APMA 0350 Homework 2

Milan Capoor

TOTAL POINTS

19.5 / 20

QUESTION 1

Problem 1 3 pts

1.1 1a **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

1.2 1b **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

1.3 1c **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

QUESTION 2

Problem 2 4 pts

2.1 2a **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

2.2 2b **0.5 / 1**

✓ **- 0.5 pts** No answer/Incorrect Answer

2.3 2c **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

2.4 2d **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

QUESTION 3

Problem 3 6 pts

3.1 3a **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

3.2 3b **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

3.3 3c **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

QUESTION 4

Problem 4 5 pts

4.1 4a **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

4.2 4b **1 / 1**

✓ **- 0 pts** Answer and work to get answer is correct

4.3 4c **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

QUESTION 5

5 Problem 5 **2 / 2**

✓ **- 0 pts** Answer and work to get answer is correct

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23 September 2022

Problem 1: Do the following ODE satisfy the assumptions of the Existence-Uniqueness Theorem from lecture? Why or why not?

a

$$\begin{cases} \frac{dy}{dt} = \frac{y}{t^2+1} \\ y(-1) = 9 \end{cases}$$

This $\boxed{does}$ satisfy the conditions because $f(t, y) = \frac{y}{t^2+1}$ is continuous and so is $f_y = \frac{1}{t^2+1}$

b

$$\begin{cases} \frac{dy}{dt} = y^2(|t| + y) \\ y(0) = 2 \end{cases}$$

This $\boxed{does}$ satisfy the conditions because $f(t, y) = y^2(|t|+y)$ is continuous and so is $f_y = y(2|t| + 3y)$

c

$$\begin{cases} \frac{dy}{dt} = \frac{1}{y+1} \\ y(1) = -1 \end{cases}$$

This $\boxed{doesnot}$ satisfy the conditions because both $f(t, y) = \frac{1}{y+1}$ and $f_y = -\frac{1}{(y+1)^2}$ are not continuous at $y = -1$, so the theorem applies

Problem 2: Consider the equation

$$y' = (y + 2)(y - 1)(y - 3)$$

a Find the equilibrium solutions

$$\boxed{y = \{-2, 1, 3\}}$$

1.1 1a 1 / 1

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Problem 2: Consider the equation

$$y' = (y + 2)(y - 1)(y - 3)$$

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1.2 1b 1 / 1

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2.1 2a **1 / 1**

✓ **- 0 pts** **Answer and work to get answer is correct**

b Draw a bifurcation diagram

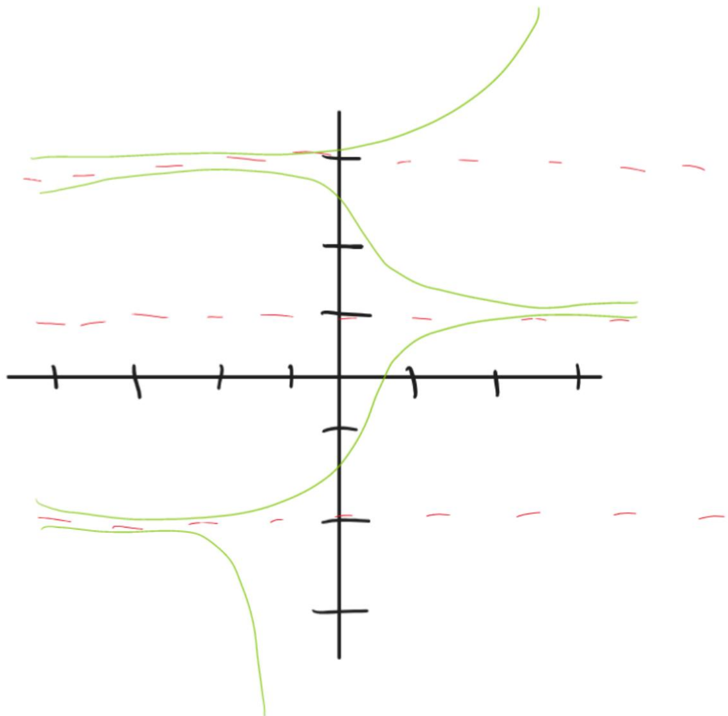
$$y' = (y+2)(y-1)(y-3)$$

y	$-\infty$	-2	1	3	∞
$y+2$	$-$	0	$+$	$+$	$+$
$y-1$	$-$	$-$	0	$+$	$+$
$y-3$	$-$	$-$	$-$	0	$+$
y'	$-$	$-$	$-$	$+$	$+$
y	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$

c Draw a sketch of the equilibrium solutions and of a couple of solutions between them

2.2 2b 0.5 / 1

✓ **- 0.5 pts** No answer/Incorrect Answer



d Classify the equilibrium solutions as stable/unstable/bistable.

- $y = -2$ - Unstable
- $y = 1$ - Stable
- $y = 3$ - Unstable

Problem 3: Solve using separation of variables

a (Implict form)

$$\frac{dy}{dt} = \frac{t \sec y}{y(e^{t^2})}$$

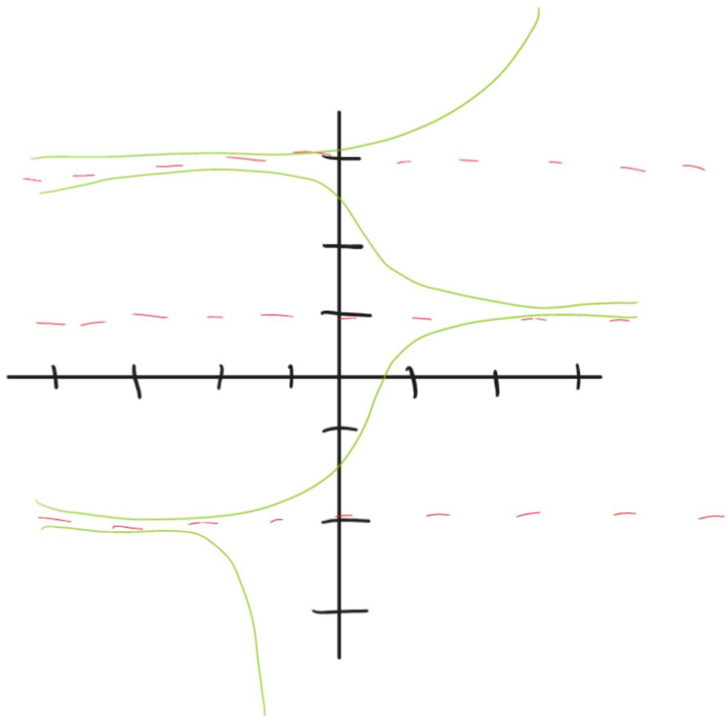
$$y \cos y \; dy = \frac{t}{e^{t^2}} \; dt$$

$$\boxed{y \sin y + \cos y = -\frac{e^{-t^2}}{2} + C}$$

b (Remember the initial condition)

2.3 2c 1 / 1

✓ - 0 pts Answer and work to get answer is correct



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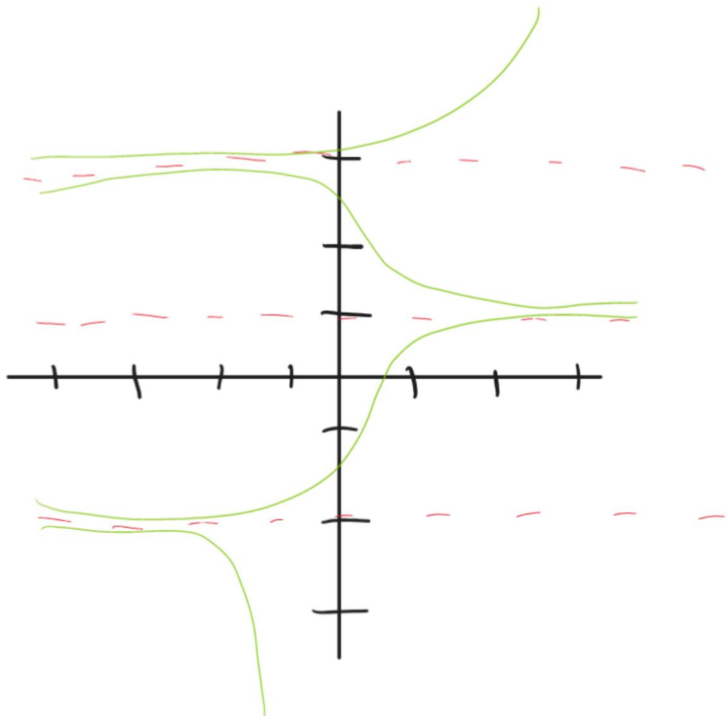
2.4 2d 1 / 1

✓ **- 0 pts** **Answer and work to get answer is correct**



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b (Remember the initial condition)

3.1 3a **2 / 2**

✓ **- 0 pts** **Answer and work to get answer is correct**

c

$$\begin{cases} \frac{dy}{dt} = \frac{2t+\sec^2 t}{2y} \\ y(0) = -5 \end{cases}$$

$$2y\ dy = 2t + \sec^2 t\ dt$$

$$y^2 = t^2 + \tan t + C$$

$$y = \sqrt{t^2 + \tan t + C}$$

$$-5 = \pm\sqrt{0 + \tan 0 + C} \implies C = 25$$

$$\boxed{y = -\sqrt{t^2 + \tan t + 25}}$$

d (Beware of hidden solutions)

$$\frac{dy}{dt} = \frac{-t(y-1)^2}{3}$$

$$\frac{1}{(y-1)^2} dy = -\frac{t}{3} dt$$

$$\frac{1}{1-y} = -\frac{1}{6}t^2 + C$$

$$\boxed{y = 1 + \frac{6}{t^2 + C}},\ \text{OR}\ \boxed{y = 1}$$

Problem 4:

a Follow the steps from lecture to solve the logistic equation (you can skip the partial fractions part)

$$\begin{cases} \frac{dy}{dt} = 0.08y(1 - \frac{y}{1000}) \\ y(0) = 100 \end{cases}$$

$$\frac{dy}{y(1 - \frac{y}{1000})} = 0.08\ dt$$

$$\ln y - \ln(1000 - y) = 0.08t + C$$

$$\ln \frac{y}{1000 - y} = 0.08t + C$$

$$\frac{y}{1000 - y} = Ce^{0.08t}$$

3.2 3b **2 / 2**

✓ **- 0 pts Answer and work to get answer is correct** c

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$$\frac{y}{1000 - y} = Ce^{0.08t}$$

3.3 3c 2 / 2

✓ **- 0 pts** **Answer and work to get answer is correct**

$$y = 1000Ce^{0.08t} - Cye^{0.08t}$$

$$y(1 + Ce^{0.08t}) = 1000Ce^{0.08t}$$

$$y = \frac{1000Ce^{0.08t}}{1 + Ce0.08t}$$

$$100 = \frac{1000C}{1 + C} \implies 1 + C = 10C \implies C = 1/9$$

$$\boxed{y = \frac{1000e^{0.08t}}{9 + e^{0.08t}}}$$

b What is the limit of the solution in (a) as $t \to \infty$?

$$\lim_{t\to\infty} \frac{1000e^{0.08t}}{9 + e^{0.08t}} = \boxed{1000}$$

c For which t do we have $y(t) = 900$? (exact and approximate value)

$$900 = \frac{1000e^{0.08t}}{9 + e^{0.08t}}$$

$$900(9 + e^0.08t) = 1000e^{0.08t}$$

$$8100 + 900e^{0.08t} = 1000e^{0.08t}$$

$$8100 = 100e^{0.08t}$$

$$81 = e^{0.08t}$$

$$\boxed{t = \frac{\ln 81}{0.08} \approx 54.9306}$$

Problem 4: Show that the function y(t) whose graph is below cannot be a solution of an ODE of the form $y'(t) = f(y(t))$

Solution: Because $f(y(t))$ is continuous and everywhere differentiable (on the region), its partials are also continuous so by the Existence/Uniqueness theorem there is only one solution to the ODE $y' = f(y)$. Thus, the given graph of $y(t)$ can not be a solution because there are values where $y(t_a) = t(t_b)$ but $y'(t_a) > 0$ and $y'(t_b) < 0$, suggesting that there would be more solutions, which is a contradiction.

4.1 4a **2 / 2**

✓ **- 0 pts Answer and work to get answer is correct**

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4.2 4b **1 / 1**

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4.3 4c 2 / 2

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5 Problem 5 **2 / 2**

✓ **- 0 pts** **Answer and work to get answer is correct**