Student Information

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Answer 1

a)

- In order to determine the size of our Monte Carlo simulation, we can use Normal approximation witht the values $\alpha = 0.01$ and $\epsilon = 0.02$.
- However, since we don't have an "intelligent guess" estimator p^*, we are going to use the following:

$$N \geq 0.25 \cdot \left(\frac{z_{\alpha/2}}{\epsilon}\right)^2 = 0.25 \cdot \left(\frac{2.575}{0.02}\right)^2 = 4144.14 \approx 4144$$

- Therefore, we see that the size of our simulation N should be greater than 4144. We can pick $N = 4145$ for our simulation.

b)

- Since we have the information that the weight of each automobile is a Gamma distributed random variable in kilograms with $\alpha = 120$ and $\lambda = 0.1$, we can use the expected value formula for Gamma distribution:

$$\mathbf{E}(X) = \frac{\alpha}{\lambda} = \frac{120}{0.1} = 1200$$

- Similarly, since the weight of each is a Gamma distributed random variable in kilograms with $\alpha = 14$ and $\lambda = 0.001$, we can use the expected value formula for Gamma distribution:

$$\mathbf{E}(X) = \frac{\alpha}{\lambda} = \frac{14}{0.001} = 14000$$

- Since the weight of an automobile and the number of automobiles passing through a bridge are independent events, we can just multiply the expected values:

$$\mathbf{E}(XY) = \mathbf{E}(X) \cdot \mathbf{E}(Y) = 1200 \cdot 60 = 72000 \text{ where } \mathbf{E}(Y) = \lambda_A = 60$$

- Similarly, the weight of a truck and the number of trucks passing through a bridge are independent events, we can just multiply the expected values:

$$\mathbf{E}(XY) = \mathbf{E}(X) \cdot \mathbf{E}(Y) = 14000 \cdot 12 = 168000 \text{ where } \mathbf{E}(Y) = \lambda_T = 12$$

Answer 2

```
N = 4145;
lambdaA = 60;  % number of automobiles
lambdaT = 12;  % number of trucks

alphaA = 120; lambdaA_2 = 0.1; % Gamma distribution lambda
alphaT = 14; lambdaT_2 = 0.001;

TotalWeight = zeros(N, 1);
for k = 1:N
  % first generate the number of passed vehicles for each type from Poisson
  numA = 0;
  numT = 0;

  % number of automobiles
  U = rand;
  F = exp(-lambdaA);
  while (U >= F)
    numA = numA + 1;
    F = F + exp(-lambdaA) * lambdaA ^ numA/gamma(numA + 1);
  end

  % number of trucks
  U = rand;
  F = exp(-lambdaT);
  while (U >= F)
    numT = numT + 1;
    F = F + exp(-lambdaT) * lambdaT ^ numT/gamma(numT + 1);
  end

  weight = 0; % total weight of vehicles for this run

  % calculate the total weight of automobiles
  weightA = 0;
  for j=1:numA
    weightA = weightA + sum(-1/lambdaA_2 * log(rand(alphaA, 1)));
  end

  % calculate the total weight of trucks
  weightT = 0;
  for j=1:numT
    weightT = weightT + sum(-1/lambdaT_2 * log(rand(alphaT, 1)));
  end
```

```
  weight = weightA + weightT;
  TotalWeight(k) = weight;
end

p_est = mean(TotalWeight>250000);
expectedWeight = mean(TotalWeight);
stdWeight = std(TotalWeight);

fprintf('Estimated probability = %f\n', p_est);
fprintf('Expected weight = %f\n', expectedWeight);
fprintf('Standard deviation = %f\n', stdWeight);
```

- I've used JDoodle online compiler to run my code and got the following results:

```
Estimated probability = 0.406031
Expected weight = 239669.053437
Standard deviation = 50944.603136
```

Figure 1: Outputs of our simulation.

- We got a standard deviation X of 50944 kilograms in our simulation. This is a high variability compared to our expected value of 239669 kilograms which means that individual X values may differ from the expected true value of X. We can say that our accuracy is not that high.
- For the limiting value of λ_T, I've tested the values that might make the probability of bridge's collapse close to 0.1 which are 8 and 9:

```
Estimated probability = 0.119662
Expected weight = 197432.125570
Standard deviation = 44485.164266
```

Figure 2: Output of our simulation when $\lambda_T = 9$.

```
Estimated probability = 0.064174
Expected weight = 183383.115777
Standard deviation = 41628.376315
```

Figure 3: Output of our simulation when $\lambda_T = 8$.

- It can be seen from the outputs, when $\lambda_T \geq 9$ the probability of bridge's collapse is greater than 0.1. Therefore, we can say that our limiting value is $\lambda_T = 8$.