Homework-4 – Part 2

1 a)

$S \to S_1 \#$
$S_1 \to 0S_1T \mid 1S_1Y \mid e$
$YT \to TY$
$Y\# \to 3$
$Y3 \to 33$
$T3 \to 23$
$T2 \to 22$
$T\# \to 2$
$\# \to e$

$w = 010112 2333$

$S \to S_1\# \to 0S_1T\# \to 01S_1YT\# \to$
$010S_1TYT\# \to 0101S_1YTYT\# \to$
$01011S_1YYTYT\# \to 01011YY\underset{\sim}{T}YT\# \to$
$01011\underset{\sim}{YT}YYT \to 01011TYY\underset{\sim}{YT}\# \to$
$01011TYY\underset{\sim}{TY}\# \to 01011TY\underset{\sim}{T}YY\# \to$
$01011TTYYY\underset{\sim}{\#} \to 01011TTYY\underset{\sim}{3} \to$
$01011TT\underset{\sim}{Y3}3 \to 01011T\underset{\sim}{T3}33 \to$
$01011\underset{\sim}{T}2333 \to 0101122333$ ✓

b)

$S \to \#aT$
$T \to e \mid YT$
$aY \to Yaa$
$\#Y \to \#$
$\# \to e$

$w = aaaa$

$S \to \#aT \to \#aYT \to \#aYYT \to \#\underset{\sim}{aY}Y \to$
$\#\underset{\sim}{Y}aaY \to \#a\underset{\sim}{aY} \to \#\underset{\sim}{a}Yaa \to \#\underset{\sim}{Y}aaaa \to$
$\#aaaa \to aaaa$

2 a) Assume that M_1 decides L hence L is decidable. Then we can built the following TM M_2 to decide the halting problem.

$L_2 = \{\langle M, w\rangle \mid M$ is a TM and M accepts $w\}$

M_2 = "On input $\langle M, w\rangle$
1. Run M_1 on $\langle M, w\rangle$ if accepts, Run M on w accept if M accepts w; reject otherwise.
2. Reject if M_1 rejects $\langle M, w\rangle$"

However, we know that M_2 can not exist due to the fact that L_2 is undecidable therefore contradiction L is undecidable

b) Assume that there exists a TM M_1 that decide L. Then we can build the following TM M_2 to decide the halting problem

M_2 = " On input $\langle M, w \rangle$

1. Use the description of M and w to construct M_3

   1.1. M_3 = " On input x

   1.2 If $x \neq w$, reject

   1.3. If $x = w$, run M on w and accept if M accepts "

2. Run M_1 on $\langle M_3 \rangle$

3. If M_1 accepts, reject ; If M_1 rejects, accept. "

$L(M_3) = \emptyset$ iff M does not accept w. If we had M_1 as a decider for L then we would have M_2 as a decider for halting problem which is a contradiction Such machine does not exist.

Bonus

$L = \{ \langle M \rangle \mid M$ is a TM and $L(M)$ is a regular language $\}$

Prove that L is undecidable

Assume that M_1 decides L. Then we can build the following TM M_2 to decide halting problem

M_2 = " On input $\langle M, w \rangle$

1. Construct the following TM M_3.

   1.1 M_3 = " On input x

   1.2 If x has the form $0^n 1^n$ accept.

   1.3. If x does not have this form, run M on input w and accept x iff M accepts w "

2. Run M_1 on $\langle M_3 \rangle$

3. If M_1 accepts, accept; if M_1 rejects, reject.

If $x = 0^n 1^n$ $L(M_3) = \{0^n 1^n\}$

If $x \neq 0^n 1^n$ $L(M_3) = \Sigma^* - \{0^n 1^n\}$

$\underbrace{L(M_3) = \Sigma^*}_{\text{regular}}$ iff M accepts w