

Some useful integrals for electrodynamics homework (cite by writing “(ieq#)”)  
 All constants of integration are omitted but implied.

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1 \qquad \int \frac{dx}{x} = \ln |x| \qquad \int \sin ax dx = -\frac{\cos ax}{a} \qquad \int \cos ax dx = \frac{\sin ax}{a}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$(1) \quad \int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b|$$

$$(2) \quad \int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$(3) \quad \int (ax+b)^{m/2} = \frac{2(ax+b)^{(m+2)/2}}{a(m+2)}$$

$$(4) \quad \int x(ax+b)^{m/2} = \frac{2(ax+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(ax+b)^{(m+2)/2}}{a^2(m+2)}$$

$$(5) \quad \int \frac{dx}{\sqrt{x^2+a^2}} = \ln \left| x + \sqrt{x^2+a^2} \right|$$

$$(6) \quad \int \frac{x dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$(7) \quad \int \sqrt{x^2+a^2} dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2} \ln \left( x + \sqrt{x^2+a^2} \right)$$

$$(8) \quad \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$(9) \quad \int \frac{x dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$(10) \quad \int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$(11) \quad \int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + \left( \frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

$$(12) \quad \int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$(13) \quad \int \sin^3 ax dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$$

$$(14) \quad \int x \cos ax dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$(15) \quad \int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \left( \frac{x^2}{a} - \frac{2}{a^3} \right) \sin ax$$

$$(16) \quad \int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$(17) \quad \int \cos^3 ax dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a}$$

$$(18) \quad \int \sin ax \cos ax dx = \frac{\sin^2 ax}{2a}$$

$$(19) \quad \int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$(20) \quad \int \sin^n ax \cos ax \, dx = \frac{\sin^{n+1} ax}{(n+1)a}, \quad n \neq -1$$

$$(21) \quad \int \cos^n ax \sin ax \, dx = -\frac{\cos^{n+1} ax}{(n+1)a}, \quad n \neq -1$$

$$(22) \quad \int \frac{\sin ax \, dx}{(p+q \cos ax)^n} = \frac{1}{aq(n-1)(p+q \cos ax)^{n-1}}$$

$$(23) \quad \int x^n \ln x \, dx = \frac{x^{n+1}}{n+1} \left( \ln x - \frac{1}{n+1} \right), \quad \text{if } n = -1, \text{ see (ieq24)}$$

$$(24) \quad \int \frac{\ln x}{x} \, dx = \frac{1}{2} (\ln x)^2$$

### Definite Integrals

$$(25) \quad \int_0^{\pi/2} \sin^2 x \, dx = \int_0^{\pi/2} \cos^2 x \, dx = \frac{\pi}{4}$$

$$(26) \quad \int_0^\infty e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$(27) \quad \int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}}, \quad n = 0, 1, 2, \dots$$

### Fourier's trick integrals

$$(28) \quad \int_0^a \sin(m\pi x/a) \sin(n\pi x/a) \, dx = \begin{cases} 0 & \text{if } m \neq n, \\ a/2 & \text{if } m = n \end{cases}$$

$$(29) \quad \int_0^a \cos(m\pi x/a) \cos(n\pi x/a) \, dx = \begin{cases} 0 & \text{if } m \neq n, \\ a/2 & \text{if } m = n \end{cases}$$

$$(30) \quad \int_{-1}^1 P_\ell(x) P_m(x) \, dx = \int_0^\pi P_\ell(\cos \theta) P_m(\cos \theta) \sin \theta \, d\theta = \begin{cases} 0 & \text{if } \ell \neq m, \\ \frac{2}{2\ell+1} & \text{if } \ell = m \end{cases}$$

### Legendre Polynomials

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = (3x^2 - 1)/2$$

$$P_3(x) = (5x^3 - 3x)/2$$

$$P_4(x) = (35x^4 - 30x^2 + 3)/8$$

$$P_5(x) = (63x^5 - 70x^3 + 15x)/8$$