \Rightarrow

eavesdropper learns nothing about k_{AB}

- TTP needed for every key exchange, knows all session keys
- In a corporate environment environment might make sense
 - Example: Kerberos system



Toy protocol: insecure against active attacks

Example: insecure against replay attacks

Attacker records session between Alice and merchant Bob

- For example a book order

Attacker replays session to Bob

- Bob thinks Alice is ordering another copy of book



Key question

Can we generate shared keys without an **online** TTP?

Answer: yes!

Starting point of public-key cryptography:

- Merkle (1974), Diffie-Hellman (1976), RSA (1977)
- Further references:
 - More recently: ID-based enc. (BF 2001), Functional enc. (BSW 2011)



Public key cryptography



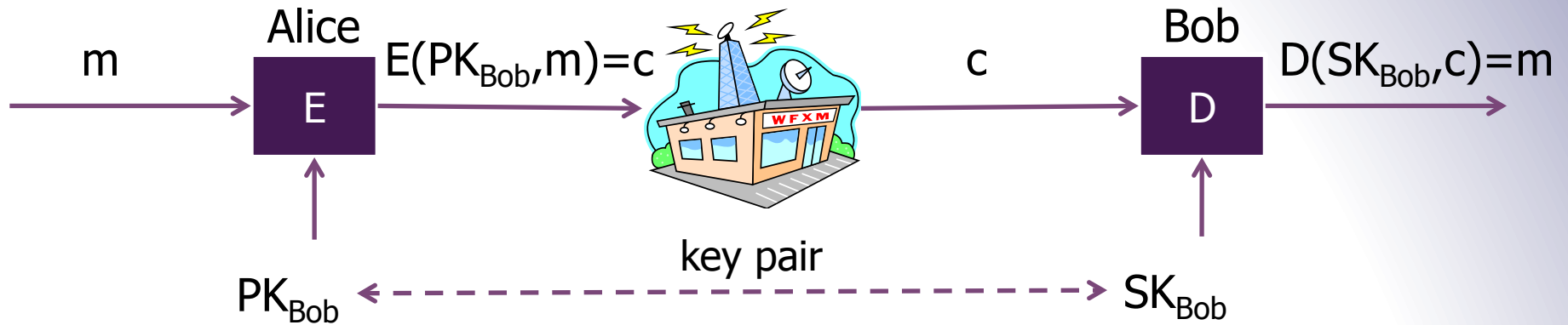
Asymmetric or Public Key Crypto

- Sender encrypts using a ***public*** key
- Receiver decrypts using a ***private*** key
- Only the private key must be kept secret!
 - Public key can be distributed at will
- Constructions generally rely on hard problems from number theory or algebra (e.g., FACT)
- Can be used for digital signatures
- Examples: RSA, El Gamal, DSA

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Public Key Encryption

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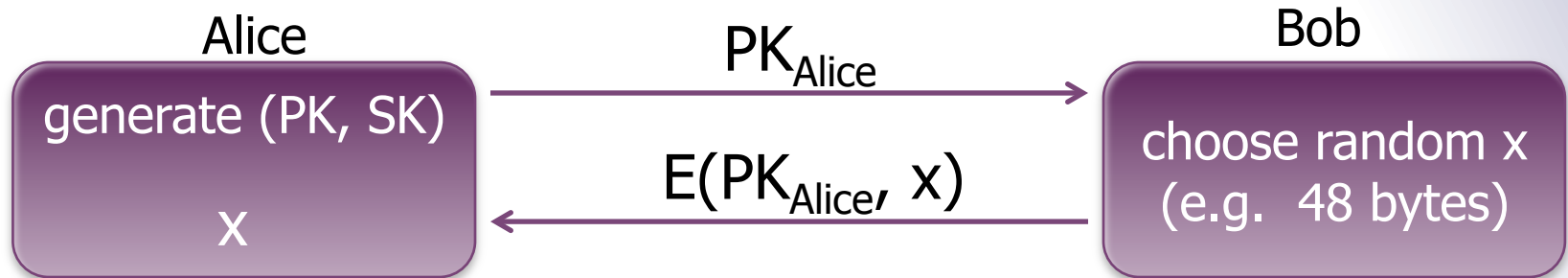
PK: public key , SK: secret key (e.g., 1024 bits)

Example: Bob generates (PK_{Bob}, SK_{Bob}) and gives PK_{Bob} to Alice

- Sometimes E is the same algorithm as D
- Fast in software or hardware implementations

Applications

Session setup (for now, only eavesdropping security)



Non-interactive applications: (e.g., Email)

- Bob sends email to Alice encrypted using PK_{Alice}
- Note: Bob needs PK_{Alice} (public key management)

Public Key Encryption

Def: a public-key cryptosys. is a triple of algs. (G, E, D)

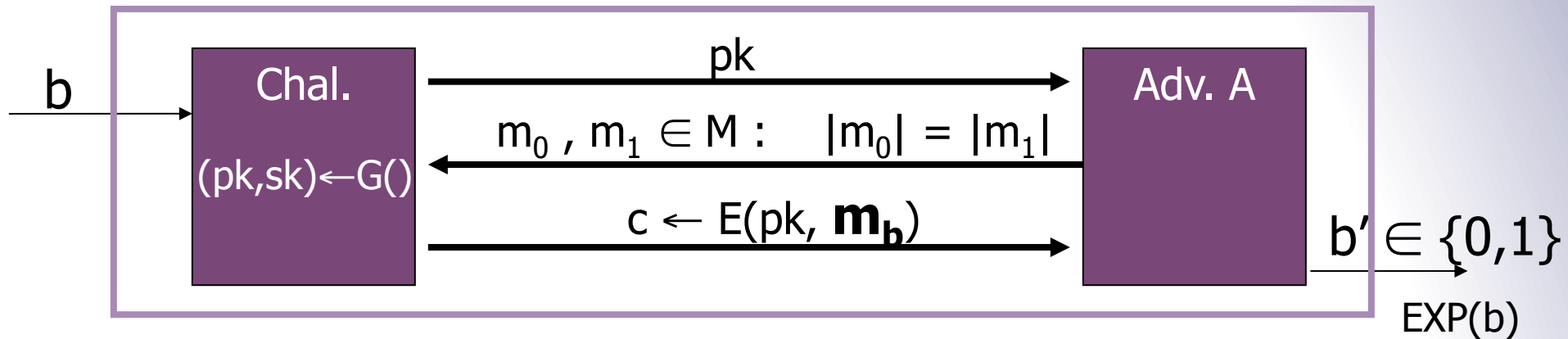
- $G()$: randomized alg. outputs a key pair (PK, SK)
- $E(PK, m)$: randomized alg. that takes $m \in M$ and outputs $c \in C$
- $D(SK, c)$: deterministic alg. that takes $c \in C$ and outputs $m \in M$ or \perp

Consistency: $\forall (PK, SK)$ output by G :

$$\forall m \in M: D(SK, E(PK, m)) = m$$

Semantic Security

For $b=0,1$ define experiments $\text{EXP}(0)$ and $\text{EXP}(1)$ as:



Def: $E = (G, E, D)$ is sem. secure if for all efficient A :

$$\text{Adv}_{\text{SS}}[A, E] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right| < \text{negligible}$$



Establishing a shared secret

Alice

$(pk, sk) \leftarrow G()$

"Alice", pk

Bob

choose random
 $x \in \{0,1\}^{128}$

"Bob", $c \leftarrow E(pk, x)$

$D(sk, c) \rightarrow x$

x shared secret

Security (eavesdropping)

Adversary sees **pk, E(pk, x)** and wants **x** $\in M$

Semantic security \Rightarrow

adversary cannot distinguish

$\{ pk, E(pk, x), x \}$ from $\{ pk, E(pk, x), rand \in M \}$

\Rightarrow can derive session key from x

Note: protocol is vulnerable to man-in-the-middle

Insecure against man in the middle

The protocol is insecure against **active** attacks

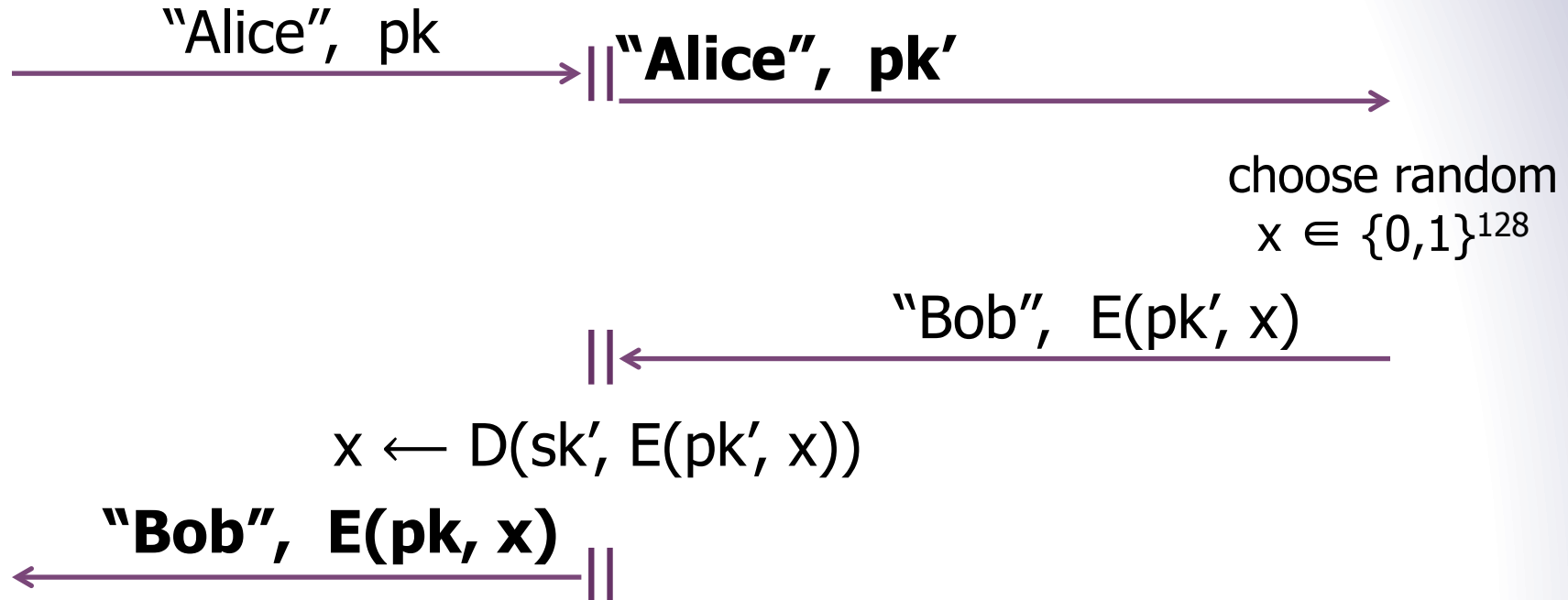
Alice

$(pk, sk) \leftarrow G()$

MiTM

$(pk', sk') \leftarrow G()$

Bob



Trade-offs for Public Key Crypto

- More computationally expensive than symmetric (shared) key crypto
 - Algorithms are harder to implement
 - Require more complex machinery
- More formal justification of difficulty
 - Hardness based on complexity-theoretic results
- A principal needs 1 private key and 1 public key
 - Number of keys for pair-wise communication is $O(n)$



RSA Algorithm

- Ron Rivest, Adi Shamir, Leonard Adleman
 - Proposed in 1979
 - They won the 2002 Turing award for this work
- Has withstood years of cryptanalysis
 - Not a guarantee of security!
 - But a strong vote of confidence
 - Further reading: Twenty years of attacks on the RSA cryptosystem, D. Boneh, Notices of the AMS, 1999
- Hardware implementations:
 - 1000 x slower than DES

RSA at a High Level

- Public and private key are derived from secret prime numbers
 - Today at least 1024 bits to ensure security (4096 bits is better)
- Plaintext message (a sequence of bits)
 - Treated as a (large!) binary number
- Encryption is modular exponentiation
- To break the encryption, conjectured that one must be able to factor large numbers
 - Not known to be in P (polynomial time algorithms)



Crypto hash functions

Hash Algorithms

- Take a variable length string
- Produce a fixed length digest

$$h: \{0,1\}^* \xrightarrow{\text{hash}} \{0,1\}^n$$

- (Non-cryptographic) Examples:

- Parity (or byte-wise XOR)
- CRC

- Realistic Example:

- The NIST Secure Hash Algorithm (SHA) takes a message of less than 2^{64} bits and produces a digest of 160 bits



Cryptographic Hashes

- Create a hard-to-invert summary of input data
- Like a check-sum or error detection code
 - Uses a cryptographic algorithm internally
 - More expensive to compute
- Sometimes called a Message Digest
- Examples:
 - Secure Hash Algorithm (SHA)
 - Message Digest (MD4, MD5)

Desired Properties

■ One way hash function

- Given a hash value y , it should be infeasible to find m s.t. $h(m)=y$

■ Collision resistance

- It should be infeasible to find two different messages m_1 and m_2 s.t. $h(m_1)=h(m_2)$

■ Random oracle property

- $h(m)$ is indistinguishable from a random n -bit value
- Attacker must spend a lot of effort to be able to modify the message without altering the hash value

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Any questions?

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