Middle East Technical University Department of Computer Engineering

CENG 280

Formal Languages and Abstract Machines

Spring 2023-2024

Homework 2

Question 1

For each of the FAs given below, construct a regular expression by eliminating states one-by-one. (While doing the conversion, at each step you will derive a smaller generalized finite automaton equivalent to the given FA. Studying example 2.3.2 in your textbook may be helpful.)

1. $A_1 = \{K_1, \Sigma_1, \Delta_1, s_1, F_1\}$ where $K_1 = \{q_0, q_1, q_2\}$, $\Sigma_1 = \{a, b\}$, $s_1 = q_0$, $F_1 = \{q_0, q_2\}$ and $\Delta_1 = \{(q_0, a, q_1), (q_0, b, q_1), (q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_0), (q_2, b, q_1)\}$

2. $A_2 = \{K_2, \Sigma_2, \Delta_2, s_2, F_2\}$ where $K_2 = \{q_0, q_1, q_2, q_3, q_4\}$, $\Sigma_2 = \{a, b\}$, $s_2 = q_0$, $F_2 = \{q_3\}$ and
$\Delta_2 = \{$
$(q_0, b, q_1), (q_0, b, q_3), (q_0, \epsilon, q_2),$
$(q_1, a, q_1),$
$(q_2, a, q_2), (q_2, b, q_2), (q_2, a, q_4),$
$(q_3, a, q_1), (q_3, \epsilon, q_4),$
$(q_4, a, q_2), (q_4, a, q_3), (q_4, b, q_4)$
$\}$

Question 2

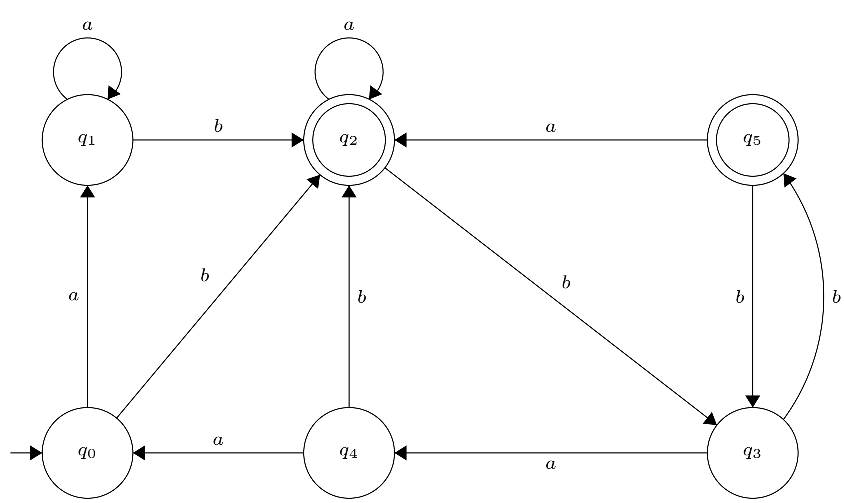

Figure 1. M_1

1. Using the state minimization algorithm you have learned, find and draw the minimal DFA that is equivalent to M_1. **Show each step of your solution clearly.**

2. Define each equivalence class of the automaton you have found at part-a with regular expressions. **Hint&Remark:** In your regular expressions, you are allowed to use the symbol L to denote all strings recognized by the automaton (e.g. $[\epsilon] = Lbaa$).

Question 3

1. Use MyHill-Nerode Theorem to **prove** that the language
   $L' = \{a^n b^m c^k d^u | m + n = k + 2u \text{ and } m, n, k, u \in \mathbb{N}\}$ is not regular.

2. Using the pumping lemma for regular languages **prove** that the language
   $L'' = \{a^m b^n | m > n \text{ and } m, n \in \mathbb{N}\}$ is not regular.

3. Using the closure properties of regular languages and the knowledge that L'' is not regular,
   prove $\overline{L''}$ is not regular.

Question 4

Given $L_1 = L(a(abb)^* \cup b)$ and $L_2 = L(a^+ \cup (ab)^+)$, using methods in the proof of the Theorem 2.3.1 in your textbook,

1. Construct an NFA that recognizes L_1.

2. Construct an NFA that recognizes L_2.

3. Construct an NFA that recognizes $L_1 \cup \overline{L_2}$.

Question 5

Give context-free grammars generating the following languages:

1. The strings over the alphabet $\{a, b\}$ with more $b's$ than $a's$.

2. $A = \{0^i 1^j 2^k \mid i + k = j \text{ and } i, j, k \geq 0\}$

3. $B = \{w \mid \text{the length of } w \text{ is odd and } w \in \{0,1\}^*\}$ and draw parse tree for string 0011100.

Question 6

Give the (context-free) languages generated by each of the given grammars:

1. 1. $G_1 = (V_1, \Sigma, R_1, S_1)$ where $V_1 = \{S_1, A\} \cup \Sigma$, $\Sigma = \{0, 1\}$ and
   $R_1 = \{S_1$ -> `0A0 | 1A1 | e`
   `A -> 0A | 1A | e` }

2. $G_2 = (V_2, \Sigma, R_2, S_2)$ where $V_2 = \{S_2, B\} \cup \Sigma$, $\Sigma = \{0, 1\}$ and
   $R_2 = \{S_2$ -> `B1B1B`
   `B -> 0B | 1B | e` }

Question 7

Given $G = (V, \Sigma, R, S)$ where $V = \{0, 1, S, A\}$, $\Sigma = \{0, 1\}$ and $R = \{S \to AS|e, A \to A1|0A1|01\}$

1. Show that G is ambiguous.
2. Given an unambiguous grammar for $L(G)$ (i.e. disambiguate the given grammar).
3. Give the leftmost derivation of the string *00111* from the grammar you have constructed at part-2 of this question, and draw the corresponding parse tree.