



$$\begin{aligned} &^2(y f(2) + 20(x^2)y_1 + e_2(x)y_2 + e_3(x)y_3 \\ &(x+1) = \left(\frac{x(x-2)}{2}\right)1 + (x(x-1))0 + \left(\frac{x(x-1)}{2}\right) \\ &)^2 = \left(\frac{(x-1)(x-2)}{2}\right)1 + (x(x-1))0 + \left(\frac{x(x-1)}{2}\right) \\ &)^2(y + 6x + x^2)^4 - (2x + x^2 + 8x)^2(y + 9x + 6)^4(x + 1) \\ &1)(x + 6)^4(x + 9)^4 \quad x(x + 6)^2(x + 2)^4 \\ &-9b + \sqrt{3}\sqrt{4a^3 + 27b^2})^{1/3} + 6x)^2(y + 10x + 8)^4x + 1 \\ &\frac{2^{1/3}3^{2/3}}{x(x+6)^2} \frac{(y+9x+6)^4}{(y+8x+4)^2} \\ &\frac{(1-i\sqrt{3})(-9b+\sqrt{3}\sqrt{4a^3+27b^2})^{1/3}}{2^{4/3}3^{2/3}x+9} \frac{(y+8x+4)^4}{(y+8x+4)^4} \end{aligned}$$

# Cryptography 1

INGI2347: COMPUTER SYSTEM SECURITY (Spring 2016)

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# Plan for today

## Lecture 4

- What's crypto? 
- Symmetric key encryption
- Trusted 3<sup>rd</sup> parties
- Public key encryption
- Crypto hash functions



# κρυπτο γραφη (Cryptography)

- Greek for “secret writing”
- Confidentiality
  - Obscure a message from eaves-droppers
- Integrity
  - Assure recipient that the message was not altered
- Authentication
  - Verify the identity of the source of a message
- Non-repudiation
  - Convince a 3<sup>rd</sup> party that what was said is accurate



# Cryptography is everywhere

## **Secure communication:**

- web traffic: HTTPS
- wireless traffic: 802.11i WPA2 (and WEP), GSM, Bluetooth

## **Encrypting files on disk:** EFS, TrueCrypt

## **Content protection** (e.g. DVD, Blu-ray): CSS, AACS

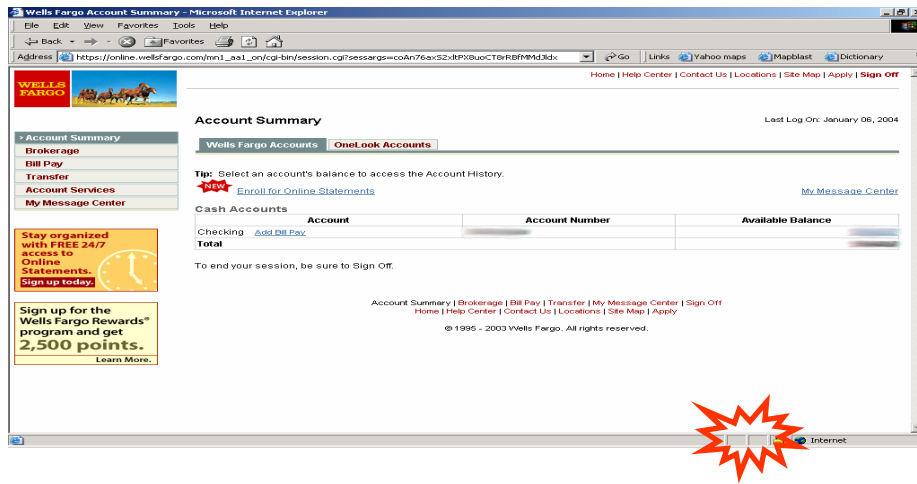
## **User authentication**

... and much much more



# Secure communication

5



Alice

HTTPS  
SSL / TLS

Bob



no eavesdropping  
no tampering



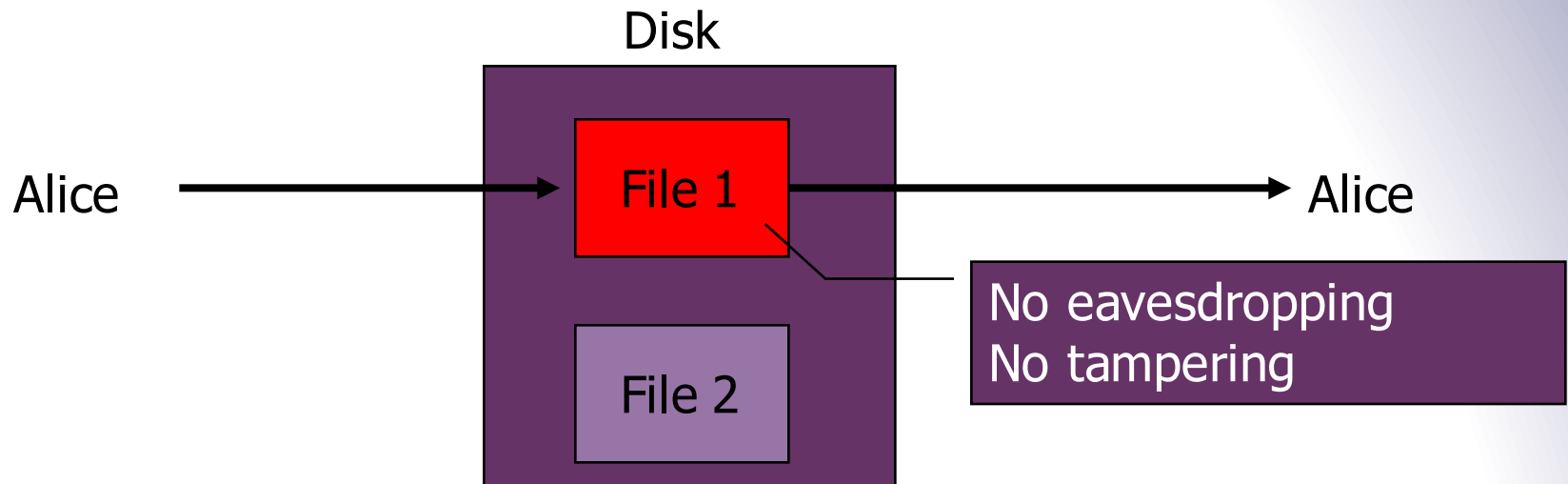
# Secure Sockets Layer / TLS

## Two main parts

1. Handshake Protocol: **Establish shared secret key using public-key cryptography**
2. Record Layer: **Transmit data using shared secret key**  
Ensure confidentiality and integrity



# Protected files on disk



Analogous to secure communication:  
Alice today sends a message to Alice tomorrow

# Terminology



## ■ Encryption algorithm

- Transforms a plaintext into a ciphertext that is unintelligible for non-authorized parties
- Usually parametrized with a cryptographic key

## ■ Asymmetric (Public) key cryptography

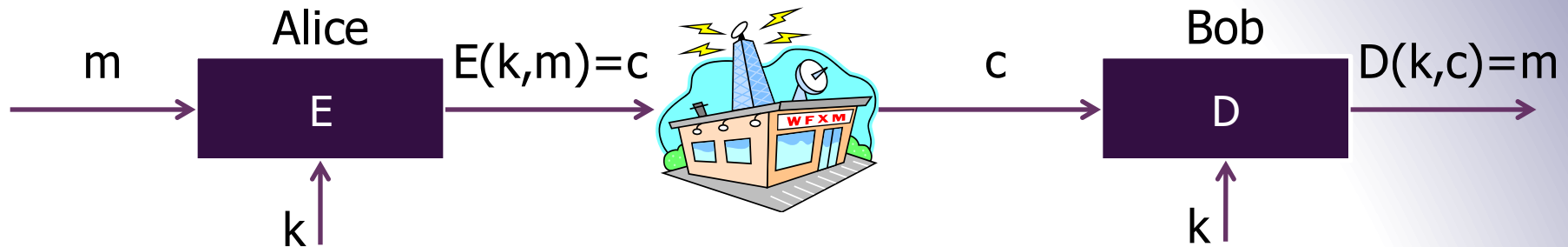
- Crypto system: encryption + decryption algorithms + key generation

## ■ Symmetric (Shared) key cryptography

- Cipher/decipher: symmetric-key encryption/decryption algorithms



# Build block: sym.-key encryption



$E, D$ : cipher

$k$ : secret key (e.g., 128 bits)

$m, c$ : plaintext, cipher text

- Same secret key for both encryption and decryption
- Encryption algorithm is publicly known
  - Never use a proprietary algorithm
- Fast in software or hardware implementations



# Use Cases

- **Single use key:** (one time key)
  - Key is only used to encrypt one message
    - encrypted email: new key generated for every email
- **Multi use key:** (many time key)
  - Key used to encrypt multiple messages
    - encrypted files: same key used to encrypt many files
    - SSL: same key used to encrypt many packets
  - Need more machinery than for one-time key

# Things To Remember

## ■ Cryptography is:

- A tremendous tool
- The basis for many security mechanisms

## ■ Cryptography is **NOT**:

- The solution to all security problems
- Reliable unless implemented and used properly
- Something you should try to invent yourself
  - many many examples of broken ad-hoc designs
- Privacy | Steganography | (Encoding|Decoding)



# Cryptography is NOT

## ■ Privacy

- Ability to control how personal information spreads in a community

## ■ Steganography

- Science of information hiding

## ■ Encoding | Decoding

- Code is a system of symbols which represent information
- Encoding transforms information into a codeword
- Decoding recovers information from a codeword

# Cryptanalysis: a rigorous science

Proves or disproves the security of a cryptosystem

## ■ The three steps:

- Precisely specify threat model
- Propose a construction
- Prove that breaking construction under threat mode will solve an underlying hard problem

## ■ Definition of break:

- Decrypting (totally or partially) a given ciphertext
- Recovering the key of the cryptosystem
- Proving a cryptosystem is less secure than what is claimed



# Symmetric-key cryptography

Assumes parties already share a secret key

# Symmetric Ciphers: definition

Def: a **cipher** defined over  $(K, M, C)$

is a pair of “efficient” algs  $(E, D)$  where

$$E : K \times M \rightarrow C, \quad D : K \times C \rightarrow M$$

$$\text{s.t. } \forall m \in M, k \in K : D(k, E(k, m)) = m$$

E is often randomized   |   D is always deterministic

# Stream vs. Block Ciphers

## ■ Stream ciphers

- Act on the plaintext one symbol at a time
- **Examples:** RC4, GSM A5-1, Bluetooth E0, CSS, ...
- High speed rate, hardware implementations very cheap
- Security analysis is not well established

## ■ Block ciphers

- Act on the plaintext in blocks of symbols
- **Examples:** DES, 3DES, AES, IDEA, Blowfish, RC5, Kasumi, Safer, ...
- Suited to software implementations on various systems
  - (e.g., 8-bit, 32-bit, 64-bit processors)
- Security analysis is well established



# Stream Ciphers: The One Time Pad (Vernam 1917)

First example of a “secure” cipher

$$M = C = \{0,1\}^n, \quad K = \{0,1\}^n$$

key = (random bit string as long the message)

$$E(k, m) = k \oplus m$$

$$D(k, c) = k \oplus c$$

msg: 0 1 1 0 1 1 1

key: 1 0 1 1 0 1 0

$\oplus$

---

CT: 1 1 0 1 1 0 1

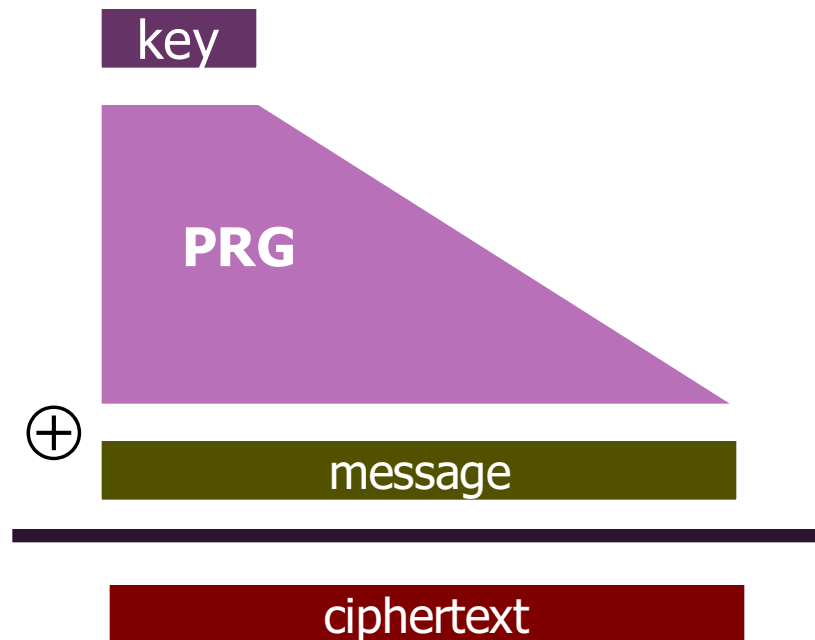
Indeed:

$$D(k, E(k, m)) = D(k, k \oplus m) = k \oplus (k \oplus m) = (k \oplus k) \oplus m = 0 \oplus m = m$$

# Stream ciphers (single key use)

**Problem:** OTP key is as long the message

**Solution:** Pseudo random key – stream ciphers



$$c \leftarrow \text{PRG}(k) \oplus m$$

# Dangers in using stream ciphers

One time key !!      “Two time pad” is insecure:

$$\begin{cases} C_1 \leftarrow m_1 \oplus \text{PRG}(k) \\ C_2 \leftarrow m_2 \oplus \text{PRG}(k) \end{cases}$$

Eavesdropper does:

$$C_1 \oplus C_2 \rightarrow m_1 \oplus m_2$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \rightarrow m_1, m_2$$

# What is a secure cipher?

Attacker's abilities: **obtains one ciphertext** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$$E(k, m) = m$$

Insecure: **leaks m**

attempt #2: **attacker cannot recover all of plaintext**

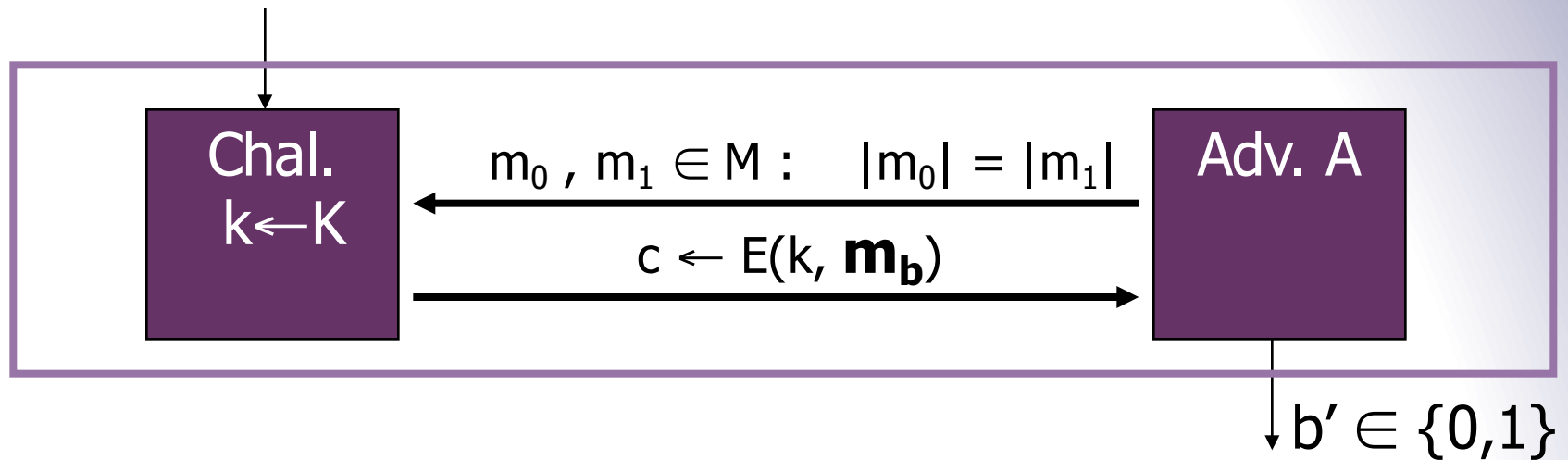
$$E(k, m_0 || m_1) = m_0 || E(k, m_1) \quad \text{Insecure: } \mathbf{leaks\ } m_0$$

Shannon's information-theoretic perfect secrecy:

**ciphertext should reveal no "info" about plaintext**

# Semantic Security (one-time key)

For  $b=0,1$  define experiments  $\text{EXP}(0)$  and  $\text{EXP}(1)$  as:



for  $b=0,1$ :  $W_b := [\text{event that } \text{EXP}(b)=1]$

$$\text{Adv}_{\text{SS}}[A, E] := \left| \Pr[W_0] - \Pr[W_1] \right| \in [0,1]$$

Sematic Security Advantage of A against E

# Semantic Security (one-time key)

Def: E is **semantically secure** if for all efficient A

$$\text{Adv}_{\text{SS}}[A, E] \quad \text{is negligible}$$

# Model of the attacker (also for PK)

## ■ Chosen-ciphertext attack (CCA)

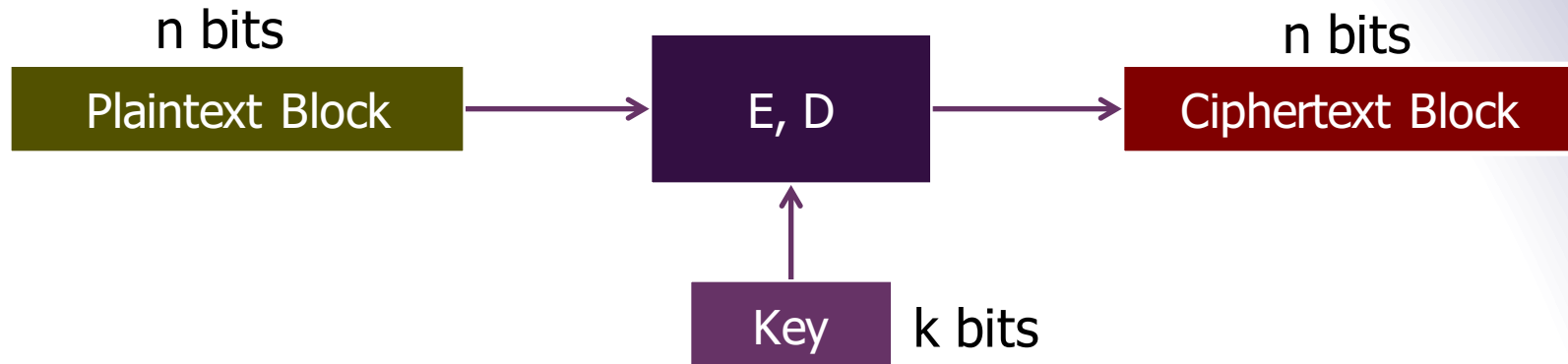
- The attacker has access to a **decryption** oracle: he can choose ciphertexts (other than the ciphertext he is challenged with) and get their corresponding plaintext

## ■ Chosen-plaintext attack (CPA)

- The adversary has access to an **encryption** oracle: he can choose plaintexts and get their corresponding ciphertexts
- More powerful than CCA



# Block Ciphers



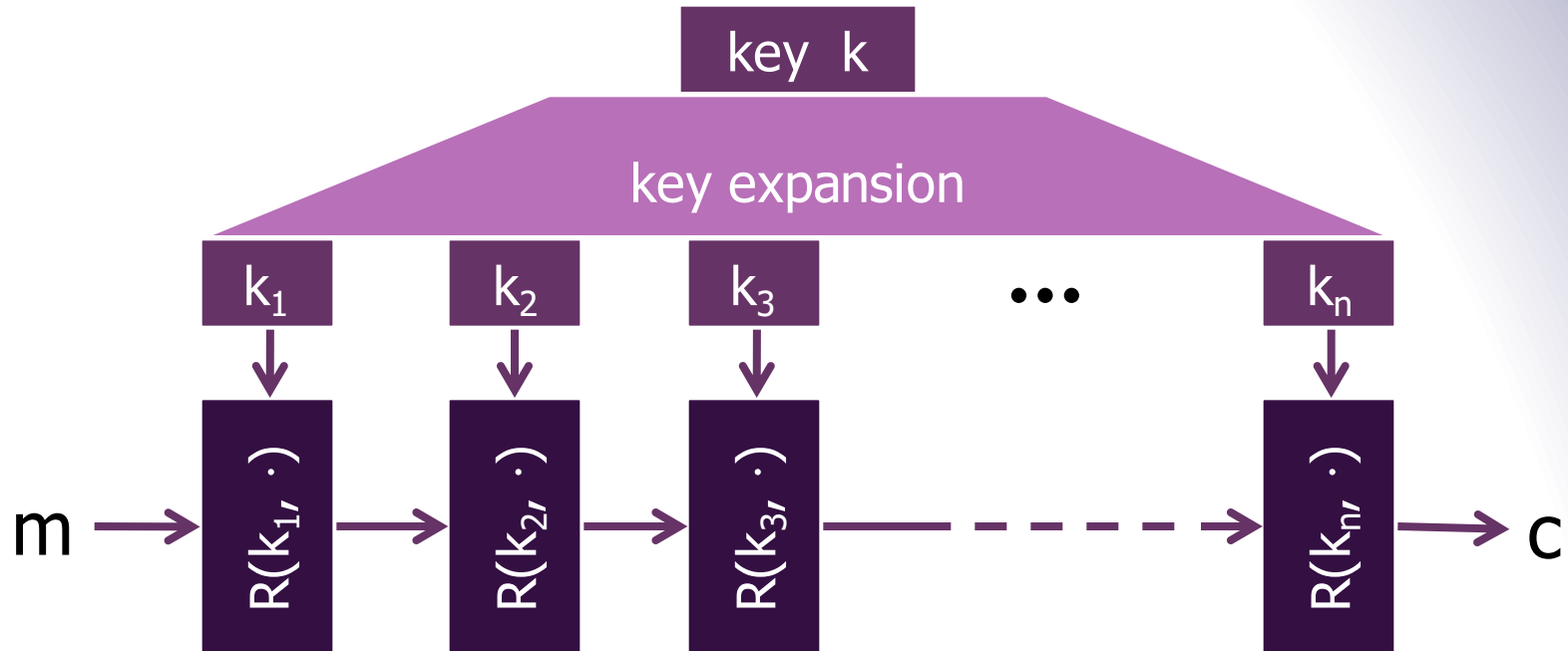
## ■ Canonical examples:

1. 3DES:  $n = 64$  bits,  $k = 168$  bits

2. AES:  $n = 128$  bits,  $k = 128, 192, 256$  bits



# Block Ciphers Built by Iteration



$R(k, m)$  is called a round function

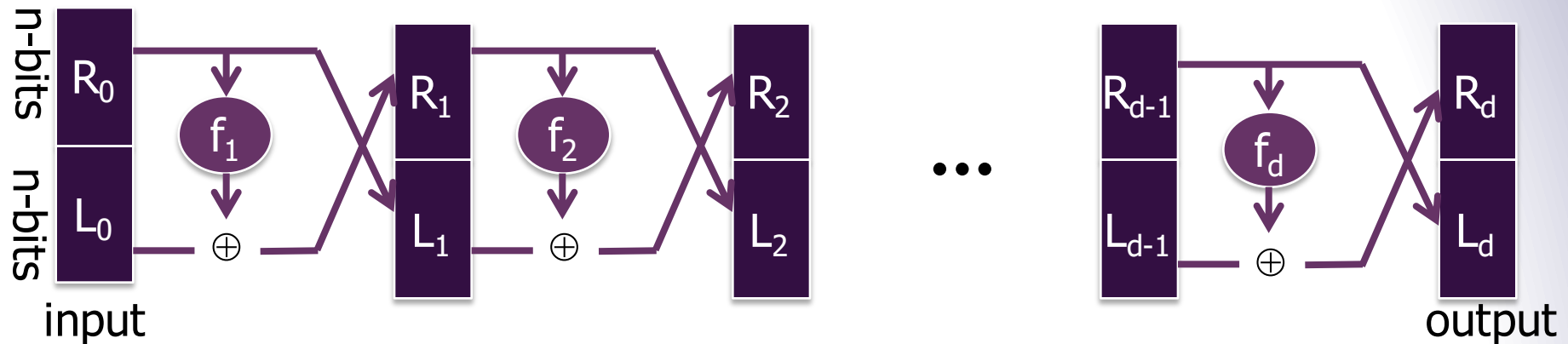
**for 3DES ( $n=48$ ),      for AES-128 ( $n=10$ )**

# Block Ciphers: DES

## (Feistel Network)

Given functions  $f_1, \dots, f_d: \{0,1\}^n \rightarrow \{0,1\}^n$

Goal: build invertible function  $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$

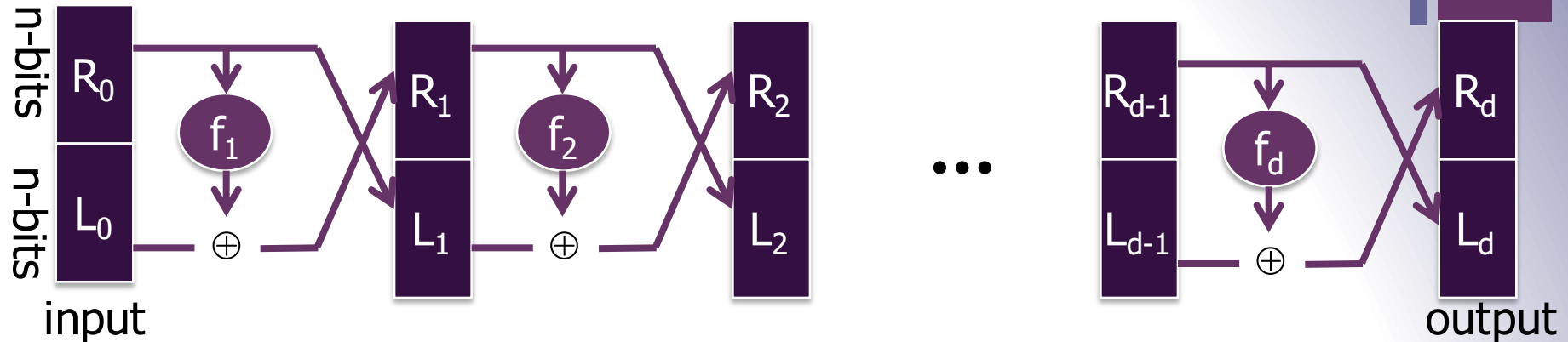


In symbols:

$$\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1} \\ L_i = R_{i-1} \end{cases}$$

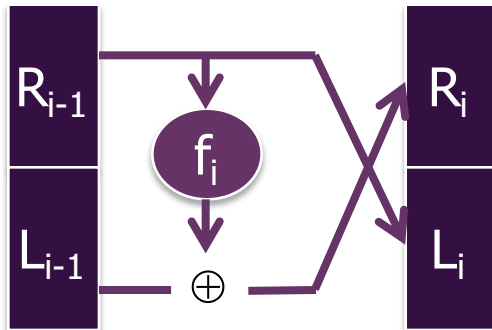
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# Feistel Network



Feistel network  $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$  is invertible

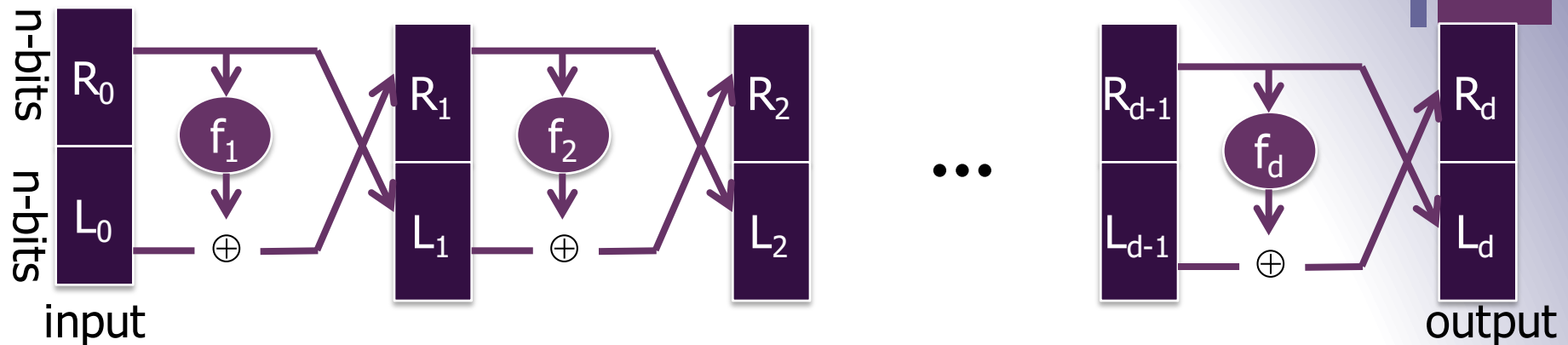
Construct inverse:



$$\left\{ \begin{array}{l} R_{i-1} = L_i \\ L_{i-1} = \end{array} \right.$$

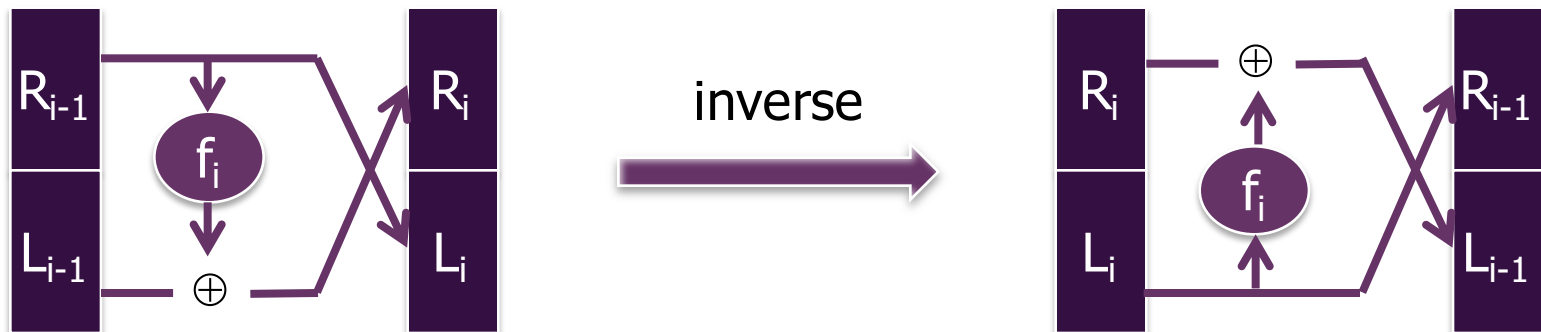
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# Feistel Network inverse construction



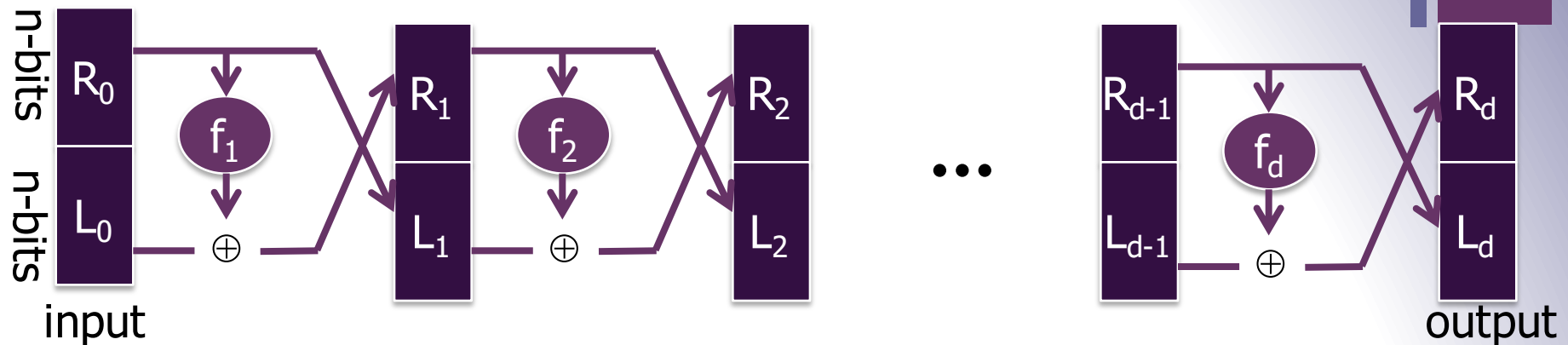
Feistel network  $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$  is invertible

Construct inverse:



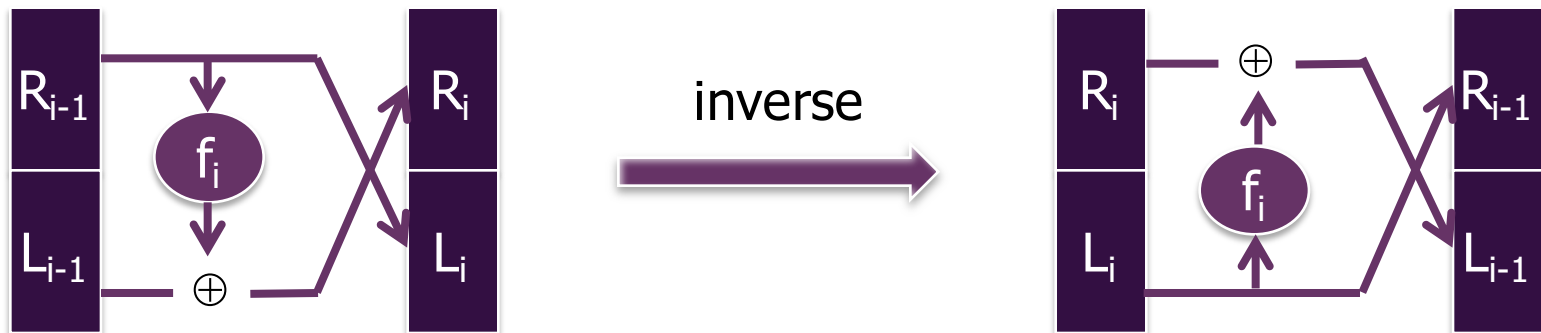
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# Feistel Network Decryption



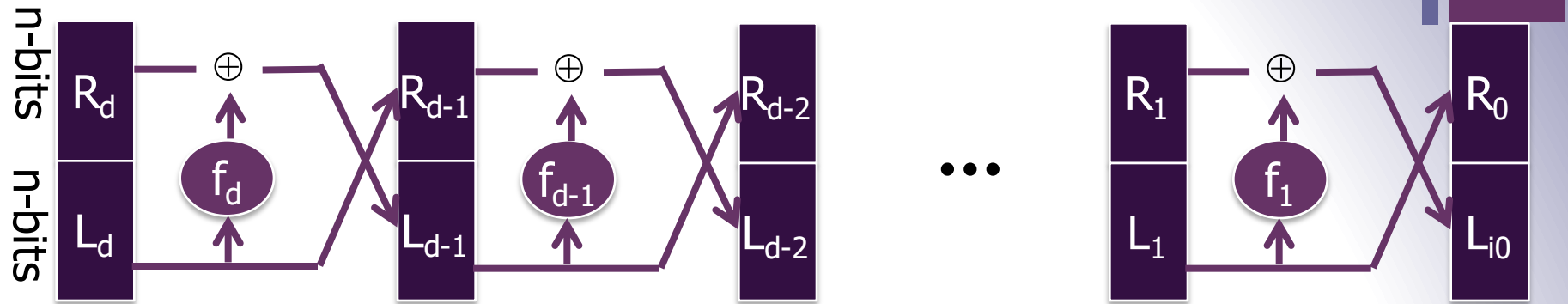
Feistel network  $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$  is invertible

Construct inverse:





# Feistel Network Decryption



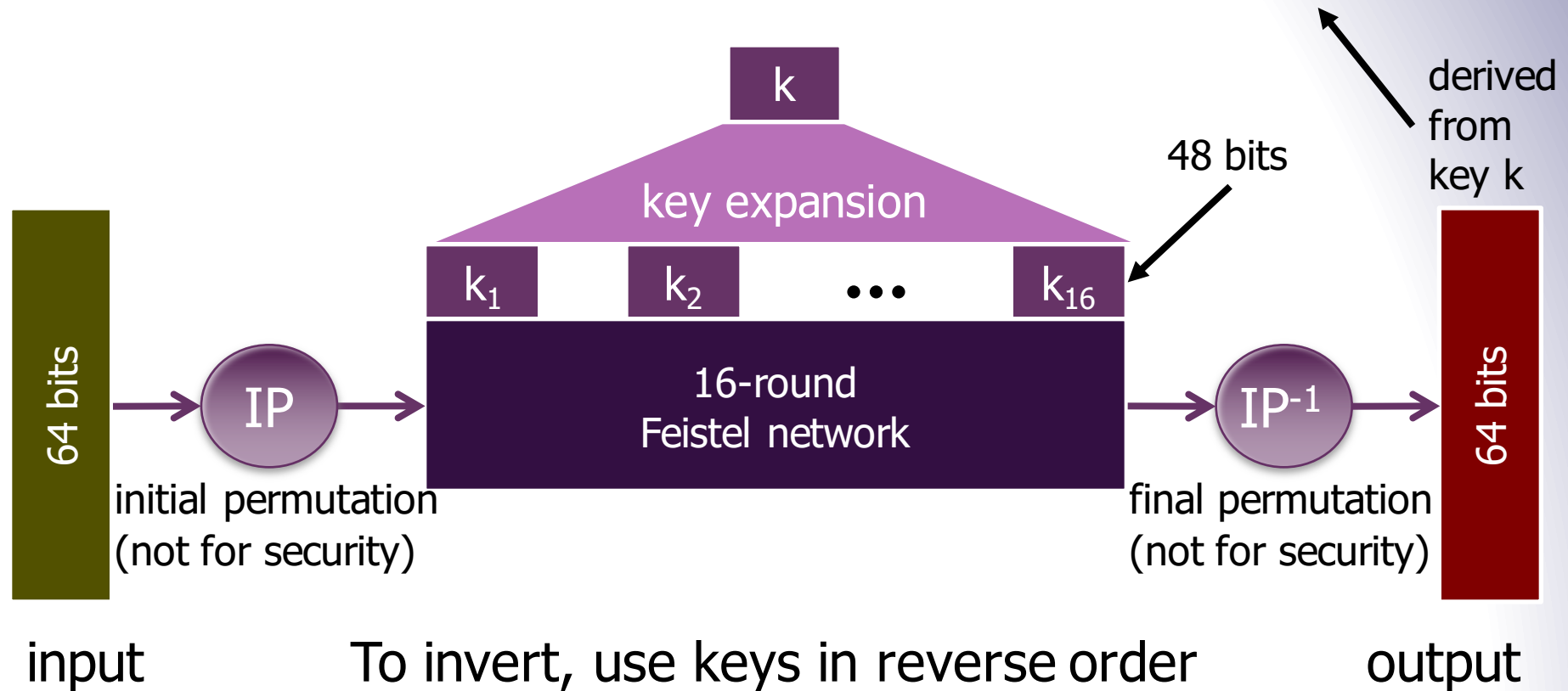
- Inversion is basically the same circuit, with  $f_1, \dots, f_d$  applied in reverse order
- General method for building invertible functions (block ciphers) from arbitrary functions
- Used in many block ciphers ... but not AES

+

# DES: 16-round Feistel Network

31

$$f_1, \dots, f_{16}: \{0,1\}^{32} \rightarrow \{0,1\}^{32}, \quad f_i(x) = \mathbf{F}(k_i, x)$$





# DES challenge

msg = "The unknown messages is: XXXX ..."

CT =  $c_1$   $c_2$   $c_3$   $c_4$  ...

**Goal:** find  $k \in \{0,1\}^{56}$  s.t.  $\text{DES}(k, m_i) = c_i$  for  $i=1,2,3$

1997: Internet search -- **3 months**

1998: EFF machine (deep crack) -- **3 days** (250K \$)

1999: combined search -- **22 hours**

2006: COPACOBANA (120 FPGAs) -- **7 days** (10K \$)

$\Rightarrow$  56-bit ciphers should not be used !! (128-bit key  $\Rightarrow 2^{72}$  days)



# Triple-DES

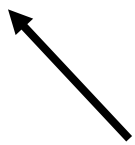
Strengthening DES against exhaustive search

- Let  $E : K \times M \rightarrow M$  be a block cipher
- Define **3E**:  $K^3 \times M \rightarrow M$  as

$$\mathbf{3E}((k_1, k_2, k_3), m) = E(k_1, D(k_2, E(k_3, m)))$$

For 3DES: key-size =  $3 \times 56 = 168$  bits.  
3×slower than DES.

$k_1 = k_2 = k_3$   
 $\Rightarrow$  single DES



# Advanced Encryption Standard (AES)

- National Institute of Standards & Technology NIST
  - Computer Security Research Center (CSRC)
  - <http://csrc.nist.gov/>
- Uses the Rijndael algorithm
  - Invented by Belgian researchers  
Dr. Joan Daemen & Dr. Vincent Rijmen
  - [http://jda.noekeon.org/JDA\\_VRI\\_Rijndael\\_V2\\_1999.pdf](http://jda.noekeon.org/JDA_VRI_Rijndael_V2_1999.pdf)
  - Adopted May 26, 2002
  - Key length: 128, 192, or 256 bits
  - Block size: 128 bits
    - If DES could be broken in 1 second, then AES would require **149 trillion years** to be broken

# Performance

Crypto++ 5.6.0 [ Wei Dai ]

AMD Opteron, 2.2 GHz ( Linux)

	<u>Cipher</u>	<u>Block/key size</u>	<u>Speed</u> (MB/sec)
stream	RC4		126
	Salsa20/12		643
	Sosemanuk		727
block	3DES	64/168	13
	AES-128	128/128	109

# Problems with Shared Key Crypto

- Compromised key means interceptors can decrypt any ciphertext they've acquired
  - Change keys frequently to limit damage
- Distribution of keys is problematic
  - Keys must be transmitted securely
  - Use couriers?
  - Distribute in pieces over separate channels?



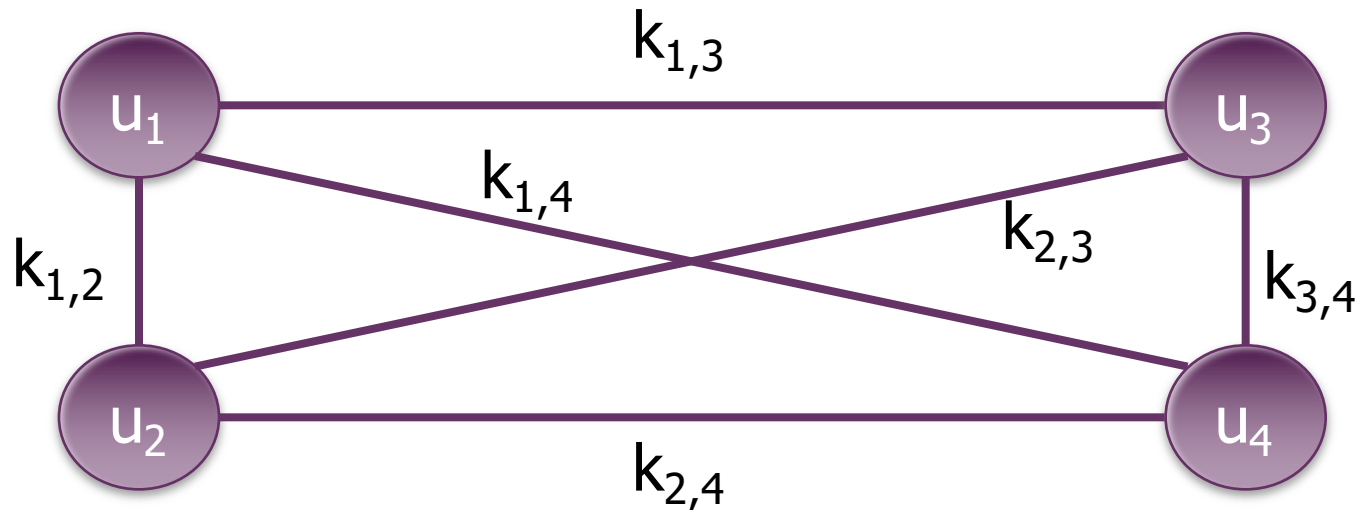
# Trusted 3<sup>rd</sup> Parties

Basic key exchange

# Key Management

Problem:  $n$  users

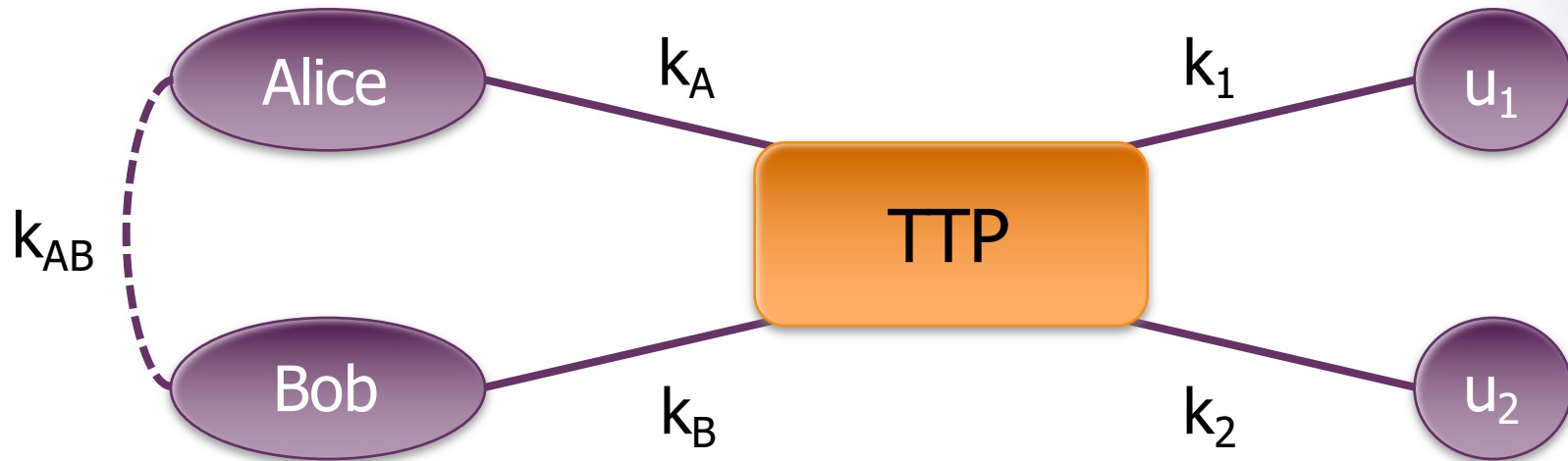
- Storing mutual secret keys is difficult



Total:  $O(n)$  keys per user ;  $O(n^2)$  keys in the system

# A better solution

## ■ Onlined Trusted 3<sup>rd</sup> Party (TTP)



## ■ Every user only remembers one key

# Generating keys: a toy protocol

Alice wants a shared key with Bob

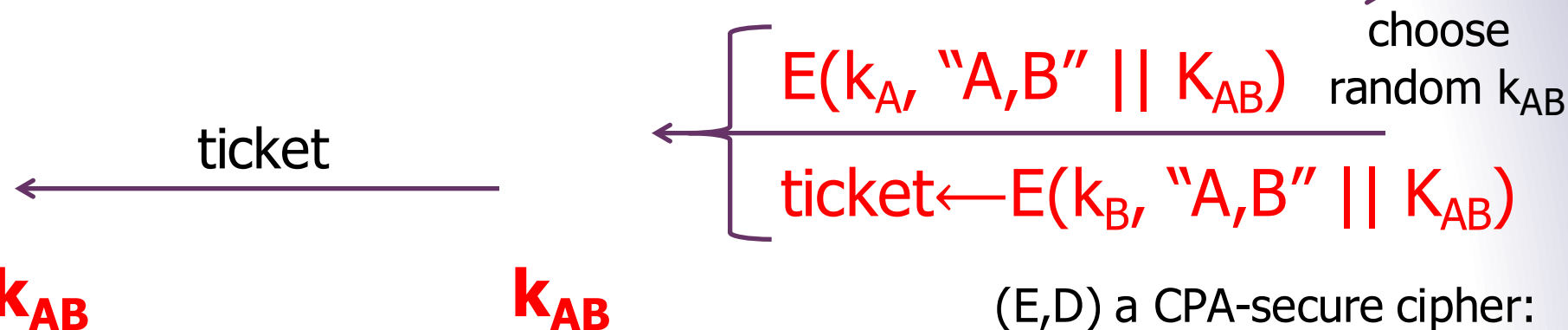
- Eavesdropping security only

Bob ( $k_B$ )

Alice ( $k_A$ )

TTP

"Alice wants key with Bob"



(E,D) a CPA-secure cipher:  
attacker cannot distinguish between  
an encrypted value and a random one



# Generating keys: a toy protocol

Alice wants a shared key with Bob

- Eavesdropping security only

Eavesdropper sees:

$$E(k_A, \text{"A, B"} \parallel k_{AB}) \ ; \ E(k_B, \text{"A, B"} \parallel k_{AB})$$

$(E, D)$  is CPA-secure  $\Rightarrow$

eavesdropper learns nothing about  $k_{AB}$

- TTP needed for every key exchange, knows all session keys
- In a corporate environment environment might make sense
  - Example: Kerberos system

# Toy protocol: insecure against active attacks

Example: insecure against replay attacks

Attacker records session between Alice and Bob

- For example a book order

Attacker replays session to Bob

- Bob thinks Alice is ordering another copy of book

# Key question

Can we generate shared keys without an **online** TTP?

Answer: yes!

Starting point of public-key cryptography:

- Merkle (1974), Diffie-Hellman (1976), RSA (1977)
- Further references:
  - More recently: ID-based enc. (BF 2001), Functional enc. (BSW 2011)

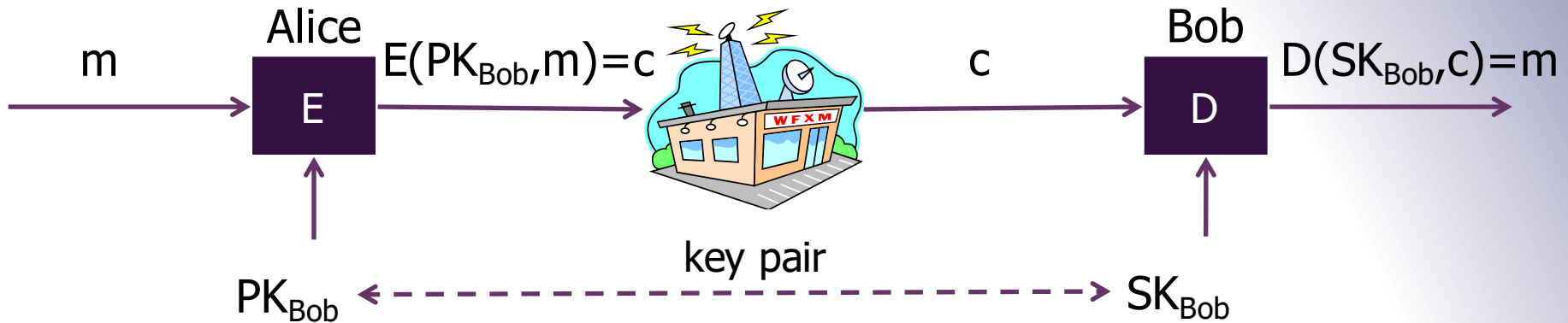


# Public key cryptography

# Asymmetric or Public Key Crypto

- Sender encrypts using a ***public*** key
- Receiver decrypts using a ***private*** key
- Only the private key must be kept secret!
  - Public key can be distributed at will
- Constructions generally rely on hard problems from number theory or algebra (e.g., FACT)
- Can be used for digital signatures
- Examples: RSA, El Gamal, DSA

# Public Key Encryption



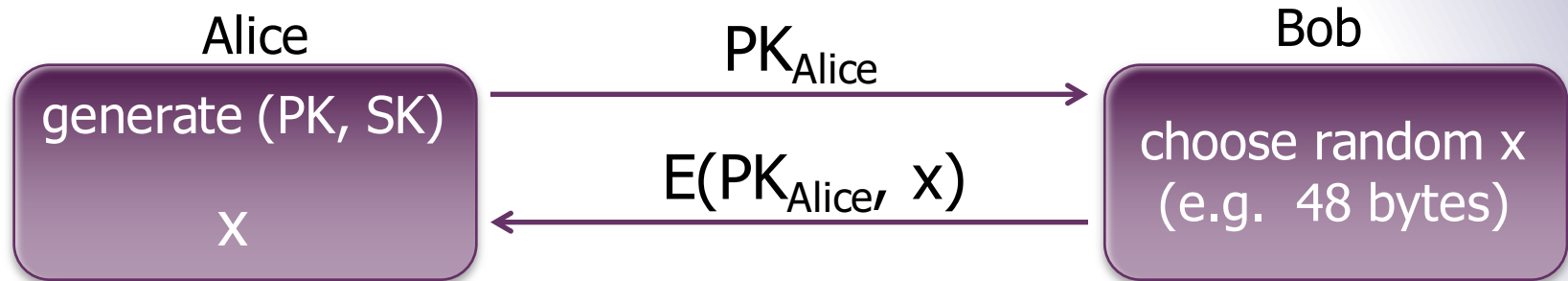
PK: public key , SK: secret key (e.g., 1024 bits)

Example: Bob generates  $(PK_{Bob}, SK_{Bob})$  and gives  $PK_{Bob}$  to Alice

- Sometimes  $E$  is the same algorithm as  $D$
- Fast in software or hardware implementations

# Applications

## Session setup (for now, only eavesdropping security)



## Non-interactive applications: (e.g., Email)

- Bob sends email to Alice encrypted using  $PK_{\text{Alice}}$
- Note: Bob needs  $PK_{\text{Alice}}$  (public key management)

# Public Key Encryption

Def: a public-key cryptosys. is a triple of algs.  $(G, E, D)$

- $G()$ : randomized alg. outputs a key pair  $(PK, SK)$
- $E(PK, m)$ : randomized alg. that takes  $m \in M$  and outputs  $c \in C$
- $D(SK, c)$ : deterministic alg. that takes  $c \in C$  and outputs  $m \in M$  or  $\perp$

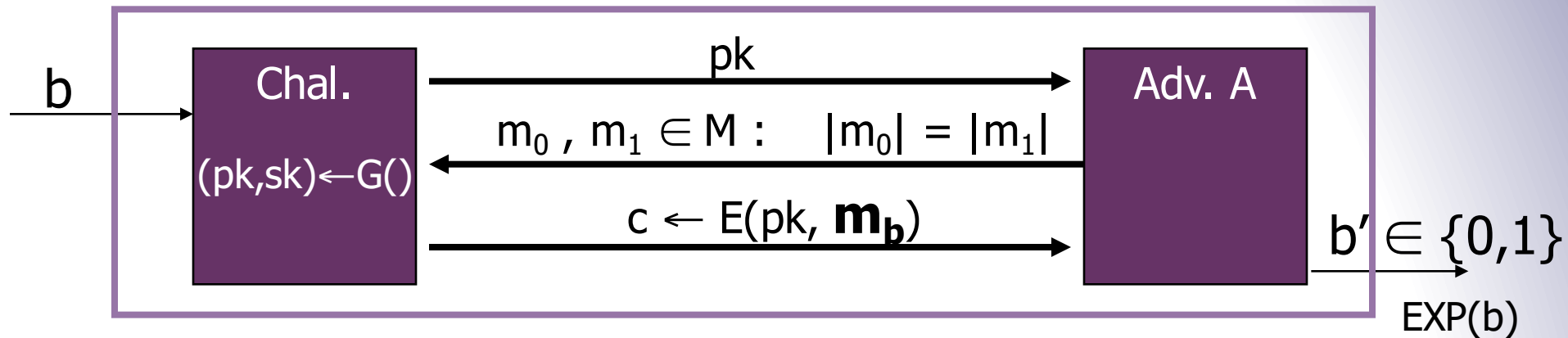
Consistency:  $\forall (PK, SK)$  output by  $G$  :

$$\forall m \in M: D(SK, E(PK, m)) = m$$



# Semantic Security

For  $b=0,1$  define experiments  $\text{EXP}(0)$  and  $\text{EXP}(1)$  as:



Def:  $E = (G, E, D)$  is sem. secure if for all efficient  $A$ :

$$\text{Adv}_{\text{SS}}[A, E] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right| < \text{negligible}$$



# Establishing a shared secret

**Alice**

**Bob**

$(pk, sk) \leftarrow G()$

"Alice",  $pk$

choose random  
 $x \in \{0,1\}^{128}$

"Bob",  $c \leftarrow E(pk, x)$

$D(sk, c) \rightarrow x$

$x$  shared secret

# Security (eavesdropping)

Adversary sees **pk, E(pk, x)** and wants **x**  $\in M$

Semantic security  $\Rightarrow$

adversary cannot distinguish

$\{ pk, E(pk, x), x \}$  from  $\{ pk, E(pk, x), rand \in M \}$

$\Rightarrow$  can derive session key from  $x$

Note: protocol is vulnerable to man-in-the-middle

# Insecure against man in the middle

The protocol is insecure against **active** attacks

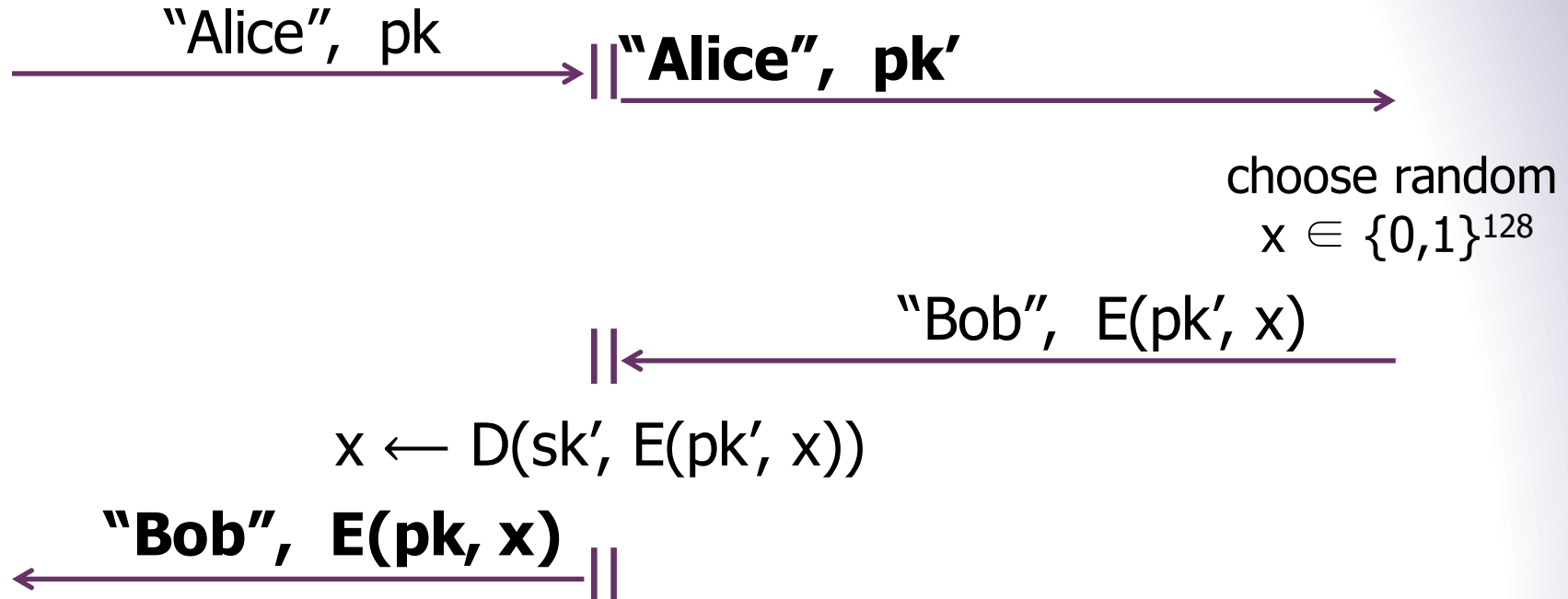
**Alice**

$(pk, sk) \leftarrow G()$

**MiTM**

$(pk', sk') \leftarrow G()$

**Bob**



# Trade-offs for Public Key Crypto

- More computationally expensive than symmetric (shared) key crypto
  - Algorithms are harder to implement
  - Require more complex machinery
- More formal justification of difficulty
  - Hardness based on complexity-theoretic results
- A principal needs 1 private key and 1 public key
  - Number of keys for pair-wise communication is  $O(n)$

# RSA Algorithm

- Ron Rivest, Adi Shamir, Leonard Adleman
  - Proposed in 1979
  - They won the 2002 Turing award for this work
- Has withstood years of cryptanalysis
  - Not a guarantee of security!
  - But a strong vote of confidence
    - Further reading: Twenty years of attacks on the RSA cryptosystem, D. Boneh, Notices of the AMS, 1999
- Hardware implementations:
  - 1000 x slower than DES

# RSA at a High Level

- Public and private key are derived from secret prime numbers
  - Today at least 1024 bits to ensure security (4096 bits is better)
- Plaintext message (a sequence of bits)
  - Treated as a (large!) binary number
- Encryption is modular exponentiation
- To break the encryption, conjectured that one must be able to factor large numbers
  - Not known to be in P (polynomial time algorithms)



# Crypto hash functions



# Hash Algorithms

- Take a variable length string
- Produce a fixed length digest

$$h: \{0,1\}^* \xrightarrow{\text{hash}} \{0,1\}^n$$

- (Non-cryptographic) Examples:

- Parity (or byte-wise XOR)
- CRC

- Realistic Example:

- The NIST Secure Hash Algorithm (SHA) takes a message of less than  $2^{64}$  bits and produces a digest of 160 bits

# Cryptographic Hashes

- Create a hard-to-invert summary of input data
- Like a check-sum or error detection code
  - Uses a cryptographic algorithm internally
  - More expensive to compute
- Sometimes called a Message Digest
- Examples:
  - Secure Hash Algorithm (SHA)
  - Message Digest (MD4, MD5)

# Desired Properties

## ■ One way hash function

- Given a hash value  $y$ , it should be infeasible to find  $m$  s.t.  $h(m)=y$

## ■ Collision resistance

- It should be infeasible to find two different messages  $m_1$  and  $m_2$  s.t.  $h(m_1)=h(m_2)$

## ■ Random oracle property

- $h(m)$  is indistinguishable from a random  $n$ -bit value
- Attacker must spend a lot of effort to be able to modify the message without altering the hash value



# Any questions?

