APMA 0350 Homework 4

Milan Capoor

TOTAL POINTS

20 / 20

QUESTION 1

1 Problem 1 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 2

2 Problem 2 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 3

Problem 3 6 pts

3.1 3a **2 / 2**

✓ **- 0 pts** Correct

3.2 3b **2 / 2**

✓ **- 0 pts** Correct

3.3 3c **2 / 2**

✓ **- 0 pts** Correct

QUESTION 4

Problem 4 6 pts

4.1 4a **2 / 2**

✓ **- 0 pts** Correct

4.2 4b **2 / 2**

✓ **- 0 pts** Correct

4.3 4c **2 / 2**

✓ **- 0 pts** Correct

APMA 0350: Homework 4

Milan Capoor

7 October 2022

Problem 1: Apply Euler's method by hand with N = 4 to find y_0, y_1, y_2, y_3, y_4 on [0, 1] where

$$\begin{cases} y' = -2y + 3t \\ y(0) = 1 \end{cases}$$

Solution:

$$h = \frac{1-0}{4} = \frac{1}{4}$$

$$\begin{aligned}
y_0 &= y(0) = 1 \\
y_1 &= y_0 + hy_0' = 1 + \frac{1}{4}(-2(1) + 3(0)) = \frac{1}{2} \\
y_2 &= y_1 + hy_1' = \frac{1}{2} + \frac{1}{4}(-2(\frac{1}{2}) + 3(\frac{1}{4})) = \frac{7}{16} \\
y_3 &= y_2 + hy_2' = \frac{7}{16} + \frac{1}{4}(-2\frac{7}{16} + 3(\frac{1}{2})) = \frac{19}{32} \\
y_4 &= y_3 + hy_3' = \frac{19}{32} + \frac{1}{4}(2\frac{19}{32} + 3\frac{3}{4}) = \frac{55}{64}
\end{aligned}$$

1 Problem 1 **4 / 4**

✓ **- 0 pts** Correct

Problem 2: Use Python to apply Euler with N = 50 on [2, 3] where

$$\begin{cases} y' = \cos y + ty \\ y(2) = 5 \end{cases}$$

Plot the points on a graph.

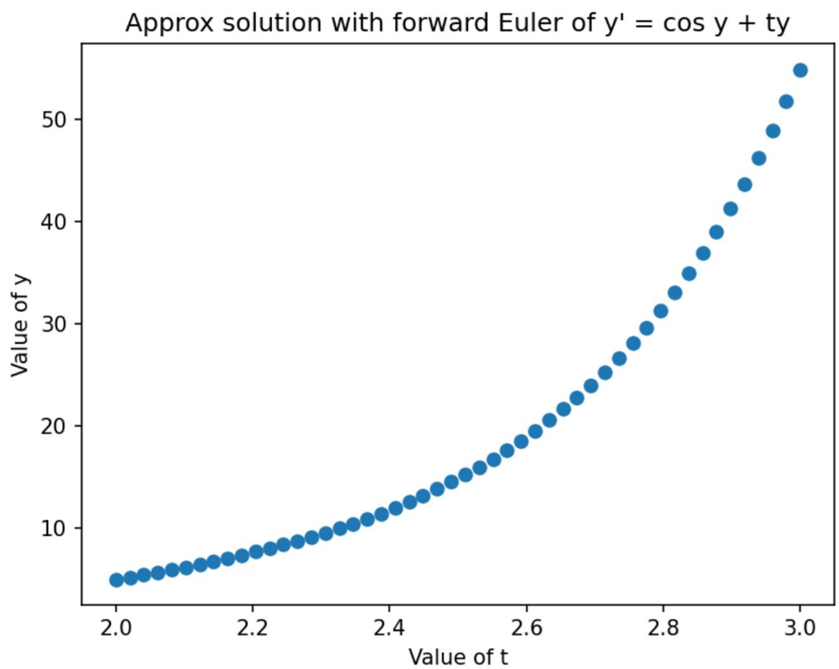
Solution:

HW 4 > ODE.py > ...

```python
from sympy import *
from matplotlib import pyplot as plt
import numpy as np

a = 2
b = 3
n = 50
y0 = 5
h = (b - a)/n

t = np.linspace(a, b, n)
y = np.zeros([n])

y[0] = y0

for i in range(1, n):
    eq = np.cos(y[i -1]) + t[i - 1] * y[i -1]
    y[i] = y[i - 1] + h * eq

print(y[-1])
plt.plot(t, y, 'o')
plt.xlabel("Value of t")
plt.ylabel("Value of y")
plt.title("Approx solution with forward Euler of y' = cos y + ty")
plt.show()
```

PROBLEMS 3 OUTPUT DEBUG CONSOLE TERMINAL JUPYTER

```
[Running] python -u "c:\Users\capoo\Documents\APMA 0350\HW 4\ODE.py"
54.908622235544165
```


Approx solution with forward Euler of y' = cos y + ty
Value of y
Value of t
50
40
30
20
10
2.0
2.2
2.4
2.6
2.8
3.0

2 Problem 2 **4 / 4**

✓ **- 0 pts** Correct

Problem 3: Use the dsolve command in Python to solve the following. Don't solve them by hand.

1.
$$y' + y = 3\cos(2t)$$

HW 4 > ODE.py > ...

```python
from sympy import *

t = symbols('t')
y = Function('y')
deq = diff(y(t), t) + y(t) - 3 * cos(2*t)
ysoln = dsolve(deq, y(t))
print(ysoln)
```

PROBLEMS 3 OUTPUT DEBUG CONSOLE ··· Code

```
[Running] python -u "c:\Users\capoo\Documents\APMA 0350\HW 4\ODE.py"
Eq(y(t), C1*exp(-t) + 6*sin(2*t)/5 + 3*cos(2*t)/5)
```

2.
$$\begin{cases} y' + 2y = 2te^{2t} \\ y(0) = 1 \end{cases}$$

HW 4 > ODE.py > ...

```python
from sympy import *

t = symbols('t')
y = Function('y')
deq = diff(y(t), t) + 2* y(t) - 2 * t * exp(2 * t)
ysoln = dsolve(deq, y(t), ics = {y(0):1})
print(ysoln)
```

PROBLEMS 3 OUTPUT DEBUG CONSOLE ··· Code

```
[Running] python -u "c:\Users\capoo\Documents\APMA 0350\HW 4\ODE.py"
Eq(y(t), (4*t - 1)*exp(2*t)/8 + 9*exp(-2*t)/8)
```

3.1 3a 2 / 2

✓ - 0 pts Correct

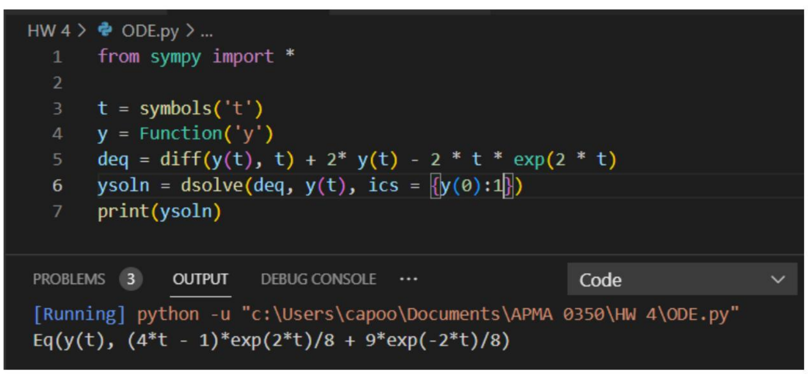
Problem 3: Use the dsolve command in Python to solve the following. Don't solve them by hand.

1.

$$y' + y = 3\cos(2t)$$

2.

$$\begin{cases} y' + 2y = 2te^{2t} \\ y(0) = 1 \end{cases}$$



3.2 3b **2 / 2**

✓ **- 0 pts** Correct

3.

$$\begin{cases} y' = 20y(1 - \frac{y}{20}) \\ y(0) = 10 \end{cases}$$

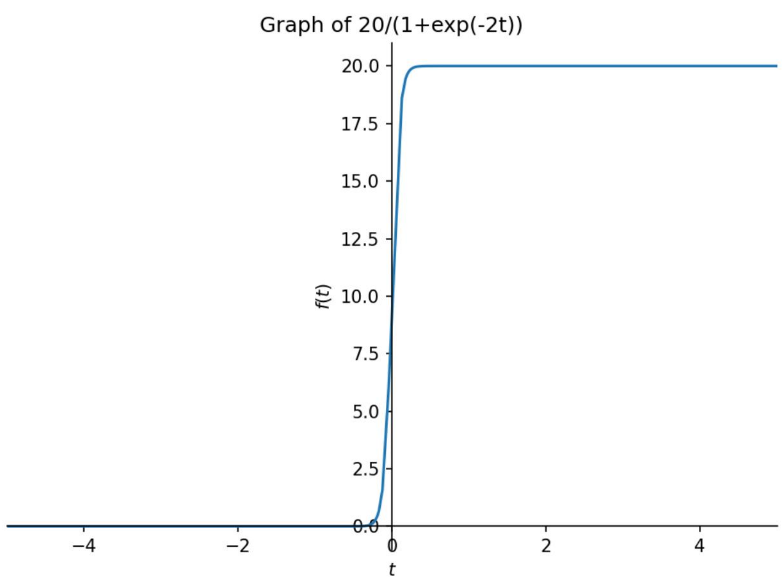
Please also plot the solution using -5 and 5 as the t limits and -1 and 21 as the y limits

HW 4 > ODE.py > ...

```python
from sympy import *
from matplotlib import pyplot as plt
import numpy as np

t = symbols('t')
y = Function('y')
deq = diff(y(t), t) - 2* y(t) * (1 - y(t)*1/20)
ysoln = dsolve(deq, y(t), ics = {y(0):10})
print(ysoln)

plotting.plot(ysoln.rhs,
    title="Graph",
    xlim=(-5, 5), ylim=(-1, 21))
```

PROBLEMS 3 OUTPUT DEBUG CONSOLE ··· Code

```
[Running] python -u "c:\Users\capoo\Documents\APMA 0350\HW 4\ODE.py"
Eq(y(t), 20/(1 + exp(-2*t)))
```



3.3 3c **2 / 2**

✓ **- 0 pts** Correct

Problem 4: Suppose a tank is filled initially (at time t = 0) with 100 gallons of fresh clean water.

Water containing 10 grams/gallon of chemical pollutants enters the tank at a rate of 2 gallons/day, and the mixture in the well-stirred tank leaves the tank at the rate of 1 gallon/day.

1. Find the amount of water W(t) as a function of time and determine the time when the amount of water in the tank reaches 200 gallons.

   Solution:

$$\begin{cases} W(0) = 100 \\ W'(t) = 2 - 1 = 1 \end{cases}$$

$$W(t) = t + C$$

$$W(0) = 0 + C = 100 \implies C = 100 \implies \boxed{W(t) = t + 100}$$

$$W(t) = 200 = t + 100 \implies \boxed{t = 100 \text{ days}}$$

2. Write down the differential equation that describes the total amount of pollutants P(t) in the tank and find the initial condition P(0) of this quantity at time t = 0.

$$P'(t) = 20 - \frac{P(t)}{W(t)}$$

$$P'(t) = 20 - \frac{P(t)}{t + 100}$$

$$P'(t) + \frac{1}{t + 100} P(t) = 20$$

$$\left(P(t)e^{\ln|t+100|}\right)' = 20e^{\ln|t+100|}$$

$$P(t)(t + 100) = \int 20(t + 100)\, dt = 10t^2 + 2000t + C$$

$$P(t) = \frac{10t^2 + 2000t + C}{t + 100}$$

$$P(0) = \frac{C}{100} = 0 \implies \boxed{C = 0}$$

$$P(t) = \boxed{\frac{10t^2 + 2000t}{t + 100}}$$

4.1 4a 2 / 2

✓ **- 0 pts** Correct

Problem 4: Suppose a tank is filled initially (at time t = 0) with 100 gallons of fresh clean water.

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$$W(0) = 0 + C = 100 \implies C = 100 \implies \boxed{W(t) = t + 100}$$

$$W(t) = 200 = t + 100 \implies \boxed{t = 100 \text{ days}}$$

2. Write down the differential equation that describes the total amount of pollutants P(t) in the tank and find the initial condition P(0) of this quantity at time t = 0.

$$P'(t) = 20 - \frac{P(t)}{W(t)}$$

$$P'(t) = 20 - \frac{P(t)}{t+100}$$

$$P'(t) + \frac{1}{t+100}P(t) = 20$$

$$\left(P(t)e^{\ln|t+100|}\right)' = 20e^{\ln|t+100|}$$

$$P(t)(t+100) = \int 20(t+100)\,dt = 10t^2 + 2000t + C$$

$$P(t) = \frac{10t^2 + 2000t + C}{t+100}$$

$$P(0) = \frac{C}{100} = 0 \implies \boxed{C = 0}$$

$$P(t) = \boxed{\frac{10t^2 + 2000t}{t+100}}$$

4.2 4b 2 / 2

✓ **- 0 pts** Correct

3. Find the total amount of pollutants in the tank at the time you determined in (a)

$$P(100) = \frac{10(100^2) + 2000(100)}{100 + 100} = \boxed{1500 \text{ grams}}$$

4.3 4c 2 / 2

✓ - 0 pts Correct