Today I will be presenting my senior capstone project that I completed in Math 55200. I will be discussing the numerical solution for a partial differential equation, specifically I will discuss the method used to solve the equation, its order, and how the order is calculated.I was given the task to solve the following initial value problem of the partial differential equation, ut + uux = 0 where u is a function of two variables, space and time, up to t = 1. This equation is also an example of nonlinear advection which is commonly found in fluid dynamics and other areas of physics. The differential equation is in the region R, which is defined as the set of all pairs (x,t) such that t is greater than or equal to zero and x is less than 2π and greater than zero. The initial condition for u is given as u(x,0) = 2.5 + sin(x), which specifies the value of u at t = 0 for all x in the interval [0,2π]. In this problem, the boundary condition is assumed to be periodic, which means the value of u at x = 2π is equal to the value of u at x = 0 for all of t. This condition is used to enforce the solution is periodic in the x direction and ensures it is well defined in the given region R.

To solve this partial differential equation, we first take a look at the hyperbolic linear advection equation, ut + aux = 0, where u is a function of space and time and a is a constant.. The numerical method used to solve the linear advection equation is the upwind scheme. The upwind scheme is a finite difference method which is first order accurate and conditionally stable. The upwind scheme is based on the idea of using the direction of wave propagation to determine in which numerical information should be propagated. For example, when the direction is positive (a > 0), the upwind scheme uses the value of u at the point to the left of the current point in the spatial grid to compute the value of u at the next time step. This would look like, + . Similarly, when the direction is negative (a < 0), the upwind scheme uses the value of u at the point to the right of the current point in the spatial grid to compute the value of u at the next time step, which would look like + . Now, this method can be extended to our nonlinear partial differential equation like so + .