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CS-171: PS4

PS4 Q2

If there are m items (or features), there are $3^m - 2^{m+1} + 1$ different association rules possible. Prove this.

For each set with m features, we can consider the different combinations of X. Different combination of X can be attained by m-choose-j for subsets of size j. These combinations of X are compounded with all possible combinations of Y for each subset X. The possible combinations of Y for each subset X can range from size 1 to size (m-j), so we add up those combinations for each size. As a result, we get the following:

$$\sum_{j=1}^{m-1}\left[\left(\binom{m}{j}\right) * \sum_{k=1}^{m-j}\left(\binom{m-j}{k}\right)\right]$$

Considering only the right summation, we can condense that into the following using the binomial theorem $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$. We replace both a and b with 1's.

$$\sum_{j=1}^{m-1}\left[\left(\binom{m}{j}\right) * (2^{m-j}-1)\right]$$

We can distribute the multiplication and the summation to the -1:

$$\sum_{j=1}^{m-1}\left[\left(\binom{m}{j}\right) * (2^{m-j})\right] - \sum_{j=1}^{m-1}\left[\left(\binom{m}{j}\right)\right]$$

Using the binomial theorem again we can get the left-hand side of 3^m and the right-hand side of $2^m - 1$ before applying the negative between the summations, and we get the proven equation.

$$3^m - 2^{m+1} + 1$$