APMA 0350 Homework 1

Milan Capoor

TOTAL POINTS

20 / 20

QUESTION 1

Problem 1 5 pts

1.1 1a **3 / 3**

✓ **- 0 pts** **Answer and work to get answer is correct**

1.2 1b **2 / 2**

✓ **- 0 pts** **Answer and work to get answer is correct**

QUESTION 2

Problem 2 5 pts

2.1 2a **1 / 1**

✓ **- 0 pts** **Correct**

2.2 2b **1 / 1**

✓ **- 0 pts** **Correct**

2.3 2c **1 / 1**

✓ **- 0 pts** **Correct**

2.4 2d **1 / 1**

✓ **- 0 pts** **Correct**

2.5 2e **1 / 1**

✓ **- 0 pts** **Correct**

QUESTION 3

Problem 3 5 pts

3.1 3a **2 / 2**

✓ **- 0 pts** **Correct**

3.2 3b **3 / 3**

✓ **- 0 pts** **Correct**

QUESTION 4

4 Problem 4 **5 / 5**

✓ **- 0 pts** **Correct**

APMA 0350: Homework 1

Milan Capoor

9 September 2022

Problem 1:

a For which values of r is $y = e^{rt}$ a solution of $y'' + 3y' - 10y = 0$?

Solution:

$$\begin{aligned}
\left(e^{rt}\right)'' + 3(e^{rt})' - 10e^{rt} &= 0 \\
r^2e^{rt} + 3re^{rt} - 10e^{rt} &= 0 \\
(e^{rt})(r^2 + 3r - 10) &= 0 \\
r^2 + 3r - 10 &= 0 \\
(r+5)(r-2) &= 0 \\
\mathbf{r} &= \mathbf{-5, 2}
\end{aligned}$$

b Determine whether $y = t^4$ is a solution of $t^3y'' + ty' - 8y = 0$

Solution:

$$\begin{aligned}
t^3(t^4)'' + t(t^4)' - 8t^4 &= 0 \\
t^3(12t^2) + t(4t^3) - 8t^4 &= 0 \\
12t^5 + 4t^4 - 8t^4 &= 0 \\
12t^5 - 4t^4 &= 0 \\
t^4(12t - 4) &= 0 \\
t &= 0, \frac{1}{3}
\end{aligned}$$

$\therefore$ $y = t^4$ is **not a solution** because it is only true for certain values.

1.1 1a 3 / 3

✓ **- 0 pts** **Answer and work to get answer is correct**

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$\therefore$ $y = t^4$ is **not a solution** because it is only true for certain values.

1.2 1b **2 / 2**

✓ **- 0 pts Answer and work to get answer is correct**

Problem 2: Find the order of each equation and state if it is linear or non-linear. If it is linear, state if it is homogeneous or inhomogeneous. No justification needed.

a. $e^t y'' + y' - t^2 y = 13t^2$

Second-order linear (inhomogeneous)

b. $(y')^2 + ty + \cos y = 0$

First-order nonlinear

c. $y''' + 10y'' + 5y' - 2y = 0$

Third-order linear (homogeneous)

d. $\cos(t)y' + ty = 0$

First-order linear (homogeneous)

e. $y' = t^3 y^2 + ty$

First-order nonlinear

Problem 3:

a Solve $y' = 3y$ with $y(1) = 2$ Solution:

$$\begin{aligned}
\left(\frac{y'}{y}\right) &= 3 \\
(\ln|y|)' &= 3 \\
\ln|y| &= 3t + C \\
y &= \pm e^{3t+C} \\
y &= Ce^{3t} \\
2 &= Ce^3 \implies C = \frac{2}{e^3} \\
y &= \frac{2}{e^3}e^{3t} = \mathbf{2e^{3t-3}}
\end{aligned}$$

2.1 2a 1 / 1

✓ - 0 pts Correct

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2.2 2b **1 / 1**

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2.4 2d 1 / 1

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2.5 2e **1 / 1**

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\end{aligned}$$

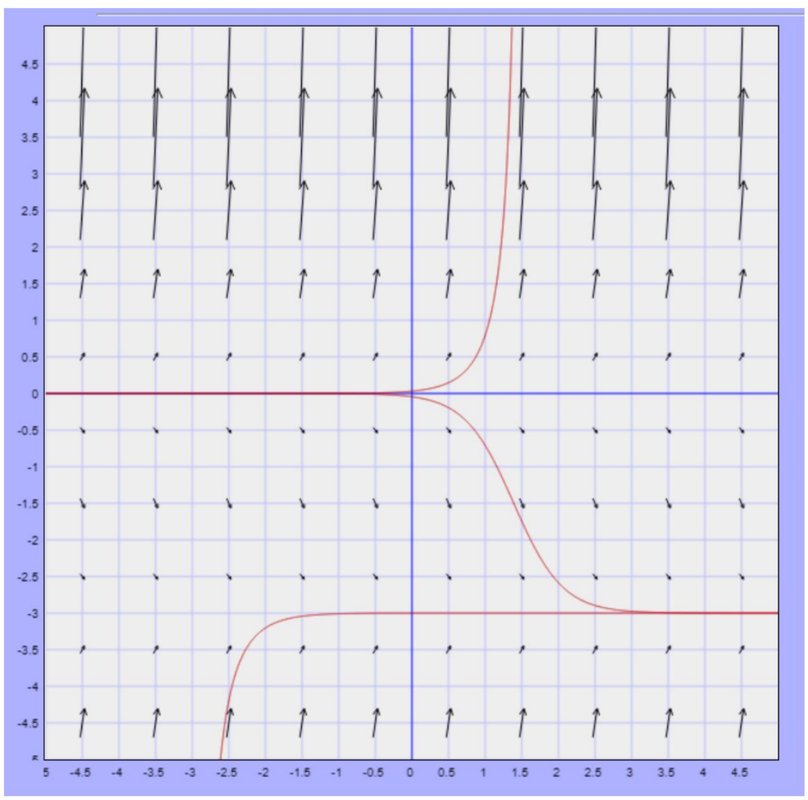
3.1 3a **2 / 2**

✓ **- 0 pts** Correct

b Solve $y' = y + 2$ with $y(0) = 3$ Solution:

$$\begin{aligned}
\left(\frac{y'}{y+2}\right) &= 1 \\
\ln|y+2| &= t + C \\
y + 2 &= \pm e^{t+C} \\
y &= Ce^t - 2 \\
3 &= C - 2 \implies C = 5 \\
y &= \mathbf{5e^t - 2}
\end{aligned}$$

Problem 4: Use the dfield app to draw the direction field of $y' = y(y+3)$. On that direction field, please click on three solutions, one in the region$y > 0$, one in the region $-3 < y < 0$, and one in the region $y < -3$.



3.2 3b 3 / 3

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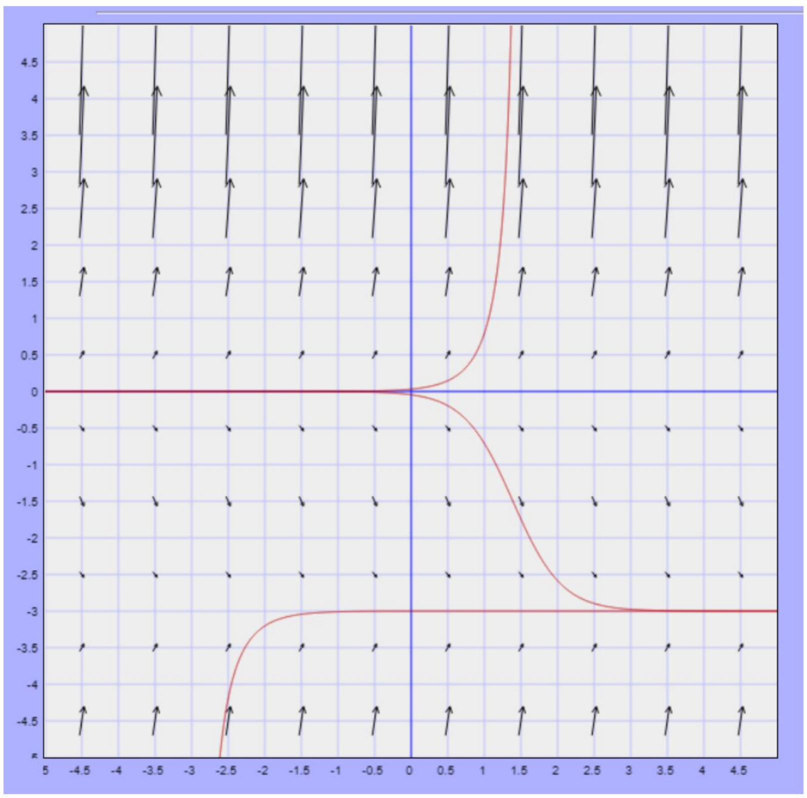

4 Problem 4 **5 / 5**

✓ **- 0 pts** Correct