2 a) $f(x,y) = 1/2\pi\sigma_x\sigma_y\sqrt{1-\rho^2} \times D(x,y)$

$$D(x,y) = \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-u_x)^2}{\sigma_x^2} + \frac{(y-u_y)^2}{\sigma_y^2} - \frac{2\rho(x-u_x)(y-u_y)}{\sigma_x\sigma_y}\right]\right.$$

$$f(y|x=x) = \frac{f(y,x)}{f_x(x)} = \frac{1/2\pi\sigma_x\sigma_y\sqrt{1-\rho^2} \times D(x,y)}{1/2\pi\sigma_x \exp\left[-\frac{1}{2}\left(\frac{x-u_x}{\sigma_x}\right)^2\right]}$$

b) $B_0 + B_1 x = \alpha_{yx} + B_{yx}x = (\mu_y - \mu_x\rho(\frac{\sigma_y}{\sigma_x}) + \rho(\sigma_y/\sigma_x)x$

$$\sigma^2 = \sigma_y^2(1-\rho^2)$$

2.50 $\sum k_i X_i = \sum\left(\frac{X_i - \bar{X}}{\sum(X_i-\bar{X})^2}\right)X_i$

$$= \sum\frac{(X_i-\bar{X})(X_i-\bar{X})}{\sum(X_i-\bar{X})^2}$$

$$= \sum\frac{(X_i-\bar{X})^2}{(X_i-\bar{X})^2} = 1$$

2.51 $b_0 = \bar{Y} - b_1\bar{X}$

$$E(b_0) = E(\bar{y} - b_1\bar{x}) = 1/n\sum E(Y_i) - \bar{x}E(b_1)$$
$$= 1/n\sum(B_0 + B_1X_1) - \bar{x}B_1$$
$$= B_0 + B_1\bar{x} - B_1\bar{x} = B_0$$

2.52 $\sigma^2(b_0) = \sigma^2(\bar{Y} - b_1\bar{X})$

$$= \sigma\bar{Y} + \bar{x}^2\sigma^2 b_1 - 2\bar{x}\,\sigma(Y, b_1)$$
$$= \sigma^2/n + \bar{x}^2\sigma^2/\sum(X_1-\bar{x})^2$$
$$= \sigma^2[1/n + \bar{x}^2/\sum(X_1-\bar{x})^2]$$

2.55 $SSR = \sum(\hat{Y}_1 - \bar{Y})^2$

$$= \sum[b_0 + b_1X_1) - \bar{Y}]^2$$
$$= \sum[\bar{Y} - b_1\bar{x} + b_1X_1 - \bar{Y}]^2$$
$$= b_1^2\sum(X_i - \bar{X})^2$$

2.58. $f(Y_1 Y_2)$ would become

$$1/2\pi\sigma_1\sigma_2 \exp\{-1/2\left[\left(\frac{Y_1-u_1}{\sigma_1}\right)^2+\left(\frac{Y_2-u_2}{\sigma_2}\right)^2\right]$$

which is just $f_1(Y_1)\cdot f_2(Y_2)$

4)

a.

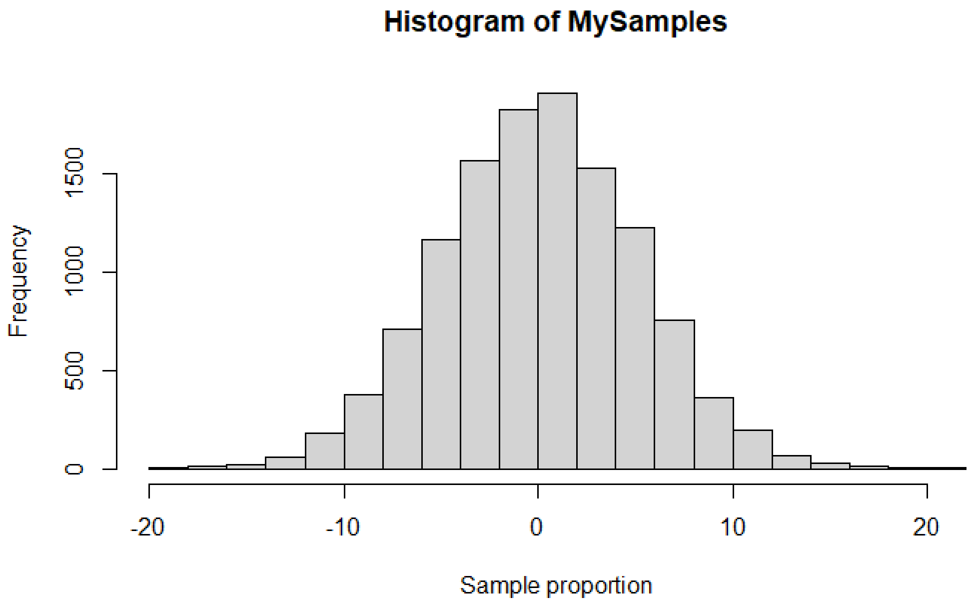
```
> x=c(4,8,10,12,16,20)
> x
[1]  4  8 10 12 16 20
> e=c(rnorm(6,0,5))
> e
[1]  -4.5737092   5.0062393  -0.2821146   1.4832258 -13.9573543
[6]  -1.4137021
> y=20+4*x+e
> y
[1] 31.42629 57.00624 59.71789 69.48323 70.04265 98.58630
> mod=lm(y~x)
> mod

Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)            x
     21.610        3.666

> pred <- predict(mod, data.frame(x = 14), interval = "confidence")
> pred
       fit      lwr      upr
 72.93058 64.25293 81.60824
```

b.

```
x=c(4,8,10,12,16,20)
x
MySamples=replicate(2000,rnorm(6,0,5))
y2=20+4*x+MySamples
t(y2)
mod2=lm(y2~x)
t(coefficients(mod2))
pred2 <- predict(mod2, data.frame(x = 14), interval = "confidence")
t(pred2)
```

c.

```
hist(MySamples, right = FALSE, xlab = "Sample proportion")
```



> mean(t(coefficients(mod2)[2,]))
[1] 4.013283
> sd(t(coefficients(mod2)[2,]))
[1] 0.3845899

Yes

d)
> 76+mean(t(coefficients(mod2)[2,]))
[1] 80.01328
> 76-mean(t(coefficients(mod2)[2,]))
[1] 71.98672
>
> mean(71.98672 < pred2 & 80.01328 > pred2)
[1] 0.9305

So this results in 93% which is pretty close to the theoretical expectation of 95%.