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Coexistence of Languages is possible

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Abstract

In this work we study the dynamics of language competition. In Abrams and Strogatz [Modeling the dynamics of language death, Nature 424 (2003) 900], the extinction of one of the competing languages is predicted, although in some case the coexistence occurs. The preservation of both languages was explained by Patriarca and Leppanen [Modeling language competition, Physica A 338 (2004) 296] by introducing the existence of two disjoint zones where each language is predominant. However, their results cannot explain the survivance of both languages in only one zone of competition. In this work we discuss their results and propose a new alternative model of Lotka–Volterra type in order to explain the coexistence of two languages.

Keywords: Populational dynamics; Language competition

1. Introduction

A model concerning the dynamics of language competition was introduced by Abrams and Strogatz [1]. Their results are matter of discussion since the extinction of one of the competing languages is predicted, although in some case the coexistence occurs as they remark. The preservation of both languages was explained

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by Patriarca and Leppanen [2] introducing the existence of two disjoint zones where each language is predominant. However, their results cannot explain the survivance of both languages in only one zone of competition. In this work we discuss their results and propose a new alternative model in order to explain the coexistence of two languages, which belongs to the family of Lotka–Volterra models [3,4].

Let us assume that two competing languages x and y are spoken by populations $\mathbb{X}(t)$ and $\mathbb{Y}(t)$ at time t. For simplicity, we suppose that x is the only attractive language (i.e., there are no motivation to shift from language x to y), and we define a coefficient c which is the rate of conversion from language y to x. Following Ref. [1], this coefficient represents the status of x, and reflects the social or economics opportunities afforded to its speakers.

We assume that the conversion is proportional to the product $\mathbb{X}\mathbb{Y}$ instead of $\mathbb{Y}\mathbb{X}^a$ with $a \in [1.05, 1.55]$, as in Ref. [1]. We will show below that this is not a critical modification of the problem, since the dynamical behavior is the same.

The main difference is the inclusion of the rates of growth of both populations, with a limitation on the size of $\mathbb{X}$ and $\mathbb{Y}$. We introduce the positive parameters α_x, α_y which includes the natality and mortality rates of each population, and the rates of growth are given by

$$\alpha_x \mathbb{X}\left(1 - \frac{\mathbb{X}}{S_x}\right) \quad \text{and} \quad \alpha_y \mathbb{Y}\left(1 - \frac{\mathbb{Y}}{S_y}\right),$$

where S_x and S_y are the carrying capacities in absence of competition ($c = 0$), which are predetermined by the conditions of the environment.

Let us observe that S_x can be reached abruptly if a great proportion of the people in $\mathbb{Y}$ decide to shift to language x. This cannot be interpreted as a loss of attractiveness of language x, since there are still advantages to the ones who speak it instead of y; in the model is reflected as a change of sign in the rate of growth, as a consequence of the competition among the members of population $\mathbb{X}$ for the resources.

Hence, we propose the following system of differential equations modeling the language competition,

$$\frac{d\mathbb{X}}{dt} = c\mathbb{X}\mathbb{Y} + \alpha_x \mathbb{X}\left(1 - \frac{\mathbb{X}}{S_x}\right) \quad \text{and} \tag{1.1}$$

$$\frac{d\mathbb{Y}}{dt} = -c\mathbb{X}\mathbb{Y} + \alpha_y \mathbb{Y}\left(1 - \frac{\mathbb{Y}}{S_y}\right), \tag{1.2}$$

which belongs to the well-known family of Lotka–Volterra models. Indeed, it is a mix of the model for two competing species (where the prey is converted into a predator after being captured, perhaps a vampire-type predator), and epidemics models (where the infected individuals never recover nor die off immediately).

2. Dynamics

There are several works concerning the dynamics of Lotka–Volterra models, we refer the interested reader to Refs. [3–5]. We sketch briefly the dynamics in our problem, in order to obtain a better insight on the role of the parameters involved. Let us find first the fixed points of the system. We have

$$\frac{d\mathbb{X}}{dt} = 0 \Longleftrightarrow \mathbb{Y} = \frac{\alpha_x \mathbb{X}}{cS_x} - \frac{\alpha_x}{c} \quad \text{or } \mathbb{X} = 0, \tag{2.3}$$

$$\frac{d\mathbb{Y}}{dt} = 0 \Longleftrightarrow \mathbb{Y} = -\frac{cS_y}{\alpha_y}\mathbb{X} + S_y \quad \text{or } \mathbb{Y} = 0. \tag{2.4}$$

Hence, we have the equilibria $(0, S_y)$ and $(S_x, 0)$ corresponding to the death of one language, and the $(0, 0)$ equilibrium which correspond to the extinction of both populations. There exists another equilibrium point whenever the intersection of the two lines belongs to the first quadrant. This condition is fulfilled if

$$S_x < \frac{\alpha_y}{c},$$

we call this inequality the *threshold condition*, due to the high similitude with epidemic models [5].

The new equilibrium

$$(x_e, y_e) = \left(\frac{\alpha_y S_x(cS_y + \alpha_x)}{c^2 S_x S_y + \alpha_x \alpha_y}, \frac{\alpha_x S_y(\alpha_y - cS_x)}{c^2 S_x S_y + \alpha_x \alpha_y}\right)$$

is stable since the trace of the linearized system is negative $(-\alpha_x/S_x - \alpha_y/S_y)$, and the determinant is positive $(\alpha_x \alpha_y / S_x S_y)$. Then, contrary to the results predicted in Refs. [1,2], the coexistence of languages is possible.

The equilibrium (x_e, y_e) goes to (S_x, S_y) when the rate of change c goes to zero. Also, the population y_e becomes negative when the threshold condition is violated, which shows that the coexistence of language is not possible in this case.

In Fig. 1 the equilibrium point (x_e, y_e) as a function of c for fixed values of S_x, S_y and α_x, α_y is depicted. As expected, y_e goes to zero when c approaches the critical value α_y/S_x in the threshold condition. However, $x_e(c)$ is not necessarily monotonic, and the $\mathbb{X}$ population is not always beneficiated for the language competition. Moreover, let us observe that the total population $x_e + y_e$ in Fig. 1 is decreasing, that is, when the parameter c growths, not only the $\mathbb{Y}$ language faces extinction, also the total population $\mathbb{X} + \mathbb{Y}$ diminishes. Since always $x_e > S_x$, we may understand the equilibrium in this case as a population $\mathbb{X}$ which decreases due to biological reasons, with a compensating contribution from the population $\mathbb{Y}$. On the other hand, since $y_e < S_y$, the population $\mathbb{Y}$ tends to grow, and the shift from language y to x left its number of members unchanged.

Surprisingly, the equilibrium $(S_x, 0)$ is not stable if the threshold condition is satisfied. That is, even when there are no interest to shift from language x to y, a small fraction $\mathbb{Y}$ is enough to develop a community, since their growth near the

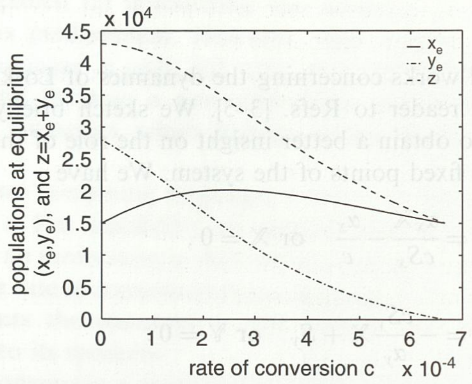

Fig. 1. The equilibrium point (x_e, y_e) as a function of c for $S_x = 1500$ and $S_y = 2800$ with $\alpha_x = \alpha_y = 10$.

equilibrium is given by

$$\frac{d\mathbb{Y}}{dt} = -cS_x\mathbb{Y} + \alpha_y\mathbb{Y} = (\alpha_y - cS_x)\mathbb{Y}\,,$$

where $\alpha_y - cS_x > 0$.

Let us note that the threshold condition synthesizes three simple conditions for the survival of a language which are as follows:

- The carrying capacity S_x is small and it is easily reached.
- The rate of growth of population $\mathbb{Y}$ high.
- There is a low rate of shift from the language y to x.

Finally, the existence of a stable equilibrium when the conversion is proportional to $\mathbb{Y}\mathbb{X}^a$ follows since the $d\mathbb{X}/dt = 0$ isocline is increasing, and the $d\mathbb{Y}/dt = 0$ isocline is decreasing, and there exists an intersection of the curves

$$\mathbb{Y} = \left(\frac{\alpha_x\mathbb{X}}{cS_x} - \frac{\alpha_x}{c}\right)\mathbb{B}x^{1-a}, \quad \mathbb{Y} = -\frac{cS_y}{\alpha_y}\mathbb{X}^a + S_y\,,$$

whenever the modified threshold condition

$$S_x^a < \frac{\alpha_y}{c}$$

is satisfied.

3. Conclusions

In the Abrams and Strogatz model, the extinction of the less attractive (weakest) language is predicted. However, although Quechua, Scottish Gaelic or Welsh could be faced to extinction, the survivance of Guarani (mainly in Paraguay), Catalan, Gallego and Vasco (in Spain) cannot be explained by split populations. In this work we addressed the question of coexistence of languages in terms of a Lotka–Volterra type model. The extinction of a language seems to be caused by an hostile environment rather than the influence of a more promising language x, except when the population $\mathbb{X}$ has a large carrying capacity S_x, combined with a low growth parameter α_y for $\mathbb{Y}$.

There exists a dynamical analogy between language competition and the propagation of infectious diseases, which suggest measures to prevent the extinction of languages. Reducing c is similar to minimize the contact rate between infectives and susceptibles. On the other hand, there are some measures which cannot be applied: splitting populations is equivalent to segregate the infected members. Here S_x could be thought as increasing the rate of removal of the infected population. It is interesting to note that S_y does not enter in the dynamic of language death directly, however, since the environment is fixed and the resources are available for populations $\mathbb{X}$ and $\mathbb{Y}$ simultaneously, any redistribution of the resources which increase S_y, must reduce S_x. Hence, increasing the social or economics opportunities afforded to $\mathbb{Y}$, is equivalent to reduce S_x.

There are still many open problems in this field and possible generalizations. The coefficient c can be allowed to vary (the situation of French in Canada); a third group must be considered, the bilingual people, since they can switch from one language to the other, taking advantage on the opportunities (for computational programming languages, they had more chances to get a job knowing two of them). Also, the time factor must be introduced in the process of learning a different language, since during this process, its status could be diminished (specially, for quickly evolving languages, as in computer science).

Moreover, there are many connections to be explored: mutatis mutandis, the model could be applied to migration problems, changing "languages" by "territory". In this context, it is easy to understand the coexistence: when a large group shift from y to x, the resources on y offers a better perspective for the remaining population.

As was pointed out by Epstein [5], it seems surprising that our models for viruses, insects or fish turn out to be interesting models of man. As he said, 'perhaps we are true Darwinians more in our heads than in our hearts'.

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