Invertible Matrix Theorem

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For a given $n \times n$ matrix A:

1. A is an invertible matrix
2. A is row equivalent to I_n
3. A has n pivot positions
4. The equation $Ax = 0$ has only the trivial solution
5. The columns of A form a linearly independent set
6. The linear transformation $x \mapsto Ax$ is one-to-one
7. The equation $Ax = b$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$
8. The columns of A span $\mathbb{R}^n$
9. The linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$
10. There is an $n \times n$ matrix C such that $CA = I$
11. There is an $n \times n$ matrix D such that $AD = I$
12. A^T is an invertible matrix
13. The columns of A form a basis of $\mathbb{R}^n$
14. Col A = $\mathbb{R}^n$
15. dim Col A = n
16. rank A = n
17. Nul A = $\{\mathbf{0}\}$

18. dim Nul A = 0
19. det $A \neq 0$
20. 0 is not an eigenvalue of A