We have seen that we can fit an SVM with a non-linear kernel in order to perform classification using a non-linear decision boundary. We will now see that we can also obtain a non-linear decision boundary by performing logistic regression using non-linear transformations of the features.

(a) Generate a data set with $n = 500$ and $p = 2$, such that the observations belong to two classes with a quadratic decision boundary between them. For instance, you can do this as follows:

```
> x1 <- runif(500) - 0.5
> x2 <- runif(500) - 0.5
> y <- 1 * (x1^2 - x2^2 > 0)
```

(b) Plot the observations, colored according to their class labels. Your plot should display X_1 on the x-axis, and X_2 on the y-axis.

(c) Fit a logistic regression model to the data, using X_1 and X_2 as predictors.

(d) Apply this model to the *training data* in order to obtain a predicted class label for each training observation. Plot the observations, colored according to the *predicted* class labels. The decision boundary should be linear.

(e) Now fit a logistic regression model to the data using non-linear functions of X_1 and X_2 as predictors (e.g. X_1^2, $X_1 \times X_2$, $\log(X_2)$, and so forth).

(f) Apply this model to the *training data* in order to obtain a predicted class label for each training observation. Plot the observations, colored according to the *predicted* class labels. The decision boundary should be obviously non-linear. If it is not, then repeat (a)-(e) until you come up with an example in which the predicted class labels are obviously non-linear.

(g) Fit a support vector classifier to the data with X_1 and X_2 as predictors. Obtain a class prediction for each training observation. Plot the observations, colored according to the *predicted class labels.*

(h) Fit a SVM using a non-linear kernel to the data. Obtain a class prediction for each training observation. Plot the observations, colored according to the *predicted class labels.*

(i) Comment on your results.