

CENG 280

Formal Languages and Abstract Machines

Spring 2023-2024

Homework 4

Question 1

By using the Basic Machines in our textbook design a TM that scans to the right until it finds two consecutive a's and then halts. The alphabet of the TM should be $\{a, b, \sqcup, \triangleright\}$

Question 2

Explain what this machine does.

$$\triangleright R \xrightarrow{a \neq \sqcup} R \xrightarrow{b \neq \sqcup} R_{\sqcup} a R_{\sqcup} b$$

Question 3

Suppose we allow the Turing Machine the ability to stay put (stay put is an operation where neither the the tape head moves nor anything gets written to the tape). Does this make the TM more powerful than the standard TM? If yes give an example language that is not recognized by standard TM but recognized by TM with stay put. Otherwise describe the equivalence of both TMs.

Question 4

For the following languages give a high level description of the corresponding TMs

- $\{ w \mid w \text{ contains an equal number of 0s and 1s} \}$
- $\{ w \mid w \text{ contains twice as many 0s as 1s} \}$
- $\{ w \mid w \text{ does not contain twice as many 0s as 1s} \}$

Question 5

Let a k -PDA be a PDA that has k stacks. Thus 0-PDA is an NFA and a 1-PDA is a conventional PDA. We already know that 1-PDAs are more powerful than 0-PDAs.

- Show that 2-PDAs are more powerful than 1-PDAs.
- Show that 3-PDAs are not more powerful than 2-PDAs. (Hint: You can show equivalence of both machines with TMs.)

Question 6

Show that the collection of recursively enumerable languages is closed under

- union
- concatenation
- star
- intersection

Question 7

Let $L = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$. Show that L is decidable.