$$\sin(2x) = 2\cos x \sin x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(-x) = \cos(x) \qquad \sin(-x) = -\sin(x)$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x) \qquad \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cosh(mx) = \frac{e^{mx} + e^{-mx}}{2} \qquad \sinh(mx) = \frac{e^{mx} - e^{-mx}}{2}$$

$$Ae^{mx} + Be^{-mx} = A\cosh(mx) + B\sinh(mx)$$

$$\int_{-\infty}^{\infty} \cos(x^2)\, dx = \int_{-\infty}^{\infty} \sin(x^2)\, dx = \sqrt{\frac{\pi}{2}}$$

$$\int \tan x\, dx = \ln|\sec x| + C$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{4}\sin(2x) + C$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{4}\sin(2x) + C$$

Simple PDEs:

$$u_x = 0 \implies u = f(y)$$

$$u_{xx} = 0 \implies u = xf(y) + g(y)$$

$$u_{xx} + u = 0 \implies u = A(y)\cos x + B(y)\sin x$$

$$cu_x + u_t = 0 \implies u(x,t) = f(x - ct)$$

$$\sinh x = -i\sin(ix) \qquad \cosh(x) = \cos(ix) \qquad \tanh x = -i\tan(ix)$$

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh(x)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Type of second order:

$a : u_{xx} \quad b : u_{xy} \quad c : u_{yy}$

$D = b^2 - 4ac$:

Elliptic ($D < 0$)

Parabolic ($D = 0$)

Hyperbolic ($D > 0$)

D'Alembert's Formula:

$$\begin{cases} u_{tt} = c^2 u_{xx} \\ u(x,0) = \phi(x) \\ u_t(x,0) = \psi(x) \end{cases} \implies u = \frac{1}{2}(\phi(x-ct)+\psi(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)\, ds$$

Cosine:

$$A_m = \frac{2}{L}\int_0^L f(x)\cos(\frac{\pi m x}{L})\, dx \qquad A_0 = \frac{1}{L}\int_0^L f(x)\, dx$$

Sine:

$$A_m = \frac{2}{L}\int_0^L f(x)\sin(\frac{\pi m x}{L})\, dx$$

Full:

$$A_m = \frac{1}{L}\int_{-L}^L f(x)\cos(\frac{\pi m x}{L})\, dx \qquad B_m = \frac{1}{L}\int_{-L}^L f(x)\sin(\frac{\pi m x}{L})\, dx \qquad A_0 = \frac{1}{2L}\int_{-L}^L f(x)\, dx$$

Complex:

$$C_m = \frac{1}{2L}\int_{-L}^L f(x)\exp(-i(\frac{\pi m x}{L}))\, dx$$

Parseval's:

$$\sum_{m=1}^{\infty} |A_m|^2 = \frac{2}{\pi}\int_0^\pi (f(x))^2\, dx$$

Polar laplace:

$$u_{rr} + \frac{u_r}{r} + \frac{u_{\theta\theta}}{r^2} = 0$$

Fundamental:

$$\Phi(x,y) = -\frac{1}{2\pi}\ln(\sqrt{x^2 + y^2})$$

Rotational invariance:

$$\begin{cases} x' = \cos(\theta)x - \sin(\theta)y \\ y' = \sin(\theta)x + \cos(\theta)y \end{cases}$$

$$\implies u_{x'x'} + u_{y'y'} = u_{xx} + u_{yy} = 0$$

Method of characteristics:

$$h'(t) = u_x \cdot x'(t) + u_t$$

Calculate $h(t)$, set $h = u(x(t), t)$, plug in initial condition, replace $x(t)$ with value of first characteristic curve, replace C with x.

Fourier Transform:

$$\widehat{f}(k) = \int_{-\infty}^{\infty} f(x)e^{ikx}\,dx$$

Inverse Fourier:

$$\mathcal{F}^{-1}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(k)e^{-ikx}\,dk$$

$$\mathcal{F}(e^{-ax^2}) = \sqrt{\frac{\pi}{a}}\exp(-\frac{k^2}{4})$$

Heat Equation:

$$\exp(-k^2Dt) = \mathcal{F}\left(\frac{1}{\sqrt{4\pi Dt}}\exp(-\frac{x^2}{4Dt})\right)$$

If $g(x) = f(x-a)$,

$$\widehat{g}(k) = e^{ika}\widehat{f}(k)$$

Convolution

$$(f*g)(x) = \int_{-\infty}^{\infty} f(x-y)g(y)\,dy$$

$$\mathcal{F}((f*g)(k)) = \widehat{f}(k)\cdot\widehat{g}(k)$$

Derivatives

$$\mathcal{F}(\frac{d^n}{dx^n}f(x)) = (-ik)^n\widehat{f}(k)$$

Completing the Square:

$$3y - \frac{(x-y)^2}{4Dt} = -\frac{(y-x-6Dt)^2}{4Dt} + 3(x+3Dt)$$

Wave Equation:

$$\frac{d^2}{dt^2}\mathcal{F}(u) = -(kc)^2\mathcal{F}(u) \implies \mathcal{F}(u) = \widehat{F}(k)e^{ikct} + \widehat{G}(k)e^{-ikct} \implies u = F(x-ct)+G(x+ct)$$

Mean-Value Formula: if $\Delta u = 0$ the average over the ball is the value at the center

Strong Max: if $\Delta u = 0 \in \Omega$, the max and min are only on $\partial\Omega$

Euler-Lagrange: Consider $h(t) = I[u+tv]$ with $v(a) = v(b) = a$ then $h'(0)$ will be minimum and u is the Euler-Lagrange. Lagrangian $L = L(p,z,x)$:

$$-(L_p(f',f,x))_x + L_z(f',f,x) = 0$$

Lagrangian $L = L(p,q,z,x,y)$:

$$-(L_p(u_x,u_y,u,x,y))_x - (L_q(u_x,u_y,u,x,y))_y + L_z(u_x,u_y,u,x,y) = 0$$