APMA 1360: Applied Dynamical Systems

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Spring 2025

1 Jan 22

Motivations - Applications + Phenomena

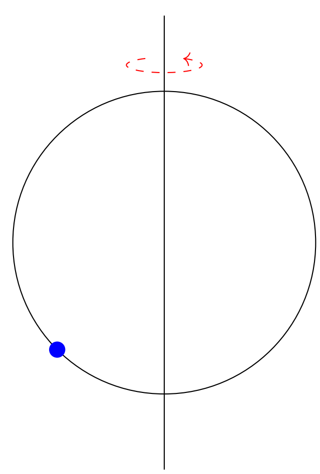
1. **Bifurcation theory:** How do systems change as parameters change?

   Examples:

   - Mechanical systems (e.g. what will happen to a bead as an apparatus is rotated at velocity ω?)
   - Chemical reactions (e.g. Belusov-Zhabotinsky reaction - oscillations in chemical reactions)
   - Tipping points (e.g. climate change, convection currents)
   - Population dynamics (e.g. predator-prey models, outbreaks)
   - Synchronization (e.g. firefly synchronous lighting, brain activity patterns)
   - Chaotic dynamics (e.g. double pendulum)
2. **Existence and Uniqueness**
3. **Dynamical theory**
4. **Chaotic dynamics**

Bifurcation Theory

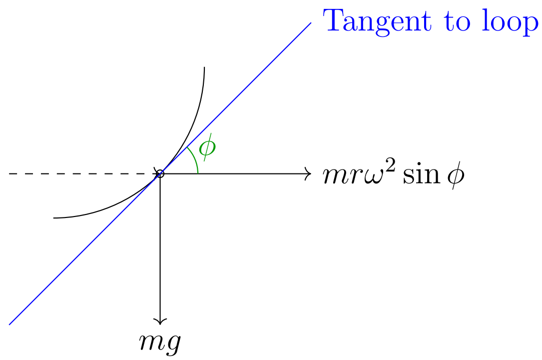
Example (Overdamped bead on loop)



Goal: What will happen to the bead as the loop is rotated at velocity ω?

We assume that the only forces on the bead are gravitation, friction, and centrifugal force.

This gives a force diagram:



From Newton's law,

$$\underbrace{mr\frac{d^2\phi}{dt^2}}_{\text{acceleration}} = -b\frac{d\phi}{dt} - mg\sin\phi + m\omega^2 r\sin\phi\cos\phi$$

Assuming $b \gg 1$, we can neglect the LHS so

$$\begin{aligned}\frac{d\phi}{dt} &= -\frac{mg}{b}\sin\phi + \frac{m\omega^2 r}{b}\sin\phi\cos\phi \\ &= \frac{mg}{b}\sin\phi\left(\frac{\omega^2 r}{g}\cos\phi - 1\right) \\ &= a\sin\phi(\mu\cos\phi - 1)\end{aligned}$$